Path Length, Dipole Field and RF Frequency Correction

Dipole Modeling

Finding the correct RF frequency for a storage ring can difficult task but has important practical implications. In the design phase, we typically start with an 'ideal' magnet lattice with a hard-edge model for the dipole magnets. One of the simplest examples for path length through the dipoles is the circular arc

$$L_{arc} = \frac{L\sin\left(\frac{\theta}{2}\right)}{\frac{\theta}{2}} \quad ,$$

where L is the straight-line length of the dipole cores and θ is the bending angle. After magnetic measurements, the 'effective' length of the iron core, L_{eff}, can be used to improve the approximation, including fringe field effects:

$$L_{arc} = \frac{L_{eff}\sin\left(\frac{\theta}{2}\right)}{\frac{\theta}{2}} \quad .$$

The next level of approximation can be found by numerical integration of magnetic measurement data. For gradient dipoles, the equation of motion for the beam trajectory is

$$\frac{d^2x}{dz^2} + k\left(1 + \left(\frac{dx}{dz}\right)^2\right)^{3/2} x = 0$$

where 'z' is the straight-line coordinate along the magnet and x is perpendicular to z. k is the normalized field gradient, $\nabla B / B\rho$. Note that z is not the same as the local coordinate 's' which follows the motion of the beam. In the small angle, paraxial approximation, $\frac{dx}{dz} \approx 0$ and we arrive at the more common equation for particle motion through quadrupoles: $\frac{d^2x}{dz^2} = kx$ with the resulting hyperbolic equations of motion.

In practice, once the equation of motion is defined, numerical integration can be used to find the k-value in the dipole magnets that yields the correct bending angle, including fringe fields. The dipole power supply current is found from magnetic measurements of k vs. I. The calculated path length provides a good starting point for the rf frequency.

With the dipole *k*-value loaded into the accelerator model, the optics can be re-matched to find the correct quadrupole k-values and power supply currents. The model should also use the effective length for the quadrupoles. Although it is difficult to generate a numerical model that will produce the correct optics from the start, *response matrix analysis* has proven effective for adjusting the optics to the nominal values.

When the storage ring comes into operation, finding the correct rf frequency remains an issue. In SPEAR 3, the rf frequency was increased about 17 kHz above the original circular arc estimate (36 dipole magnets with steel length 145 cm) to account for the effect of fringe fields and the effect of the non-paraxial beam trajectory. In practice, the optimum frequency was found to lie about 16 kHz above the circular arc estimate but changes with tunnel temperature.

Experimentally finding the correct rf frequency can be done with the sextupole centering technique (measure tune vs. rf frequency for several sextupole k-values) or by analyzing the strength of the horizontal corrector magnets. In this section, we will investigate projecting the orbit into the dispersion component and optimization of the corrector magnet settings in more detail.

MOMENTUM COMPACTION AND DISPERSION

When the storage ring rf frequency is changed, the electron beam moves horizontally onto a dispersion orbit given by

$$x = \eta \frac{dp}{p}$$

where η is the dispersion function and the energy shift dp/p is given by the momentum compaction factor

$$\frac{dp}{p} = \frac{1}{\alpha}\frac{dl}{L} = -\frac{1}{\alpha}\frac{df}{f}.$$

Substituting,

$$x = -\frac{\eta}{\alpha}\frac{df}{f}.$$

To estimate the sensitivity, take a storage ring with peak dispersion η=0.2 m, α=0.001 and a 500 MHz rf system. If we increase the rf frequency by 1 kHz, the beam energy goes up by 0.2% and the beam moves by -400 μm at points at points of peak dispersion. Since we now have master oscillators controllable to 1 Hz (x=400 nm in this example) and accurate orbit correction systems, the question becomes one of finding the correct rf frequency for storage ring operation.

RF COMPONENT IN THE ORBIT

One way to correct the rf frequency is to measure the closed orbit distortion and project it onto a known dispersion orbit. In general, the COD contains both a 'betatron' component and an 'rf' component:

$$x = x_\beta + x_{rf}$$

where

x = total COD
x_β = betatron component
x_{rf} = rf component .

To extract the rf component, we project the orbit perturbation onto a measurement of the orbit response to a known rf-frequency change, the column vector Δx_{rf}. The projection generates a scalar proportional the required rf-frequency change. Normalizing,

$$\boldsymbol{f} = \left(\frac{\boldsymbol{x} \cdot \Delta \boldsymbol{x}_{rf}}{\Delta \boldsymbol{x}_{rf} \cdot \Delta \boldsymbol{x}_{rf}} \right) \qquad \text{(scalar value result of projection)}$$

Note that if $x = \Delta x_{rf}$ then $f = 1$, i.e. the entire orbit perturbation is due to an rf-frequency perturbation. Furthermore, we assume $x_\beta \cdot \Delta x_{rf} = 0$ which is a good (but not perfect!) approximation since betatron orbits tend to be oscillatory with the betatron period, and the rf-orbit is predominately DC.

To compute the rf frequency shift required to correct the orbit we have

$$\partial f_{rf} = -f(\Delta f_{rf})$$

where Δf_{rf} is the change in frequency used to generate the measured dispersion orbit, Δx_{rf}. The residual betatron component, $x_\beta = x - f\,\Delta x_{rf}$, is corrected with standard methods for orbit control.

MATLAB Example – Projection orbit into dispersion vector
>>edit rf_1

AUGMENTED RESPONSE MATRIX

An alternative way to correct the rf orbit is to add column vector Δx_{rf} to the response matrix prior to inversion. The rf-frequency is treated as a 'corrector' but care is required to find to proper scaling between corrector magnets currents and the master oscillator control signal. In effect, weighted least-squares is required to weight the rows of the response matrix.

$$\boldsymbol{R} = \{\boldsymbol{R}_{corrector} : \Delta \boldsymbol{x}_{rf}\}$$
$$\boldsymbol{\theta} = \boldsymbol{R}^{-1}\boldsymbol{x}$$
$$\boldsymbol{\theta} = \{\boldsymbol{\theta}_{corrector} : \Delta \boldsymbol{f}_{rf}\}$$

RF COMPONENT IN THE CORRECTORS

One problem with standard rf-frequency correction methods is that there still may be 'errors' masked by the horizontal corrector magnets - how effective do corrector magnets offset rf frequency errors? Imaging again we increase the rf frequency by 1 kHz. The beam energy increases and the orbit moves in a few hundred microns. Can the corrector magnets put the beam back on the original orbit? If the response matrix has sufficient rank (the orbit perturbation lies in the column space of R) we can steer the beam through the BPMS - but what does the corrector pattern look like and what happens in-between the BPMS?

Arguably, if we reduce the average corrector strength then the magnetic flux encircled by the beam, and consequently the beam energy, is reduced back toward the original value. In practice, this is indeed the case, a corrector pattern with a strong DC-component will compensate rf-frequency variations. But the rf-frequency still defines the total path length so the compensation is not exact. At best, we can correct the orbit at the BPM locations but there will be path length variations between BPM locations to satisfy the path length constraint.

MATLAB Example – Correctors required to correct RF orbit
>>edit rf_2

Turning the argument around, we can try to adjust the rf-frequency so as to minimize the DC-component in the corrector pattern. In a well-aligned storage ring the result should be relatively accurate. A straightforward procedure would be turn on the horizontal orbit feedback system and adjust the rf-frequency until the DC-component of the corrector magnets is zero. Most (if not all) singular values will be required since the 'DC-orbit shift' does not project well into the column space of a typical response matrix.

A further refinement of this technique was developed at the SLS. As before, each feedback cycle calculates the projection coefficient 'f' and, *if f is of sufficient magnitude*, removes the rf component (δf_{rf}). If f is too small, no rf-correction is made but we don't want the betatron portion of the feedback system to act on the rf-orbit contribution.

To reject the rf component from the corrector set, the feedback system first calculates the incremental corrector pattern each cycle: $\Delta\theta = R^+ x$. Prior to applying the new corrector pattern, $\theta_{new} = \theta_{old} + \Delta\theta$, however, the feedback algorithm removes components of the corrector pattern that act on the rf-orbit perturbation. To remove the rf-component from the corrector pattern, we project θ_{new} into the corrector vector $\Delta\theta_{rf}$ required to compensate for an rf-frequency perturbation. In practice, $\Delta\theta_{rf}$ can be measured by changing the rf-frequency, measuring the orbit shift Δx_{rf}, and then calculating

$$\Delta\theta_{rf} = R^+ \Delta x_{rf}. \qquad \text{(corrector pattern required to offset rf-frequency error)}$$

Mathematically, the new corrector pattern with the rf-component removed is

$$\theta'_{new} = \theta_{new} - \frac{\theta_{new} \cdot \Delta\theta_{rf}}{\Delta\theta_{rf} \cdot \Delta\theta_{rf}}.$$

By systematically removing the corrector pattern associated with rf-errors the functionality of the rf-error and betatron feedback components remain decoupled.