Lecture 5:
RF and Space Charge Emittance in Guns
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Goal:

The students will gain an understanding of two important sources of emittance in RF guns.

Objectives:

The students will

(i) Identify correlated and uncorrelated emittances

(ii) Calculate correlated impulses from RF fields; projected emittances

(iii) Calculate correlated impulses from space charge effects; projected emittances

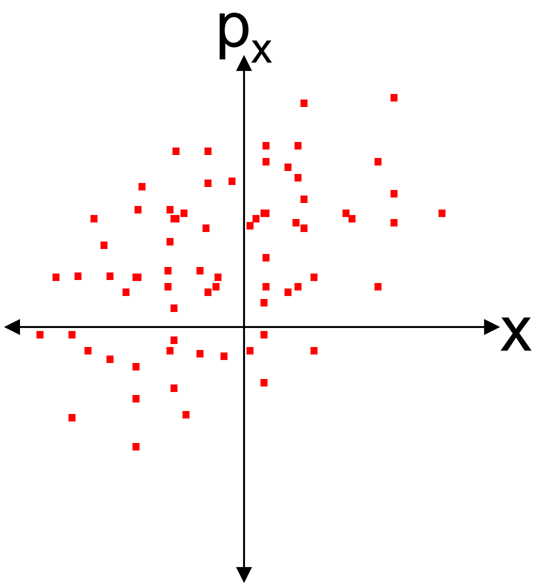
(iv) Discuss correlations btw rf and s-c effects, and higher-order effects

Overview

- Phase space concept
- Sources of emittance
  - Correlated vs. non-correlated
- Sketch of dynamics in gun

Particle Distribution and Phase Space

We often deal with beams that contain a large number of electrons – 10^9-10^{10} or more.



Each particle has individual coordinates (x,y,z,p_x,p_y,p_z) at a given time t.

When we include the collection of particles that comprise a beam, this set makes up the beam's *phase space*.

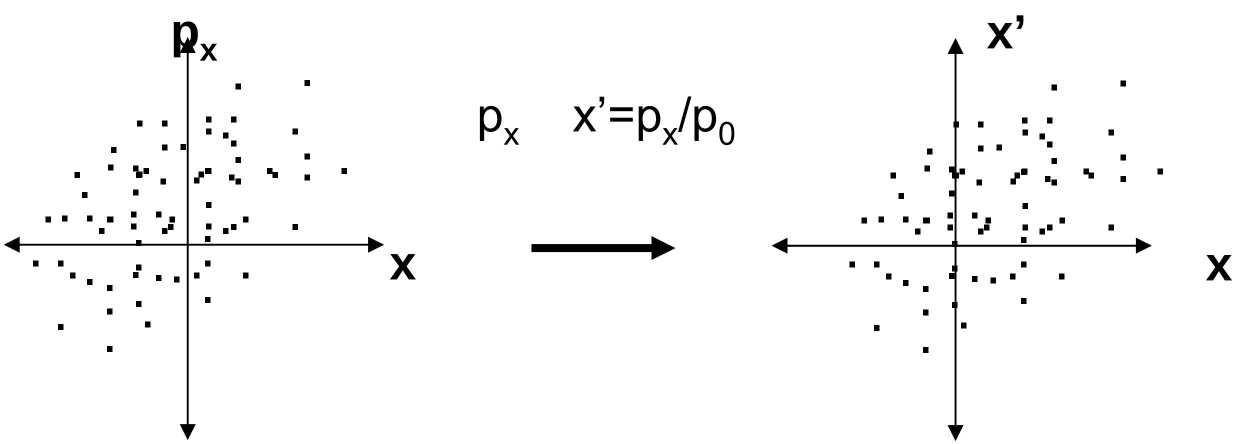
It is difficult to visualize a 6D distribution, so we often resort to descriptions in the 2D subspaces: (x,p_x), (y,p_y), (z,p_z)

Normalizing the momenta

Typical beam distributions that we see in practice have a particle average momentum value that is much, much larger than other momenta. This is what signifies a ‘beam’ rather than a single component ‘plasma’.

Thie larger momentum component defines the ‘longitudinal’ direction, while the other components lie in the ‘transverse’ directions. It is usual to use z for the longitudinal coordinate and p_z for the corresponding momentum.

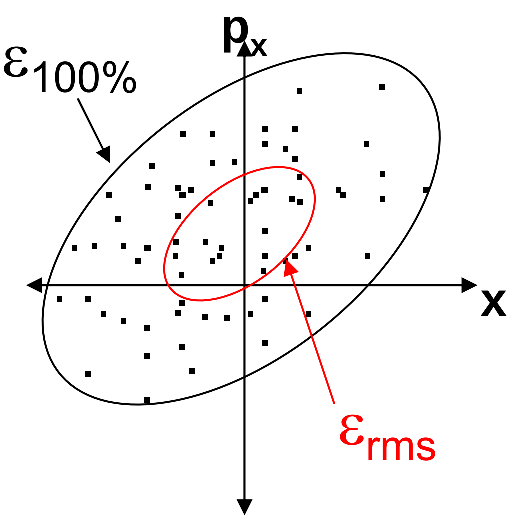
Since (for a beam) we are assuming that $p_z >> p_x$ or p_y and that $<p_z> = p_0$ is, say, always positive, it may make sense to normalize all other momenta by p_0. This defines what we mean by *trace space*.



Taking measures from the distribution

Phase space distributions have many degrees of freedom

We want a measure of beam quality that is useful yet reflects the underlying dynamics. 'Emittance' is an exact measure for linear forces and elliptical distributions (bi-linear forms).



The emittance is a measure of the phase space area occupied by a beam. We define an ellipse (usually rotated) that contains all the particles. The emittance in this case is the ellipse area divided by π. More commonly, we use the 'rms emittance' measure defined by (where <> indicates an average over the ensemble of particles)

$$\varepsilon_x^{rms} = \sqrt{\langle p_x^2 \rangle \langle x^2 \rangle - \langle p_x x \rangle^2}$$

Under conditions of linear forces in all directions, the emittance of a beam remains a conserved, invariant quantity. It measure the quality of a beam and how well a focusing system can bring the beam spot down to a single point.

Normalized Emittance

When beams are accelerated or decelerated as a whole, the average momentum p_0 changes. In fact, all the longitudinal momenta change.

Conservation of momentum, however, dictates that the transverse momentum components do not change unless acted upon by a transverse impulse.

We define *normalized* and *unnormalized* emittances to reflect the changes occuring when the beam energy is varying.

The normalized emittance remains constant if (and only if) p_0 varies.

$$\varepsilon_x^{rms} = \sqrt{\left\langle p_x^2 \right\rangle \left\langle x^2 \right\rangle - \left\langle p_x x \right\rangle^2}$$

$$= p_0 \sqrt{\left\langle x'^2 \right\rangle \left\langle x^2 \right\rangle - \left\langle x' x \right\rangle^2}$$

$$= p_0 \left[\varepsilon_x^{rms} \right]_{unnormalized}$$

Equivalent emittance measures

Sometimes other measures of transverse beam quality are useful.

$$\varepsilon_{4D} = \frac{1}{4}\left[\left\langle r^2\right\rangle\left\langle r'^2 + (r\varphi')^2\right\rangle - \left\langle rr'\right\rangle^2 - \left\langle r^2\varphi'\right\rangle^2\right] \qquad \varepsilon_{2D} = \sqrt{\varepsilon_{4D}}$$

$$\varepsilon_r = \frac{1}{2}\sqrt{\left\langle r^2\right\rangle\left\langle r'^2\right\rangle - \left\langle rr'\right\rangle^2}$$

$$\varepsilon_x = \sqrt{\left\langle x^2\right\rangle\left\langle x'^2\right\rangle - \left\langle xx'\right\rangle^2}$$

Axisymmetric, uncoupled beams:

$$\varepsilon_x = \varepsilon_y = \varepsilon_r = \varepsilon_{2D}$$

With any angular momentum coupling

$$\varepsilon_{4D} = \frac{1}{4}\left[\left\langle r^2\right\rangle\left\langle r'^2 + (r\varphi')^2\right\rangle - \left\langle rr'\right\rangle^2 - \left\langle r^2\varphi'\right\rangle^2\right] = \varepsilon_r^2 + \varepsilon_t^2$$

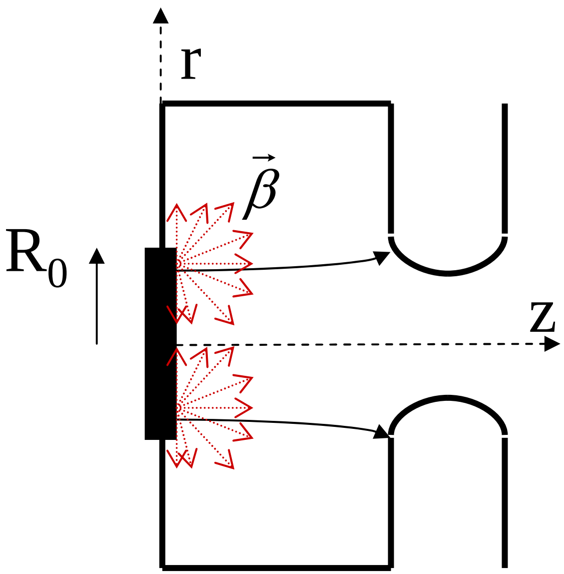
$$\varepsilon_t = \frac{1}{2}\sqrt{\left\langle r^2\right\rangle\left\langle (r\varphi')^2\right\rangle - \left\langle r^2\varphi'\right\rangle^2} \Rightarrow \varepsilon_t = \frac{1}{2}\sqrt{\left\langle l^2\right\rangle - \left\langle l\right\rangle^2} \qquad l = xy' - x'y = r^2\varphi'$$

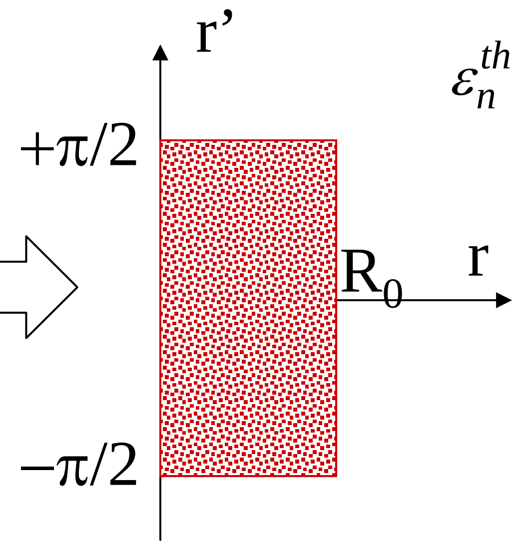
Ref. Nagaitsev and Shemyakin

Thermal emittance model

Courtesy, L. Serafini

Thermal emittance @ photo-cathode (real Liouvillian emittance)





$$\varepsilon_n^{th} = \frac{\langle\beta\gamma\rangle}{2}\sqrt{\langle r^2\rangle\langle r'^2\rangle - \langle rr'\rangle^2}$$

$$\langle r^2\rangle \equiv \frac{\int_0^{R_0} r^3 dr}{\int_0^{R_0} r dr}$$

$$\langle x^2\rangle = \frac{\langle r^2\rangle}{2}$$

$$\langle\beta\gamma\rangle \cong \langle\beta\rangle \cong \sqrt{2T_e/m_e c^2} = 2\cdot 10^{-3}\sqrt{T_e[eV]}$$

$$\varepsilon_n^{th} = \frac{\pi\langle\beta\rangle R_0}{4\sqrt{6}}$$

$$\varepsilon_n^{th}\ [mm\cdot mrad] = 0.64\ \ R_0[mm]\ \sqrt{T_e[eV]}$$

in absence of any channeling mechanism

Correlated Emittance Contributions

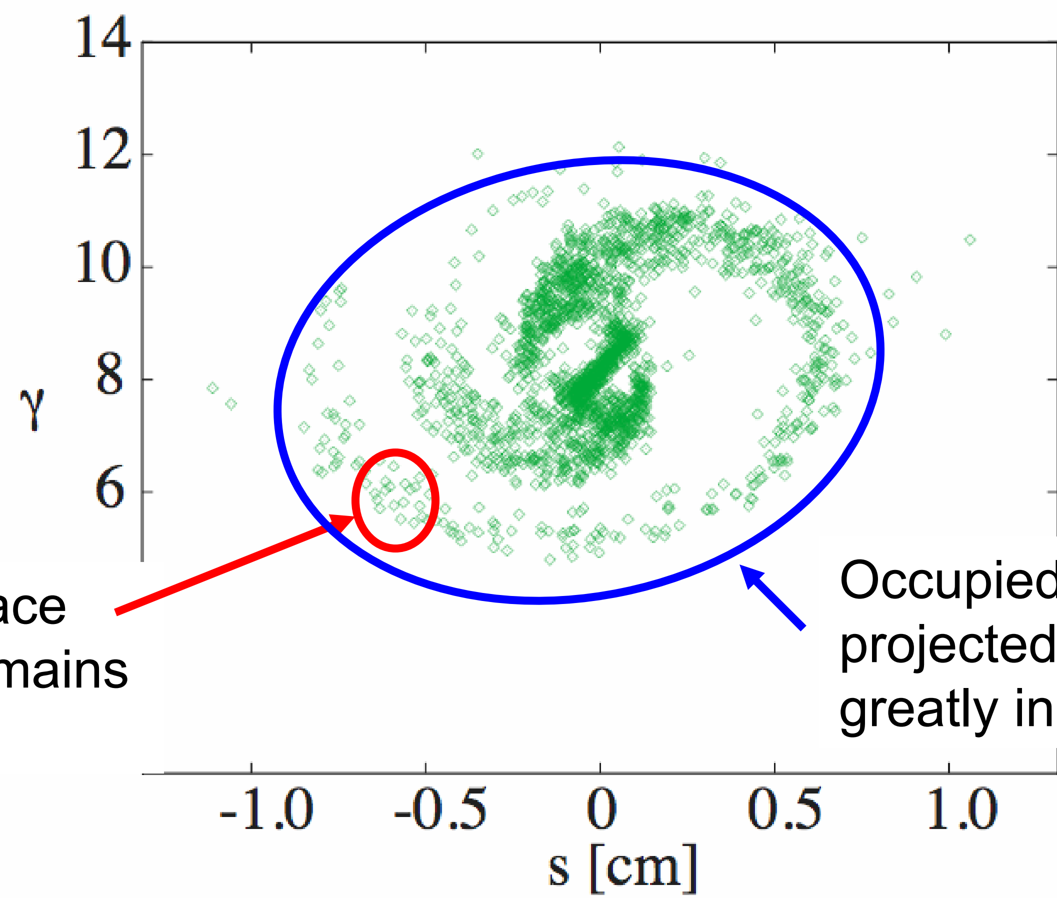
We will study various forces that introduce correlations or distortions to both the longitudinal and transverse phase space distributions.

The scale length of these forces is much larger than the Debye length for a core beam electron.

Electrons will not be scattered out of their phase space ‘cells’ - no diffusion across cell boundaries - even as the cells themselves are stretched and distorted. This is *Liouville’s Theorem* for conservative systems.

$$\varepsilon_{total} = \sqrt{\varepsilon_{thermal}^2 + \varepsilon_{dynamics}^2}$$

Adiabatic capture and bunching in a relativistic klystron



Local phase space volume (cell) remains constant

Occupied volume in projected space space greatly increases.

Longitudinal vs. Transverse Forces

For relativistic beams, changes in transverse momenta can occur over much smaller beamline distances than changes in longitudinal momenta

There is a natural distinction between longitudinal and transverse dynamics, and the associated phase space distributions.

$$\frac{d\vec{p}}{dt} = \frac{d}{dt}\left(\gamma\vec{\beta}mc\right) = \vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$$

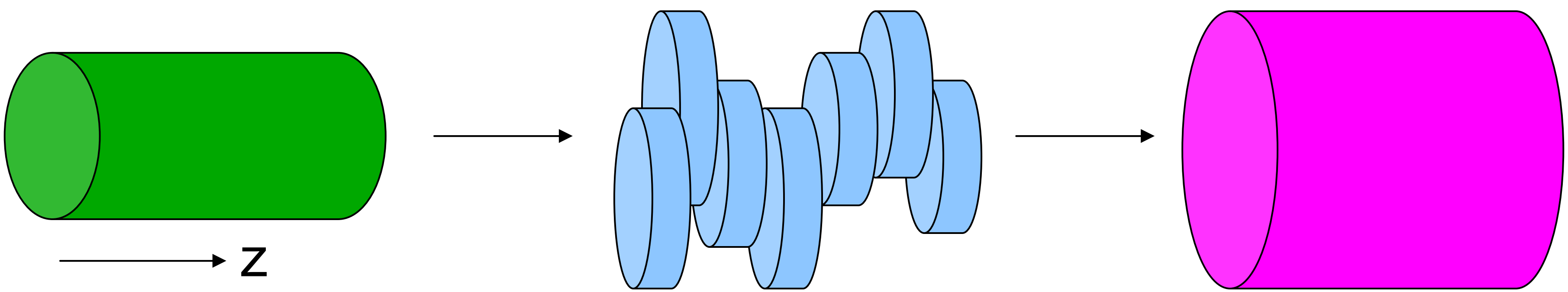
$$\begin{aligned} \vec{F} \perp \vec{v} \quad &\Rightarrow \quad \frac{d}{dt}\vec{p}_\perp = \gamma mc \frac{d}{dt}\vec{\beta}_\perp \\ \vec{F} \parallel \vec{v} \quad &\Rightarrow \quad \frac{d}{dt}\vec{p}_\parallel = \gamma^3 mc \frac{d}{dt}\vec{\beta}_\parallel \end{aligned} \quad \Rightarrow \quad \begin{aligned} \frac{d}{dt}\vec{\beta}_\perp &\propto \frac{1}{\gamma} \\ \frac{d}{dt}\vec{\beta}_\parallel &\propto \frac{1}{\gamma^3} \end{aligned}$$

Slice picture

Disregarding the effects of longitudinal-transverse correlations for now, we can visualize the longitudinal phase space segmented into *slices*.

Slices have approximately the same emittance (~thermal emit.) but may be displaced or distorted with respect to each other.

Adding the effects of these displacements and distortions gives us a *projected* emittance larger than the thermal emittance.

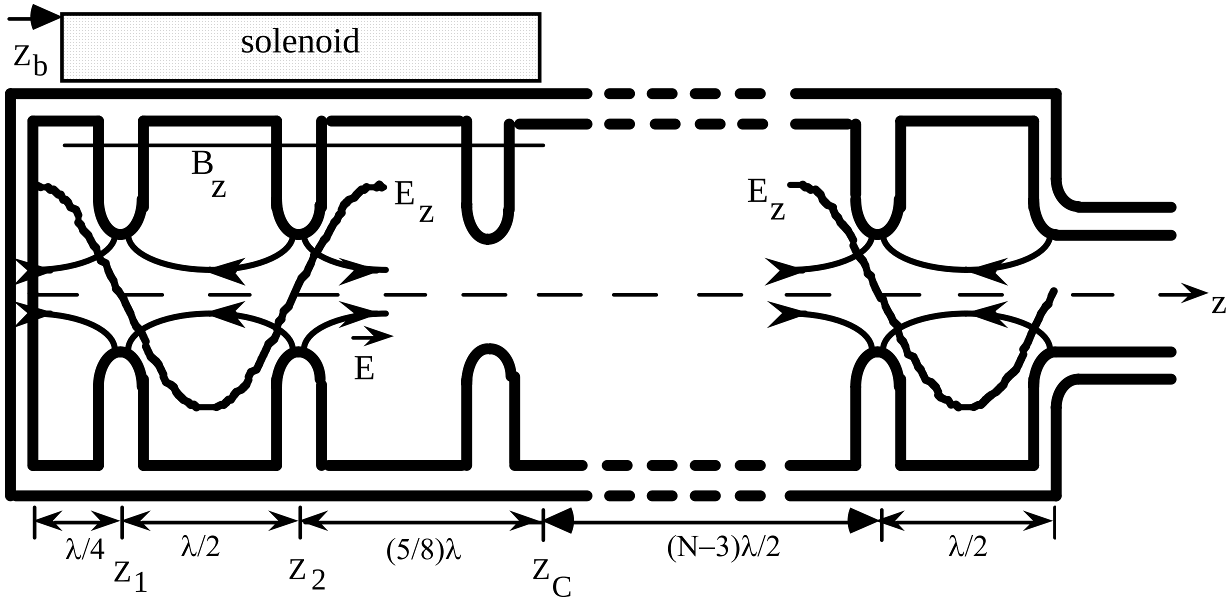


Displaced slices → Larger effective emittance

Beam dynamics due to RF fields

- Review of acceleration
- RF effects on longitudinal phase space
  - Energy
  - Energy spread
- RF effects on transverse phase space
  - RF Kick
  - Emittance
  - Optimization
- Scaling

Cavity Fields



On-axis expansion of the $TM_{010\text{-}\pi}$ standing mode

E_0 = the peak field at the cathode
$k \equiv 2\pi/\lambda = \omega/c$
a_n = spatial harmonic coefficients functions of cavity geometry

$$E_z = \mathcal{E}_z(r,z)\cdot\sin(\omega t+\varphi_0) \quad ; \quad \mathcal{E}_z(r,z) = E_0 \sum_{n=1,odd}^{\infty} a_n \cos(nkz)$$

$$E_r = \mathcal{E}_r(r,z)\cdot\sin(\omega t+\varphi_0) \quad ; \quad \mathcal{E}_r(r,z) = \frac{kr}{2} E_0 \sum_{n=1,odd}^{\infty} n\cdot a_n \sin(nkz) \quad ; \quad a_1 = 1$$

$$B_\theta = B_\theta(r,z)\cdot\cos(\omega t+\varphi_0) \quad ; \quad B_\theta(r,z) = c\,\frac{kr}{2}\,\mathcal{E}_z(r,z)$$

Courtesy, L. Serafini

Cavity Fields II

The longitudinal electric field for a pill box cavity is

$$E_z^{mnp}(r,z) = E_0 J_m(k_{mn} r)\cos(m\theta)\cos\left(\frac{2p\pi z}{\lambda}\right)e^{i\omega\frac{z}{c}}$$

using Euler's relation, $e^{i\theta} = \cos\theta + i\sin\theta$, and integrating gives the beam voltage in the z-dimension:

$$V_z^{mnp}(r,z) = eE_0 J_m(k_{mn} r)\cos(m\theta)\int_0^z \cos\left(\frac{2p\pi z'}{\lambda}\right)\sin\left(\frac{\omega z'}{c}+\phi\right)dz'$$

Consider the pi-mode (m=0,n=0,p=1) for a one and a half cell gun, and making the usual assumption that m=0 and n=0 as well, then the familiar relation for the gun field is obtained:

$$E_z = E_0 \cos kz \sin(\omega t + \phi_0) \ \ , \ \ k = \frac{\omega}{c}$$

Courtesy, L. Serafini

Longitudinal Dynamics

The longitudinal dynamics are found by solving the relativistic, longitudinal force equation:

$$\frac{d}{dt}[\gamma\beta mc]=\frac{d}{dt}\left[\frac{\beta}{\sqrt{1-\beta^2}}mc\right]=qE(z)\cos kz\sin(\omega t+\phi)$$

This has been done analytically by Kim, (see references), who obtained good agreement between his formulas and numerical integration. Simplifying the field description to

$$E_z=E_0\cos kz\sin(\omega t+\phi_0)\quad,\quad k=\frac{\omega}{c}$$

and defining the dimensionless parameter $\alpha=\dfrac{eE_0}{2mc^2k}$

The longitudinal equation can be integrated approximately, yet with good accuracy, to

$$\gamma=1+\alpha\left[kz\sin\phi+\frac{1}{2}\left(\cos\phi-\cos(\phi+2kz)\right)\right]$$

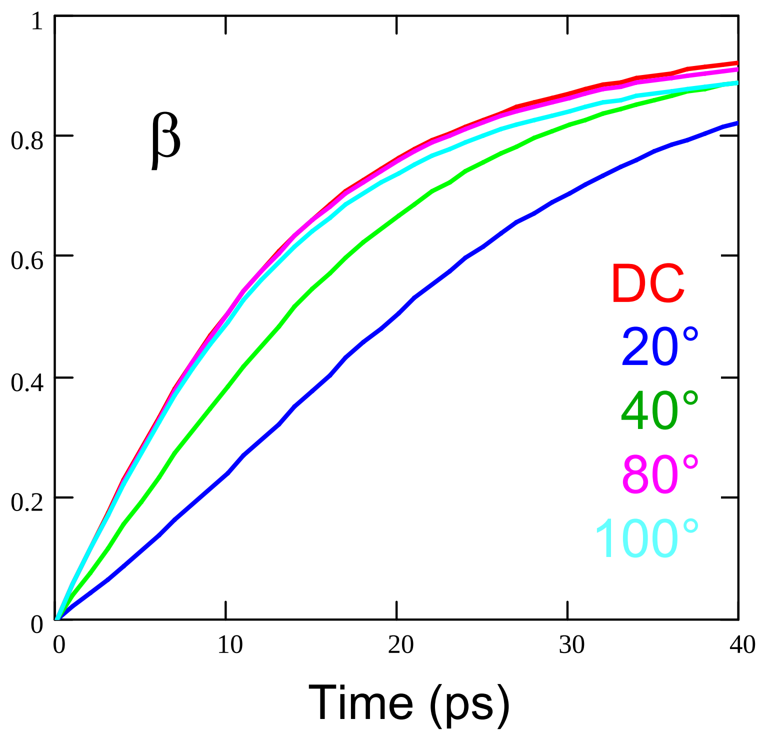
$$=1+\alpha\left[(n+1/2)\pi\sin\phi+\cos\phi\right],\quad for\ \ n\ \ full\ \ cells$$

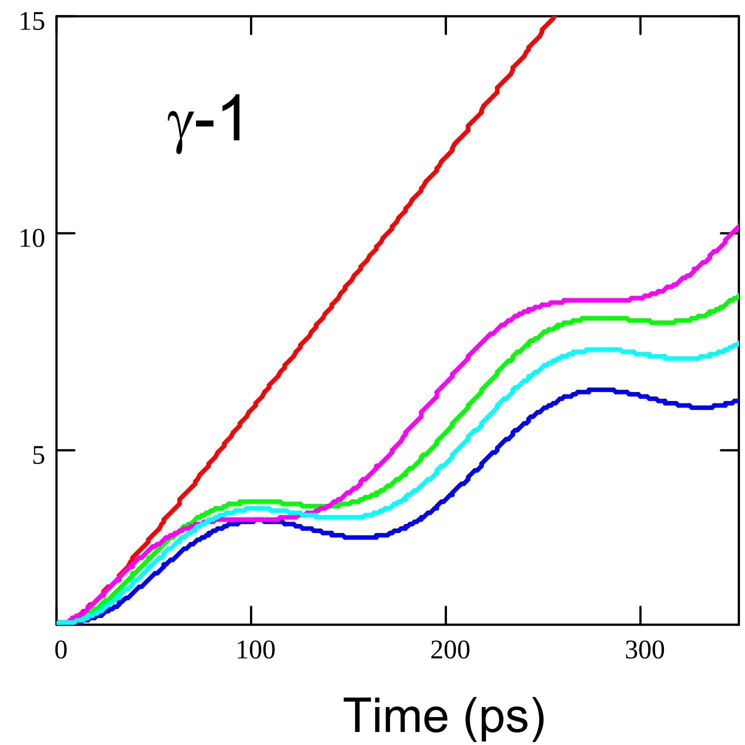
The cavity phase has the asymptotic value $\phi=\omega t-kz+\phi_0\rightarrow\phi_e=\dfrac{1}{2\alpha\sin\phi_0}+\phi_0$

Evolution of Particle Velocity and Kinetic Energy

Numerical solutions to the longitudinal rf force equation as well as the result for a dc field of the same strength are plotted below. The cathode field is 100 MV/m and the rf frequency is s-band, 2.856GHz.

Because of the changing time-dependence of the field, the energy during acceleration oscillates and reaches a lower energy than that for constant field. The two are similar in the first cm.





Bunch length and compression

Particles are emitted from the cathode with a spread in time that is equivalent to a spread in phase

$$\Delta\phi_0 = \omega\Delta t = kc\Delta t \qquad \phi_{head} < \phi_{tail}$$

arriving at the gun exit with the asymptotic phase,

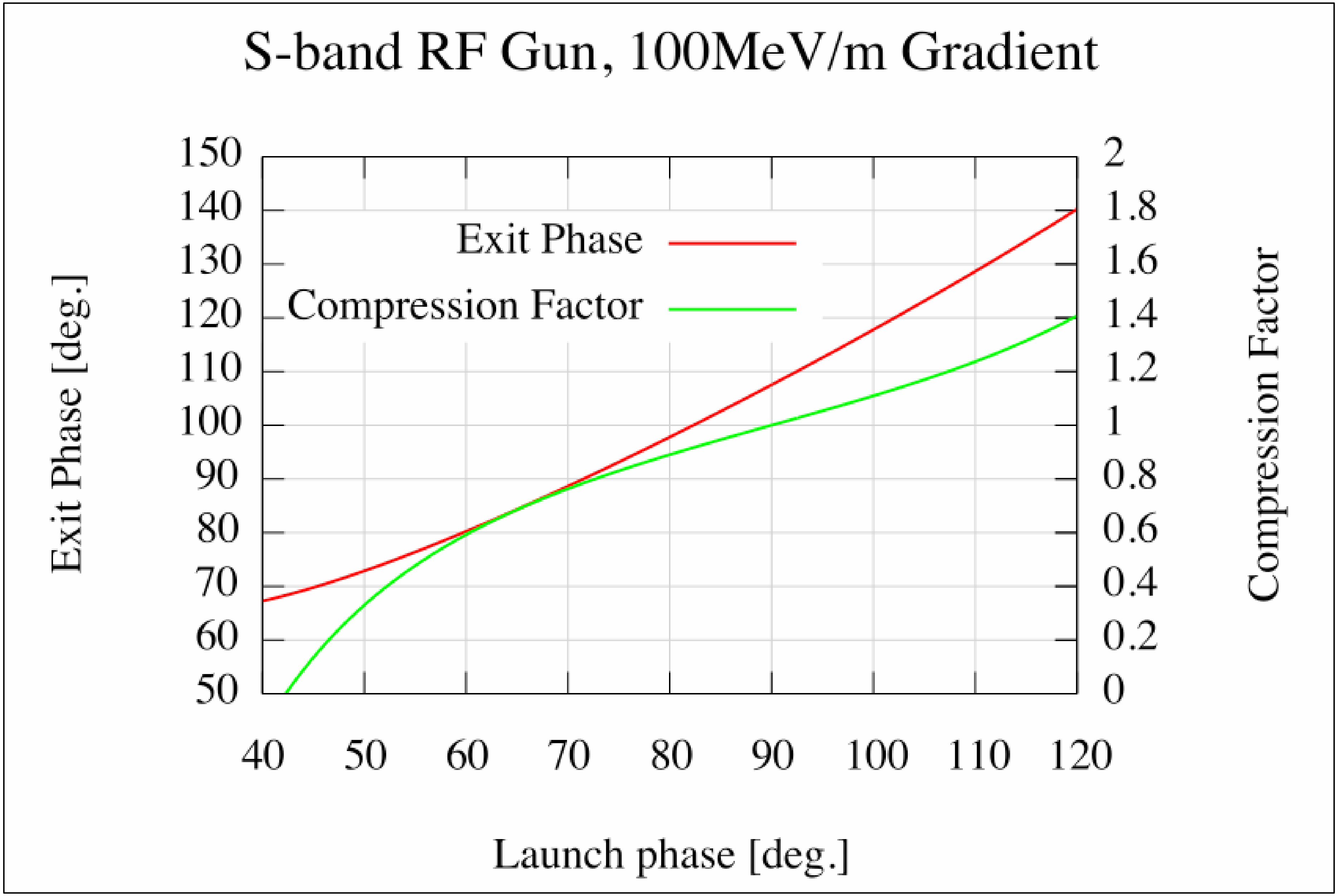
$$\Delta\phi_e = \frac{1}{2\alpha\sin(\phi_0 + \Delta\phi_0)} + \Delta\phi_0$$

We see that the bunch has been compressed by the factor

$$\frac{\Delta\phi_e}{\Delta\phi_0} = 1 - \frac{\cos\phi_0}{\alpha\sin^2\phi_0}$$

The rms bunch length at the cavity exit is found to be

$$\sigma_z = \frac{\sigma_\phi}{k} = \frac{1}{k}\left\langle\left(\Delta\phi_e\right)^2\right\rangle^{1/2}$$


S-band RF Gun, 100MeV/m Gradient
Exit Phase
Compression Factor
150
140
130
120
110
100
90
80
70
60
50
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
40
50
60
70
80
90
100
110
120
Exit Phase [deg.]
Compression Factor
Launch phase [deg.]

RF induced energy spread

The energy spread induced by the rf field can be calculated from

$$\gamma = 1 + \alpha\left[(n+1/2)\pi\sin\phi + \cos\phi\right] \quad \Rightarrow$$

$$\langle\gamma\rangle + \Delta\gamma = 1 + \alpha\left[(n+1/2)\pi\sin(\langle\phi\rangle + \Delta\phi) + \cos(\langle\phi\rangle + \Delta\phi)\right]$$

It will be shown later the transverse emittance can be minimized by setting

$$\langle\phi\rangle = \phi_e = \pi/2$$

Thus, $\Delta\gamma = -\alpha\Delta\phi - \frac{1}{2}(\gamma_e - 1)(\Delta\phi)^2 + \frac{\alpha}{3!}(\Delta\phi)^3 - \cdots \quad , \quad \gamma_e = \langle\gamma\rangle_{exit}$

The rms energy spread is $\sigma_\gamma = \left\langle(\Delta\gamma)^2\right\rangle^{1/2} = \alpha\left\langle(\Delta\phi)^2\right\rangle^{1/2} = \alpha k \sigma_z$

Longitudinal emittance - RF

Kim uses the following definition of longitudinal emittance:

$$\varepsilon_z = \frac{1}{k}\sqrt{\left\langle(\Delta\gamma)^2\right\rangle\left\langle(\Delta\phi)^2\right\rangle - \left\langle(\Delta\gamma)\right\rangle^2\left\langle(\Delta\phi)\right\rangle^2}$$

We assume (for simplicity here) that the particles have a gaussian distribution of phase f(z) ~ exp[-(kΔz)^2/2]

$$\boxed{\varepsilon_z = \sqrt{3}(\gamma_e - 1)k^2\sigma_z^3}$$

While not strictly true, this approximation is fairly accurate, and presents a reasonable scaling.

Transverse dynamics in RF field

Discussion of the transverse particle dynamics begins with the Lorentz force:

$$F_r = e(E_r - \beta c B_\theta)$$

The field components for the axisymmetric RF fields are derivable from the Maxwell equations. The transverse components are related to the longitudinal electric field via:

$$E_r = -\frac{r}{2}\frac{\partial}{\partial z}E_z \qquad cB_\theta = \frac{r}{2c}\frac{\partial}{\partial t}E_z$$

With our representation of the axial rf electric field

$$E_z = E(z)\cos kz \sin(\omega t + \phi_0)$$

the force on a particle can be shown to be

$$F_r = er\left\{-\frac{1}{2}\left(\frac{dE(z)}{dz}\right)\cos kz \sin(\omega t+\phi_0) - \frac{1}{2c}\frac{d}{dt}\big(E(z)\sin kz\cos(\omega t+\phi_0)\big) + \frac{\beta}{2}\frac{dE(z)}{dz}\sin kz\cos(\omega t+\phi_0)\right\}$$

RF induced transverse kick at gun exit

We make a further approximation to the rf gun field description

Assume: $E(z)=E_0\theta(z_f-z)$ and that E(z=0)=0 and E(z=z_f)=0 (electric field is zero at cathode and outside of gun)

$$p_r=\gamma\beta r'=\frac{1}{mc}\int F_r dt=\frac{1}{mc^2}\int F_r\frac{dz}{\beta}$$

$$=-\frac{er}{2}\int_0^{t_f}E_0\delta(z-z_f)\cos kz\sin(\omega t+\phi_0)dt$$

$$-\frac{er}{2c}\left(E(z)\sin kz\cos(\omega t+\phi_0)\right)\Big|_0^{t_f}$$

$$+er\int_0^{t_f}\frac{\beta}{2}E_0\delta(z-z_f)\sin kz\cos(\omega t+\phi_0)dt$$

$$p_r=p_{r0}+\frac{eE_0}{2mc^2}r\left[\beta\sin kz_f\cos(\omega t_f+\phi_0)-\cos kz_f\sin(\omega t_f+\phi_0)\right]$$

Ignore thermal emittance, assume (as was already done) that β=1, and use the exit phase, ϕ_e,

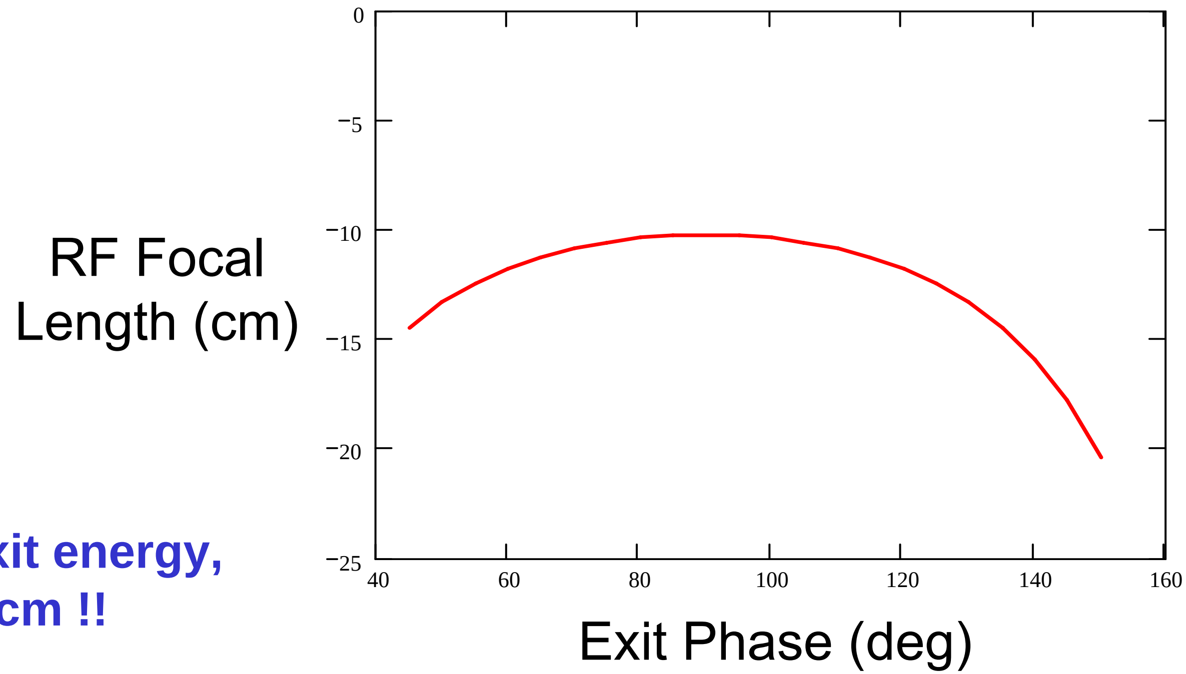
$$\boxed{\Delta p_r=\gamma\Delta r'=r\frac{eE_0}{2mc^2}\sin(kz_f-\omega t_f-\phi_0)=r\frac{eE_0}{2mc^2}\sin(\phi_e)}$$

Effective focal length of RF kick

Switch to Cartesian coordinates and compute the effective focal length:

$$p_x = \beta\gamma x' = \frac{eE_0}{2mc^2} x \sin\phi_e \qquad x' = x\frac{eE_0}{2\beta\gamma mc^2}\sin\phi_e = -\frac{x}{f_{rf}}$$

$$f_{rf} = -\frac{2\beta\gamma mc^2}{eE_0 \sin\phi_e}$$



For 100MV/m and 5 MeV exit energy, the focal length is only 10 cm !!

Transverse emittance - rf

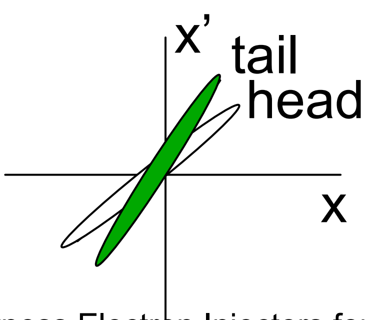
$$\varepsilon_x = \sqrt{\langle p_x^2\rangle\langle x^2\rangle - \langle p_x x\rangle^2} = \frac{eE_0}{2mc^2}\langle x^2\rangle\sqrt{\langle \sin^2\phi_e\rangle - \langle \sin\phi_e\rangle^2}$$

$$\phi_e = \langle\phi\rangle + \Delta\phi$$

$$\varepsilon_x^{rf} = \frac{eE_0}{2mc^2}\langle x^2\rangle\sqrt{\left[\left\langle(\Delta\phi)^2\right\rangle^2 - \frac{1}{3}\left\langle(\Delta\phi)^4\right\rangle\right]\cos^2\langle\phi_e\rangle + \frac{1}{4}\left[\left\langle(\Delta\phi)^4\right\rangle - \left\langle(\Delta\phi)^2\right\rangle^2\right]\sin^2\langle\phi_e\rangle}$$

Emittance is minimum when the exit phase is 90 degrees, where the change in rf-focal length with phase is zero. To first order, all slices see the same kick.

Time-dependent emittance growth



If the distribution in phase is Gaussian of width σ_ϕ, then the normalized emittance is

$$\varepsilon_x^{rf} = \frac{eE_0}{2mc^2}\frac{\langle x^2\rangle\sigma_\phi^2}{\sqrt{2}}$$

Space charge fields

We consider two different regimes for the electron distribution in the rest frame of the electron bunch

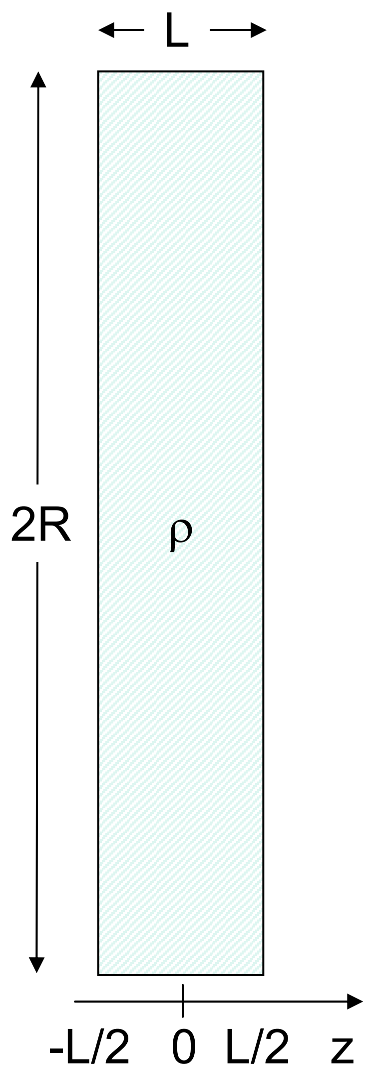
'Pancake' beam ($L_z << L_r$)

Here the longitudinal electric field is proportional to the surface charge density, while the radial electric field is inversely proportional to L_r^2.

'Cigar' beam ($L_z >> L_r$)

Here the transverse electric field is proportional to the line charge density and inversely proportional to the transverse dimension, while the longitudinal electric field is inversely proportional to L_z^2

Pancake Beam



In the beam rest frame (or close to the cathode)

$$\int_S D \cdot dS = \int_V \rho dV$$ Gauss's Law

$$\pi\varepsilon R^2 E_z \approx \rho\pi R^2 z$$ $2R >> L$

$$E_z \approx \frac{\rho z}{\varepsilon} = \frac{Qz}{\pi\varepsilon R^2 L}$$ $2R >> L$

We will happily neglect fringe fields in our treatment

Self-field scaling

For relativistic beams, the electrostatic field in the beam rest frame becomes an electric *and* magnetic field in the laboratory frame, via a Lorentz transformation:

$$E_x^{lab} = \gamma E_x^{beam} \quad , \quad B_y^{lab} = \gamma \frac{v_z}{c^2} E_x^{beam} \quad , \quad E_z^{lab} = E_z^{beam}$$

The self-force on a beam particle is found from the Lorentz force

$$F_x = e(E_x - v_z B_y)_{lab} = \frac{e}{\gamma} E_x^{beam} \quad , \quad F_z = eE_z^{beam}$$

Impulses derived from self-fields

To calculate the net transverse and longitudinal impulses to the beam from self-field forces, we make first make the assumption that these forces are much smaller than the rf forces in the gun. We can calculate the impulses from the Lorentz force

$$\Delta(\gamma\beta x') = \frac{1}{mc}\int F_x dt = \frac{1}{mc^2}\int \frac{O(\gamma)}{\gamma^2\beta}dz$$

$$\Delta\gamma = \frac{1}{mc}\int F_z dt = \frac{1}{mc^2}\int \frac{O(\gamma^2)}{\gamma^2\beta}dz$$

Near cathode:
$O(\gamma^2) \approx O(\gamma) \approx O(1)$

We make further progress by integrating the self-field forces along the rf induced trajectories via

$$\frac{d\gamma}{dz} \approx \frac{eE_0 \sin\phi_0}{mc^2} \Rightarrow \begin{aligned} \Delta(\gamma\beta x') &\approx \frac{1}{eE_0 \sin\phi_0} E_{s-c,x} \int \frac{d\gamma}{\gamma^2\beta} \\ \Delta\gamma &\approx \frac{1}{eE_0 \sin\phi_0} E_{s-c,z} \int \frac{d\gamma}{\gamma^2\beta} \end{aligned}$$

$$\int_1^{\gamma_e} \frac{d\gamma}{\gamma^2\beta} = \left[\frac{\pi}{2} - \sin^{-1}\left(\frac{1}{\gamma_e}\right)\right] \approx \frac{\pi}{2}$$

$$\boxed{\{\Delta(\gamma\beta x'), \Delta\gamma\} \approx \frac{\pi/2}{eE_0 \sin\phi_0}\{E_{s-c,x}, E_{s-c,z}\}}$$

Rest frame fields

Space charge emittance factors

We can factor out the bunch distribution dependence by defining the line density of the bunch center and the reduced space-charge field

$$n_0 = \int \rho(\vec{r}_\perp, z=0) d^2 r_\perp \qquad \vec{E}_{s-c} = \frac{n_0}{4\pi\varepsilon_0}\vec{e}_{s-c}$$

The longitudinal and transverse emittances induced by the self-field forces

$$\boxed{\varepsilon_{x,z}^{s-c} \approx \frac{\pi/4}{\alpha k \sin\phi_0}\frac{I}{I_0}\mu_{x,z}(A)} \qquad I_0 = \frac{4\pi\varepsilon_0 mc^3}{e} = 17kA$$

$$\mu_x = \sqrt{\left\langle \left(e_x^{sc}\right)^2 \right\rangle \left\langle x^2 \right\rangle - \left\langle e_x^{sc} x \right\rangle^2} \;,$$

$$\mu_z = \sqrt{\left\langle \left(e_z^{sc}\right)^2 \right\rangle \left\langle (\Delta z)^2 \right\rangle - \left\langle e_z^{sc} \Delta z \right\rangle^2}$$

are the space charge *'emittance factors'*

Scaling

The emittance factors introduce the distribution dependence onto the self-field impulses. These factors can be calculated for any beam distribution.

For the simple case of a tri-gaussian distribution with aspect ratio $A=\sigma_x/\sigma_z$ the emittance factors are found to be approximately

$$\boxed{\begin{aligned}\mu_x(A) &= \frac{1}{3A+5}\\ \mu_z(A) &= \frac{1.1}{1+4.5A+2.9A^2}\end{aligned}}$$

Correlations in emittance evolution

The total emittance growth induced by rf forces and self-field forces is a quadrature sum

$$\varepsilon_x^{rf,sc} = \sqrt{\left(\varepsilon_x^{rf}\right)^2 + \left(\varepsilon_x^{sc}\right)^2 + 2\left(\varepsilon_x^{rf}\right)\left(\varepsilon_x^{sc}\right)J_x}$$ and similar in z

of the individual contributions as well as a correlation term that describes the possible interaction between self-field and rf forces.

Kim evaluates these effects and finds the longitudinal correlation to be small in general, while the transverse correlation can be significant. In general, however, we have the triangle inequality result

$$0 < J_x < 1 \;\Rightarrow\; \sqrt{\left(\varepsilon_x^{rf}\right)^2 + \left(\varepsilon_x^{sc}\right)^2} < \varepsilon_x^{rf,sc} < \varepsilon_x^{rf} + \varepsilon_x^{sc}$$

which places bounds on our estimate of emittance growth.

Comparison with simulation

Travier analyzed Kim's model by comparing it with Parmela simulations and found it was valid when:

$$\alpha = \frac{eE_0 k}{2mc^2} \geq 0.9 \text{ and } \phi \geq \frac{\pi}{4}$$

Travier also derived the following empirical scaling laws for Gaussian distributions, from Parmela calculations, and extended the range of validity to launch angles of only a few degrees.

$$\varepsilon_{rf} = 4\sqrt{2}\pi^3 \frac{\alpha}{c} f^3 \sigma_x^2 \sigma_b^2 \qquad \alpha = \frac{eE_0}{2kmc^2}$$

$$\varepsilon_{sc}(\phi_0) = \frac{c^2}{8\alpha f \sin\phi_0} \frac{Q}{I_A} \frac{1}{(3\sigma_x + 5c\sigma_b)}$$

evaluated at gun exit

$$\phi_\infty = \phi_0 + \frac{1}{2\alpha \sin\left(\phi_0 + \frac{\pi}{6\sqrt{\alpha}}\right)} + \frac{\pi}{15\alpha}$$

Summing all the contributions

At the exit of the rf gun, we must include all possible contributions to the beam's emittance. This includes the rf-induced and self-field induced (plus any possible correlation) terms as well as the initial thermal emittance picked up during emission.

$$\varepsilon_n = \sqrt{\varepsilon_{thermal}^2 + \varepsilon_{rf}^2 + \varepsilon_{sc}^2 + 2\varepsilon_{rf}\varepsilon_{sc}J_{rf-sc}}$$

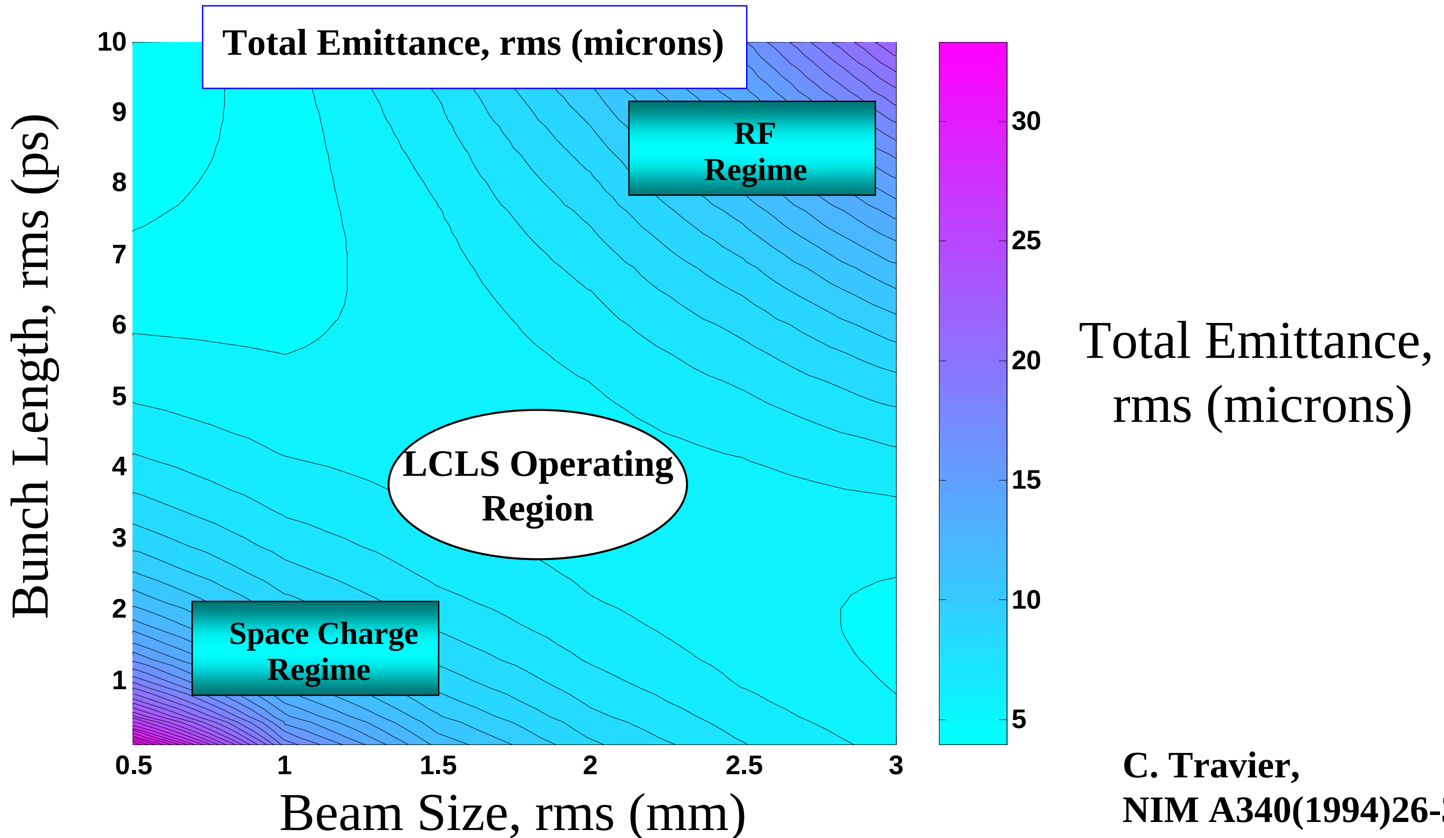
The thermal emittance is assumed to be smaller than the other contributions. The deviations in particle trajectories from the thermal velocity spread is much smaller than either the rf or self-field forces, so that additional correlations are negligible and the thermal emittance can be simply added in quadrature.

Summary of design parameters

Travier summarized the scaling laws in practical units

Optimization of Operating Regime

RF guns typically operate in the slightly space charge dominated regime, using compensation to reduce SC emittance



Contour plot of the total emittance at the gun exit in the plane of bunch size and length. The bunch charge is 1 nC.

C. Travier, NIM A340(1994)26-39; D. H. Dowell et al., SLAC-PUB-10851 & Proc. 2003 FEL Conf.

Further emittance optimization

Early rf gun designers realized the limitations of their gun designs to produce beams with very small transverse emittances. Quickly, however, many ideas were proposed to eliminate particular sources of emittance growth. These ideas are collectively known as *emittance compensation*

Space-charge techniques use external focusing elements to remove transverse phase spaced correlations due to space-charge forces. We will discuss in the next lecture.

RF techniques in structures seek to introduce additional spatial harmonics to the axial electric field (at the same rf frequency), or to introduce additional modes at frequencies harmonic to the fundamental.

Identification of RF contributions

In a 1992 paper (NIM A318(1992)301) Serafini examined more deeply the sources of emittance growth due to time-dependent rf forces. He saw two main effects

– Linear and non-linear correlations imparted by the loss of symmetry of the rf gun and resulting in a time or phase-dependent transverse kick.

– Spherical aberrations arising from the non-linear variation of the axial electric field with increasing radius.

The 2nd effect can be minimized or eliminated by appropriate shaping of the exit iris.

Reduction of RF emittance by symmetrizing RF fields

The time-dependent kick experienced by beam particles as they exit the gun was found to be

$$\Delta p_r = \gamma \Delta r' = r \frac{eE_0}{2mc^2} \sin(\phi_e)$$

Serafini suggested using harmonic frequencies to eliminate the time dependent RF emittance:

$$\begin{aligned} \mathrm{E_z(z,t)} &= \mathrm{E_0(z)\cos(kz)\sin(\omega t + \phi_0)} \\ &\quad + \mathrm{E_n(z)\cos(nkz)\sin(n(\omega t + \phi_n))} \end{aligned}$$

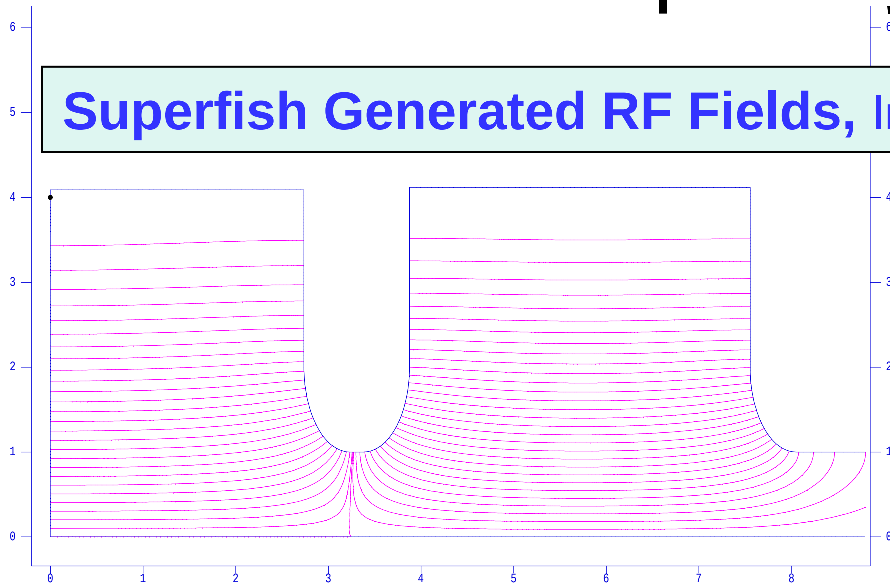
He showed that the RF emittance vanishes to 4th order and the gun energy is also linearized to 4th order when:

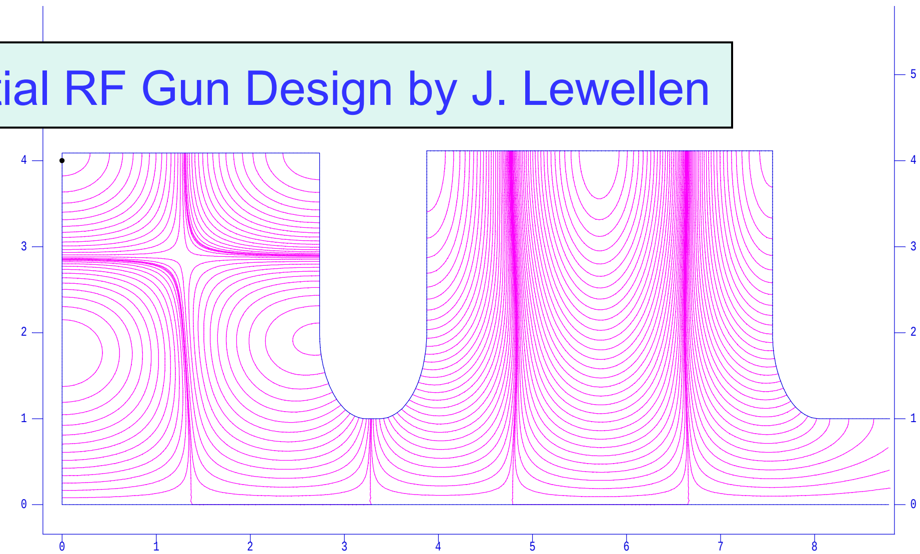
$$\mathrm{E_n} = (-1)^{(n-3)/2}\, \mathrm{E_0} / \mathrm{n}^2$$

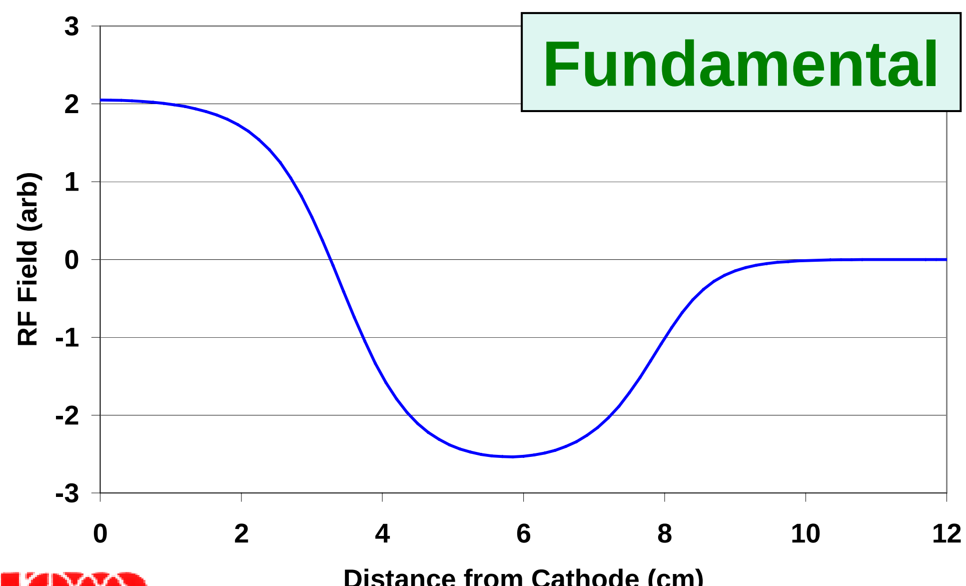
for gun exit phase $\langle\phi_e\rangle = \pi/2$ and n = 3, 7, 11, …

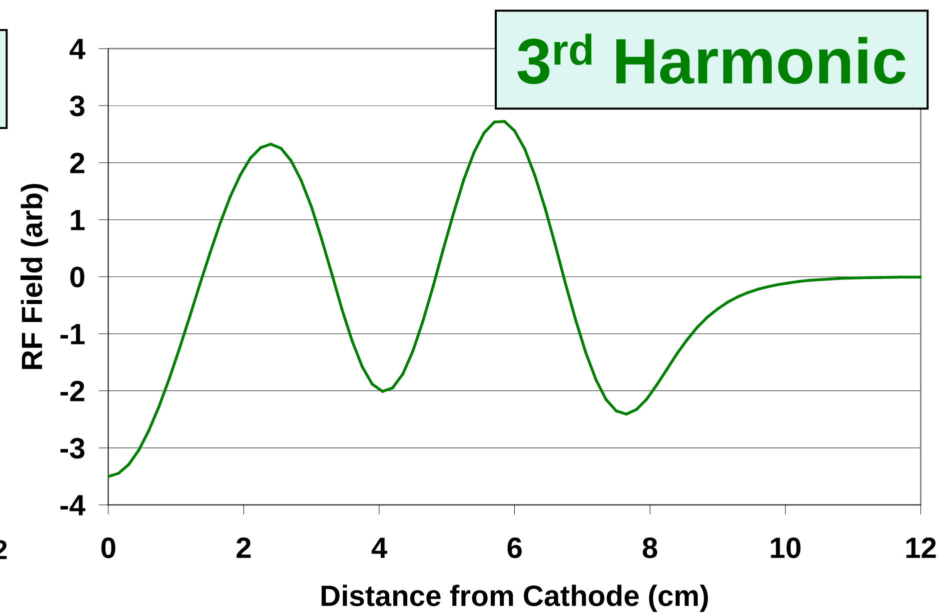
Two frequency gun proposal

Superfish Generated RF Fields, Initial RF Gun Design by J. Lewellen





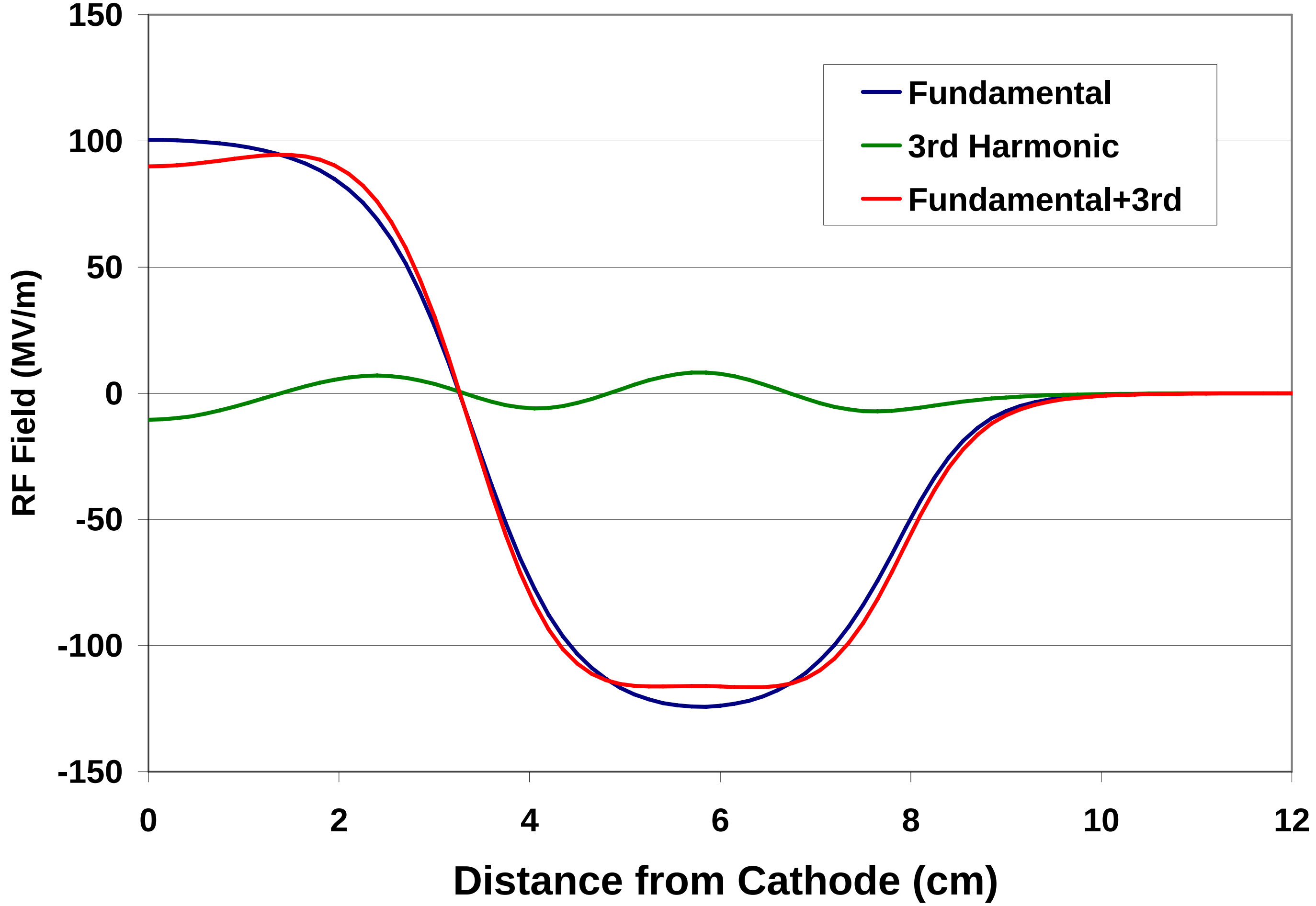




D. H. Dowell et al., SLAC-PUB-10851 & Proc. 2003 FEL Conf.

Field linearization

Superimposed Superfish RF Fields Used in 2f Parmela



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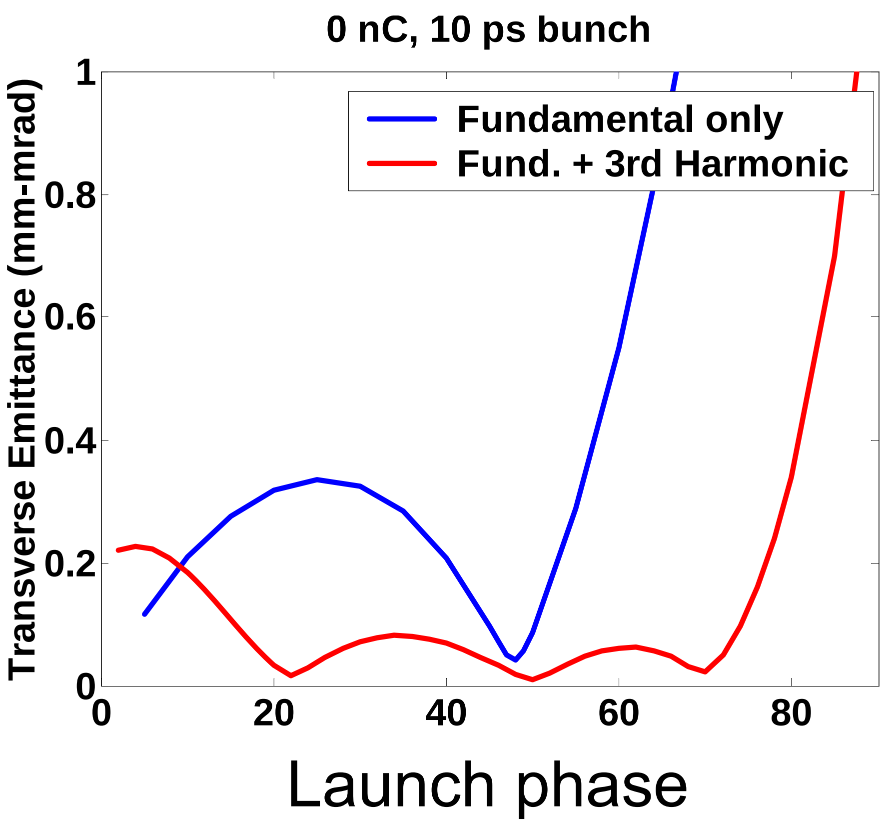
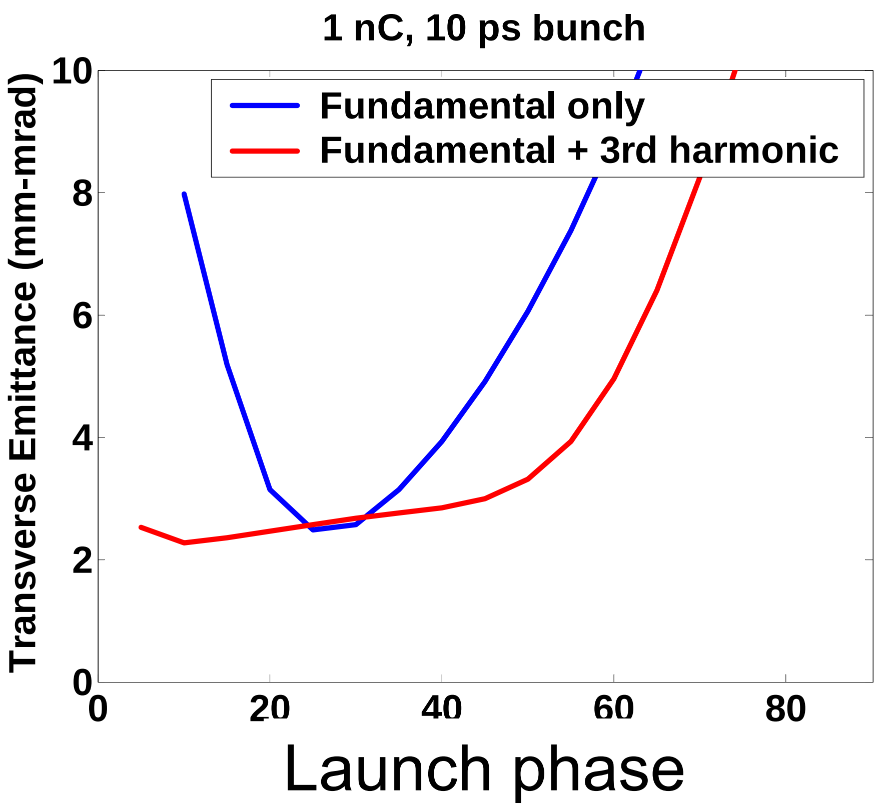
Emittance compensation

Short Bunch, Space Charge Dominated Regime

RF Emittance



SC & RF Emittance



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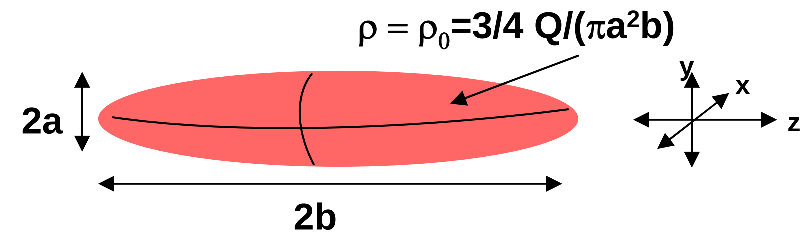
Linearization of forces by pulse shaping

Space charge emittance factors reduce to zero when the self-field forces are linear with position within the bunch.

Beam Frame potential

$$\nabla^2 \Phi_b = -\rho_b / \varepsilon_0$$



$$\boxed{\Phi_b(r,z) = \frac{-\rho_0}{2\varepsilon_0}\left(\frac{1-M_E}{2} r^2 + M_E z^2\right)} \qquad M_E = \frac{1-\xi^2}{\xi^2}\left(\frac{1}{2\xi}\ln\frac{1+\xi}{1-\xi} - 1\right) \qquad \xi^2 = 1 - \left(\frac{a}{b}\right)^2$$

Beam Frame Electric Fields and Forces

$$E_{bz} = -\partial_z \Phi_b = \left(\frac{\rho_0}{\varepsilon_0} M_E\right) z \qquad E_{br} = -\partial_r \Phi_b = \left(\frac{\rho_0}{2\varepsilon_0}\left[1 - M_E\right]\right) r$$

Linear in all directions!

Looking towards space-charge emittance compensation

- Need for emittance compensating elements
  - Rf defocusing at iris exit (single mode guns)
  - Space charge correlations

- Emittance compensating elements can be
  - Solenoid at low energy
  - RF ponderomotive focusing
  - Acceleration at high gradient

Summary

- Higher order effects:
  - Higher order RF multipoles
  - Nonlinear fields (rf, sc)
  - Transient effects (mode beating)
  - Longitudinal velocity shear
  - Wakefield effects

References

S. Nagaitsev and A. Shemyakin, FERMILAB-TM-2107 (May 2000).

K.-J. Kim, NIM **A275** (1989), 201.

C. Travier, NIM **A340** (1994), 26.

L. Serafini, NIM **A318** (1992), 301.

L. Serafini, NIM **A340** (1994), 40.

HW Problems

1. Compare L-band and S-band 1.5-cell guns
   1. The S-band gun (f=2856 MHz) has a 100MeV/m gradient. Assuming tat the alpha parameter is the same, what is the gradient for the L-band (1300 MHz) gun?
   2. Exit energy, launch phase, and compression factor? To produce a 2.5ps rms bunch duration at the exit, what should be the incident laser pulse duration?
   3. RF kick for both cases for an electron at r=1mm.
   4. Rf transverse emittance growth. Assume sigx=1mm.
   5. Identical charge (1nC), estimate emittance growth
   6. Discuss the apparent disparities in the two sources of emittance growth for the two gun frequencies.

HW Problems

2. An important source of coupling between space charge emittance growth and rf induced emittance occurs through the beam radius. Temporal modulations or fluctuations in the drive laser intensity can map onto the instantaneous beam current. Space charge forces force the spot size to track the current.

   Assume the longitudinal dependence of the beam radius is square root-parabolic in s=t-z/v. The charge distribution is uniform. Give functions of I(s) and R(s)

   1. Express the rms bunch duration in terms of the total pulse duration. For a bunch of total charge Q.
   2. What is the rms spot size in terms of the maximum radius?
   3. Calculate the rms rf emittance growth. Need to go back to the definition to maintain the correlations. Assume <phi_exit>=90°

Final Exam Problems

1. Calculate space charge emittance (x and z) growth for ellipsoidal beam distribution.
   1. Use the formulae for the beam self-fields, the definitions of rms emittance.
2. Calculate rf emittance growth for phi_exit near to but ≠ 90°. Assume gaussian beam, and keep only lowest order terms.
3. DC gun exit defocusing from fringe fields. 400keV beam. 15MeV/m gradient. I=10A.