Problems for Physics 575
Accelerator Physics and Technologies for Linear Colliders
Winter 2002
Chapter 5 – Damping Rings

1. Determine the optimum value of the dispersion in the dipole of the theoretical minimum emittance (TME) lattice cell. The dispersion goes as $\eta = \eta_0 + s^2/(2\rho)$ where $s = 0$ is the center of the dipole. In other words, determine the optimum value of η_0 in terms of the bending radius ρ and the length of the dipole L. Basically, solve for η_0 in

$$\frac{d}{d\eta_0}\int_0^{L/2} \mathcal{H}(s) = 0 \tag{1}$$

where $\mathcal{H} = \beta\eta'^2 + 2\alpha\eta\eta' + \gamma\eta^2$ and where

$$\begin{aligned} \eta'(s) &= s/\rho \\ \beta(s) &= \beta_0 + s^2/\beta_0 \\ \alpha(s) &= -s/\beta_0 \\ \gamma(s) &= 1/\beta_0 \end{aligned} \tag{2}$$

By symmetry one only has to integrate from 0 to $L/2$. Note that a lot of the terms will drop out if you do the η_0 derivative before integrating.

The optimum for β_0 is more difficult to work out. Please don't try it.

2. The present NLC damping ring design uses damping wigglers and TME cells to achieve $\gamma\epsilon = 3\times10^{-6}$ m-rad and an effective damping time of $\tau_x = 1.7$ ms. The old SLC damping rings are FODO-cell based, and achieved $\gamma\epsilon = 2.1\times10^{-5}$ m-rad and an effective damping time of $\tau_x = 3.0$ ms with 20 FODO cells (40 dipoles) and operating at 1.2 GeV.

   Design a FODO-cell only ring (with no damping wigglers) to achieve the same values of $\gamma\epsilon$ and τ_x as the NLC damping ring. Use the following scaling relations to determine the energy E and the number of cells N_{cell} or dipoles N_{dip} of the new ring based on the results of SLC damping ring.

Scaling relations for normalized emittance:

$$\gamma\epsilon \sim E^3\theta_{\rm dip}^3$$
$$\gamma\epsilon \sim E^3/N_{\rm dip}^3$$

Scaling relation for damping time:

$$\tau_x \sim \frac{T_0}{E^3 I_2} \sim \frac{\rho^2}{E^3}, \tag{3}$$

where the revolution time T_0 and the radiation integral I_2 scaling of

$$T_0 \sim \rho \text{ and}$$
$$I_2 \sim 1/\rho$$

were used in the last relation.

You must assume that ρ/E stays constant, i.e. magnetic field stays the same to get a unique solution. Note that the answer is not practical to implement. That is why damping wigglers are included in the NLC design.