Lecture 5
Perturbations

[1] Halbach, K., FIRST ORDER PERTURBATION EFFECTS IN IRON-DOMINATED TWO-DIMENSIONAL SYMMETRICAL MULTIPOLES, “Nuclear Instruments and Methods”, Volume 74 (1969) No. 1, pp. 147-164.
[2] Halbach, K., and R. Yourd, TABLES AND GRAPHS OF FIRST ORDER PERTURBATION EFFECTS IN IRON-DOMINATED TWO-DIMENSIONAL SYMMETRICAL MULTIPOLES, LBNL Internal Report, UCRL-18916, UC-34 Physics, TID 4500 (54thEd.), May 1969.

- The subject of Perturbations is covered in chapter 4 of the text and is one of the more important subjects covered in this course. This is because the performance of an accelerator lattice is dominated by the quality and reproducibility of the magnets fabricated/installed in the lattice.
- Perturbations are characterized by the multipole content of a magnet. A good magnet is characterized by the harmonic content of its integrated field.
  - A perfect dipole is characterized by $F_1 = C_1 z$
  - A perfect quadrupole is characterized by $F_2 = C_2 z^2$
  - A perfect sextupole is characterized by $F_3 = C_3$

- Tables of coefficients and expressions are presented which make it possible to compute the magnitude of error multipoles resulting from fabrication, assembly and pole excitation errors associated with magnet manufacture.
  - Pole excitation errors can result from shorted coil turns, errors in winding the coils or other sources.
    - The other sources of pole excitation errors are poles which differ in length from other poles.
    - Pole excitation errors can also be introduced *intentionally* in order to produce trim fields, those fields which do not normally exist in a particular yoke geometry.
- Physics requirements normally specify the maximum amplitude of the various multipole errors. As a magnet designer, these multipole errors must be translated into fabrication/assembly tolerances. Examples of tolerance calculations are given in Section 4.3.5 of the text.

Effect of Mechanical Fabrication Errors on Error Multipole Content

- In the previous lecture, we showed that the field distribution in a magnet can be characterized by a function of the complex variable, *z*. In particular;

$$F = C_N z^N + \sum_{n \neq N} C_n z^n$$

$$C_N z^N = Fundamental\ Field\ Component$$

$$ErrorFields = \sum_{n \neq N} C_n z^n$$

Random Multipole Errors Introduced by Pole Excitation

and Pole Placement Errors

- Random multipole errors are introduced if the poles are improperly excited or assembly errors which displace poles are introduced. If one can identify these errors, one can predict the multipole content of the magnet. The means for calculating these errors are summarized in two papers published by Klaus Halbach. The first paper describes the derivation of the relationships, the second computes and tabulates the coefficients used to calculate the multipole errors from the perturbations derived in the first paper.

- A portion of a table from UCRL-18916 by Halbach and Yourd is reproduced below. This table is for quadrupoles, N=2.

n	$\frac{n}{N}\frac{\Delta C_n(j)}{i\varepsilon}$	$\frac{n}{N}\frac{\Delta C_n(rd)}{i\varepsilon}$	$\frac{n}{N}\frac{\Delta C_n(ad)}{\varepsilon}$	$\frac{n}{N}\frac{\Delta C_n(r)}{\varepsilon}$
1	1.99 • 10-1	-4.25 • 10-1	7.46 • 10-2	1.76 • 10-1
2	2.50 • 10-1	-5.16 • 10-1	2.14 • 10-1	5.00 • 10-1
3	1.57 • 10-1	-2.88 • 10-1	2.88 • 10-1	6.60 • 10-1
4	0	6.76 • 10-2	2.31 • 10-1	5.00 • 10-1
5	-2.05 • 10-2	1.08 • 10-1	1.08 • 10-1	1.91 • 10-1
6	0	-4.45 • 10-2	2.87 • 10-2	0
7	1.61 • 10-2	-1.04 • 10-2	1.04 • 10-2	-3.06 • 10-2
8	0	1.28 • 10-2	1.56 • 10-2	0
9	-1.90 • 10-3	1.25 • 10-2	1.25 • 10-2	7.53 • 10-3
10	0	6.37 • 10-3	5.81 • 10-3	0

- This table of coefficients is used to estimate the error multipole due to excitation, radial, azimuthal and rotation errors in the location of a pole on the horizontal axis. The imaginary term in the denominator for the excitation (j) and radial (rd) terms indicate that errors in these quantities introduce skew terms. The tabulated coefficients are computed for poles *centered on the positive horizontal axis*. For the normal multipole magnet (one whose axis is rotated away from the horizontal axis), the expression for calculating the error multipole normalized to the desired fundamental, evaluated at the pole radius, is given by;

$$\frac{H_n^*}{H_N^*} = \frac{n}{N} \Delta C_n e^{-in\beta}$$

where β is the angle of the pole(s) which have been perturbed.

Example Calculation

- Suppose we construct a 35 mm radius quadrupole whose first pole is radially offset by 1 mm. What is the effect on the n=3 (sextupole) multipole error.

$$\varepsilon_{rd} = \frac{\Delta\,\text{azimuthal position}}{\text{r}_0} = \frac{1}{35}$$

$$\frac{n}{N}\frac{\Delta C_n(rd)}{i\varepsilon} = -0.288 \qquad \text{where } n=3.$$

$$\frac{H_n^*}{H_N^*} = \frac{n}{N}\Delta C_n e^{-in\beta} = \frac{n\Delta C_n}{Ni\varepsilon}\times i\varepsilon_{rd} e^{-in\beta} = -0.288\times i\frac{1}{35}e^{-i3\frac{\pi}{4}}$$

$$= -8.23\times 10^{-3} e^{i\frac{\pi}{2}} e^{-i\frac{3\pi}{4}} = -8.23\times 10^{-3} e^{-i\frac{\pi}{4}}$$

Meaning of the Result

- The calculation means that the *n=3* (sextupole) error multipole *normalized to the fundamental field at the pole radius* (35 mm) is approximately 0.8% due to the radial displacement of the first pole at π/4 by 1 mm.
- Carrying the calculation further to determine the phases;

$$\left(\frac{H_{3x} - iH_{3y}}{-iH_{2y}}\right)_{@35mm} = -8.23\times10^{-3} e^{-i\frac{\pi}{4}}$$

$$= -8.23\times10^{-3}\left[\cos\left(-\frac{\pi}{4}\right) + i\sin\left(-\frac{\pi}{4}\right)\right]$$

- Further simplification;

$$\left(\frac{H_{3x} - iH_{3y}}{-iH_{2y}}\right)_{@35mm} = \left(\frac{iH_{3x} + H_{3y}}{H_{2y}}\right)_{@35mm}$$

$$= -8.23\times10^{-3}\left[\frac{\sqrt{2}}{2} - i\frac{\sqrt{2}}{2}\right]$$

$$\left(\frac{H_{3y} + iH_{3x}}{H_{2y}}\right)_{@35mm} = 5.82\times10^{-3}\left[-1+i\right]$$

This means that the real component of the sextupole error is out of phase with the fundamental field and that a positive skew component of the sextupole field also exists.

Evaluation at the Required Good Field Radius

- We recall that the field for an n multipole varies as z^{n-1}. Therefore if the good field radius is r_0=30 mm, the $n=3$ normalized multipole error can be evaluated at this radius.

$$\left(\frac{H_{3y}+iH_{3x}}{H_{2y}}\right)_{@30mm} = \left(\frac{H_{3y}+iH_{3x}}{H_{2y}}\right)_{@35mm} \left(\frac{30}{35}\right)^2 \Big/ \left(\frac{30}{35}\right)$$

$$= \left(\frac{H_{3y}+iH_{3x}}{H_{2y}}\right)_{@35mm} \left(\frac{30}{35}\right)$$

Other Errors

- The coefficient table can be used for other errors.

$$\varepsilon_j = \frac{\Delta \, \text{excitation}}{\text{excitation}}$$

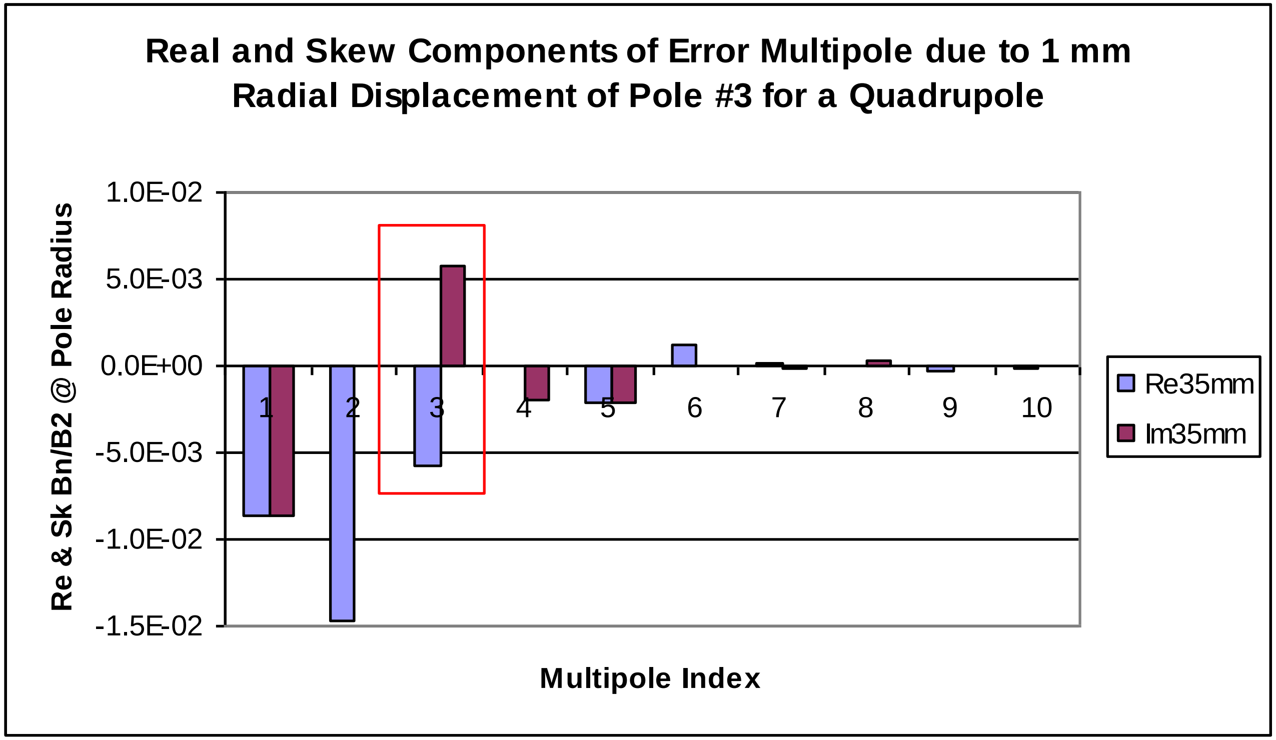
$$\varepsilon_{rd} = \frac{\Delta \, \text{radial position}}{\text{r}_0}$$

$$\varepsilon_{ad} = \frac{\Delta \, \text{azimuthal position}}{\text{r}_0}$$

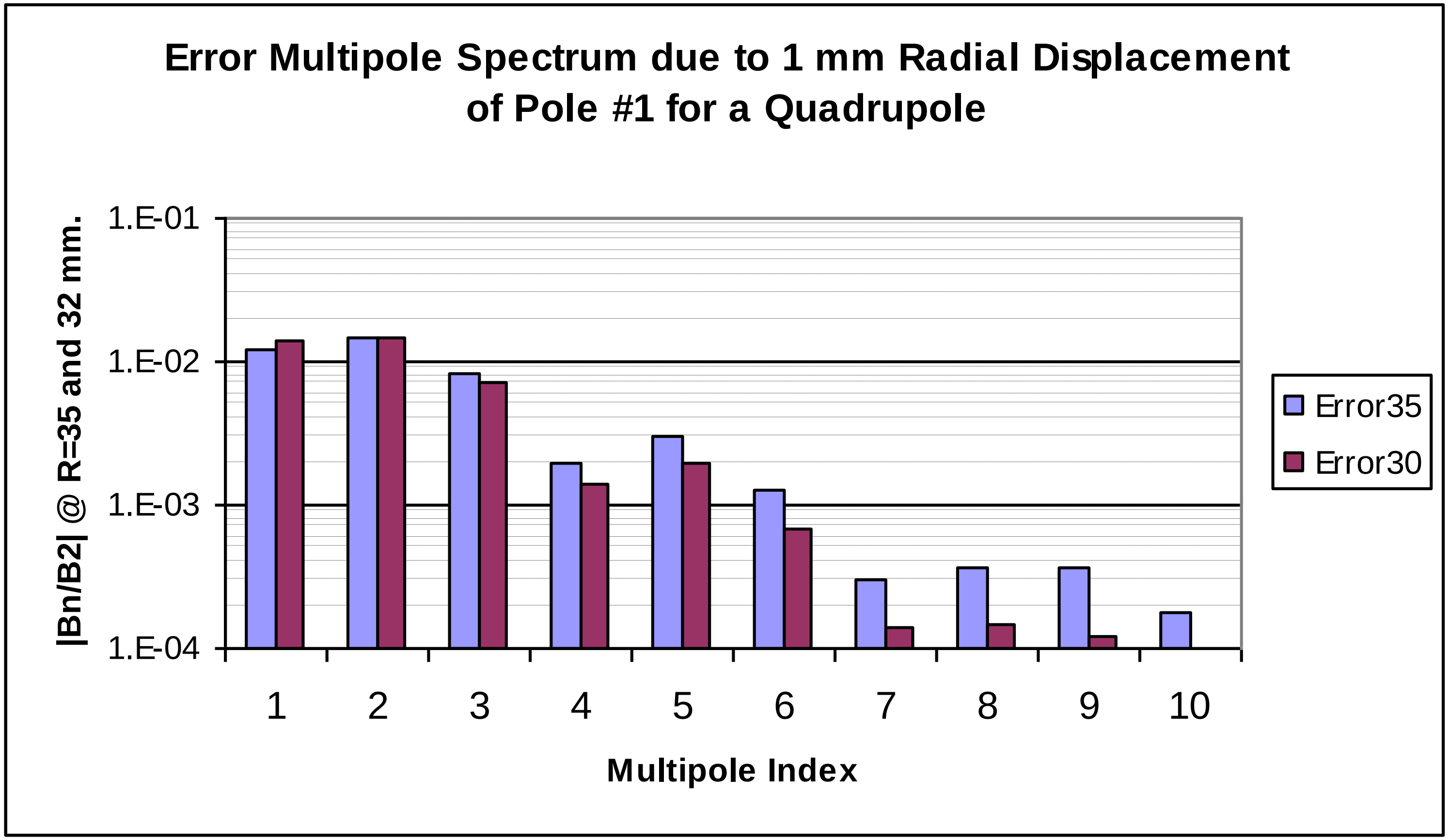
$$\varepsilon_{\rho} = \Delta \mathit{pole} \, \text{rotation} \left(\text{radians} \right)$$

The Error Multipole *Spectrum*

- In general, the table of coefficients includes entries for *all* the multipole indices. Therefore, although the sample calculation was performed *only* for the sextupole error field component, all the multipole errors due to the radial misalignment of the first pole exist.
- These errors include the error in the fundamental (n=2) as well as the n=1 dipole field.



Error Amplitudes as a Function of Radius



Lesson

- The lesson from this sample calculation is *not* the detailed calculation of the multipole error, but the estimate of the mechanical assembly tolerances which must be met in order to achieve a required field quality.
- In general, the coefficient is <0.5. Therefore in order to achieve a field error at the pole radius of 5 parts in 10000 (a typical multipole error tolerance), the following tolerance illustrated in the calculation must be maintained.

$$\left|\frac{H_n^*}{H_N^*}\right| = Coefficient \times \frac{\Delta}{PoleRadius} \qquad \Rightarrow$$

$$\Delta \leq \left|\frac{H_n^*}{H_N^*}\right|_{required} \frac{PoleRadius}{Coefficient} = 5\times 10^{-4} \times \frac{35mm}{0.5}$$

$$= 0.035mm = 0.0014inch$$

A very small error.

Another Lesson
The Magnet Center

- We note that *all* the multipole errors are introduced by mechanical assembly errors. In particular, we look in detail at the *dipole* error term introduced by assembly errors.
- For the *pure* quadrupole field, the expression for the complex function is;

$$F(z) = C_2 z^2$$

If the magnet center is shifted by an amount Δz, the expression becomes;

$$F(z) = C_2 (z + \Delta z)^2$$

$$H^* = iF'(z) = i2C_2(z + \Delta z) = i2C_2 z + i2C_2 \Delta z$$

The first term in this expression is the quadrupole field (a linear function of *z*). The second term is a constant and, therefore, is the dipole field.

- Rewriting the expression as the sum of two fields;

$$H^* = H_2^* + H_1^* = i2C_2 z + i2C_2(\Delta x + i\Delta y)$$

$$H_1^* = H_{1x} - iH_{1y} = i2C_2(\Delta x + i\Delta y)$$

$$\frac{H_{1x} - iH_{1y}}{|H_2^*|} = \frac{i2C_2(\Delta x + i\Delta y)}{2C_2|z|}$$

Evaluating the quadrupole field at the pole radius, r_0;

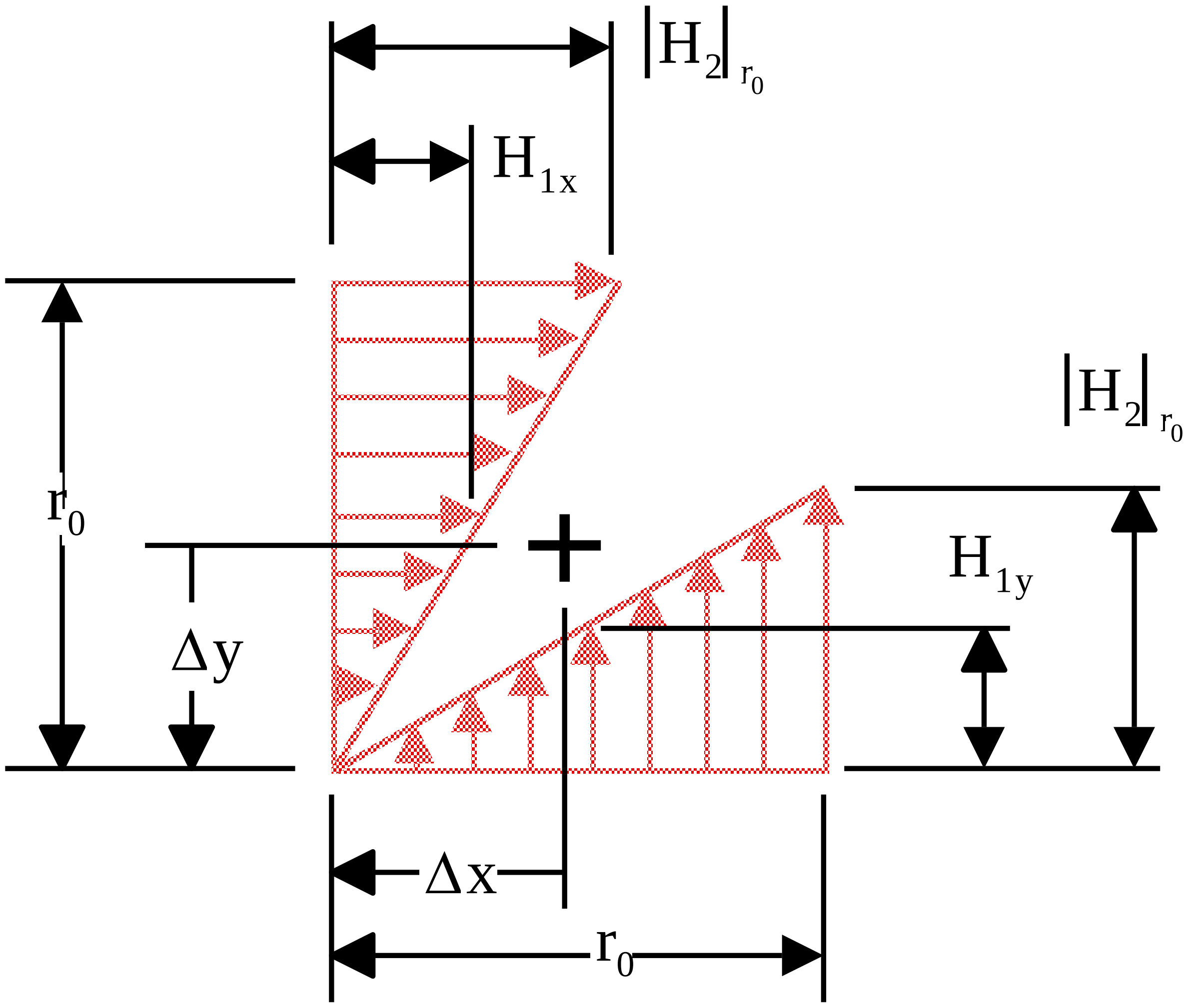
$$\frac{H_{1x} - iH_{1y}}{|H_2^*|_{@r_0}} = \frac{H_{1x}}{|H_2^*|_{@r_0}} - i\frac{H_{1y}}{|H_2^*|_{@r_0}} = \frac{i(\Delta x + i\Delta y)}{r_0} = i\frac{\Delta x}{r_0} - \frac{\Delta y}{r_0}$$

Equating the real and imaginary parts of the expression, the magnetic center shift can be evaluated from the dipole field.

$$\Delta x = -r_0 \frac{H_{1y}}{|H_2|_{r_0}}$$

$$\Delta y = -r_0 \frac{H_{1x}}{|H_2|_{r_0}}$$

- These relationships may be more easily visualized with a figure



Effect of a Pole Excitation Error on the Magnetic Center

- One of the many issues faced by the NLC project is the stability of the magnetic center of adjustable hybrid permanent magnet quadrupoles.
- A sample calculation is made to compute the required pole excitation precision.

$$\frac{nC_n(j)}{Ni\varepsilon} = 1.99\times 10^{-1}$$ For n = 1 (dipole error)

where $$\varepsilon_j = \frac{\Delta\,\text{excitation}}{\text{excitation}}$$

$$\frac{H_n^*}{H_N^*} = \frac{n}{N}\Delta C_n e^{-in\beta}$$

- Suppose we have a 1% error on the excitation of a single pole in the first quadrant.

$$\frac{H_1^*}{H_2^*} = \frac{n}{N} \Delta C_n e^{-in\beta} = 0.199 \times 0.01 i\, e^{-i\frac{\pi}{4}} \approx 0.002 i \left(\cos\frac{\pi}{4} - i \sin\frac{\pi}{4} \right)$$

$$\frac{H_{1x} - iH_{1y}}{-iH_{2y}} = \frac{H_{1y} + iH_{1x}}{H_{2y}} = 0.002i(0.707 - i0.707) = i0.0014 + 0.0014$$

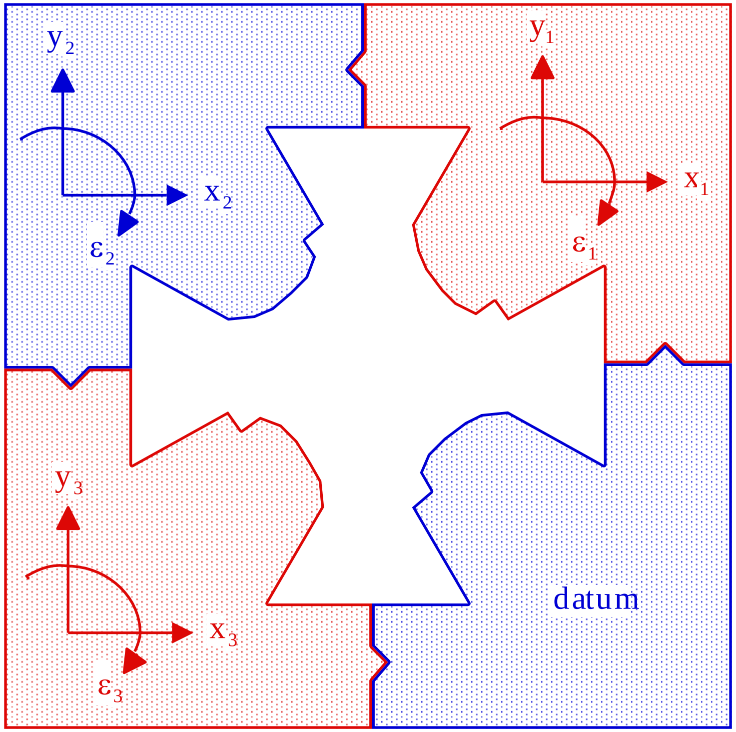
Equating the real and imaginary terms,

$$\frac{H_{1y}}{H_{2y}} = 0.0014 \qquad \Rightarrow \qquad \Delta x = -r_0 \frac{H_{1y}}{|H_2|_{r_0}} = -0.0014 r_0$$

$$\frac{H_{1x}}{H_{2y}} = 0.0014 \qquad\qquad \Delta y = -r_0 \frac{H_{1x}}{|H_2|_{r_0}} = -0.0014 r_0$$

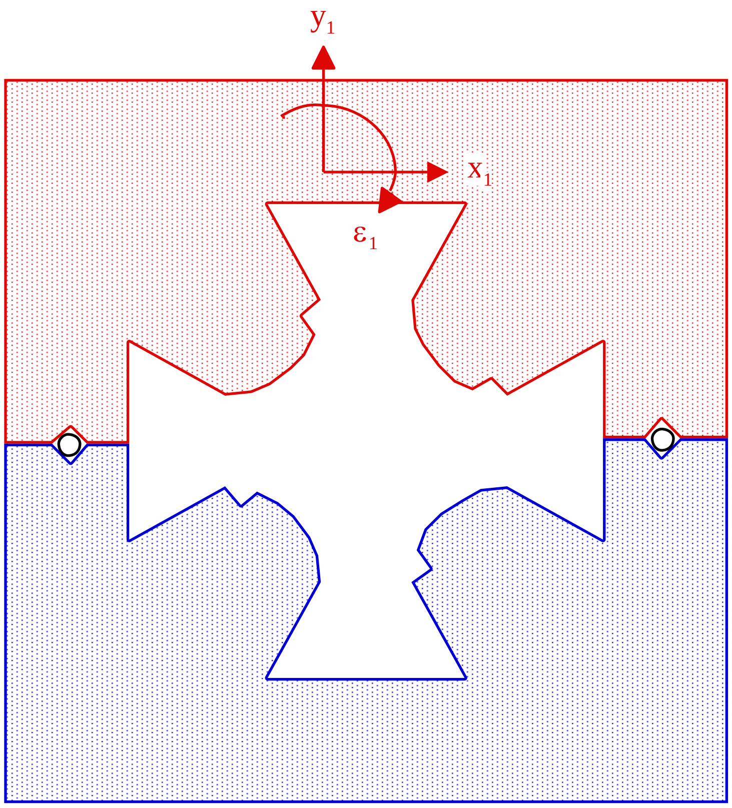
The Four Piece Magnet Yoke

- The ideal assembly satisfies the rotational symmetry requirements so that the only error multipoles are allowed multipoles, n=6, 10, 14 ... However, *each* segment can be assembled with errors with three kinematic motions, x, y and ε (rotation). Thus, combining the possible errors of the three segments with respect to the datum segment, the core assembly can be assembled with errors with 3x3x3=27 degrees of freedom.



The Two Piece Magnet Yoke

- This assembly has the advantage that the two core halves can be assembled kinematically with *only* three degrees of freedom for assembly errors. Thus, assembly errors are more easily measured and controlled.



Coefficients for a Two-Piece Quadrupole

n	$\frac{n}{N}\frac{C_n}{i\Delta x}$	$\frac{n}{N}\frac{C_n}{\Delta y}$	$\frac{n}{N}\frac{C_n}{\varepsilon_\rho}$
1	0	0	2.49 • 10-1
2	3.02 • 10-1	-7.30 • 10-1	0
3	0	0	-9.33 • 10-1
4	-9.56 • 10-2	-3.27 • 10-1	0
5	0	0	-2.70 • 10-1
6	-4.06 • 10-2	-6.29 • 10-2	0
7	0	0	-4.33 • 10-2
8	1.81 • 10-2	2.20 • 10-2	0
9	0	0	1.06 • 10-2
10	8.22 • 10-3	9.01 • 10-3	0
11	0	0	5.12 • 10-3
12	-3.77 • 10-3	-3.94 • 10-3	0
13	0	0	-1.31 • 10-3
14	-1.74 • 10-3	-1.78 • 10-3	0
15	0	0	-9.41 • 10-4
16	8.14 • 10-4	8.23 • 10-4	0

Two Piece Quadrupole Error Computations

- The computations of the multipole error fields due to assembly errors of the two piece quadrupole are similar but simpler than the computations for the four piece quadrupole. In the expressions below, *h* is the pole radius.
- The error terms are evaluated at the pole radius.

$$\frac{H_n^*}{H_N^*} = \frac{n}{N} C_n \qquad \Delta x = \frac{\delta x}{h} \qquad \Delta y = \frac{\delta y}{h} \qquad \varepsilon_\rho = \text{Rotational Perturbation}$$

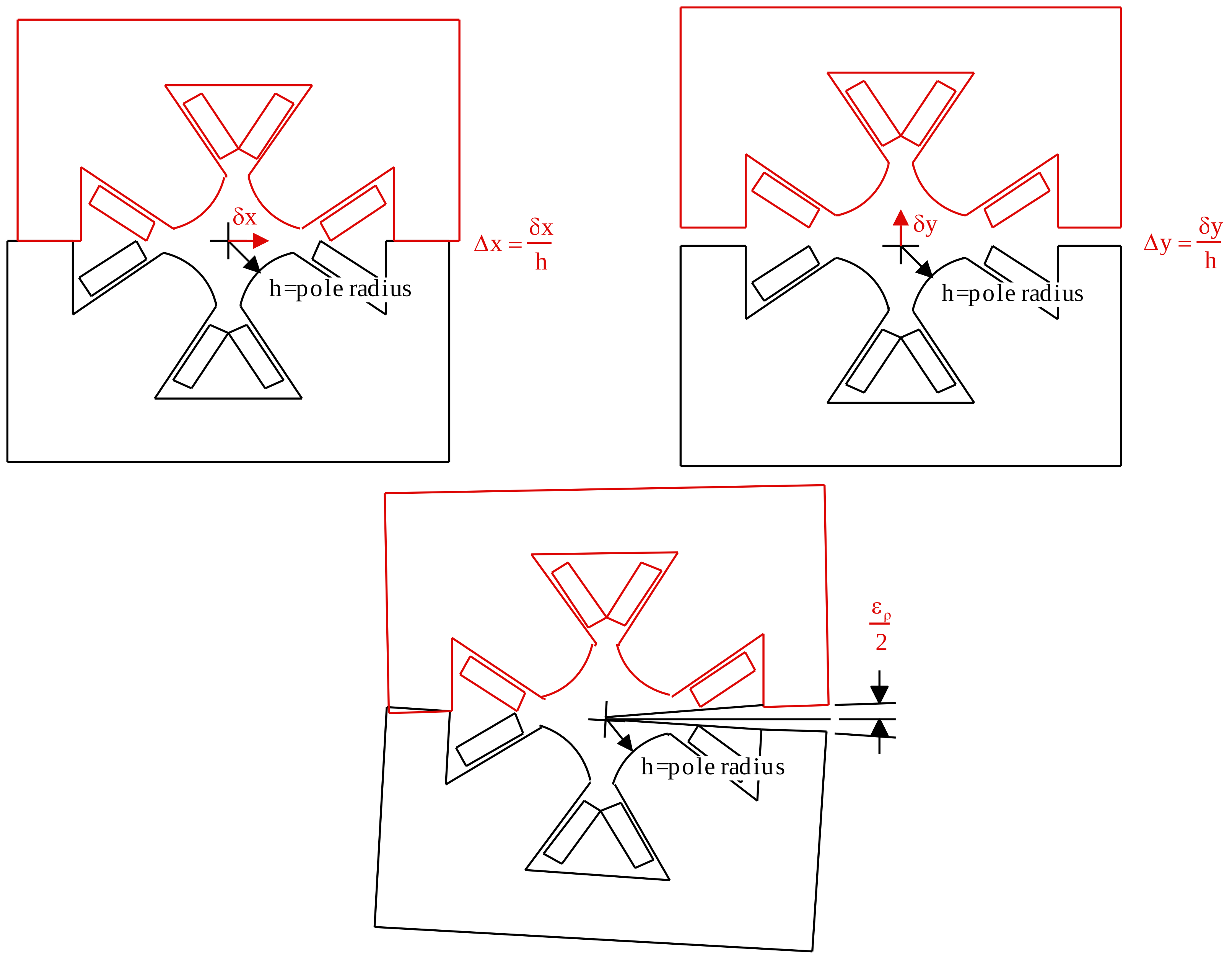
$$\left(\frac{H_n^*}{H_N^*} \right)_{\Delta x} = Coefficient \times i \frac{\delta x}{h}$$

Why *i*?

$$\left(\frac{H_n^*}{H_N^*} \right)_{\Delta y} = Coefficient \times \frac{\delta y}{h}$$

$$\left(\frac{H_n^*}{H_N^*} \right)_{Rotation} = Coefficient \times \varepsilon_\rho$$

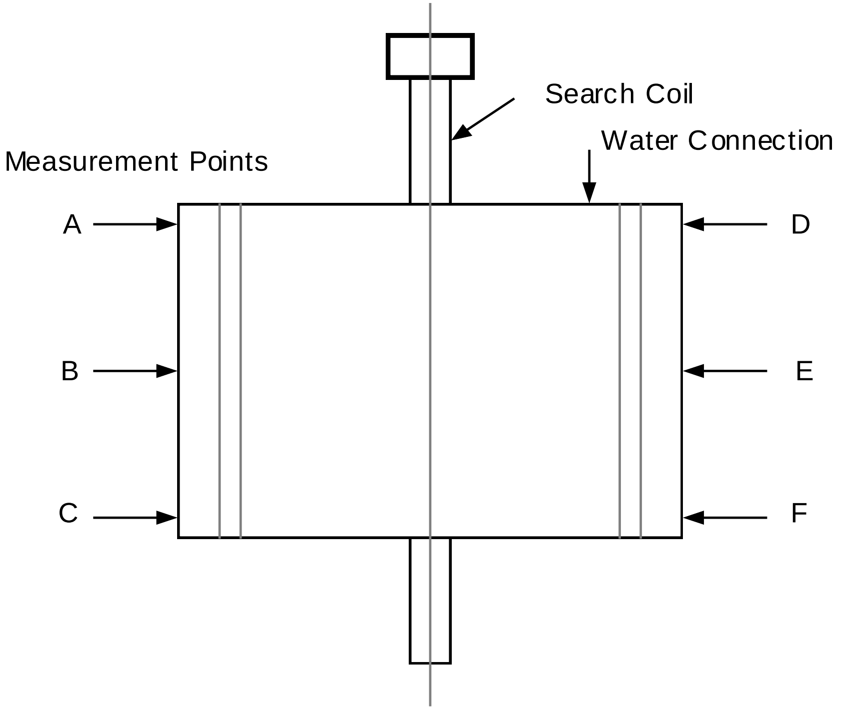
- Referring to the table;
  - The shear motion of the top half of the magnet with respect to the bottom introduces skew even multipole errors.
  - The vertical motion introduces real even multipole errors.
  - The rotational motion introduces real odd multipole errors.



δx
Δx = δx/h
h=pole radius
δy
Δy = δy/h
h=pole radius
ε_ρ/2
h=pole radius

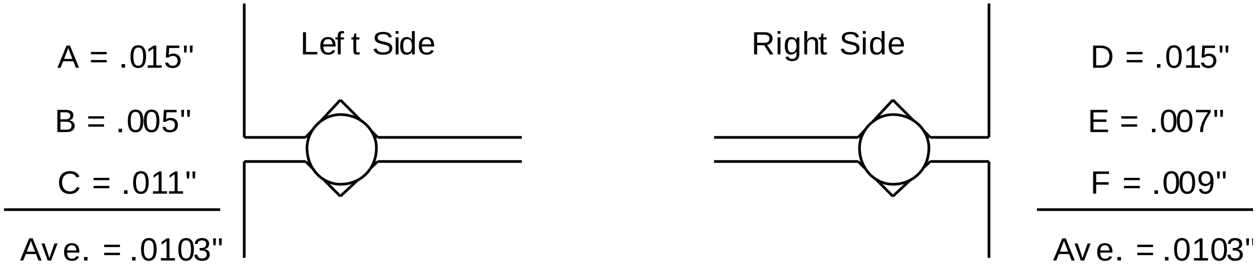
Experiments

- Computations using the coefficients for the two piece magnet have been compared to experiments where the upper half of a magnet was intentionally displaced with respect to its nominal position.

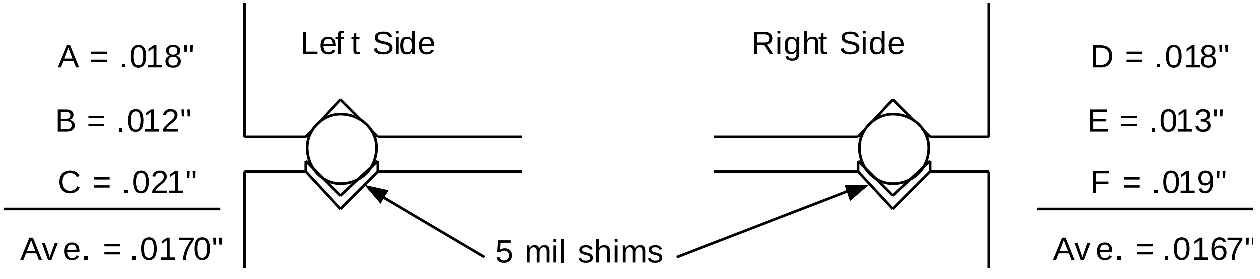


Vertical Perturbation

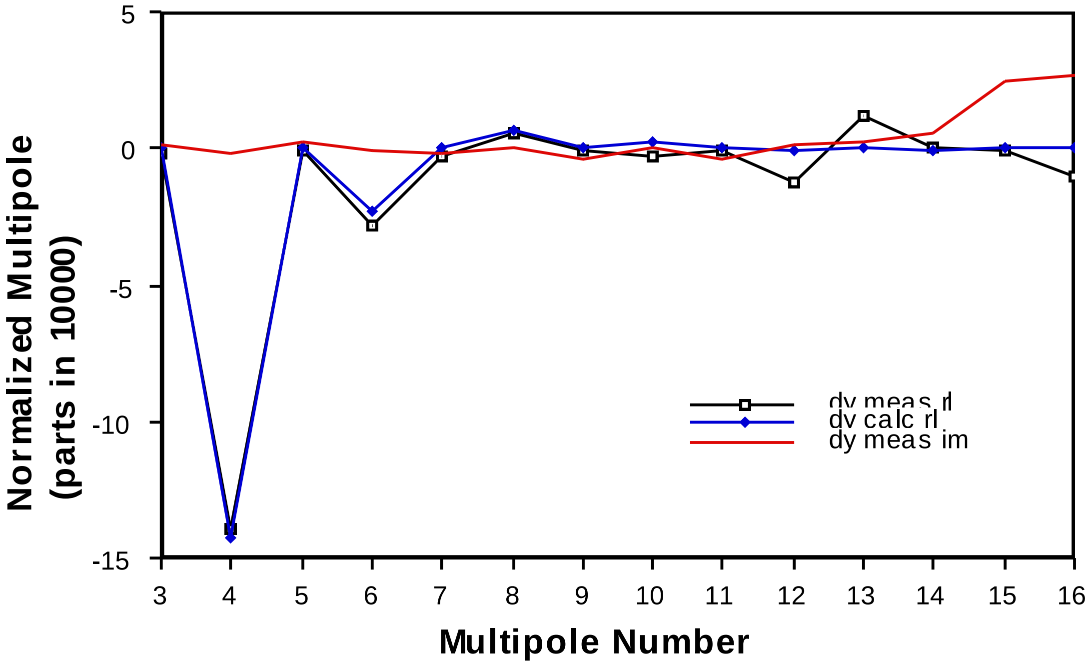
Normal Assembly



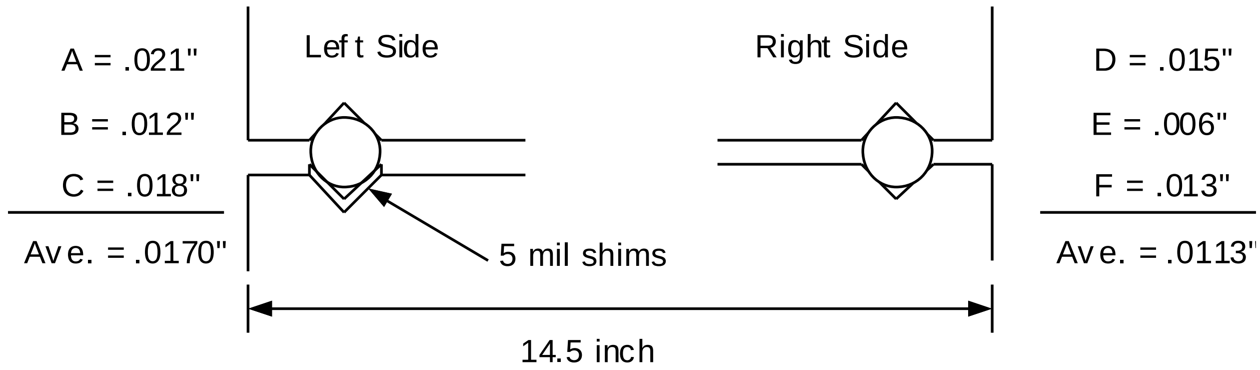
Vertical Perturbation



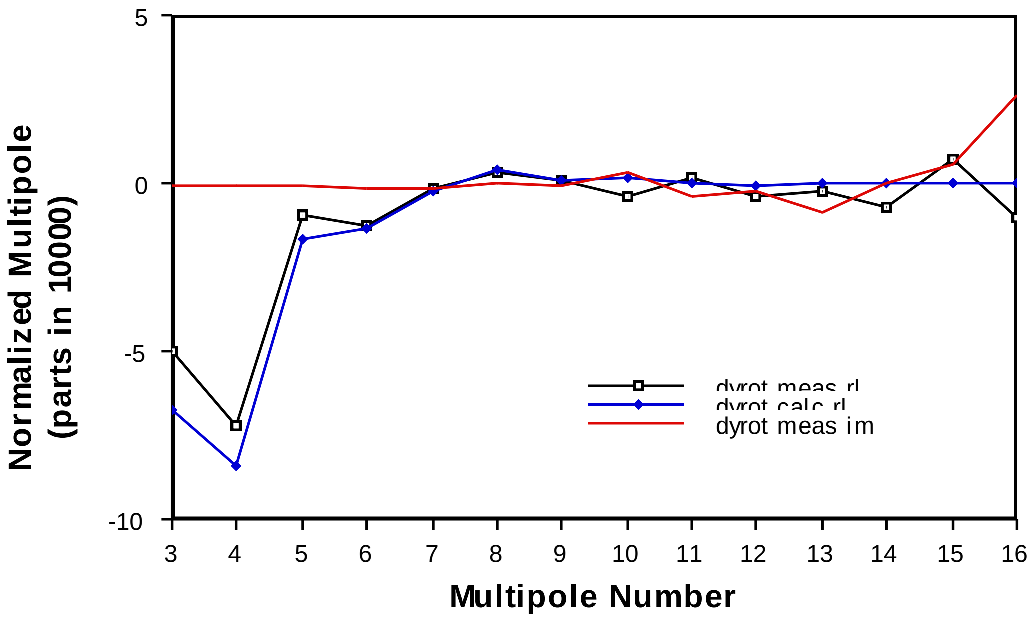
for Vertical Perturbation



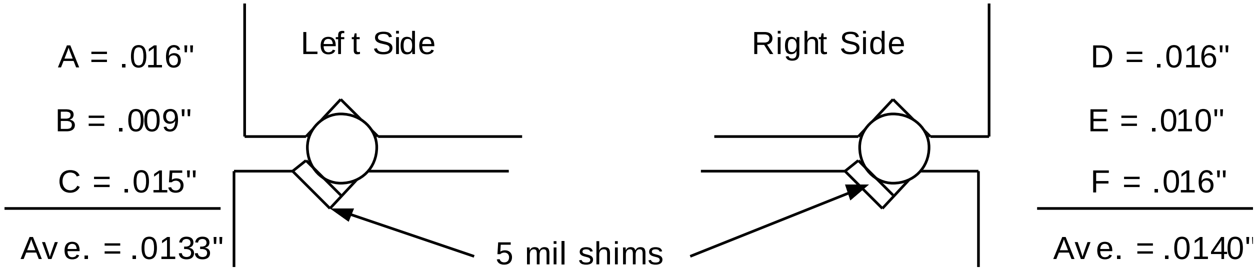
- Vertical and Rotational Motion



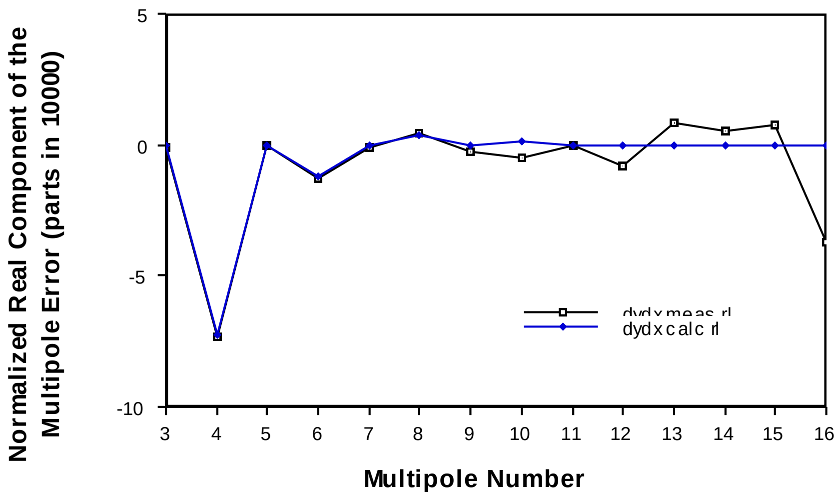
for Vertical and Rotational Perturbation



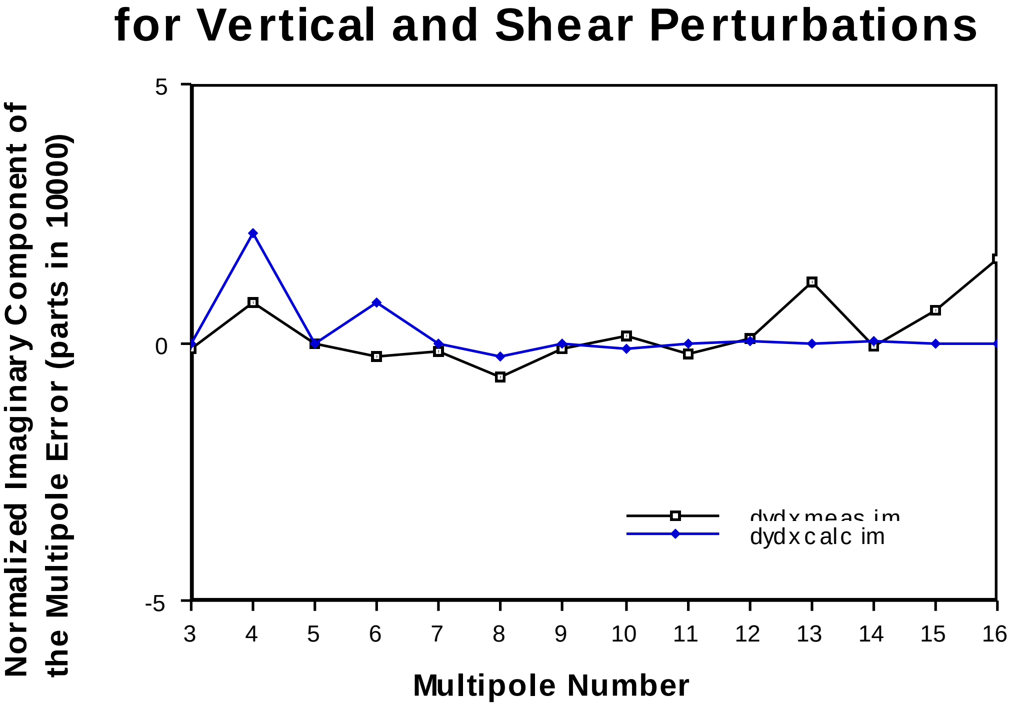
• Shear and Vertical Motion



for Vertical and Shear Perturbation



- Skew term due to vertical and shear perturbations.



Other Applications

- In crowded lattices, there is often insufficient room to place all the desired magnetic elements. An occasional solution to this problem, employed both at ALS and at APS, is to provide *trim* windings on a sextupole yoke in order to obtain *horizontal and vertical steering fields* and *skew quadrupole fields* without introducing a sextupole field. (The controls sextupole fields want to be independent of the horizontal and vertical steering and the skew quadrupole controls.) The design of such trim windings and the evaluation of the field quality which results when employing these techniques exploit Klaus Halbach's perturbation coefficients.

- The table of coefficients for N=3 (sextupole) for pole *excitation* error, ε, is reproduced from Klaus Halbach's perturbation paper.

$$\frac{H_n^*}{H_N^*} = \frac{n}{N}\Delta C_n e^{-in\beta}$$

$$\frac{H_n^*}{H_N^*} = i\varepsilon \frac{n}{N}\frac{\Delta C_n}{i\varepsilon} e^{-in\beta} = i\varepsilon\, x_n e^{-in\beta}$$

where $$\varepsilon = \frac{\Delta \mathrm{NI}_{\mathrm{pole}\beta}}{\mathrm{NI}_{\mathrm{sextupole}}}$$

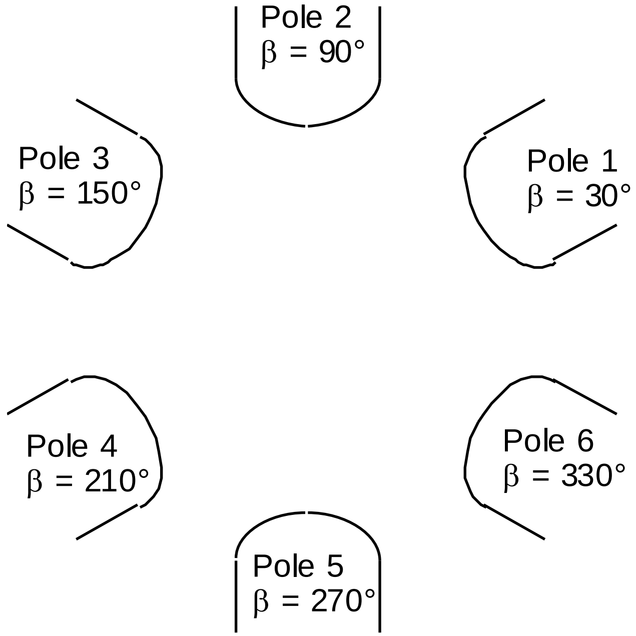
$$\beta = \text{Angle of Perturbed Pole}$$

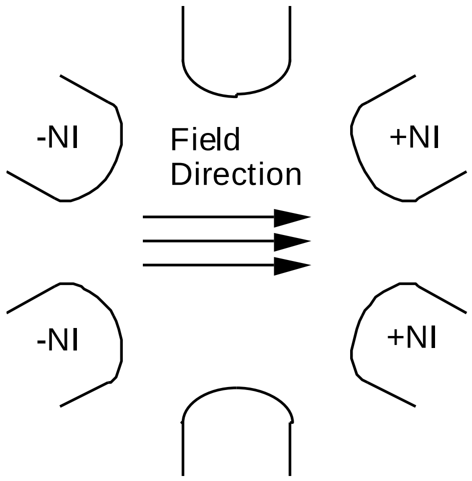
and $$x_n = Coefficient$$

n	$x_n = \frac{n}{N}\frac{\Delta C_n}{i\varepsilon}$
1	9.79E-02
2	1.56E-01
3	1.67E-01
4	1.33E-01
5	7.09E-02
6	
7	-1.34E-02
8	-1.07E-02
9	
10	9.13E-03
11	9.72E-03
12	
13	-1.01E-03
14	-1.18E-03
15	
16	1.63E-03
17	2.07E-03
18	
19	-1.12E-04
20	-1.70E-04
21	
22	3.25E-04
23	4.65E-04
24	

Vertical Steering Trim (Horizontal Flux Lines)

- Vertical steering trim (horizontal flux lines) is achieved in a sextupole yoke by exciting the four horizontal poles.





- The formulation of the expressions follows:

$$\frac{H_n^*}{H_N^*} = \frac{H_{nx} - iH_{ny}}{-iH_3} = \frac{H_{ny} + iH_{nx}}{H_3}$$

$$= \sum_{poles} i\varepsilon\, x_n e^{-in\beta_j} = \varepsilon\, x_n \sum_{poles} ie^{-in\beta_j}$$

$$\sum_{poles} ie^{-in\beta_j} = i\left[\begin{array}{l} \cos\frac{n\pi}{6} - \cos\frac{5n\pi}{6} - \cos\frac{7n\pi}{6} + \cos\frac{11n\pi}{6} \\ -i\left(\sin\frac{n\pi}{6} - \sin\frac{5n\pi}{6} - \sin\frac{7n\pi}{6} + \sin\frac{11n\pi}{6}\right) \end{array}\right]$$

$$= \left[\begin{array}{l} \sin\frac{n\pi}{6} - \sin\frac{5n\pi}{6} - \sin\frac{7n\pi}{6} + \sin\frac{11n\pi}{6} \\ i\left(\cos\frac{n\pi}{6} - \cos\frac{5n\pi}{6} - \cos\frac{7n\pi}{6} + \cos\frac{11n\pi}{6}\right) \end{array}\right]$$

- Equating real and imaginary terms for the horizontal steering trim;

$$\frac{H_{ny}}{H_3} = \varepsilon\, x_n \operatorname{Re}\left(\sum_{poles} i e^{-in\beta_j} \right)$$

$$\frac{H_{nx}}{H_3} = \varepsilon\, x_n \operatorname{Im}\left(\sum_{poles} i e^{-in\beta_j} \right)$$

- Tabulating the results;

n	$x_n = \frac{n}{N}\frac{\Delta C_n}{i\varepsilon}$	$\mathrm{Re}\sum ie^{-in\beta_j}$	$\mathrm{Im}\sum ie^{-in\beta_j}$	$\frac{B_{xn}}{\varepsilon B_3}$
1	9.79E-02	0	3.4641	0.3391
2	1.56E-01	0	0	0
3	1.67E-01	0	0	0
4	1.33E-01	0	0	0
5	7.09E-02	0	-3.4641	-0.2456
6	0	0	0	0
7	-1.34E-02	0	-3.4641	0.0464
8	-1.07E-02	0	0	0
9	0	0	0	0
10	9.13E-03	0	0	0
11	9.72E-03	0	3.4641	0.0337
12	0	0	0	0
13	-1.01E-03	0	3.4641	-0.0035
14	-1.18E-03	0	0	0
15	0	0	0	0
16	1.63E-03	0	0	0
17	2.07E-03	0	-3.4641	-0.0072
18	0	0	0	0
19	-1.12E-04	0	-3.4641	0.0004
20	-1.70E-04	0	0	0
21	0	0	0	0
22	3.25E-04	0	0	0
23	4.65E-04	0	3.4641	0.0016
24	0	0	0	0

Required Vertical Steering Trim Excitation

- From the table:

$$\frac{H_{nx}}{\varepsilon H_3} = 0.3391 \quad \Rightarrow \quad \frac{H_{1x}}{H_3} = \frac{B_{1x}}{B_3} = 0.3391\varepsilon = 0.3391\frac{\Delta NI}{NI_{sextupole}}$$

$$B' = \int B'' dx = B'' x$$

$$B = \int B'' x dx = \frac{B'' x^2}{2} \qquad \text{and} \qquad NI_{sextupole} = \frac{B'' h^3}{6\mu_0}$$

$$B_{@h} = B_3 = \frac{B'' h^2}{2}$$

- Substituting;

$$\frac{B_{1x}}{B_3} = \frac{B_{1x}}{\frac{B''h^2}{2}} = 0.3391\frac{\Delta NI}{\frac{B''h^3}{6\mu_0}}$$

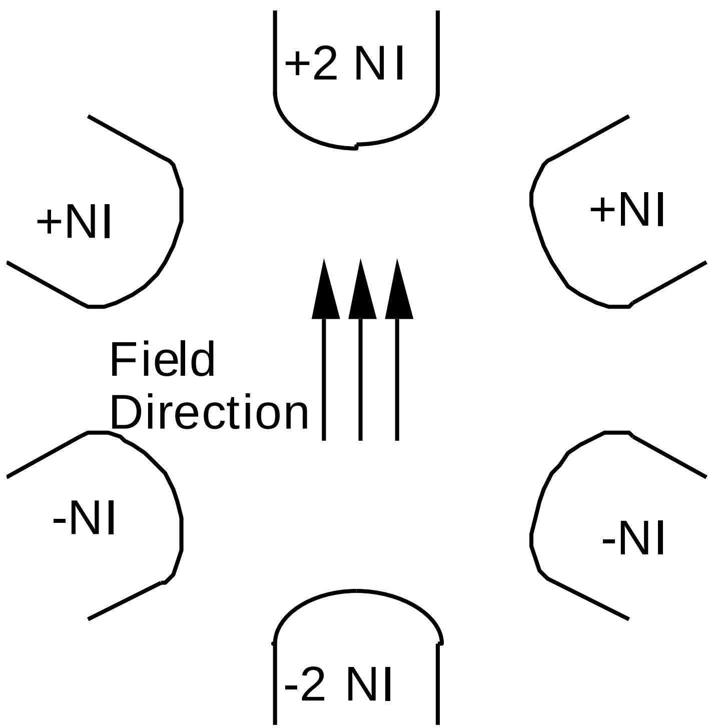
$$\Delta NI = NI_{VerticalSteering} = \frac{B_{1x}h}{3\times 0.3391\mu_0}$$

From the table, we note that the n=5 multipole error is >70% of the fundamental (horizontal dipole) field at the pole radius.

n	$\frac{B_n}{B_{1x}}$
1	1
2	0
3	0
4	0
5	-0.7242
6	0
7	0.1369
8	0
9	0
10	0
11	0.0993
12	0
13	-0.0103
14	0
15	0
16	0
17	-0.0211
18	0
19	0.0011
20	0
21	0
22	0
23	0.0047
24	0

Horizontal Steering Trim (Vertical Flux Lines)

- Horizontal steering trim can be achieved by exciting all six of the sextupole poles.



The excitation of the vertical poles is twice the excitation of the horizontal poles.

- Again, we can formulate the field in terms of the excitations of the various poles.

$$\frac{H_{ny} + iH_{nx}}{H_3} = \varepsilon\, x_n \sum_{poles} ie^{-in\beta_j}$$

$$\sum_{poles} ie^{-in\beta_j} = \begin{pmatrix} \sin\frac{n\pi}{6} + 2\sin\frac{n\pi}{2} + \sin\frac{5n\pi}{6} \\ -\sin\frac{7n\pi}{6} - 2\sin\frac{3n\pi}{2} - \sin\frac{11n\pi}{6} \end{pmatrix}$$

$$+ i\begin{pmatrix} \cos\frac{n\pi}{6} + 2\cos\frac{n\pi}{2} + \cos\frac{5n\pi}{6} \\ -\cos\frac{7n\pi}{6} - 2\cos\frac{3n\pi}{2} - \cos\frac{11n\pi}{6} \end{pmatrix}$$

- Tabulating the results,

n	$x_n = \frac{n}{N}\frac{\Delta C_n}{i\varepsilon}$	$\mathrm{Re}\sum ie^{-in\beta_j}$	$\mathrm{Im}\sum ie^{-in\beta_j}$	$\frac{B_{yn}}{\varepsilon B_{3y}}$	$\frac{B_n}{B_{1y}}$
1	9.79E-02	6	0	0.5874	1
2	1.56E-01	0	0	0	0
3	1.67E-01	0	0	0	0
4	1.33E-01	0	0	0	0
5	7.09E-02	6	0	0.4254	0.7242
6		0	0	0	0
7	-1.34E-02	-6	0	0.0804	0.1369
8	-1.07E-02	0	0	0	0
9		0	0	0	0
10	9.13E-03	0	0	0	0
11	9.72E-03	-6	0	-0.0583	-0.0993
12		0	0	0	0
13	-1.01E-03	6	0	-0.0061	-0.0103
14	-1.18E-03	0	0	0	0
15		0	0	0	0
16	1.63E-03	0	0	0	0
17	2.07E-03	6	0	0.0124	0.0211
18		0	0	0	0
19	-1.12E-04	-6	0	0.0007	0.0011
20	-1.70E-04	0	0	0	0
21		0	0	0	0
22	3.25E-04	0	0	0	0
23	4.65E-04	-6	0	-0.0028	-0.0047
24		0	0	0	0

Required Horizontal Steering Trim Excitation

- From the table:

$$\frac{H_{ny}}{\varepsilon H_3} = 0.5874 \quad \Rightarrow \quad \frac{H_{1y}}{H_3} = \frac{B_{1y}}{B_3} = 0.5874\varepsilon = 0.5874\frac{\Delta NI}{NI_{sextupole}}$$

$$B_{@h} = B_3 = \frac{B''h^2}{2} \quad \text{and} \quad NI_{sextupole} = \frac{B''h^3}{6\mu_0}$$

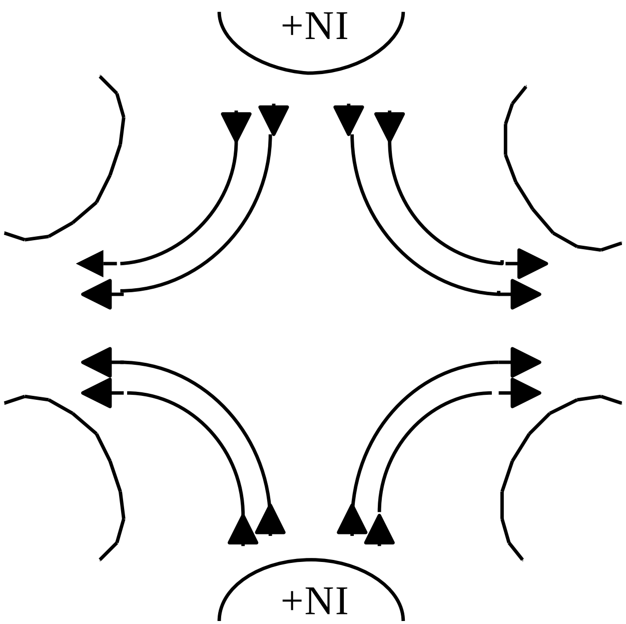
- Substituting;

$$\frac{B_{1y}}{B_3} = \frac{B_{1y}}{\frac{B''h^2}{2}} = 0.5874\frac{\Delta NI}{\frac{B''h^3}{6\mu_0}}$$

$$\Delta NI = NI_{horizontalSteering} = \frac{B_{1y}h}{3 \times 0.5874\mu_0}$$

Skew Quadrupole Trim (Horizontal Flux Lines)

- Skew quadrupole trim field can be achieved by exciting two of the sextupole poles.



Again, we can formulate the field in terms of the excitations of the various poles.

$$\frac{H_{ny} + iH_{nx}}{H_3} = \varepsilon\, x_n \sum_{poles} ie^{-in\beta_j}$$

$$\sum_{poles} ie^{-in\beta_j} = \left(\sin\frac{n\pi}{2} + \sin\frac{3n\pi}{2} \right) + i\left(\cos\frac{n\pi}{2} + \cos\frac{3n\pi}{2} \right)$$

- Tabulating the results,

n	$x_n = \frac{n}{N}\frac{\Delta C_n}{i\varepsilon}$	$\mathrm{Re}\sum ie^{-in\beta_j}$	$\mathrm{Im}\sum ie^{-in\beta_j}$	$\frac{B_{xn}}{\varepsilon B_{3y}}$	$\frac{B_{xn}}{B_{2x}}$
1	9.79E-02	0	0	0	0
2	1.56E-01	0	-2	-0.3120	1
3	1.67E-01	0	0	0	0
4	1.33E-01	0	2	0.2660	-0.8526
5	7.09E-02	0	0	0	0
6		0	-2	0	0
7	-1.34E-02	0	0	0	0
8	-1.07E-02	0	2	-0.0214	0.0686
9		0	0	0	0
10	9.13E-03	0	-2	-0.0183	0.0585
11	9.72E-03	0	0	0	0
12		0	2	0	0
13	-1.01E-03	0	0	0	0
14	-1.18E-03	0	-2	0.0024	-0.0076
15		0	0	0	0
16	1.63E-03	0	2	0.0033	-0.0104
17	2.07E-03	0	0	0	0
18		0	-2	0	0
19	-1.12E-04	0	0	0	0
20	-1.70E-04	0	2	-0.0003	0.0011
21		0	0	0	0
22	3.25E-04	0	-2	-0.0007	0.0021
23	4.65E-04	0	0	0	0
24		0	2	0	0

Required Skew Quadrupole Trim Excitation

- From the table:

$$\frac{H_{ny}}{\varepsilon H_3} = 0.3120 \quad \Rightarrow \quad \frac{H_{1y}}{H_3} = \frac{B_{1y}}{B_3} = 0.3120\varepsilon = 0.3120\frac{\Delta NI}{NI_{sextupole}}$$

$$B_{@h} = B_3 = \frac{B''h^2}{2} \qquad \text{and} \qquad NI_{sextupole} = \frac{B''h^3}{6\mu_0}$$

- Substituting;

$$\frac{B_{2x}}{B_3} = \frac{B'h}{\frac{B''h^2}{2}} = 0.3120\frac{\Delta NI}{\frac{B''h^3}{6\mu_0}}$$

$$\Delta NI = NI_{SkewQuadrupole} = \frac{B'h^2}{3\times 0.3120\mu_0}$$

Predicted n=4, 8 and 10 Multipole Errors

- Applying the perturbation theory, the multipole errors normalized to the skew quadrupole field, evaluated at the pole radius, *h*, can be computed.

$$\left(\frac{B_4}{B_{2x}}\right)_{@h} = 0.853 \qquad \left(\frac{B_8}{B_{2x}}\right)_{@h} = 0.069 \qquad \left(\frac{B_{10}}{B_{2x}}\right)_{@h} = 0.059$$

Magnetic measurements were performed on the SPEAR3 production sextupole magnets with skew quadrupole windings. These measurements were evaluated at the required good field radius, 32 mm. The predicted normalized multipole errors at 32 mm can be computed.

$$\left(\frac{B_n}{B_{2x}}\right)_{@r} = \left(\frac{B_8}{B_{2x}}\right)_{@h} \left(\frac{r}{h}\right)^{n-2}$$

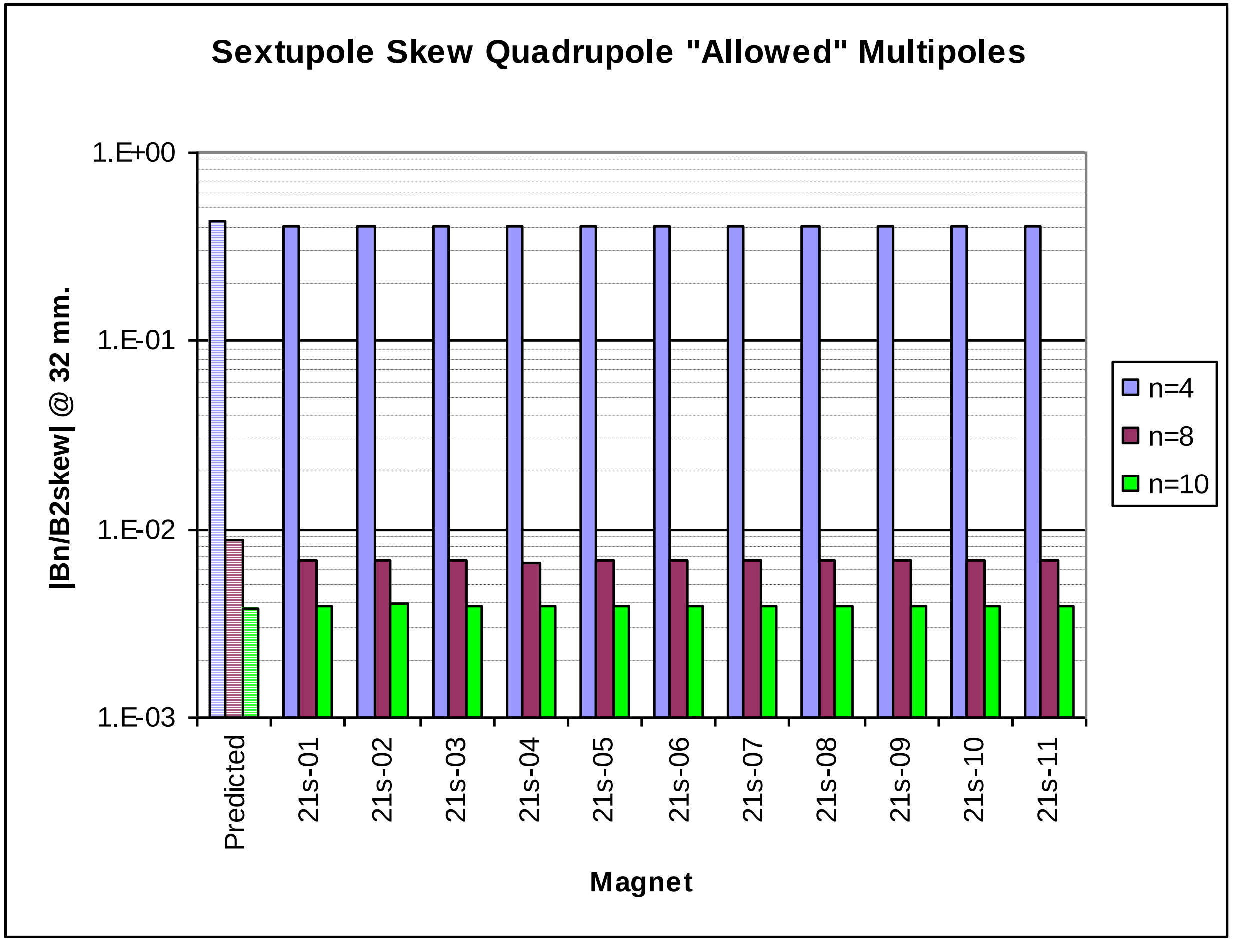
$$\left(\frac{B_4}{B_{2x}}\right)_{@32mm} = 0.853 \times \left(\frac{32}{45}\right)^2 = 0.431$$

$$\left(\frac{B_8}{B_{2x}}\right)_{@32mm} = 0.069 \times \left(\frac{32}{45}\right)^6 = 0.00892$$

$$\left(\frac{B_{10}}{B_{2x}}\right)_{@32mm} = 0.059 \left(\frac{32}{45}\right)^8 = 0.00386$$

- These prediction are compared with measurements.

Sextupole – Skew Quadrupole Field Error Measurements



Closure

- In many ways, this is one of the most important lectures. It is important that the student understands the chapter on Perturbations since successfully translating the performance of the mathematical design to the magnets manufactured and installed in a synchrotron requires that mechanical manufacturing and assembly errors translates into field errors which can threaten the performance of the synchrotron.
- Understanding the impact of mechanical fabrication and assembly errors on the magnet performance and thus, the physics impacts of these errors, can provide the understanding so that mechanical tolerances can be properly assigned.

Lecture 6

- The material covered in lecture 6 is covered in chapter 5 of the text.
- Please read this chapter prior to the next lecture. Homework covered in this chapter will be assigned.