Pinhole Cameras - Operation and Analysis

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- Use x-rays to minimize diffraction effects
- Reflective/refractive optics not available
- Simple components
X-ray Pinhole Camera - Schematic

CARTOON of PINHOLE CAMERA

SPEAR circa 1975

DIAMOND circa 2005

Cu

1 W

10 mW

0.8°

Al (3 mm)

Pinhole (25 µm²)

45 W

(outside vacuum)

d = 4 m

D = 8 m

Magnification ≈ 2
SPEAR3 Pinhole, Filters, YaG Screen and Camera

Magnification lens

YAG Screen

Si Mirror

Intensity Filters

Slits for resolution function measurement
What does the Pinhole Camera See?

1. Horizontal Plane

- Electron beam distribution
  \[ \rho(x,x') = \frac{1}{2\pi \varepsilon_o} e^{-\left(\gamma_o x^2 + 2\alpha_o xx' + \beta_o x'^2\right)/2\varepsilon_o} \]

- Photon beam distribution
  \[ \rho(x,x') = \frac{1}{2\pi \varepsilon} e^{-\left(\gamma x^2 + 2\alpha xx' + \beta x'^2\right)/2\varepsilon} \] (new Twiss, new ellipse)

Searchlight sweeping across pinhole integrates over \( x' \)

\[ \rho(x) = \frac{1}{\sqrt{2\pi \beta \varepsilon}} e^{-x^2/2\beta \varepsilon} \] (simple result)

where \( \beta \varepsilon = \beta_o \varepsilon_o + (\eta_o \delta)^2 \) → fit to Gaussian, compute \( \varepsilon_o \)
BESSY-II Pinhole Array
(Peatmann and Holldack)

Finite beam divergence

Constant Image Intensity

Pixel number

Intensity (grey levels)
SPEAR3 Example – three different operational modes

Measurement

Theory

intensity [au]

x-position [μm]

intensity [au]

x-position [μm]
What does the Pinhole Camera See?

2. Vertical Plane

- Sweeping searchlight no longer integrates over angles

- Pinhole projected onto source casts a ‘shadow’ in phase space: \( y' = \frac{y}{d_1} \)

\[
\left( \gamma y^2 + 2\alpha y y' + \beta y'^2 \right) / 2\epsilon \rightarrow \frac{y^2}{2\epsilon\beta} \cdot \left[ 1 + \left( \frac{\alpha - \beta}{d_1} \right)^2 \right]
\]

\[(\epsilon\beta)_{\text{eff}} = \frac{\epsilon\beta}{1 + F}\]

\[\rho(y_i) = Ae^{-y_i^2/(2(\epsilon\beta)_{\text{eff}})}\]

at the screen (neglecting diffraction)
What size pinhole?

If the pinhole is large, ray-optic ‘spread’ dominates

$$\sigma \approx \frac{w \cdot (d_1 + d_2)}{d_1}$$

If the pinhole is small, diffraction dominates...
Evaluating the Source Size

\[
(\sigma_{\text{image}})^2 = \left(\frac{\sigma_{\text{source}}}{\text{demag}}\right)^2 + (\sigma_{\text{blur}})^2 + (\sigma_{\text{diffraction}})^2
\]

\[
\sigma_{\text{blur}} = \frac{w}{\sqrt{2\pi}} \frac{(L_1 + L_2)}{L_1}
\]

\[
\sigma_{\text{diffraction}} = \frac{\sqrt{12}}{4\pi} \frac{\lambda L_2}{w}
\]

L₁ Source – pinhole 2.34 m
L₂ Pinhole - screen 1.2 m
Demagnification 1.95:1
W pinhole size 63 μm
Diffraction Effects – monochromatic beam

\[ F(x_i | x_o, y_i | y_o) = \frac{B}{\lambda rs} \int \int_{aperture} e^{ik(r+s)} dx dy \]
Point Source Diffraction (cont’d)

\[ F = \oint_{\text{aperture}} e^{ik(r+s)} \, dx \, dy \quad \text{field integral over pinhole aperture} \]

\[ = U_x V_y \quad \text{rectangular aperture - surface integral is separable} \]

\[ I_x = \text{intensity} = U_x^* U_x \]

\[ I_y = V_y^* V_y \]

expand ‘r’ and ‘s’ to 2\textsuperscript{nd} order - the field integrals look like

\[ U_x = \int_{x-\text{aperture}} \exp ik \left[ \frac{(x_o-x)^2}{2d_1} + \frac{(x_i-x)^2}{2d_2} \right] \, dx \]
Distributed Source Field Pattern

\[
I(x_i|x_o) = \text{intensity} = U^* U \quad \text{is for a point source}
\]

- Integrate again over the source distribution

\[
I(x_i) = \int \rho(x_o) \cdot I(x_i|x_o) dx_o
\]

where \( \rho \sim e^{-x^2/2\sigma^2_x} \) (Gaussian distribution)

- Jack reduced the double integral to a single integral

\[
I(x) = \int \frac{1}{e^{-Az^2}} \sin Bz(1-z) \cdot \cos Cxz \cdot dz
\]

A, B and C are physical system parameters
‘Polychromatic’ Diffraction

Sands-121

\( dN/dE \): Photons/sec/keV

FIG. 42—Normalized power spectrum \( S \) and photon number spectrum \( F \) of synchrotron radiation.

➤ Integrate intensity pattern over photon spectrum

C. Limborg - SSRL
Putting it all together...

*fresnel.m* valid from geometric to diffraction regimes

\[ A_y = 5 \text{ micron (diffraction)} \]

\[ A_y = 40 \text{ micron (optimum)} \]

\[ A_y = 250 \text{ micron (geometric)} \]

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Application to measurement

SPEAR3: $M=0.6$, $w=30\,\mu m$, 8keV

- magnification, wave optics, chromatics
- 0.1% coupling

Electron beam source size

Spot size on screen

Hardening of the spectrum
Summary

- Pinhole cameras effective in the x-ray regime
- System construction fairly straight-forward
- Power loading considerations
- Optimize aperture size
- Data analysis relies on comparison of model with measurement <code>fresnel.m</code>
- Effective for measuring small spot sizes