Fluctuation-Based Bunch Length Experiments

US Particle Accelerator School
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- Motivation
- Time Domain Measurements
- Interferometer-based Measurements
- Frequency-based Measurements
- Introduction to USPAS Simulator

Motivation

- Alan derived theoretical basis for using statistical fluctuations to measure pulse length

- Each electron is an independent ‘radiator’ with a random, granular distribution along the bunch (shot noise)

- Sometimes the phase of wavepackets overlap, sometimes they don’t

- The *mean and variance* (moments) in the signal yields pulse length (Alan)

- Measurements can be made in the time domain or frequency domain

- We will review some experiments and introduce the USPAS simulator

Time Domain View

Sum electric field emission from individual electrons

$$E(t) = \sum_{k=1}^{N} e(t - t_k)$$

where emission times t_k are random, Gaussian-distributed numbers

$$f(t) = \frac{1}{\sqrt{2\pi\sigma_t}} e^{-t^2/2\sigma_t^2}$$

Each wavepacket $e(t)$ is centered at random time t_k

Wavepackets superimpose to produce more or less field at time t

The electromagnetic field intensity is E^*E

Total pulse energy $\int E^* E dt$ is therefore random in time.

Frequency Domain View

Total electric field has a spectral content

$$\tilde{E}(\omega) = \tilde{e}(\omega) \sum_{k=1}^{N} e^{i\omega t_k}$$ (sifting theorem)

Phasors can add up to 'spike' at frequencies ω

Shot-noise in wavepacket emission causes the spikes

Width of each spike is inversely proportional to the bunch length

By Parseval's theorm, the energy in each pulse is $\int \tilde{E}^* \tilde{E} d\omega$

In the frequency domain still have shot-to-shot fluctuations

Start with Time Domain Measurements

Mother Nature has been kind to us...

Under the right conditions

the spread in signal fluctuations is proportional
to the convolution of the pulse envelop averaged over many shots

If the pulse is Gaussian bunch length measurement straightforward

$$\sigma^2 \approx \int I(t)I(t')dtdt'$$

central equation for this course

We will develop some terminology for the USPAS simulator

Coherence Length and Coherence Time

For time domain measurements band-limit the radiation

This increases the coherence length of the individual wavepackets

$$f = c/\lambda$$

$$\delta f = -\delta\lambda c/\lambda^2$$

$$\delta t = 1/\delta f = \lambda^2/c\delta\lambda$$

For 633nm light and a 1nm bandpass filter

$$\delta t = \lambda^2/c\delta\lambda = \frac{\left(633*10^{-9}\right)^2}{\left(3*10^8\right)\left(1*10^{-9}\right)} = 1.3\,ps$$

Coherence time of wavepacket results from the finite emission time

For a 15ps bunch, the ‘mode number’ M~ 15.

Intensity Fluctuations

Goodman, _Statistical Optics_ Chapter 6

Average Value
$$\overline{W} = \int_{-T}^{T} \overline{I}(t)dt$$

Variance
$$\sigma_W^2 = E\left[\left(\int_{-T}^{T} I(t)dt\right)^2\right] - \overline{W}^2$$

$$= \int_{-T}^{T}\int \overline{I(t)I(t')}dtdt' - \overline{W}^2$$

$$\sigma_W^2 = \int_{-T}^{T}\int \Gamma_I(t-t')dtdt' - \overline{W}^2$$

where Γ is the autocorrelation function of *I(t)*

in terms of fields $\Gamma_I(\tau) = E\left\{e(t)e^*(t)e(t+\tau)e^*(t+\tau)\right\}$

'fourth order correlation'

Intensity Variance (cont'd)

$$\sigma_W^2 = \int_{-T}^{T}\int \Gamma_I(t-t')\,dt\,dt' - \overline{W}^2$$

$$\Gamma_I(\tau) = E\left\{e(t)e^*(t)e(t+\tau)e^*(t+\tau)\right\}$$ 'fourth order correlation'

But from interferometry $\Gamma_I(\tau) = I^2 \cdot \left(1 + |\gamma(\tau)|^2\right)$

Then $\sigma_W^2 = \overline{W}^2 \frac{1}{T}\int |\gamma(\tau)|^2 d\tau$

$$\frac{\overline{W}^2}{\sigma_W^2} = \left(\frac{1}{T}\int |\gamma(\tau)|^2 d\tau\right)^{-1} = M$$ (same as before)

$$M = \frac{1}{\frac{1}{T}\int |\gamma(\tau)|^2 d\tau} = \frac{\tau_{pulse}}{\tau_{coh}}$$ is the number of modes-per-pulse!

→ measurement of *W*, σ_W with known τ_c yields τ_{pulse}

M: The ratio of Pulse Time to Coherence Time

Goodman, *Statistical Optics* Chapter 6

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$$M = \frac{1}{\frac{1}{T}\int |\gamma(\tau)|^2 d\tau} = \frac{\tau_{pulse}}{\tau_{coh}}$$

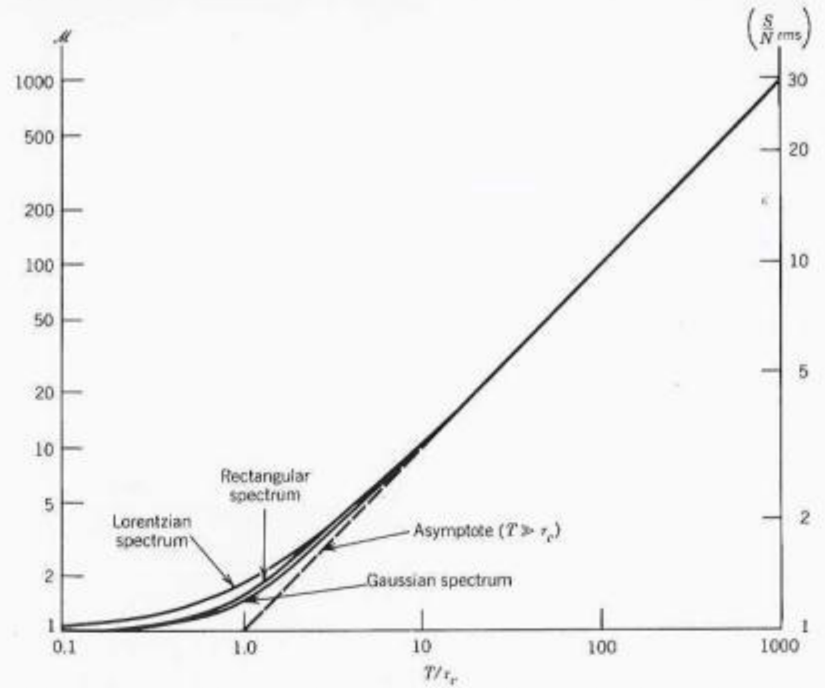
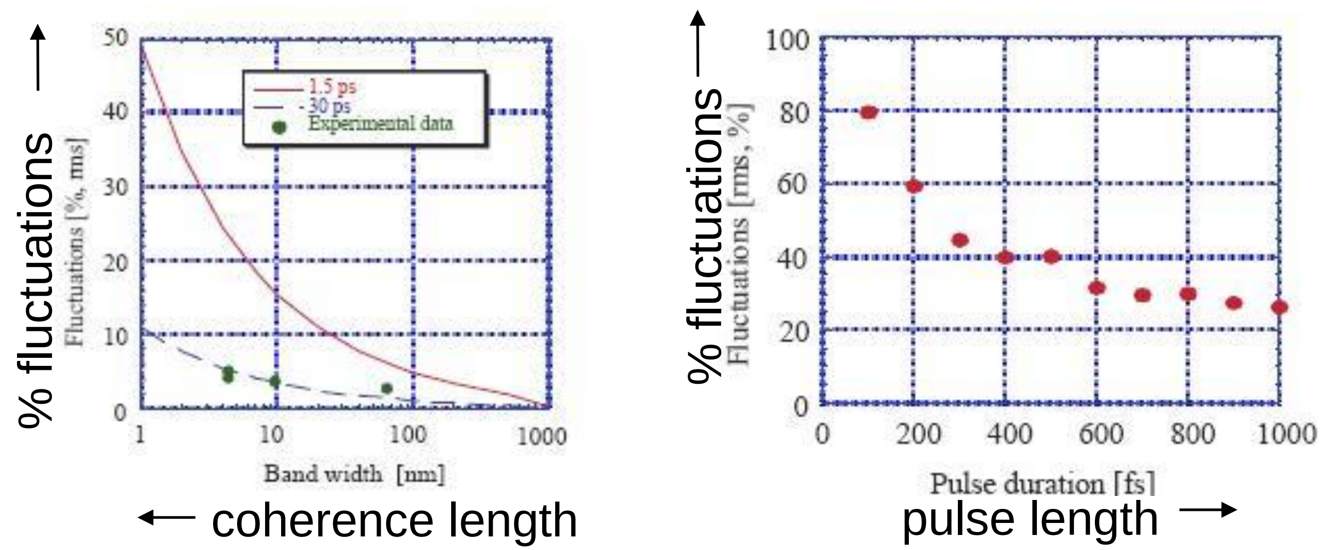


Figure 6-1. Plots of 𝓜 versus T/τ_c, exact solutions for Gaussian, Lorentzian, and rectangular spectral profiles.

Modes-per-pulse: Experimental Evidence, U. Tokyo

$$\frac{\overline{W}^2}{\sigma_W^2} = \left(\frac{1}{T} \int \left| \gamma(\tau) \right|^2 d\tau \right)^{-1} = M$$



Time-Domain Measurements (cont'd)

go back to simper form...

$$\delta^2 = \frac{\sigma_W^2}{\overline{W}^2} = \int_{-T}^{T}\int I(t)I(t')dtdt'$$ fluctuations proportional to intensity correlation

For Gaussian statistics and bandpass filter $$\delta^2 = \frac{1}{\sqrt{1+4\sigma_\tau^2\sigma_\omega^2}}$$

Expanding $$\delta^2 \approx \frac{1}{2\sigma_\tau\sigma_\omega}$$

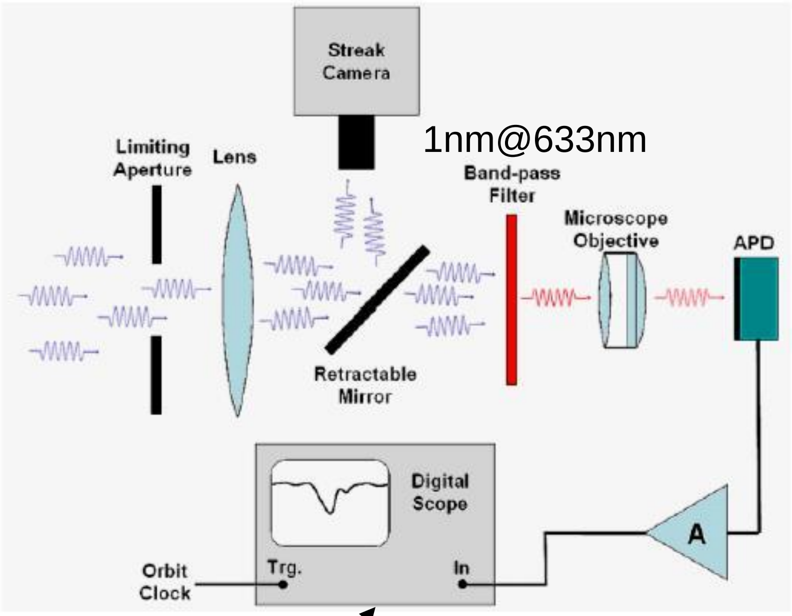
For $$\sigma_{coh} \approx 1/\sigma_\omega$$

We get $$\delta^2 \approx \frac{\sigma_{coh}}{\sigma_\tau} = \frac{1}{M}$$

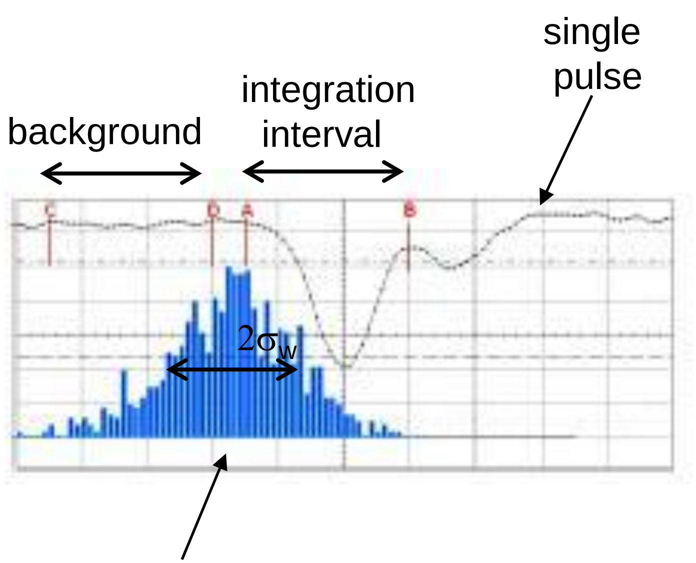
Make the coherence length long to reduce the number of modes M

Recent Time-Domain Measurements at Berkeley

Intensity fluctuations, F. Sannibale, et al



LeCroy 3GHz BW, 20Gsample/s
calculate average value of AB, CD
5000 samples @ 1.5MHz



histogram of pulse energy

$$\delta^2 = \frac{\sigma_W^2}{\overline{W}^2} = \int_{-T}^{T}\int I(t)I(t')dtdt'$$

Berkeley Measurements (cont'd)

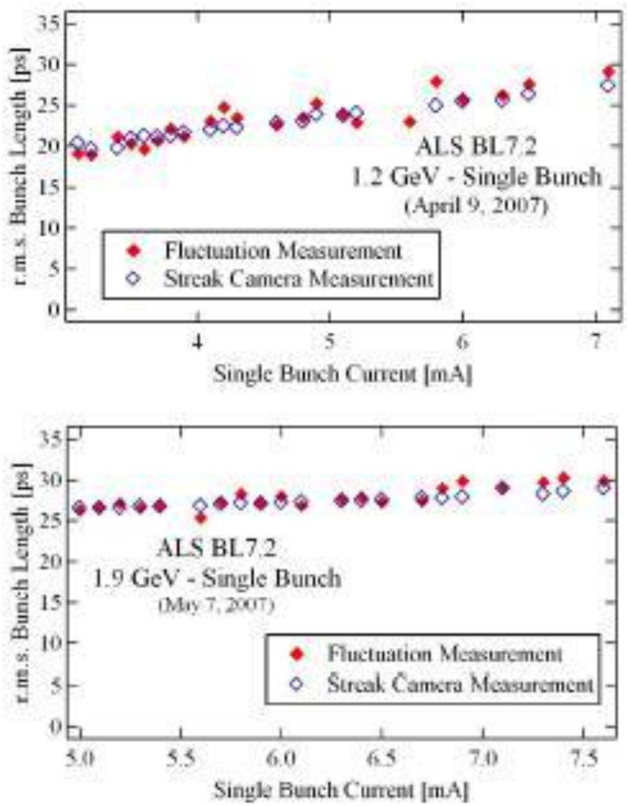


Figure 3: Examples of fluctuation and streak-camera bunch length measurements at the ALS for different beam parameters.

$$\delta^2 = \sqrt{1+\frac{\sigma_\tau}{\sigma_{\tau,c}}}\sqrt{1+\frac{\sigma_x}{\sigma_{x,c}}}\sqrt{1+\frac{\sigma_y}{\sigma_{y,c}}}$$

$\sigma_{x/y,c}$ are transverse coherence sizes

-related to transverse EM modes at 633nm

-radiation process, including diffraction

-ratios about 2 and 0.1

- also shot noise, photodiode noise

Fluctuations in Interference Visibility Pattern

Landmark paper : Zolotorev and Stupakov (1996)

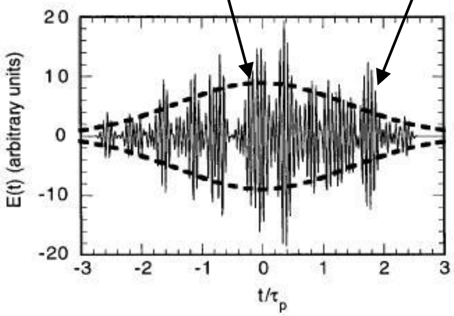
Measure fluctuations in the coherence function of the incoherent electric field

$$\Gamma(\tau) = \int E(t)E^*(t-\tau)dt$$

Utilizes a two-beam interferometer to measure $\Gamma(\tau)$

In simulation, the electric field is represented by $E(t) = A(t)e(t)$

pulse envelop   random process



Visibility Fluctuations (cont'd)

Field coherence function is $\Gamma(\tau) = \int E(t)E^*(t-\tau)dt$

Average value $<\Gamma(\tau)>= K(\tau)\int A(t)A^*(t-\tau)dt \approx K(\tau)\int I(t)dt$

where $K(\tau) = < e(t)e^*(t-\tau) >$ is the autocorrelation function of *e(t)*

Flucutation $d_\Gamma(\tau) = \left\langle \left|\Gamma(\tau) - < \Gamma(\tau) >\right|^2 \right\rangle = \left\langle \left|\Gamma(\tau)\right|^2 \right\rangle - \left|\left\langle \Gamma(\tau) \right\rangle\right|^2$

$$d_\Gamma(\tau) = \int \left|K(\tau)\right|^2 d\tau \cdot \int I(t)I(t-\tau)d\tau$$

If *I(t)* is Gaussian, can solve for d_Γ

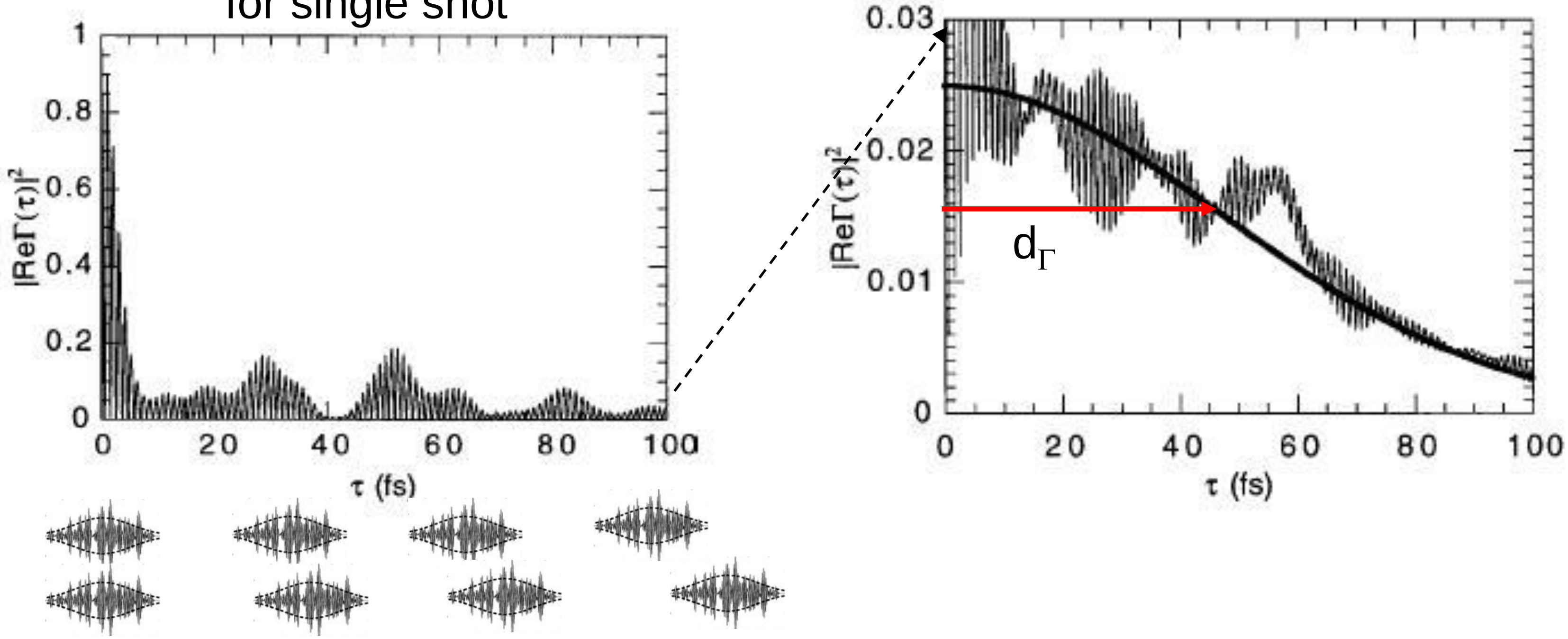
Visibility Fluctuations (cont'd)

Use a two-beam interferometer to measure $\Gamma(\tau) = \int E(t)E^*(t-\tau)dt$

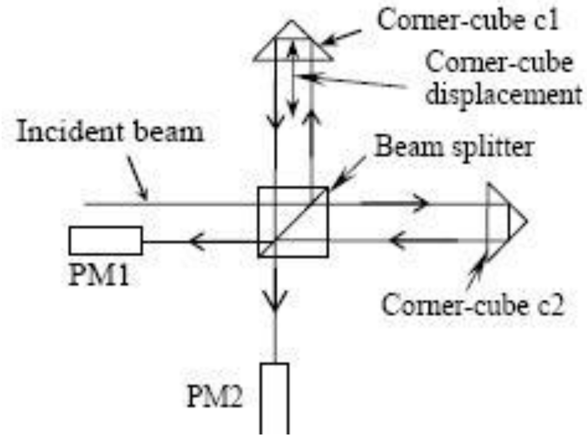
as a function of delay time τ

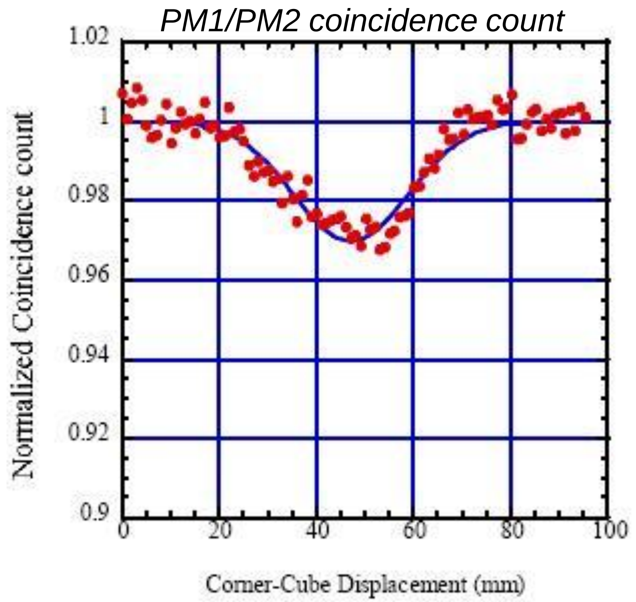
calculated |Re{correlation}|^2 for single shot

correlation average over many shots



Mitsuhashi used Michelson Intensity Interferometer





Frequency Domain Analysis

Can also analyze fluctuations in the frequency domain

Integrate the power spectrum of each pulse over frequency to find energy

$$\varepsilon = \int P(\omega) d\omega$$

The average energy is $\langle \varepsilon \rangle = \int \left(P(\omega) \right) d\omega$

And the variance is $$\frac{\langle \Delta\varepsilon^2 \rangle}{\langle \varepsilon \rangle^2} = \frac{1}{\langle \varepsilon \rangle^2} \iint \left\langle \left[P - \langle P \rangle \right] \cdot \left[P' - \langle P' \rangle \right] \right\rangle d\omega d\omega'$$

or $$\boxed{\frac{\langle \Delta\varepsilon^2 \rangle}{\langle \varepsilon \rangle^2} = \frac{1}{\langle \varepsilon \rangle^2} \iint \left| \langle PP' \rangle - \langle P \rangle \langle P' \rangle \right| d\omega d\omega'}$$

Need to compute <P> and 4th order field correlation <PP’> to evaluate variance

Frequency Domain Analysis (cont'd)

$$\frac{\langle \Delta\varepsilon^2 \rangle}{\langle \varepsilon \rangle^2} = \frac{1}{\left(\int T d\omega\right)^2} \iint T T' \left| f(\omega - \omega') \right|^2 d\omega d\omega'$$

filter

bunch transform

For a Gaussian filter and a Gaussian bunch

$$\frac{\langle \Delta\varepsilon^2 \rangle}{\langle \varepsilon \rangle^2} = \frac{1}{\sqrt{1 + 4\Delta\omega^2 \sigma_t^2}}$$

(same as time-domain analysis)

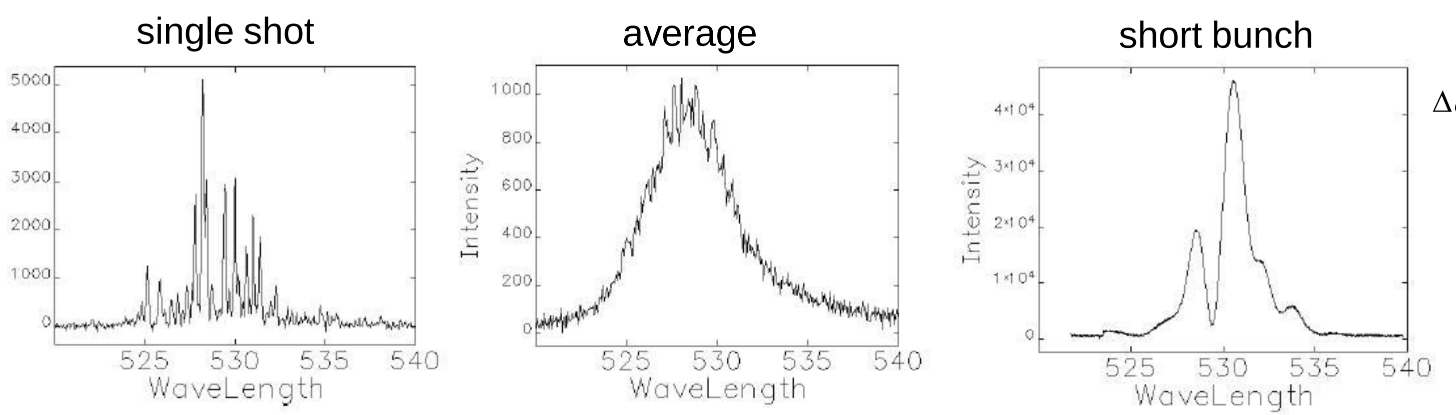
filter characteristic

bunch characteristic

Broad-band Frequency Domain Experiments

Use a spectrometer to observe spikes in single-shot spectrum

Sajaev, Argonne Nat'l Labs



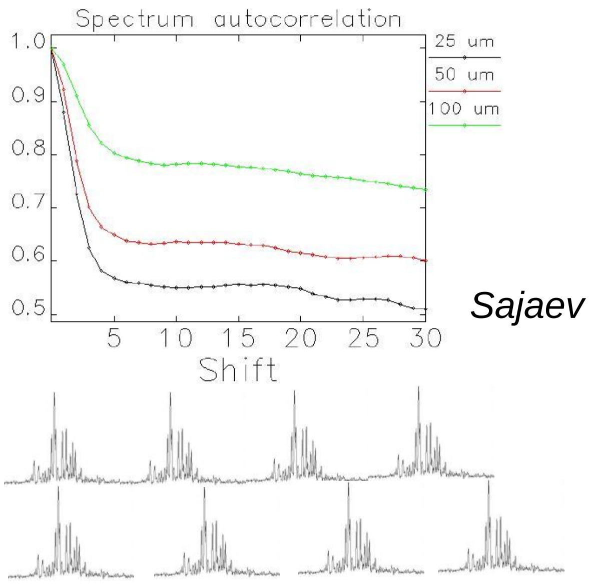
$$\Delta\omega \propto \frac{1}{\sigma_b}$$

$$\sum_{k,l,m,n=1}^{N} \left\langle e^{i\omega(t_k - t_l) + i\omega'(t_m - t_n)} \right\rangle$$

phasor sum makes 'spikes'

Single-Shot Frequency Domain Experiments

Fourier transform of bunch length is related to autocorrelation of spectrum



Sajaev

USPAS Simulator – Pulse Energy Fluctuations

Each pulse of light is a superposition of randomly-phased ‘wavepackets’

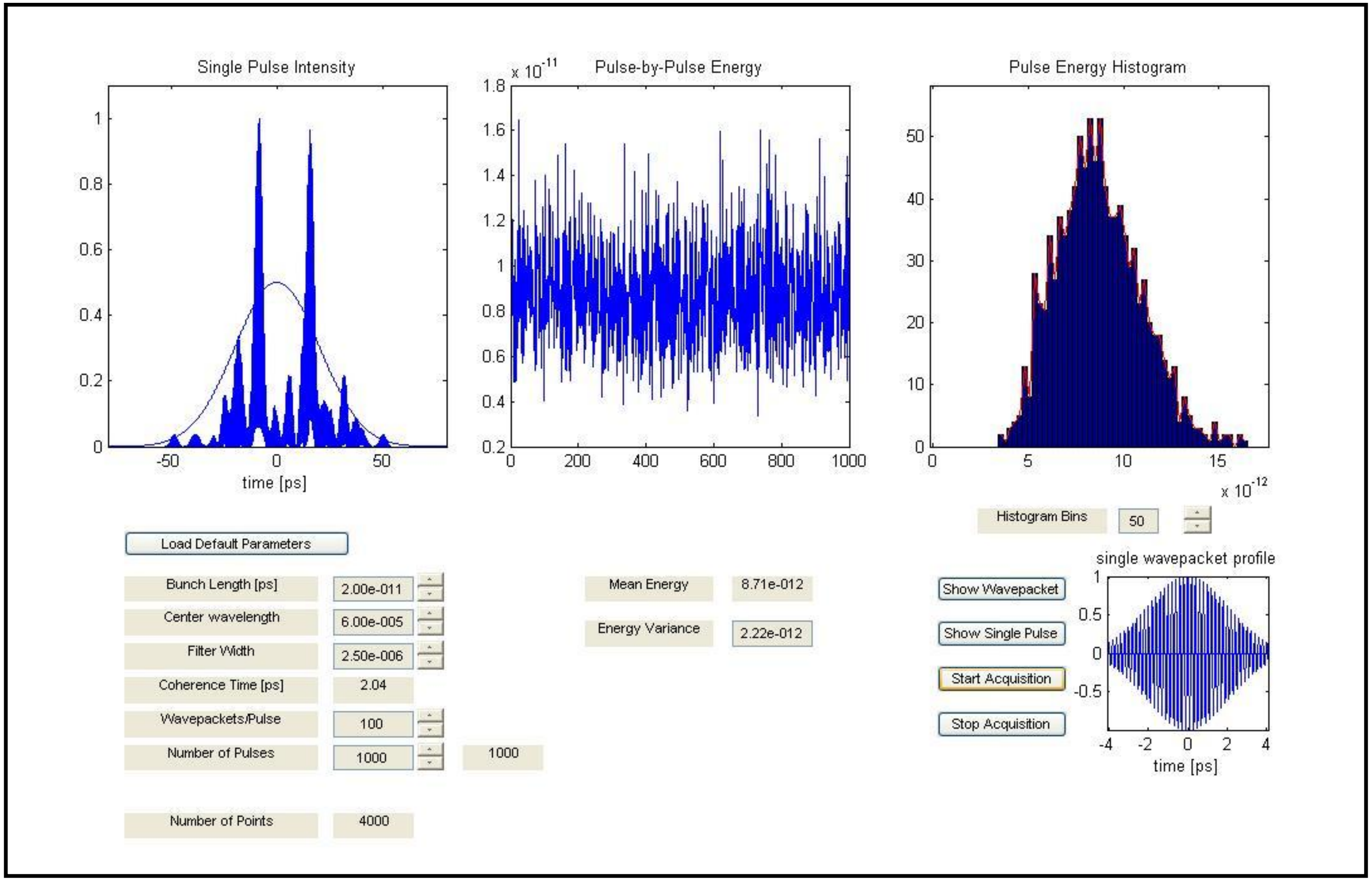
Simulator generates wavepackets at random times t_k

Computes wavepacket superposition and resulting intensity E*E

Records statistics of shot-to-shot photon beam energy $U = \int E^* E dt$
to deduce pulse length

Very much like Sinnabale experiment and USPAS laboratory
but you ‘see’ effects not physically observable

Simulator for Pulse-Energy Fluctuations



USPAS Simulator (cont'd)

Part I: Photon beam properties
Calculate wavelength, energy, photon flux, etc.

Part II: Coherence properties
Coherence length with BP filter, etc

Part III: Time-base calculations for simulator code
Need simulate with 1um radiation

Part IV: The simulator interface

Part V: Wavepackets
Study as a function of wavelength, bandwidth, etc

Part VI: Study pulse-to-pulse statistics as a function of
bunch length, filter width, etc

Independent study

Summary Fluctuation Techniques

- Wavepacket emission is a statistically random process

- In the time domain
  use a filter to make coherence length~bunch length
  look for fluctuations in shot-to-shot intensity

- Fluctuations in interferometer visibility pattern

- In the frequency domain
  use a spectrometer to observe fluctuations in spectra

- Simulator for this afternoon