

# Iron Dominated Electromagnets

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# Introduction

- Because of the proliferation of light source synchrotrons, which store and accelerate medium charged energy particles with stiffness  $B\rho$  approximately equal to 10 T-m, there is new interest in iron dominated magnets whose maximum field amplitude is limited by iron saturation characteristics.
- The most important properties of magnets designed and fabricated for this application is the need for high quality and for magnet to magnet reproducibility sharing the same family.
  - These requirements reflect the requirements for these accelerators to store high current beams for several hours.
  - The long beam storage life requires magnets whose fields are not contaminated by fields other than the desired fields.
    - For dipoles, the field must be constant throughout its volume.
    - For quadrupoles, the field must be linearly distributed throughout its volume.
    - For sextupoles, the field must be quadratically distributed throughout its volume.
- This class is given for those involved in the design, fabrication and measurement of magnets whose properties satisfy the demanding requirements of this and other applications demanding high quality reproducible magnets.

# Scope

- Accelerator magnets include:
  - superconducting magnets,
  - magnets using permanent magnet material,
  - fast pulsed magnets
  - specialized magnets such as
    - septa used for injection and extraction.
  - The scope of these lectures will be limited to *conventional room temperature, iron dominated accelerator magnet design.*

# Schedule

- The schedule for this one week half week course calls for two lecture sessions per day, one in the morning and the second in the afternoon.
  - Contractual obligations require  $>20$  hours of contact during this period. Therefore, each day will include five hours of lecture and lab (whew!).
- 10 lectures have been organized for this course. Also, it is anticipated that at least one afternoon computer laboratory session will be scheduled in order to familiarize the student with POISSON, the two dimensional magnetostatic code used by many magnet scientists engineers and designers.
- Also, for those taking the course for credit, the last morning will be scheduled for a final examination.
  - Unfortunately, the economies of flying require me to leave Cornell early on Friday morning to catch a flight. Therefore, someone from the USPAS staff will be assigned to monitor the test and mail it to me for scoring.
- Therefore, this tight schedule makes it necessary to schedule some of the sessions for more than one lecture.

# Purpose

- The purpose of this course in conventional magnet design is to transfer the skills and knowledge required to understand the design, fabrication and measurement of *high quality, conventional iron dominated, modest field* accelerator magnets to the next generation of magnet design scientists and engineers.
- It is an opportunity to collect and accumulate the *tools, practices and mathematical expressions* used in magnet design.

# History and Structure

- This course has evolved over the years.
  - The first courses, taught at Stanford and Duke University, covered a far broader scope than the limited scope of this course.
  - The first courses were taught by Drs. Klaus Halbach and Ross Schlueter and Jack Tanabe and involved complex theory, including mathematical transformations and the design and fabrication of permanent magnet structures.
  - A later course was taught at the Chinese Academy of Science by Dr. Ross Schlueter and myself.
- Subsequent courses, taught by myself at Rice, University of Arizona and UC Santa Barbara, isolated a portion of the broader course to cover iron dominated electromagnets.
  - This course was based on the notes of an internal training course at Lawrence Livermore National Laboratory.
  - The lecture notes for this course have evolved over the repeated courses.
  - These lecture notes have been reorganized and published as a book, “Iron Dominated Electromagnets”, intended to supplement the lectures and the notes for this course.
- The lectures roughly parallel the different chapters in the book and, in order to maximize the benefits of this course, the student is strongly urged to read the chapters prior to the scheduled lecture.

# What This Course Will Cover

## Part One

- This course will describe the design of electromagnets whose fields are shaped by the iron yoke.
- The physics of particle beam optics are briefly discussed, if only to illustrate the requirements for the magnets.
- Magnet types and their functions are discussed.
- Forces on particle beams are described and conventions for determining polarities and means of electrically connecting the magnets to achieve the desired polarities are discussed.

- General principles for the visualization of two dimensional magnetic fields are described.
- These visualization tools are useful since they provide means for describing the field shapes when the shapes of the yoke and coils are generally known without the necessity of going into the mathematics of magnetic fields.
- Despite these visualization tools, it is always useful to understand the mathematics of magnetic fields.
  - Mathematical functions are developed from the two dimensional second order differential equations, derived from Maxwell's electromagnetic equations, which describe the magnetic fields and their errors.
  - Although the two dimensional expressions are developed, it is claimed (without proof) that the expressions holds also for the two dimensional fields integrated in the third dimension over the domain where the fields are non-zero.
  - The properties of the mathematical expressions are exploited in order to characterize properties of the magnetic fields which result from the symmetries of the magnetic design.



- The properties of the mathematical expressions are further exploited and result in developing a tool, *conformal mapping*, which can be used to extend knowledge about a simple magnet geometry, the dipole, to more complex magnet types, the quadrupole and sextupole magnets.
- The mathematical expressions are also useful for understanding the fundamentals of one of the most useful means of magnet analysis, rotating coil measurements of various magnet types.

- Stoke's Theorem is applied to Maxwell's non-homogeneous differential equations and result in expressions to determine the currents required to produce desired magnetic fields.
- These expressions are used to select specific coil configurations and to lay out the general shape of the two dimensional magnet cross section.
- The engineering parameters of selected coils are computed to ensure that the final magnet design satisfies power supply and facilities constraints (ie. the hydraulic cooling capabilities of the facility).

- *Perturbation Theory*, a method of relating mechanical and electrical tolerances and errors to the parameters provided by the physics tolerances of the magnets (multipole errors) of the synchrotron lattice is discussed.
- Perturbation Theory is further exploited to design trim coil configurations required to introduce trim fields in cores designed for other fields.

- *POISSON*, a two dimensional magnetostatic code, originally written by Dr. Klaus Halbach and Ron Holsinger and maintained at Los Alamos by James Billen, is described.
- Example *POISSON* problems are presented in order to expose the student to this program and to teach the student means used to exploit some of its features.
- At least two computer lab sessions will be held so that the student can practice using this tool.

- Some time will be spent discussing rotating coil magnetic measurements. These discussions will be brief and general since the subject is complex and the student can obtain most of what is needed to understand the technique by careful reading of the subject in the chapter written on this subject.
- The concept of magnetic stored energy is used to introduce magnetic forces, the requirements power supplies to control slowly changing fields in a magnet. (Slow meaning field variations in msec.)
- Extremely fast cycled magnets ( $\ll$  msec.) which require understanding transmission line theory is not covered.
- Eddy currents, although covered in the book, will not be discussed.

# Part Two

- Part Two of this course is less mathematical and is meant for the engineer and designer translating the mathematical principles into manufactured hardware.
- The principles described in this part of the course is equally important to the mathematics since, if they are not properly observed, can result in magnets whose performance described by the mathematics to fall short of the physics specifications required for the synchrotron lattice.

- Techniques for the manufacture of the yoke are described.
  - These techniques include shaping the ends of the poles to trim the fringe fields so that the field integral maintains the field quality of the two dimensional field.
- Techniques for the manufacture and testing of the coils are described.
- Means of assembling the coils to the cores and making the electrical connections among the coils and connecting the power supplies are described.
- Techniques of fiducialization, installation and alignment of magnets are described.

# Skills

- The student will need to understand the mathematics of complex variables.
- The student should understand some of the physics concepts which determine the parameters which determine good synchrotron lattice performance.
- It is useful if the student understands some of the principles of electrical circuit design.
- Understanding the capabilities and limitations of shop manufacturing techniques is useful in order to complete the spectrum of understanding magnet design from concepts to final product.



# Lecture One

- The first lecture describes the basic magnet types, describes forces on a particle beam, defines polarities and determines means of electrically connecting electro-magnets to achieve the desired polarities.
- An introduction to the orthogonal analog model as a means of visualizing the magnetic fields is presented. This tool is used to determine some properties of different magnet geometries.
- Much of the discussion will be devoted to understanding the properties of a dipole magnet.
  - This discussion will be generalized in later lectures to generalize the knowledge gained about the design of dipole magnets into other magnet types.
- The *Orthogonal Analog model* is used as a tool to visualize the magnetic field and highlights differences among the two types of dipole magnets used in accelerators, *window frame* and *H style* dipole magnets.
- Much of the material for this lecture is covered in Chapters one and three (section 3.1.3) of the book.

# Magnet Types

- Dipoles
- Quadrupoles
- Sextupoles
- Specialized Magnets (*to be covered later*)
  - Correctors
    - Horizontal/Vertical Steering Magnets
    - Skew Quadrupoles
  - Injection Magnets (to be discussed later)
    - Septa
    - Bumps and Kickers

# Polarities

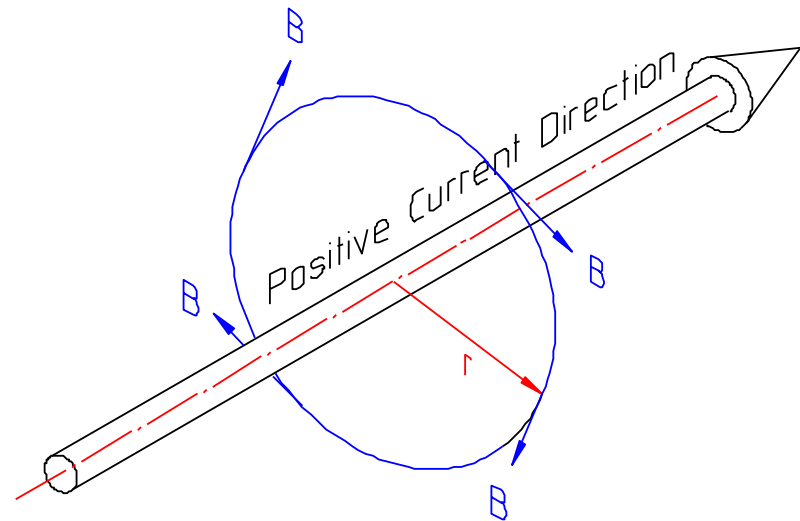
- Electrical and Beam Currents
  - Polarities are defined for positive charges. A positive direction of positive current is indicated by the arrow in the following figures.
- The positive pole of a magnet is determined by applying the right hand rule to the direction of positive current.
- Magnetic flux flows from the positive pole of a magnet to the negative pole.
- The force on a positive beam direction is determined by a vector expression observing the right hand rule.
- The polarities illustrated in the figures for the three types of magnets create forces for positive beam currents (positrons, protons or heavy ion nuclei) *into the screen*.
- In order to achieve the illustrated polarities for the various magnets, the magnet power supplies must be connected so that the positive terminal of the power supply leads connects to the current *in* coil terminal and the negative power supply leads connects to the current *out* coil terminal.
- Polarities become confusing if the particle beam is negatively charged (ie. electrons). The force rule becomes a *left hand* rule, although the flux direction from the positive magnetic pole follows the *right hand* rule.

- For this basic introduction, although the mathematical relationships have not yet been developed, the well known relations between current and magnetic fields are used to illustrate certain principles.

# Integral form of the Magnetic Field Equation

- The solution to Maxwell's equations can be written in integral form.  $I = \oint \vec{H} \cdot d\vec{l}$
- Applying the line integral *vector* equation to the illustrated case;
- The right hand rule applies for the field direction.

$$I = \oint \vec{H} \cdot d\vec{l}$$
$$= \frac{B}{\mu_0} 2\pi r$$



# Magnetic Forces on a Particle Beam

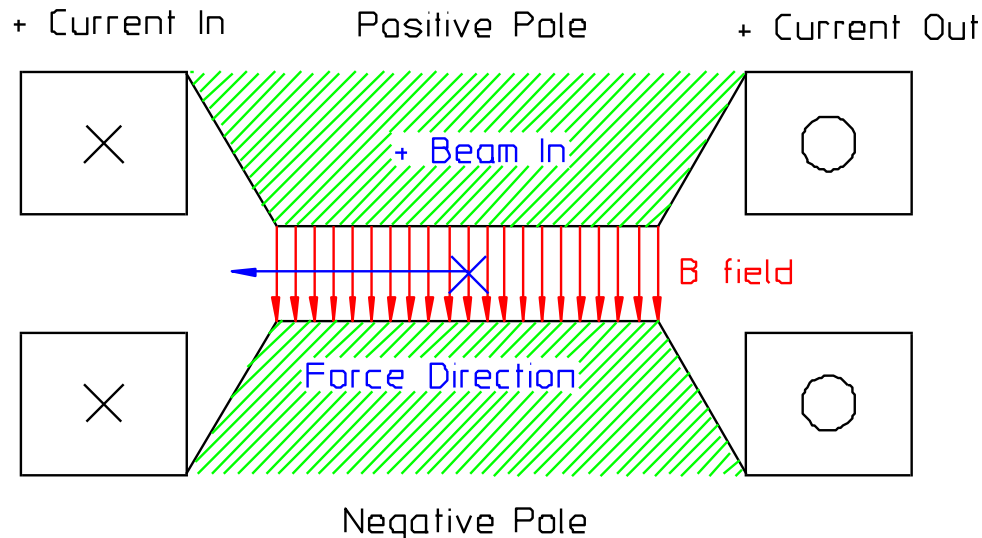
- We use the mks system of units on the vector equation.
- The “right hand” rule is used for vector directions.

$$\vec{F} = e\vec{B} \times \vec{v}$$

$$\begin{aligned}\vec{F} &= \text{coulombs} \times \text{Tesla} \times \frac{m}{\text{sec}} = \text{coulombs} \times \frac{\text{Webers}}{m^2} \times \frac{m}{\text{sec}} \\ &= \frac{V - \text{sec}}{m} \times \frac{\text{coulomb}}{\text{sec}} = \frac{V - \text{Amp} - \text{sec}}{m} = \frac{\text{Watt} - \text{sec}}{m} \\ &= \frac{\text{joules}}{m} = \frac{\text{Newton} - m}{m} = \text{Newton}(\text{force})\end{aligned}$$

# The Dipole Magnet

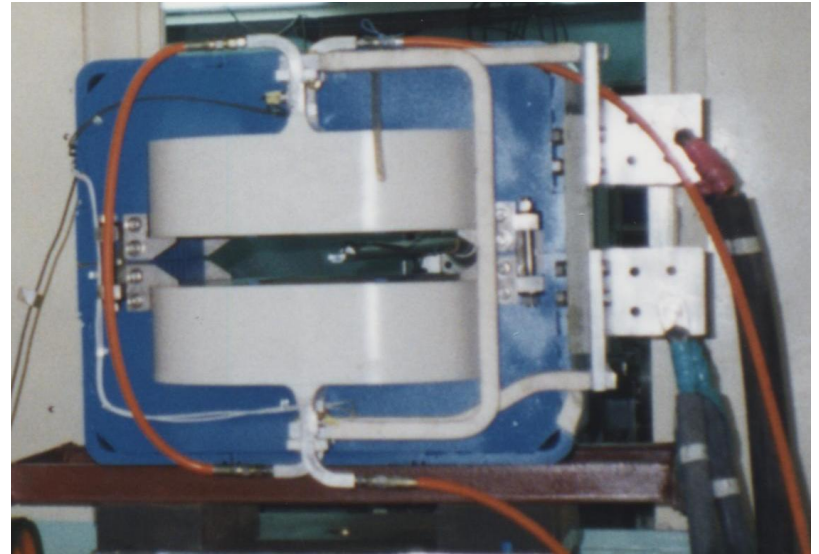
- The dipole magnet has two poles, a constant field and steers a particle beam. Using the right hand rule, the positive dipole steer the rotating beam toward the left.



# Dipoles



SPEAR3 Gradient Magnet

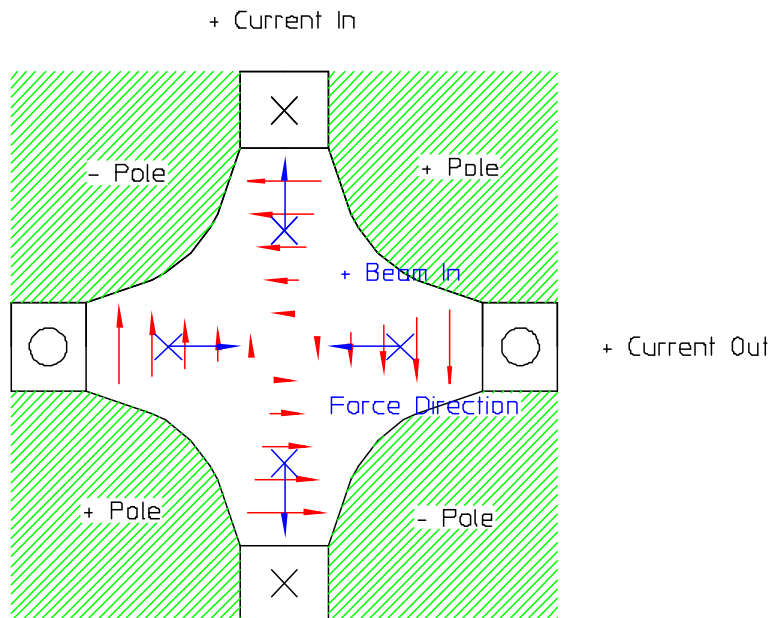


PEPII Low Energy Positron  
Ring Dipole Magnet

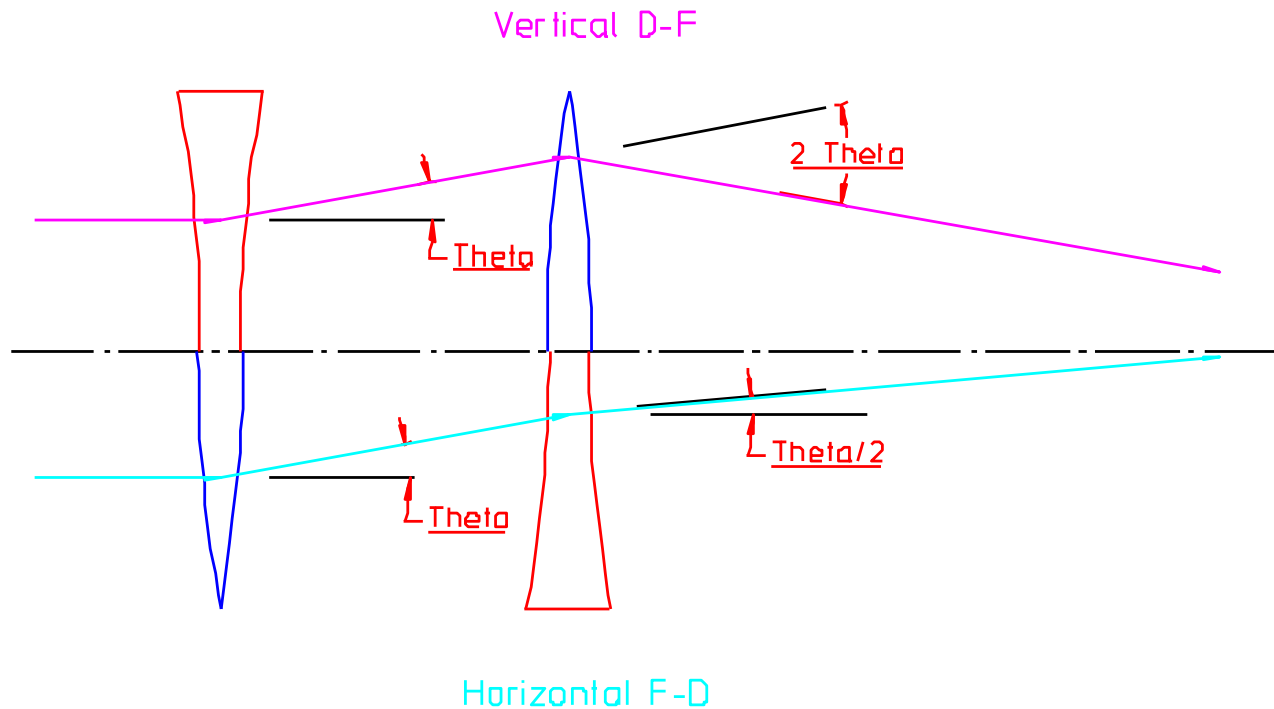


# The Quadrupole Magnet

- The Quadrupole Magnet has four poles. The field varies *linearly* with the distance from the magnet center. It focuses the beam along one plane while defocusing the beam along the orthogonal plane. An *F* or focusing quadrupole focuses the particle beam along the *horizontal* plane.



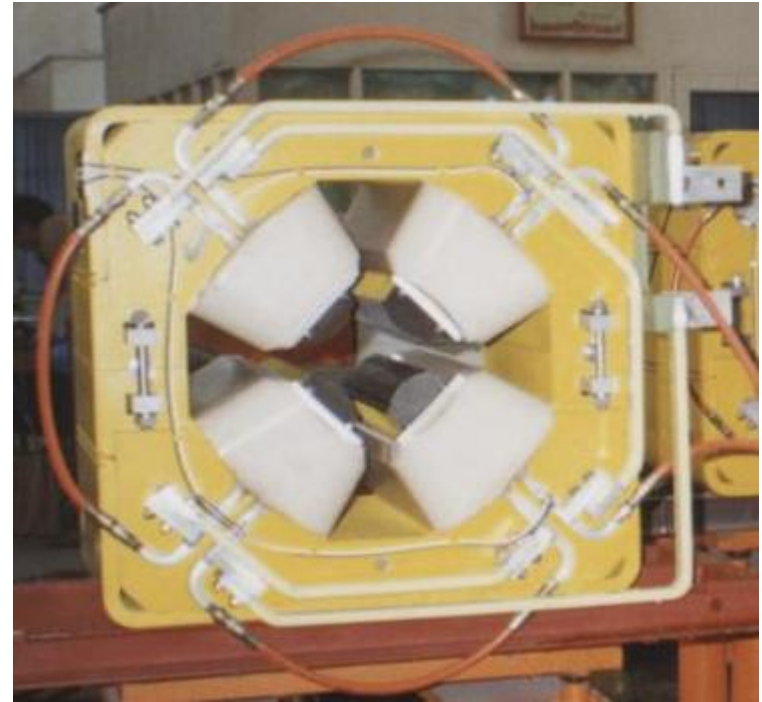
- A series of F-D or D-F magnets will focus the beam in *both* planes.



# Quadrupole Magnets



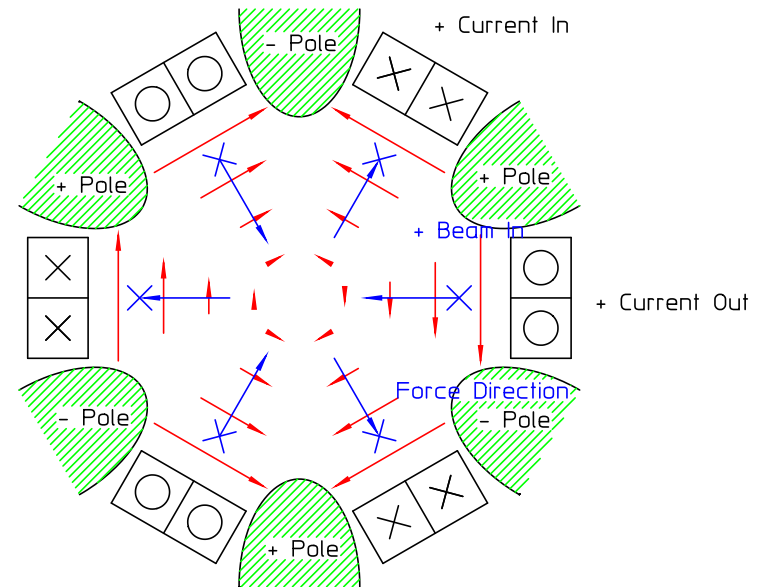
SPEAR3 Quadrupole



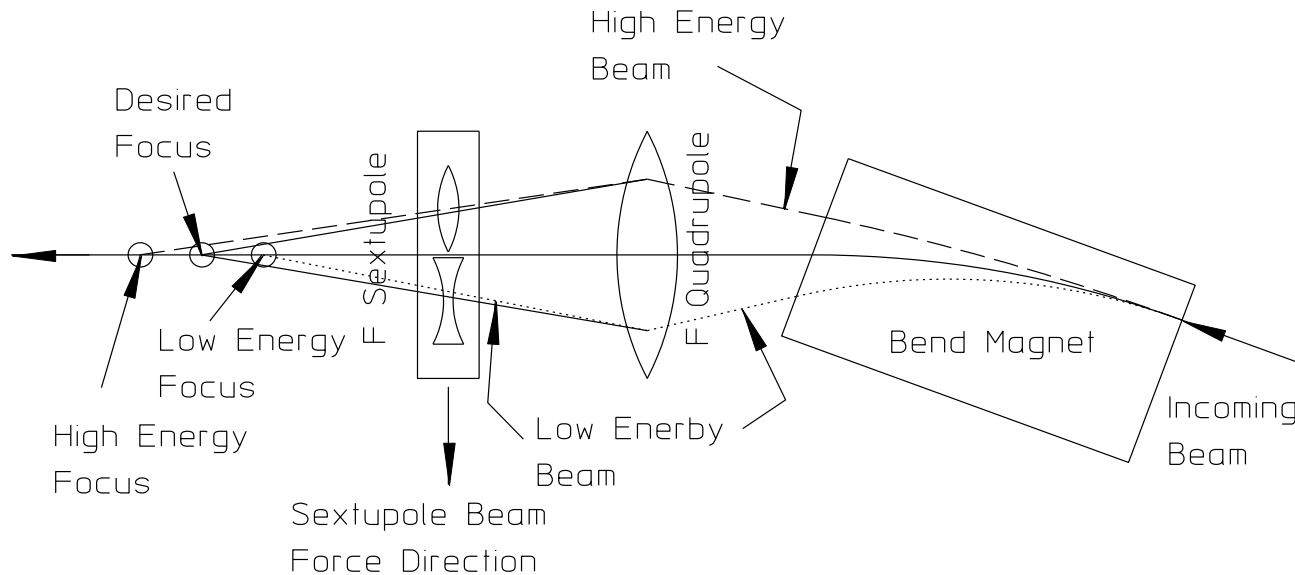
PEP-II Low Energy Ring  
Quadrupole

# The Sextupole Magnet

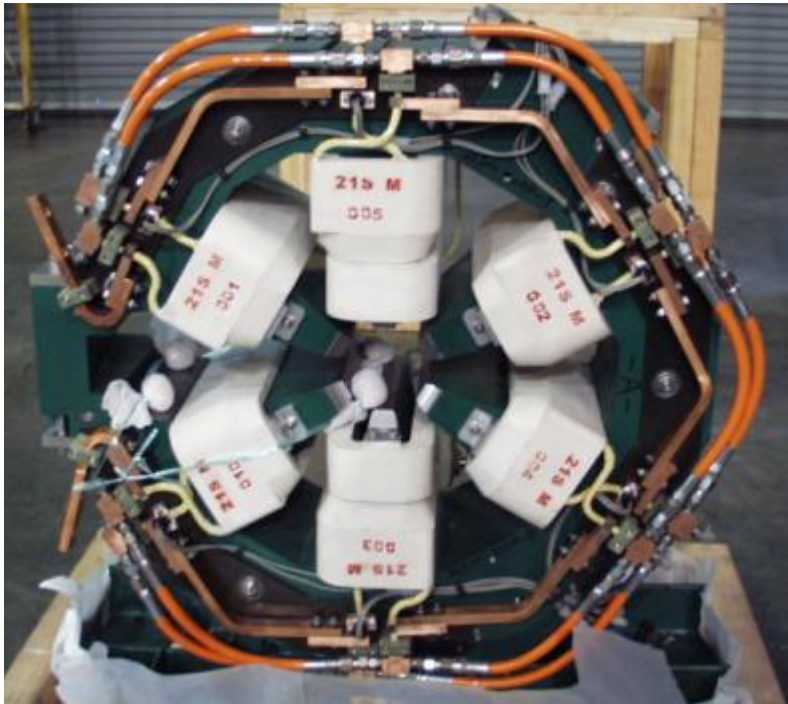
- The Sextupole Magnet has six poles. The field varies *quadratically* with the distance from the magnet center. It's purpose is to affect the beam at the edges, much like an optical lens which corrects chromatic aberration. An *F* sextupole will steer the particle beam toward the center of the ring.
- Note that the sextupole also steers along the 60 and 120 degree lines.



- The function of the sextupole magnet is to correct for momentum spread (the charged beam analogue to the optical chromaticity).
- In the following figure, a charged particle beam with differences in energy is bent by a dipole. Lower energy beam is bent more than the higher energy beam. Thus, when bent by the following quadrupole, the lower energy beam is focussed at a shorter focal length than the higher energy beam.
- The function of the sextupole, located behind the quadrupole, is to defocus the lower energy beam and focus the higher energy beam so that the focal length of the optical system is restored for all energy beams.

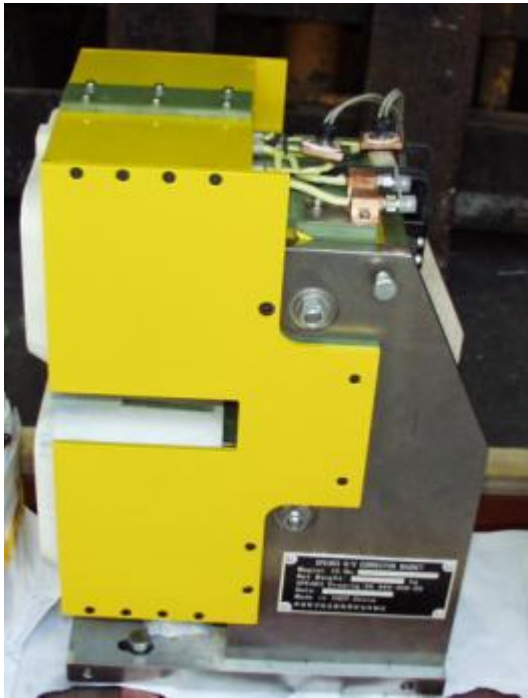


# Sextupole Magnet



- SPEAR3 Sextupole Magnet
  - This magnet, in addition to the six coils exciting the six poles, two coils are added to the top and bottom poles.
  - These coils are added as trims to create a trim field for the magnet. In this case, the trim fields are skew quadrupole fields.
    - The skew quadrupole field is a quadratically distributed field which is horizontal, rather than vertical on the horizontal axis.
  - Trim coils are often added to magnet whose yokes are designed to provide the main field. The trim coils can provide horizontal and vertical steering as well as skew quadrupole.
  - Trim coils are added, especially when the lattice perimeter is limited and there is limited room for separate corrector magnets.

# Correctors



- SPEAR3 Corrector
  - SPEAR3 is an upgrade of an existing accelerator and is a storage ring synchrotron used to exploit high energy photons which are produced when relativistic particles are bent in a magnetic field.
  - Because of this application, the corrector has a yoke with a horizontal gap to provide clearance for the photon beamlines.
    - Although the horizontal bending can be conventionally produced with a coil, the vertical bending (produced by horizontal fields) must be provided by pole face windings whose current distribution is tailored in order to enhance the field uniformity

# Alternate Definition of Polarities

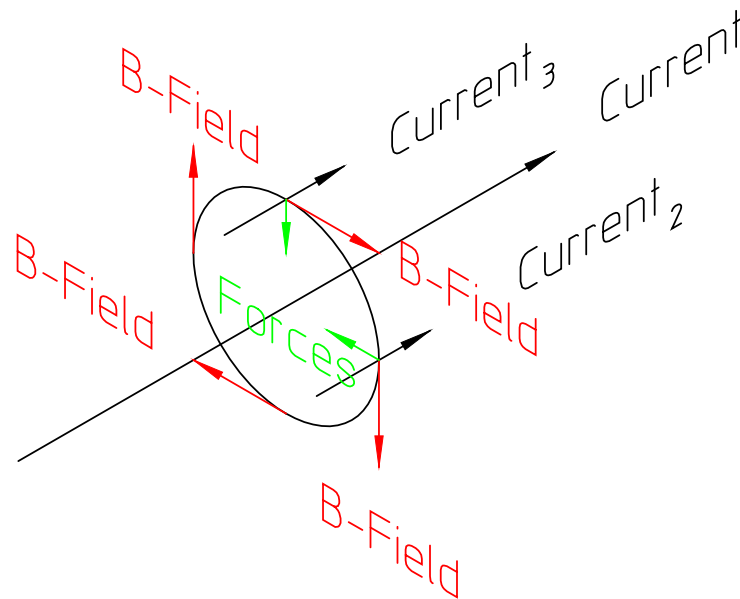
- A number of things have to be remembered in order to install magnets and their power supply connections with appropriate polarities. Primarily, one needs to satisfy the physics needs of the magnets and bend the beam in the desired direction for dipoles and sextupoles and to either focus or defocus the beam (along the horizontal centerline) for the quadrupole.
- Particle beams can be positively or negatively charged. One must remember that power supply conventions define positive current flow from the positive to negative terminals.



- All these factors are confusing and it is quite easy to make design errors and misconnect busses among coils in a magnet or make installation errors and misconnect power supply cables to magnets.
- Both types of errors are quite common and happen often during the installation and connection of accelerators magnets.
- The following uses an alternate method of determining the proper magnet polarity and requires remembering only one principal and its corollaries.

# Magnetostriction

The forces on parallel currents is illustrated in the following figure. The force on a charge moving with a given velocity through a magnetic field is expressed with the vector equation,  $\underline{F} = e\underline{v} \times \underline{B}$



# Alternate Polarity Definition

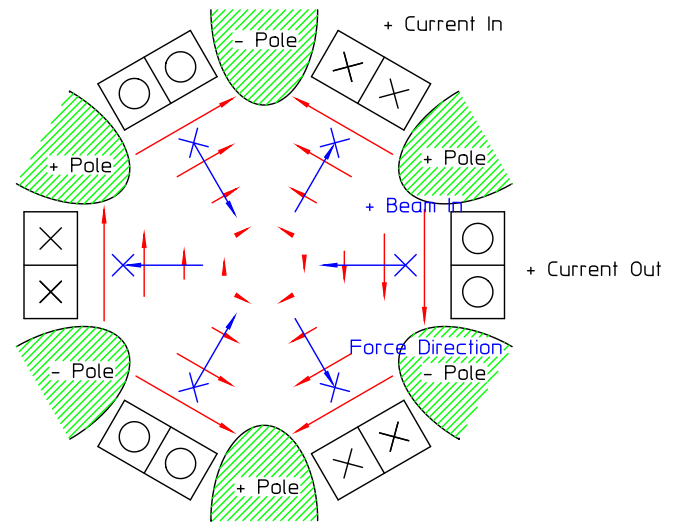
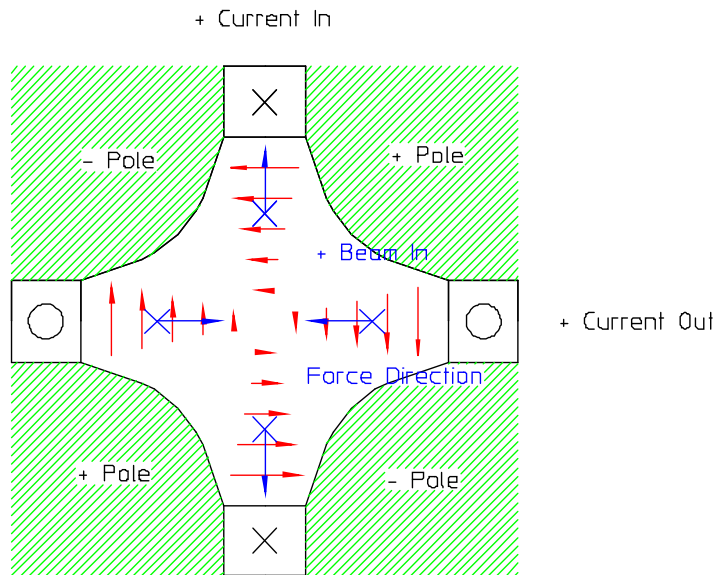
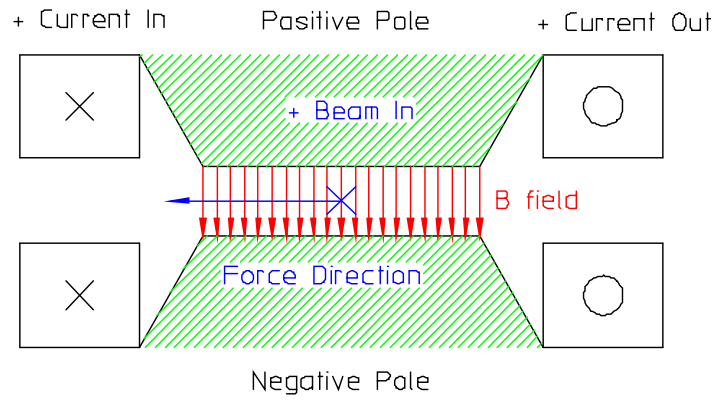
- Currents with the same charge travelling in the same direction *attract*.

- **Corollaries:**

Currents with opposite charge travelling in the same direction *repel*.

Currents with the same charge travelling in the opposite direction *repel*.

Currents with the opposite charge travelling in the opposite direction *attract*.

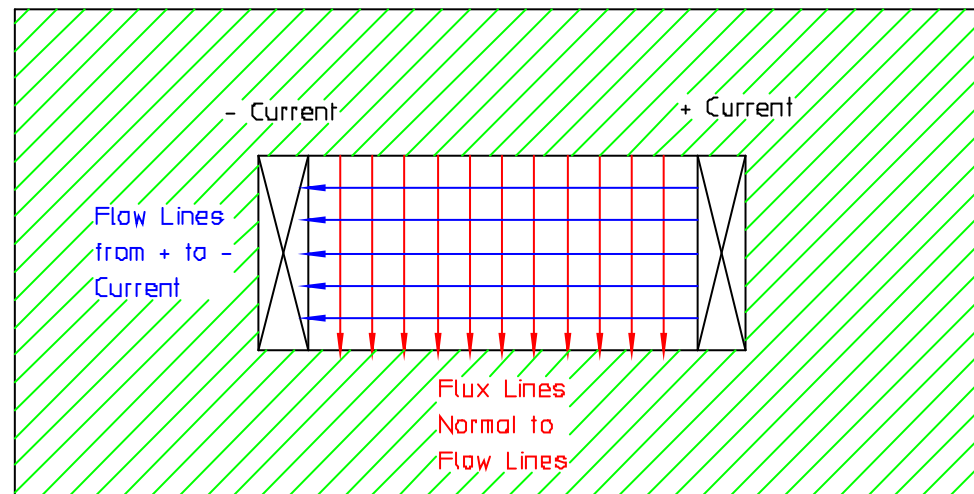


# Orthogonal Analog Model

- The name of the method for picturing the field in a magnet is called the *Orthogonal Analog Model* by Klaus Halbach. This concept is presented early in the lecture in order to facilitate visualization of the magnetic field and to aid in the *visualization* of the *vector* and *scalar* potentials.

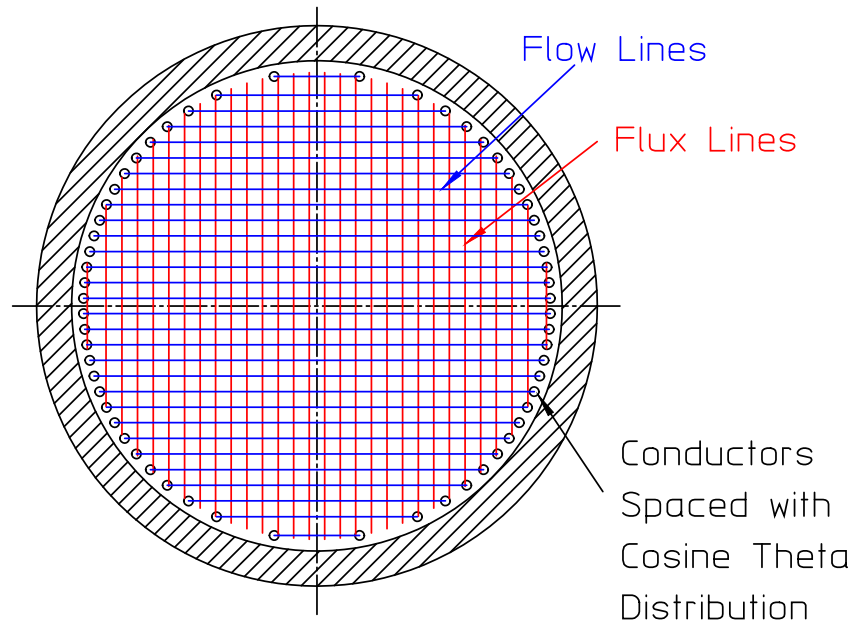
- A window frame dipole magnet is illustrated in order to demonstrate the principles.
  - *Flow Lines* go from the + to - Coils.
  - *Flux Lines* are *ortho-normal* to the *Flow Lines*.
  - Iron Surfaces are impervious to *Flow Lines*.
- Applying the *model* to the *Window Frame Dipole*, it can be seen that the field distribution in this geometry is very uniform.

"Window Frame" Magnet



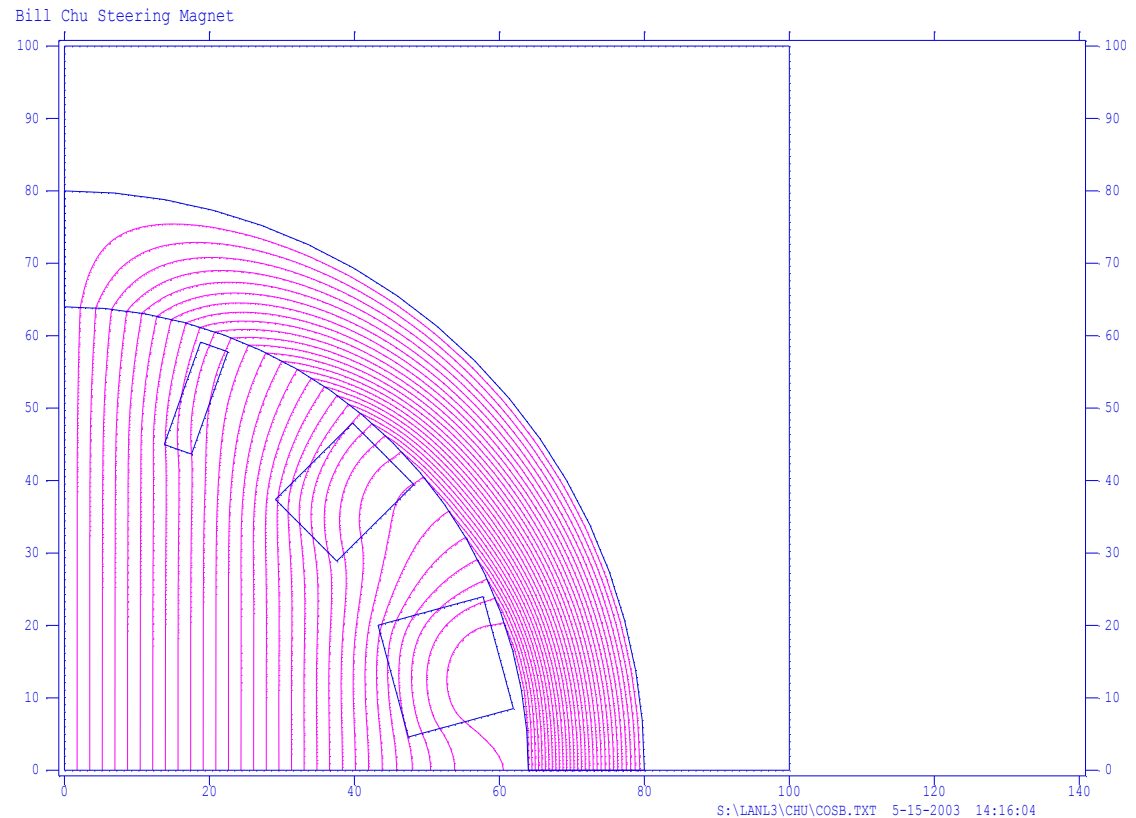
# Cosine $\Theta$ Dipole

- Using the Orthogonal Analog Model and requiring the flux lines to be vertical and uniformly spaced in the circular yoke, the flow lines, connecting the current filaments, must be horizontal and also be uniformly spaced.
- It can be shown that the current filaments satisfy a Cosine  $\Theta$  distribution.
  - This is an example of a magnet whose field is shaped by a tailored current distribution.



# Three Block Cosine $\Theta$ Distribution

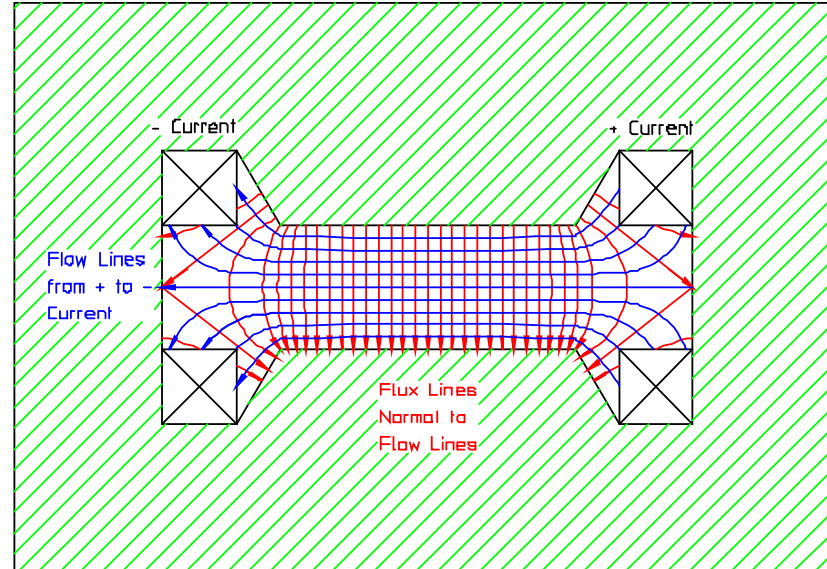
- When dividing the conductors among three blocks, approximating the cosine  $\Theta$  distribution, the POISSON plot looks like the illustration.





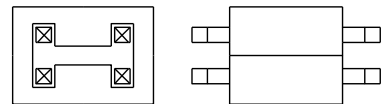
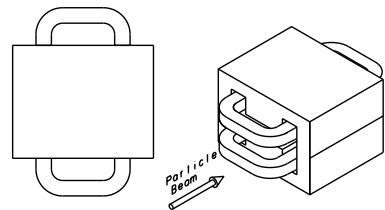
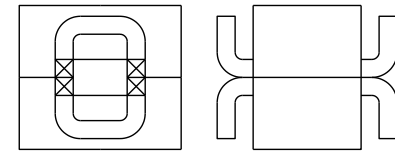
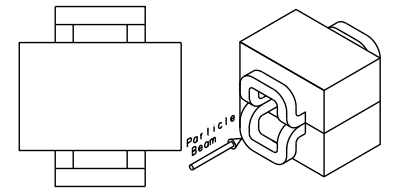
- Application of the Orthogonal Analog model to the *H magnet* geometry reveals several field properties;
  - The field falls off near the edges of the pole.
  - The field at the pole corner is high and likely to saturate.

"H" Magnet



# Choice of Dipole Type

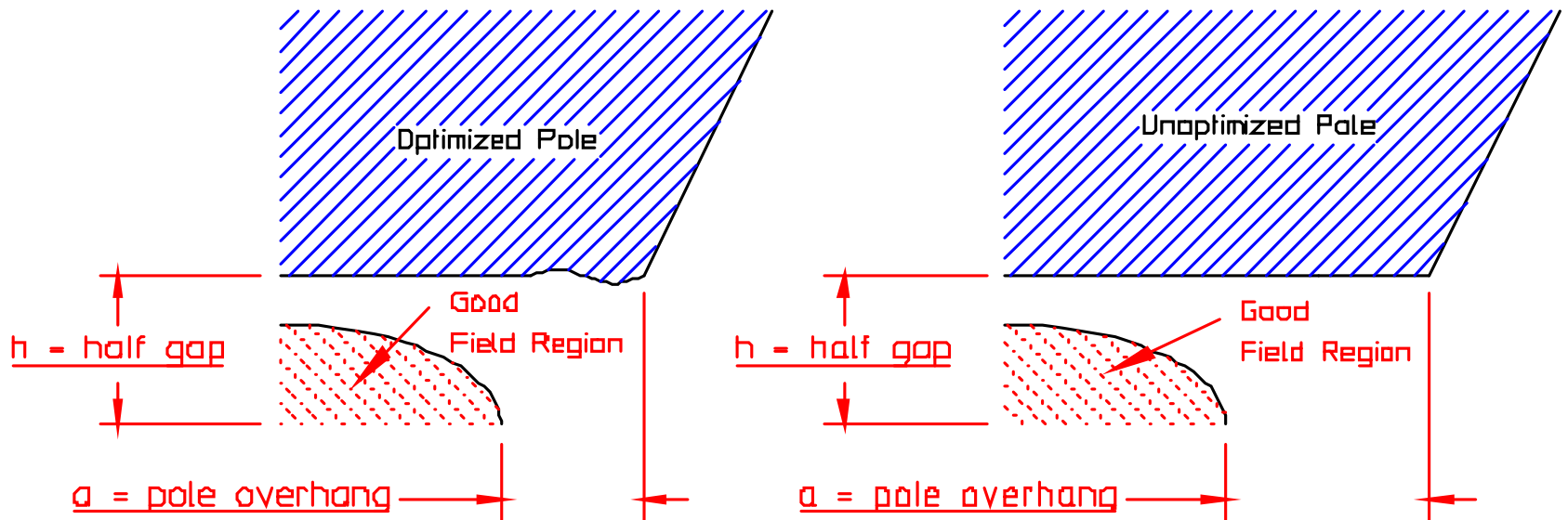
- Why build H dipoles?
  - *Window Frame*  
Magnets require  
*Saddle Coils* wound on  
two axes.
  - *H* Magnets can use  
*Pancake Coils* wound  
on a single axis.



# *H Magnet* Field Uniformity

- In general, the two dimensional magnet field quality can be improved by the amount of excess pole beyond the boundary of the good field region.
- The amount of excess pole can be reduced, for the same required field quality, if one optimizes the pole by adding features (bumps) to the edge of the pole.

- The relation between the field quality and "pole overhang" are summarized by simple equations for a window frame dipole magnet with fields below saturation.



$$\left(\frac{\Delta B}{B}\right)_{\text{optimized}} = \frac{1}{100} \exp[-7.17(x - 0.39)]$$

$$x_{\text{optimized}} = \left(\frac{a}{h}\right)_{\text{optimized}} = -0.14 \ln \frac{\Delta B}{B} - 0.25$$

$$x = \frac{a}{h} = \frac{\text{"pole overhang"}}{\text{half gap}}$$

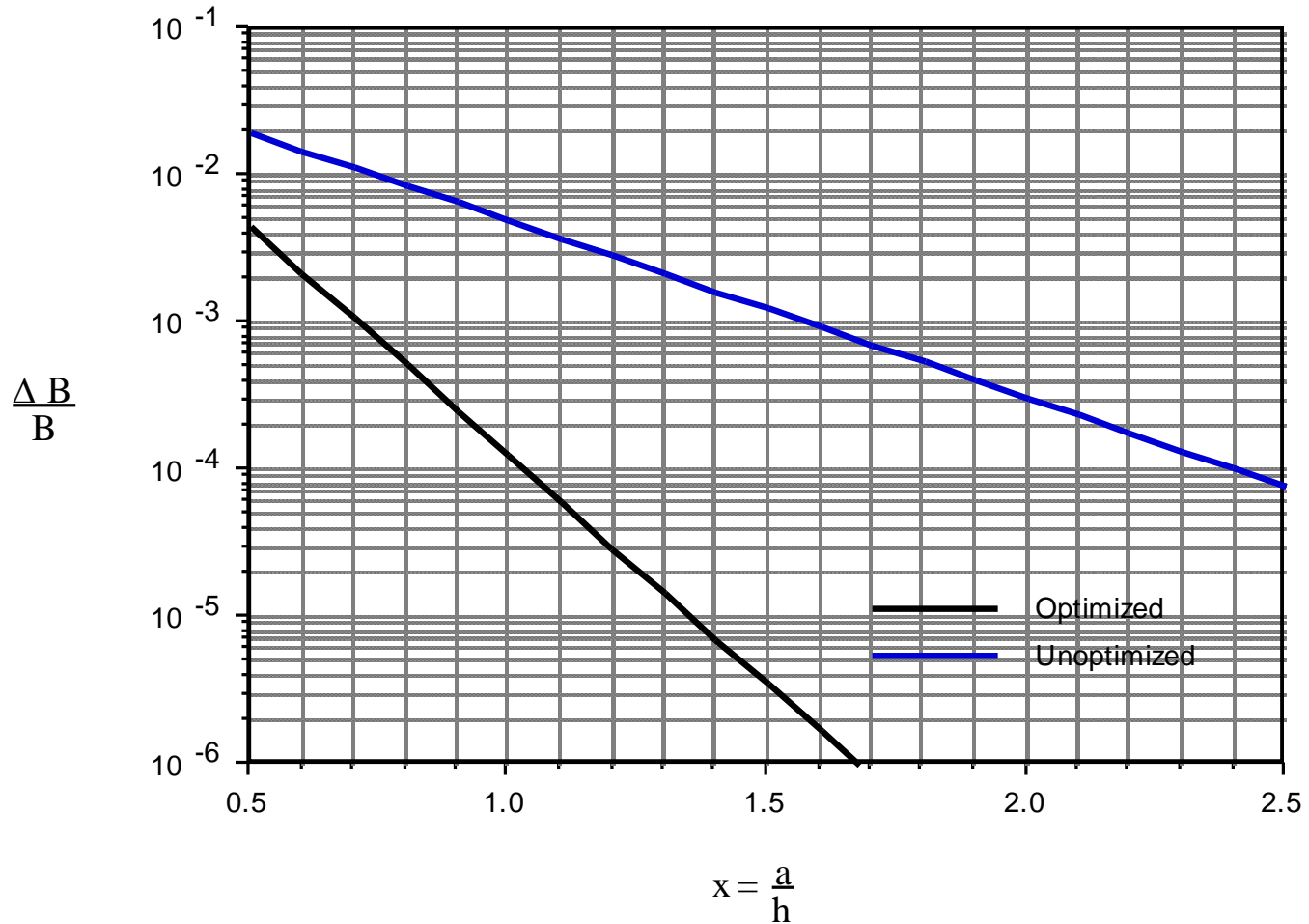
$$\left(\frac{\Delta B}{B}\right)_{\text{unoptimized}} = \frac{1}{100} \exp[-2.77(x - 0.75)]$$

$$x_{\text{unoptimized}} = \left(\frac{a}{h}\right)_{\text{unoptimized}} = -0.36 \ln \frac{\Delta B}{B} - 0.90$$

These expressions are very important since they give general rules for the design of window frame dipole designs. It will be seen later that these expressions can also be applied to quadrupole and gradient magnets.

- Graphically:

Dipole Field Quality  
as a Function of Pole Overhang



## Lecture 2

- Lecture 2 will cover the mathematical characterization of the two dimensional magnetic field and discuss functions which describe the three basic magnet types, the dipole, quadrupole and sextupole.
- Chapter Two of the book should be read.