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Response Matrix Measurements and Application to Storage Rings

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INTRODUCTION

This course covers orbit correction and optics correction in storage rings. By the end of the week you have the theoretical background, mathematical skills and hands-on experience to specify, operate or build one of these systems for a real or simulated accelerator.

PART I: ORBIT CORRECTION

The course starts with basic concepts from the field of orbit correction, mathematical concepts from linear algebra and an introduction to MATLAB. It is assumed you have some exposure to linear algebra including vectors, matrices, vector basis sets, projection into basis sets and eigenvectors. At the heart of the theory are least-squares fitting and singular value decomposition (SVD) of a matrix. Least-squares and SVD are covered in some detail. As these ideas are introduced we will connect each with orbit correction. This will help you improve your physical intuition for the mathematics, and the mathematics will help to see what is going on behind the scenes of orbit correction.

After reviewing linear algebra and least-squares fitting, the course moves to the eigenvector/eigenvalue decomposition of a matrix. The object is to introduce linearly independent basis sets that can be used for expansion of a waveform or function. For our application we will use SVD to expand the closed orbit on one set of orthogonal eigenvectors and expand the corrector set on another set of basis functions.

From the theory of eigenvectors we go straight to SVD, a more generalized factorization of a matrix that is numerically more robust and can handle all sizes of matrices and rank deficient matrices. SVD factorization produces the four fundamental ‘subspaces’ of the matrix A (outlined in linear algebra section).

We will go through the matrix mechanics of SVD extensively, forward and backward. The goal is two-fold: first and foremost to understand orbit control and second to use orbit control to provide a concrete example of SVD mechanics and the fundamental subspaces in linear algebra. The conceptual framework of linear algebra takes some of the guesswork out of SVD and least-squares fitting that are required for more advance orbit control and data analysis. SVD will provide you with a powerful linear algebra tool for other applications you may encounter. Our mantra is ‘use SVD any time you want to invert a matrix.’

Time permitting, we will demonstrate how the ‘null’ vectors generated by SVD have application to orbit correction. The optics modeling section of the course will also rely on concepts from response matrix theory, least-squares fitting and SVD so we have a strong overlap between the two main topics of the course.

The examples and homework problems rely heavily on MATLAB, but don't worry about writing reams of code. For the orbit control section, the examples in class and the homework come with a full set of code so you can 'play' with it instead of generating the routines by hand. You are given the answers to the problems! The goal is to understand the concepts but not get bogged down in the computer syntax. Your part will be to write short explanations and/or analytical formulas that say what the code is doing, – we want to know what you think. Grading will be heavily curved to don't let your inhibitions get in the way of your curiosity.

BASIC ORBIT CORRECTION
corrector-to-bpm response matrix
orbit correction algorithms
global orbit correction
local orbit correction

LINEAR ALGEBRA
action of a matrix
response matrix as a system of equations
input/output interpretation
fundamental subspaces of A (SVD)
(domain, range, column Ax=b, Ax=0)
orthogonality of subspaces
subspace diagrams

LEAST-SQUARES
Ax=b is over-constrained/over-determined
design matrix A
projection of vector on line
multivariable least-squares
different ways to look at the solution
projection matrices
orbit control applications
pseudo-inverse (SVD)

EIGENVALUE/EIGENVECTOR PROBLEM
finding the eigenvalues
spectrum of eigenvalues
finding the eigenvectors
expansion on the eigenbasis
similarity transformation
connection with SVD

SINGULAR VALUE DECOMPOSITION

non-square matrices
singular value decomposition of a matrix: $A=UWV^T$
U are the eigenvectors of A^TA
V are the eigenvalues of AA^T
W and the spectrum of singular values
expansion of vectors on the eigenbases U (orbits), V (correctors)
re-visit subspaces of A and SVD construction of subspaces
forward action of SVD: $Ax = UWV^Tx = b$
pseudoinverse
reverse action of SVD: $x = VWU^Tb$ (orbit correction)

RF-FREQUENCY CONTROL

finding the correct model rf frequency
projection of the orbit into dispersion
rf-component and betatron component
rf-component in the corrector pattern

USE OF NULL VECTORS

corrector null vectors do not move beam at BPM sites
corrector ironing