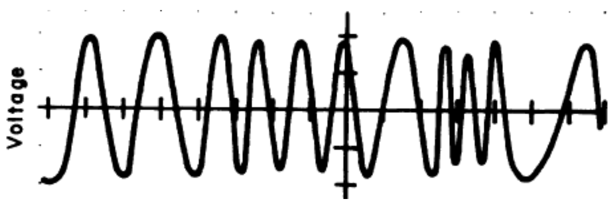
Part 2: optical and fiber physics

Coherence

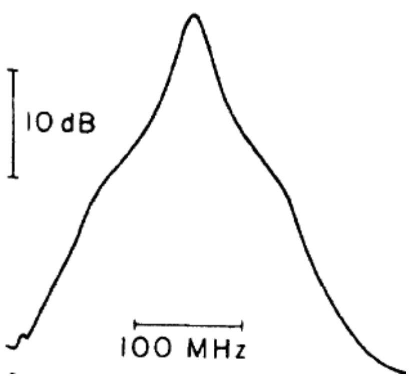
- The main advantage of lasers is, they emit coherent light, in contrast to incandescent or fluorescent sources
- Coherence is the correlation between phase of the wave at two points. “Incoherent” means large uncertainty in relative phase. (Thus, repeatable modulation still implies coherence.)
- An amount of incoherence is a deviation from perfect phase linearity



$$V_F(t) = A_0 \exp\{i[\omega_0 t + \phi(t)]\}$$

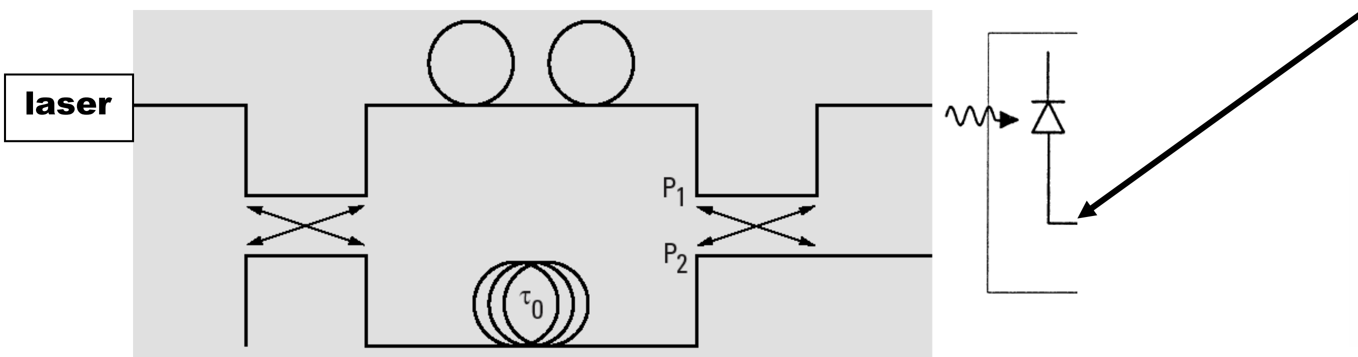
$$\tau_0 = \tau_{ave} = 1/\Delta\nu$$

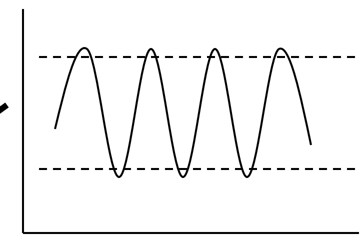
$$\ell_t = c\tau_0 = c/\Delta\nu = \lambda^2/\Delta\lambda$$



Coherence and interferometers

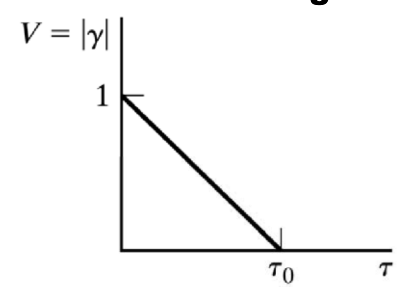
– **Temporal coherence: unequal-arm Michelson or Mach-Zehnder interferometer interferes wave at one time with same wave at later time**
  - **Detect interference term with photodiode, measures correlation**
  - **See fringes vanish as time separation is increased**





$$V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}$$

Fringe visibility



$$\mathbf{E}_P = \mathbf{E}_1(\mathbf{r}_1, t) + \mathbf{E}_2(\mathbf{r}_2, t+\tau)$$

$$I_P = \langle \mathbf{E}_P \cdot \mathbf{E}_P^* \rangle = \langle (\mathbf{E}_1 + \mathbf{E}_2) \cdot (\mathbf{E}_1^* + \mathbf{E}_2^*) \rangle$$

$$= I_1 + I_2 + 2\,\mathrm{Re}\langle E_1 E_2^* \rangle \quad \text{(polarizations assumed to be the same)}$$

Degree of coherence

$$\gamma_{12}(\mathbf{r}_1, \mathbf{r}_2, \tau) \equiv \frac{\langle E_1(\mathbf{r}_1, t) E_2^*(\mathbf{r}_2, t+\tau) \rangle}{\sqrt{I_1 I_2}}$$

$$|\gamma(\tau)| = \left\langle e^{j\Delta\phi(t,\tau)} \right\rangle = e^{-\frac{1}{2}\sigma^2_{\Delta\phi}(\tau)}$$

$$\Delta\phi(t,\tau) = \phi(t) - \phi(t-\tau)$$

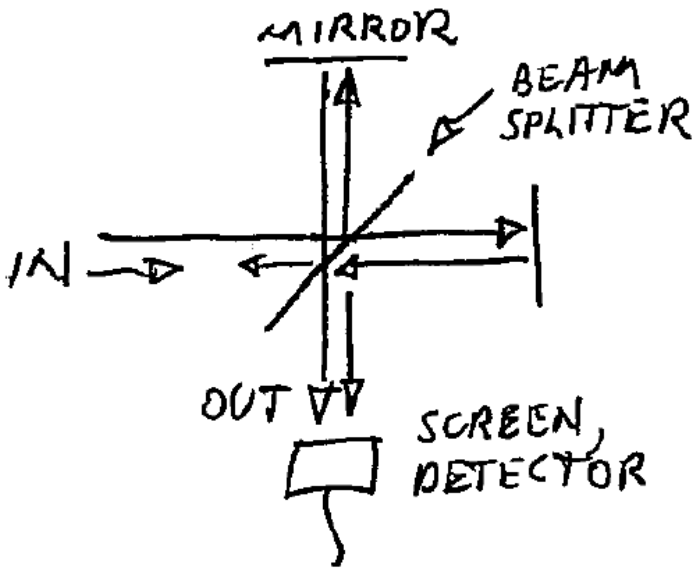
$$|\gamma(\tau)| = e^{-\pi^2\tau^2 \sigma^2_{\bar{v}}(\tau)}$$

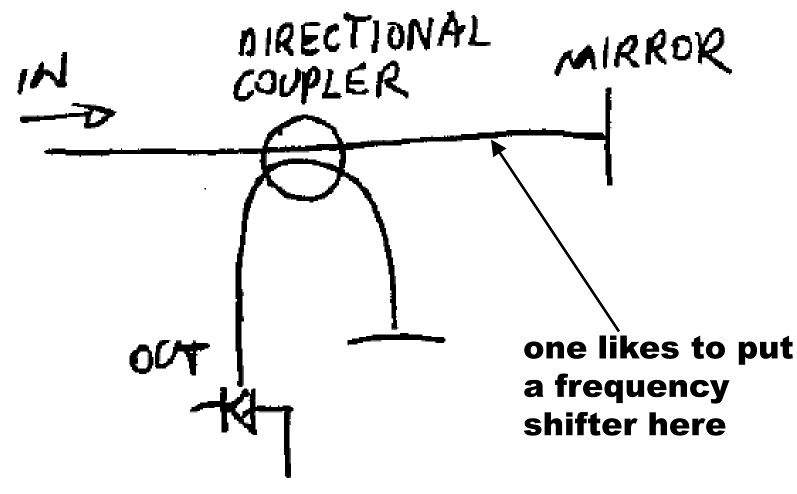
$$\text{If } I_1 = I_2 \quad \Rightarrow \quad V = |\gamma_{12}|$$

$$|\gamma_{12}(\tau)| = 1 - \frac{\tau}{\tau_0}$$

Michelson interferometer

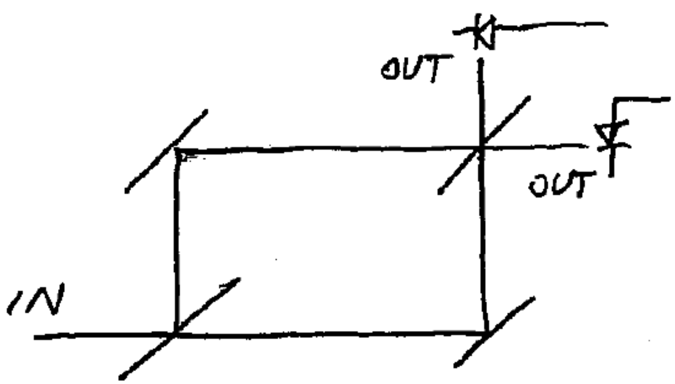
- Waves from the two arms must have same polarization for maximum signal
  - Faraday rotator mirrors are typically used
- With long coherence length laser, reference arm can be short for improved stability
- Add a frequency shifter in one arm, and the resulting beat signal is RF (frequency shifting, or heterodyne interferometer)
  - Phase comparison with local oscillator for frequency shifter
  - Better SNR, resolves direction ambiguity faster than dithering
  - Typical of commercial, free-space interferometers

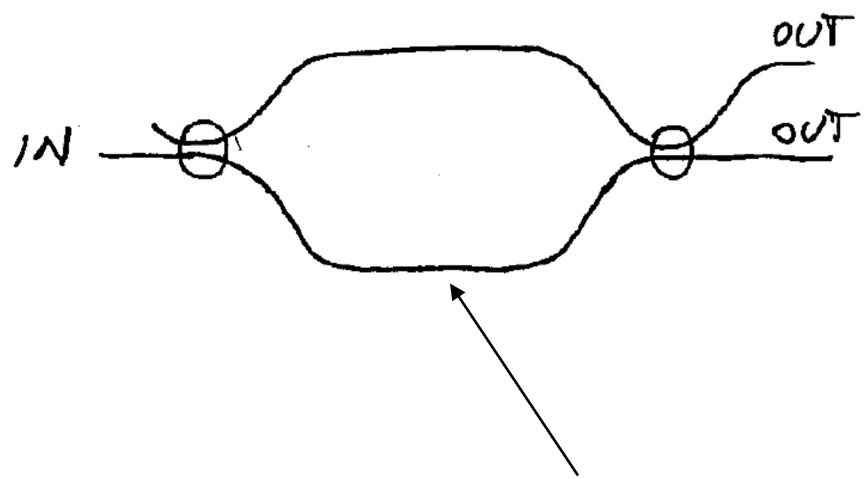




Mach-Zehnder interferometer

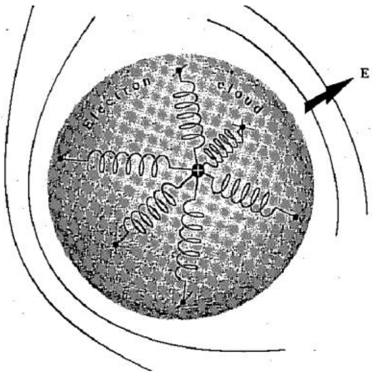
- Wave is split in two, propagated in two paths of generally different delay
- Two outputs, one which emits when in-phase, the other when out-of-phase by pi
- Looks like the Michelson interferometer unfolded about the mirrors
- We will make one of these and measure the coherence of a laser





one likes to put
a frequency
shifter here

Not-nonlinear optics



$$\boldsymbol{D} = \varepsilon_0 \boldsymbol{E} + \boldsymbol{P}.$$

$$\boldsymbol{P} = \varepsilon_0 \chi \boldsymbol{E} \quad \rightarrow \quad \boldsymbol{D} = \varepsilon_0 (1+\chi) \boldsymbol{E} = \varepsilon \boldsymbol{E} \quad \rightarrow \quad \varepsilon = \varepsilon_0 (1+\chi),$$

$$\varepsilon_r = \frac{\varepsilon}{\varepsilon_0} = n^2, \quad n^2 = 1 + \chi(\omega) = 1 + N\varpi(\omega),$$

$$n = n' - jn'', \quad \varepsilon = \varepsilon_0 \varepsilon_r = \varepsilon' - j\varepsilon'' = \varepsilon_0 (\varepsilon_r' - j\varepsilon_r''),$$

$$\varepsilon_r = n^2 \quad \rightarrow \quad \varepsilon_r' = n'^2 - n''^2 \quad \text{and} \quad \varepsilon_r'' = 2n'n''.$$

- **Typically treat dielectric material as many simple harmonic oscillators**
- **Get complex response that gives Polarization vector, adds to total Displacement**
- **Susceptibility gives polarization response, is complex number**
  - **Can be expressed as the index of refraction (relevant to optics)**
  - **Or as dielectric constant (relevant to microwaves, RF)**

Absorption and refractive index

- k is the complex wave vector, n the complex index of refraction
- The imaginary part of the index corresponds to changes in amplitude, while the real part corresponds to changes in phase

$$\boldsymbol{k} = \boldsymbol{k}' - j\boldsymbol{k}'' \quad \text{with} \quad |\boldsymbol{k}'| = n'\omega/c \quad \text{and} \quad |\boldsymbol{k}''| = n''\omega/c,$$

$$\boldsymbol{E} = \boldsymbol{E}_0 e^{j\omega(t-z/c)} = \boldsymbol{E}_0 e^{-n''\omega z/c} e^{j\omega(t-n'z/c)} = \boldsymbol{E}_0 e^{-k''z} e^{j(\omega t - k'z)},$$

The intensity will change with distance as

$$I = I_0 e^{-2k''z} = I_0 e^{-\alpha z} \quad \text{with} \quad \alpha = 2k''.$$

This can also be expressed in terms of the susceptibility:

$$n'' = \frac{\chi''}{2}; \quad \alpha = 2k'' = \chi''\frac{\omega}{c},$$

If the susceptibility is derived in terms of the atomic energy levels, one gets a very useful relation between this and the density of ground state and excited atoms. Note that when (N0-N1) is positive, there is absorption, but it can be zero or negative (for transparency or gain!)

$$\chi''(\nu) = \frac{c^3}{16\pi^2\nu^3} n'^2 \frac{1}{\tau_{\text{radiative}}} f(\nu)(N_0 - N_1)$$

Dispersion

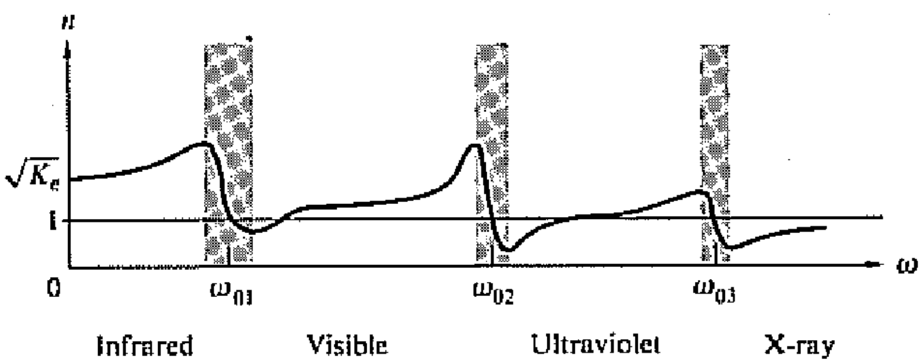
- **Index characterized by resonances nearby**
- **fj is the oscillator strength of the jth resonance**
  - **Relates to quantum energy levels, transition probability**

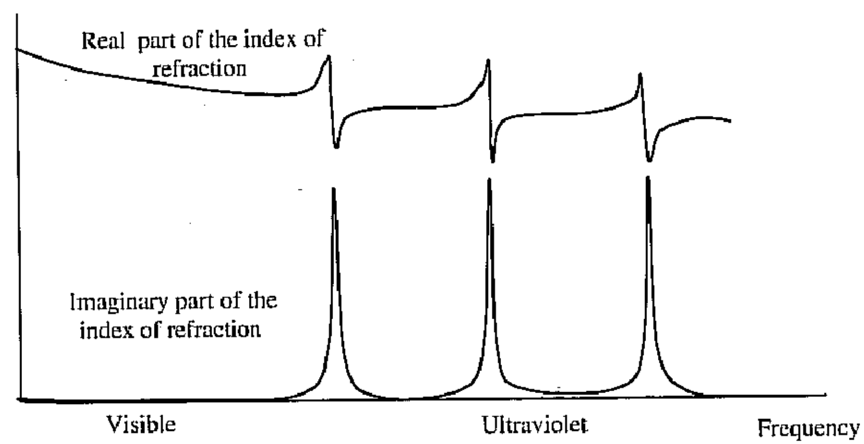
$$n^2(\omega) = 1 + \frac{Nq_e^2}{\epsilon_0 m_e} \sum_j \left(\frac{f_j}{\omega_{0j}^2 - \omega^2} \right)$$

Kramers-Kronig relates the real and imaginary parts of the susceptibility

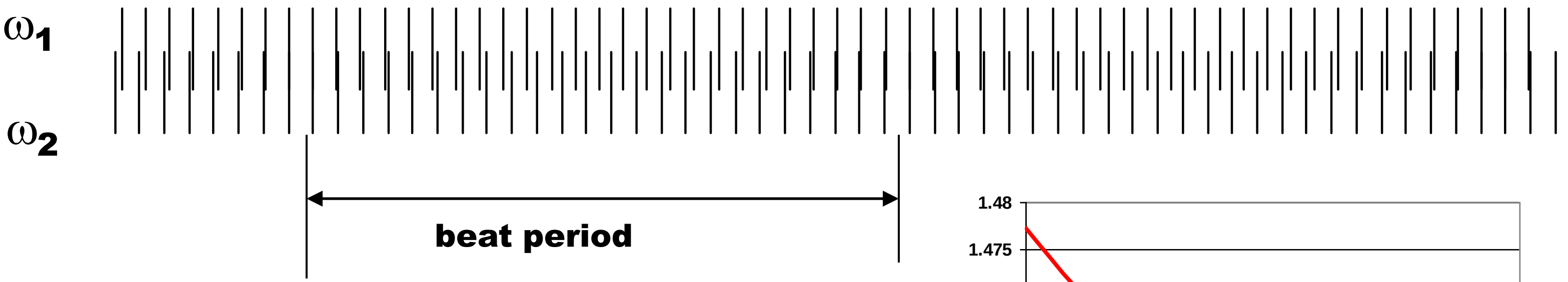
$$\chi'(\omega) = \frac{2}{\pi} \int_0^{+\infty} \frac{\omega' \chi''(\omega')}{(\omega'^2 - \omega^2)} d\omega' \quad \text{and} \quad \chi''(\omega) = -\frac{2\omega}{\pi} \int_0^{+\infty} \frac{\chi'(\omega')}{(\omega'^2 - \omega^2)} d\omega'$$

That's why one looks like the derivative of the other!

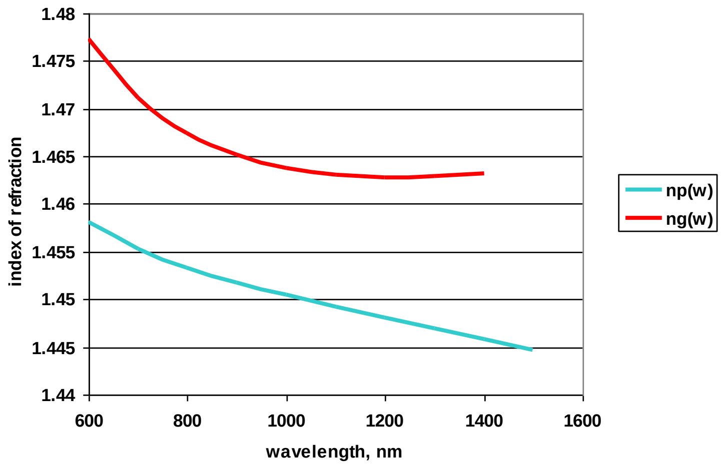




Group vs. phase velocity



$$n_g = n_p + \omega \frac{dn_p}{d\omega}$$



- RWV page 261
- Group velocity is the velocity of any modulation
- As modulation sideband relative phases shift, the “beat” shifts in Vernier fashion
  - Thus, if there is dispersion, the group and phase velocities must be different

Material dispersion and the Sellmeier equation

- **Sellmeier equation computes phase index based on**
  - **This is temperature dependent**
  - **Fit to measured data**
- **Telecom is concerned about group velocity dispersion (GVD), as they only detect the envelope**

$$n_g = n + \varpi \frac{dn}{d\varpi}$$

and also

$$\frac{dn_g}{dT} \neq \frac{dn}{dT}$$

the wavelength and temperature dependent index:

$$n^2 = A(T) + \frac{B(T)}{(1 - C(T)/\lambda^2)} + \frac{D(T)}{(1 - E(T)/\lambda^2)}$$

Ghosh et al, Journal of Lightwave Technology 12, 1338 (1994)

TABLE I

SELLMEIER COEFFICIENTS FOR FUSED SILICA (FS), ALUMINOSILICATE (AS), AND VYCOR GLASSES AT ROOM TEMPERATURE AND AT A HIGHER TEMPERATURE WHICH IS 471°C FOR FS AND 526°C FOR AS AND V GLASSES. $n^2 = A + B/(1 - C/\lambda^2) + D/(1 - E/\lambda^2)$

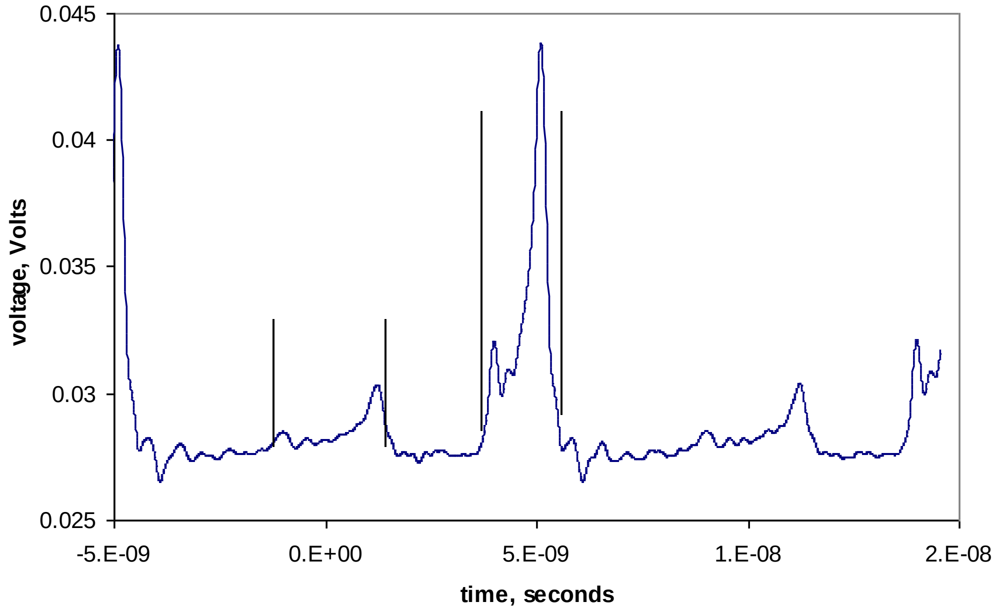
Glass & E= 100.0	Temp. (©C)	Sellmeier Coefficients				Expt. accuracy & sources	Our fit RMS error
		A	B	C	D	<- - - - - 10^{-5} - - - - ->	
Fused Silica	26	1.3121622	0.7925205	1.0996732×10^{-2}	0.9116877	±21±9.6	9.5
(SiO_2)	471	1.3148367	0.8034391	1.1248041×10^{-2}	0.9119589	[10], [11]	
Fused Silica	20	1.3107237	0.7935797	1.0959659×10^{-2}	0.9237144	2.8-1.2 [8]	1.6
SiO_2	20.5	1.3156569	0.7901384	1.0993430×10^{-2}	1.0248690	±0.3 [6]	0.5
	45.2	1.3066410	0.7994875	1.0919460×10^{-2}	0.9598566	±0.3 [6]	0.4
alumino-silicate	28	1.4136733	0.9503994	1.3249011×10^{-2}	0.9044591	±21±9.6	3.4
	526	1.5205253	0.8556252	1.5205234×10^{-2}	0.9092824	[10]	4.4
Vycor	28	1.2754213	0.8271916	1.0653107×10^{-2}	0.9384236	±21±9.6	4.1
Glass	526	1.3488048	0.7695233	1.1884981×10^{-2}	0.9460697	[10]	5.1

putting in the known values,

$$\Rightarrow \quad n_g(T) - n(T) \cong 1\%$$

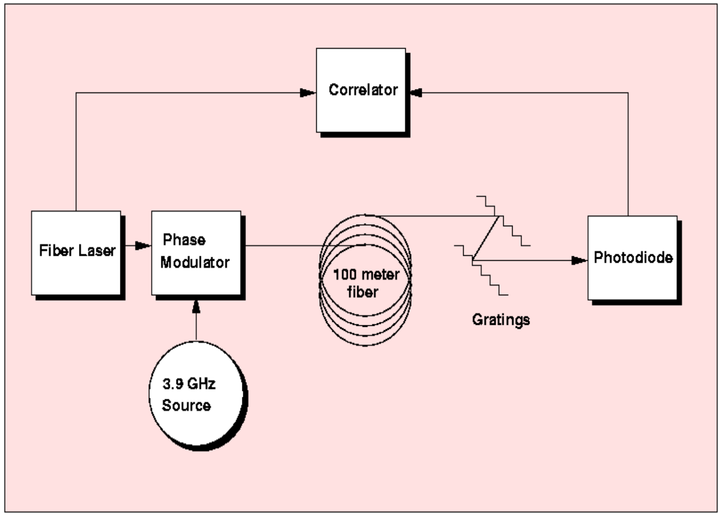
We will see the importance of this later

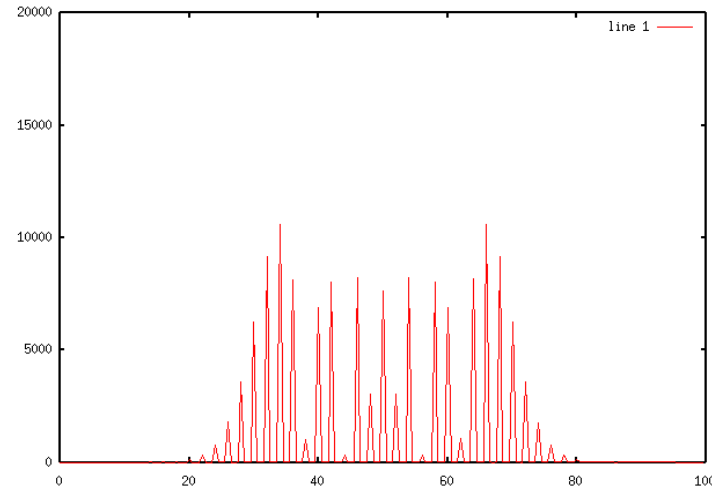
Example of pulse spreading

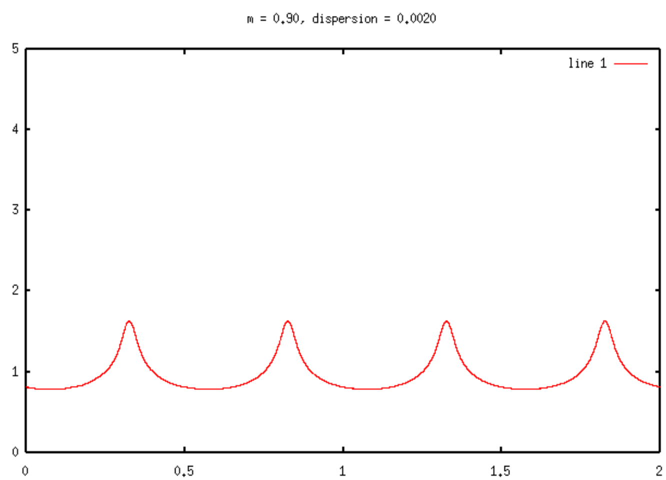


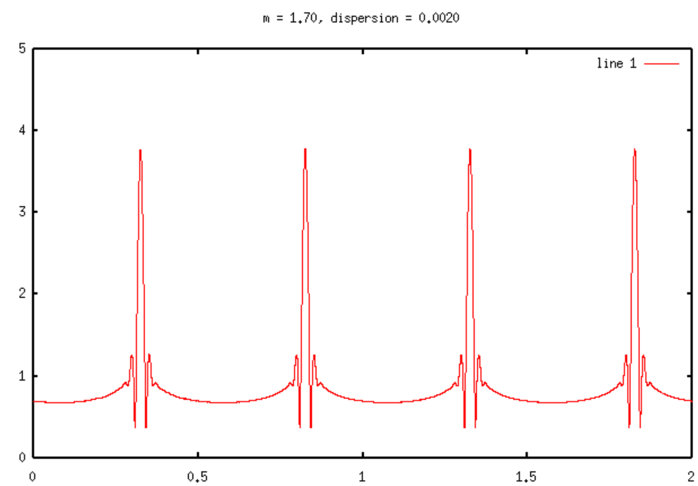
- 100fs pulse propagated through 1 and 2km spools
- Lab exercise to see if this makes sense

PM turns into AM via dispersion





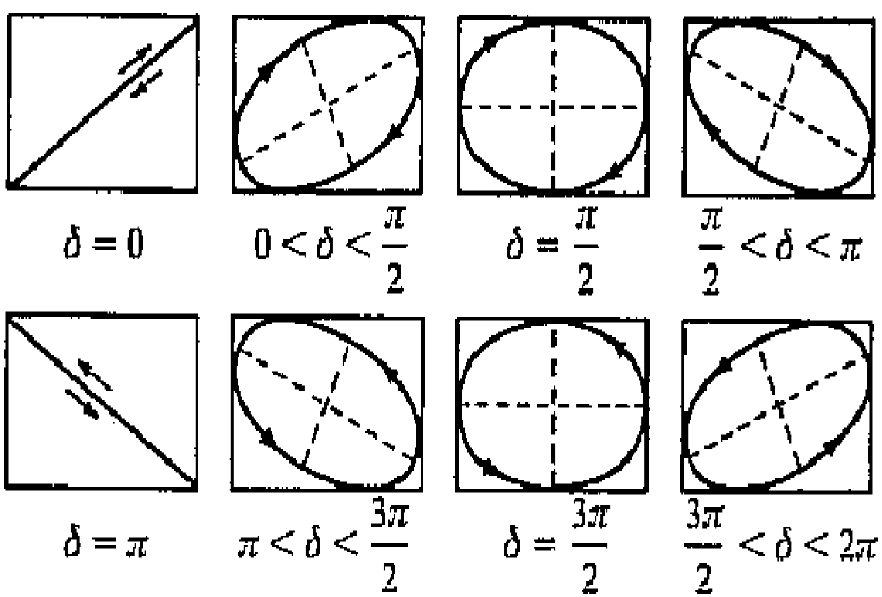


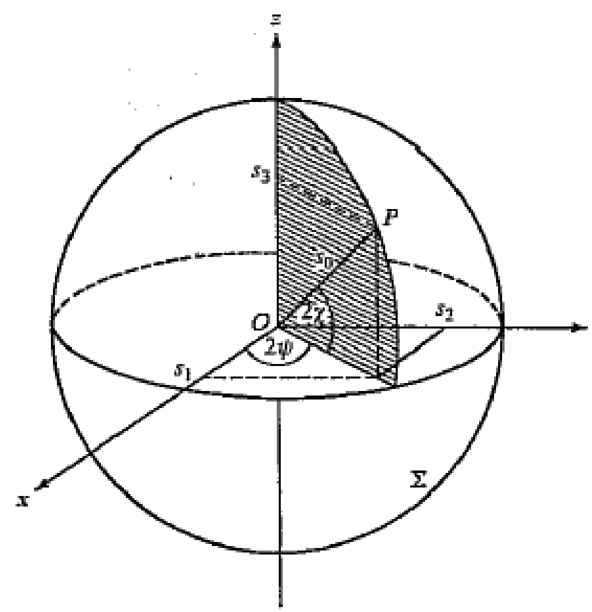


Polarization

$$\frac{E_y}{E_x} = \frac{a_2}{a_1} e^{i(\delta_1 - \delta_2)} = \frac{a_2}{a_1} e^{-i\delta},$$

- Two "polarization modes", x and y wave vectors
- Their relative phase and amplitude determines polarization state
- Several possible representations
  - Poincare sphere
    - A "globe" of possible states
    - Typical polarimeters display this in real time





Stokes vectors

- Polarimeters measure these directly with photodiodes and polarizing beamsplitters
- They then display the elipse and the Poincare sphere
- a1 and a2 are the amplitudes of the E fields
- δ is the phase difference between the two components
- All together, these uniquely specify the polarization

$$s_0 = a_1^2 + a_2^2,$$

$$s_1 = a_1^2 - a_2^2,$$

$$s_2 = 2a_1 a_2 \cos\delta,$$

$$s_3 = 2a_1 a_2 \sin\delta.$$

$$s_0^2 = s_1^2 + s_2^2 + s_3^2. \tag{44}$$

The parameter s_0 is evidently proportional to the intensity of the wave. The parameters s_1, s_2, and s_3 are related in a simple way to the angle ψ ($0 \leqslant \psi < \pi$) which specifies the orientation of the ellipse and the angle χ ($-\pi/4 \leqslant \chi \leqslant \pi/4$) which characterizes the ellipticity and the sense in which the ellipse is being described. In fact the following relations hold:

$$s_1 = s_0 \cos 2\chi \cos 2\psi, \tag{45a}$$

$$s_2 = s_0 \cos 2\chi \sin 2\psi, \tag{45b}$$

$$s_3 = s_0 \sin 2\chi. \tag{45c}$$

Born and Wolf, Principles of Optics

Jones matrices

$$\mathbf{J} \equiv \mathbf{E}(t) = \begin{bmatrix} E_x(t) \\ E_y(t \end{bmatrix} = \begin{bmatrix} E_{0_x} e^{i\phi_x} \\ E_{0_y} e^{i\phi_y} \end{bmatrix}$$

- **2x2 matrix opeates on 2-vector of polarization in x and y**
- **Multiply matrices for concatenated polarizing elements**

Polarization	Jones Vector
linear horizontal	$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$
linear vertical	$\begin{bmatrix} 0 \\ 1 \end{bmatrix}$
linear $+45°$	$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$
linear -45°	$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$
circular, right-handed	$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -i \end{bmatrix}$
circular, left-handed	$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ i \end{bmatrix}$

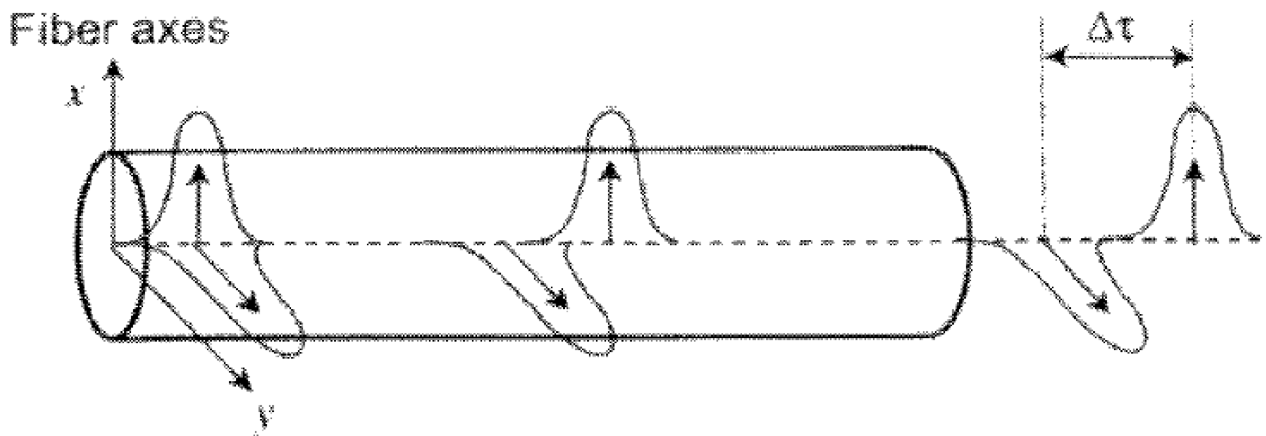
Optical Element	Jones Matrix
linear horizontal polarizer	$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$
linear vertical polarizer	$\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$
linear polarizer at $+45°$	$\frac{1}{2} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$
linear polarizer at -45°	$\frac{1}{2} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$
quarter-wave plate, fast axis vertical	$e^{i\pi/4} \begin{bmatrix} 1 & 0 \\ 0 & -i \end{bmatrix}$
quarter-wave plate, fast axis horizontal	$e^{i\pi/4} \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$
circular polarizer, right-handed	$\frac{1}{2} \begin{bmatrix} 1 & i \\ -i & 1 \end{bmatrix}$
circular polarizer, left-handed	$\frac{1}{2} \begin{bmatrix} 1 & -i \\ i & 1 \end{bmatrix}$

Applications of polarization formalism

- Applications of polarization matrices
  - Half and quarter wave retarders
  - Polarization controller
  - Faraday rotator
  - Faraday rotator mirror
- Stress optic effect causes birefringence (waveplate)
  - Can be useful, but also a perturbation
    - Makes polarization-sensitive components useless, unless…
  - Polarization-maintaining fiber (PM), with high birefringence
    - Stress birefringence is small compared with intrinsic
  - Polarizing fiber (PZ)
    - Doesn’t guide one polarization

Polarization mode dispersion

- In general, fast and slow axes exist
- Polarization drifts due to changes in stress
- Signal will shift from fast to slow axis and back, causing timing shifts
- Averages down to some value due to random “cells”



Single-mode optical fiber

- Optical waveguides made from transparent material with index step
- Boundary condition imposed by change in index yields modes, as described in RWV
  - For our purposes, step index, single-mode fiber is most relevant
- Transverse modes will be ignored, waves treated as one-dimensional
- V-number
- Numerical aperture
- Cutoff wavelength
- Core size
- Index difference

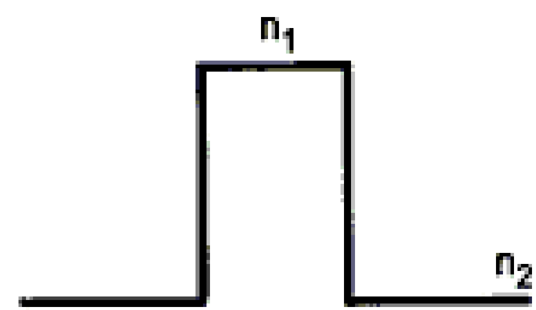
Single mode condition

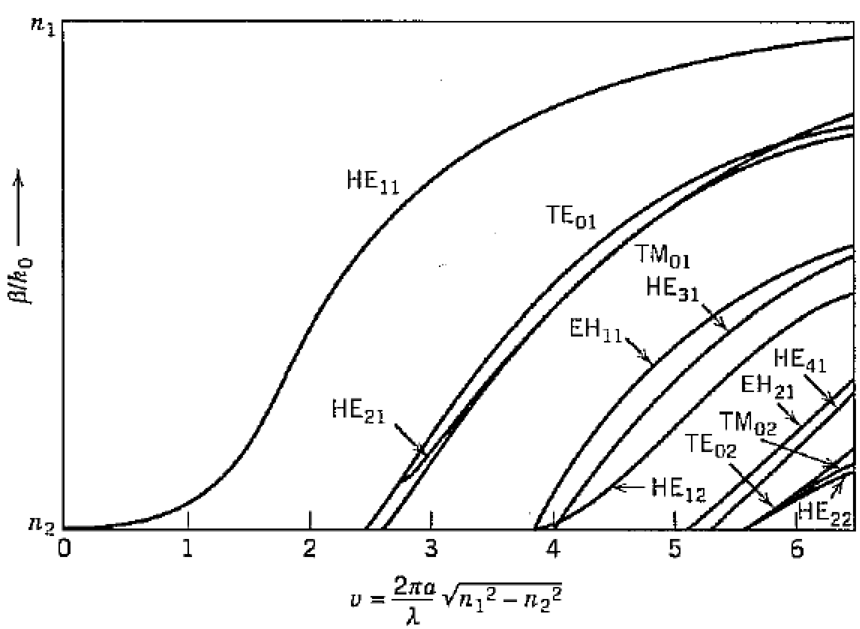
- RWV p771
- They don't spend much time on step index, unfortunately

$$v = \sqrt{u^2 + w^2} = \frac{2\pi a}{\lambda}\sqrt{n_1^2 - n_2^2}$$

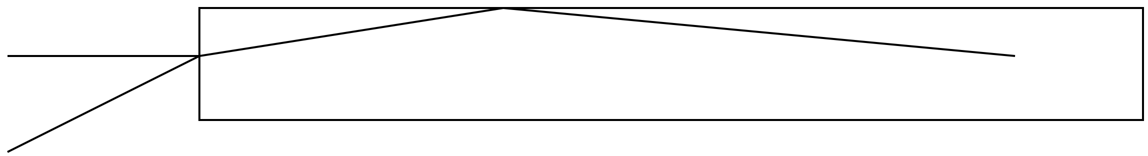
$$V = \frac{\pi d\,(NA)}{\lambda} \simeq 2.405 \text{ at cut-off}$$

$\Delta_{typical} = (n_1 - n_2)/n < 0.4\%$





Mode field and numerical aperture

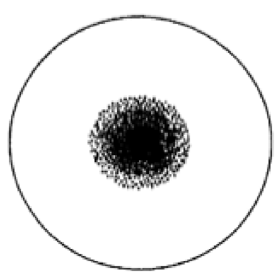


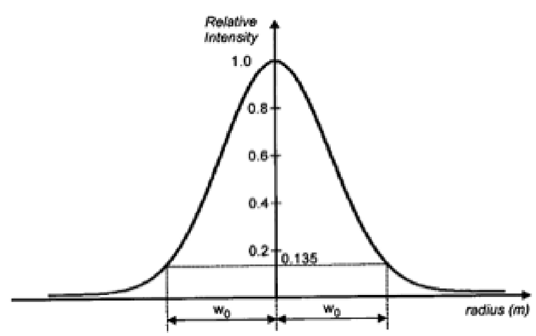
- For a multimode fiber, one can define an acceptance angle by finding the ray that is almost not guided
  - Total internal reflection quits working at too-high an angle
- This is not quite valid for single-mode fiber, as diffraction dominates, but NA is useful to know to determine index difference

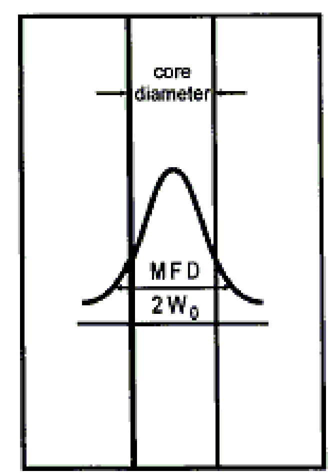
$$NA = \sqrt{[(n_1)^2 - (n_2)^2]}$$

Mode field diameter is 1/e^2 of intensity, an approximately Gaussian distribution

$$I(r) = I(0)\exp(-2r^2/w_0^2),$$







Waveguide dispersion

- Depends on variation of field distribution with wavelength
- Can be varied by changing fiber geometry
- Strong enough effect to cancel material dispersion, shift the dispersion zero
  - Dispersion shifted fiber (DSF): waveguide dispersion adjusted so that total dispersion minimized (or nearly so) at telecom wavelengths
  - Dispersion compensating fiber (DCF): dispersion over-compensated by waveguide dispersion, so that when concatenated with normal fiber, dispersion is cancelled
  - Dispersion managed fiber: alternating pieces of normal fiber and DCF, to maintain minimum overall dispersion while allowing pulse to spread periodically (reduces nonlinear effects)