Units

We will use MKS units in this course.

meter
second
kilogram

volt
ampere
ohm
electron-volt

Electrical units can be confusing.   We will break down some of the units (farad, henry, tesla) and see what's inside them.

Maxwell's Equations in MKS units.

$$\nabla \cdot D = \rho$$

$$\nabla \cdot B = 0$$

Why this asymmetry to the first equation?

$$\nabla \times E = -\dot{B}$$

$$\nabla \times H = J + \dot{D}$$

$$D = \epsilon_0 E + P$$
$$B = \mu_0 H + M$$

$$\mu_0 = 4\pi \cdot 10^{-7} \, Henries/meter$$
$$\epsilon_0 = 8.85 \cdot 10^{-12} \, Farads/meter$$

$$Force = q(E + v \times B)$$

$$Z_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} = 377\,\Omega$$

Impedance of free space. **(What does this mean?)**

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

Let's Look at electrical units, with some memory aids

Start with a simple equation that uses that quantity.

$$Q = \boldsymbol{C}\,V \qquad [farad] = \frac{coulomb}{volt} = \frac{amp\cdot sec}{volt} = \frac{sec}{ohm}$$

$$V = \boldsymbol{L}\,\dot{I} \qquad [henry] = \frac{volt\cdot sec}{amp} = ohm\cdot sec$$

$$\dot{\boldsymbol{B}}\,A = V \qquad [tesla] = \frac{volt\cdot sec}{m^2}$$

$$V = I\,\boldsymbol{R} \qquad [ohm] = \frac{volt}{amp}$$

$$\nabla\times\boldsymbol{H} = J \qquad [H] = \frac{amp}{m}$$

$$B = \boldsymbol{\mu_0}\,H \qquad [\mu_0] = \frac{henries}{meter} = \frac{B}{H} = \frac{volt\cdot sec}{amp\cdot m} = \frac{ohm\cdot sec}{m}$$

$$\mu_0\cdot\boldsymbol{\epsilon_0} = \frac{1}{c^2} \qquad [\epsilon_0] = \frac{farads}{meter} = \frac{amp\cdot sec}{volt\cdot m}$$

$$\boldsymbol{Z_0} = \sqrt{\frac{\mu_0}{\epsilon_0}} \qquad [Z_0] = \frac{volt\cdot sec}{amp\cdot m}\cdot\frac{volt\cdot m}{amp\cdot sec} = \sqrt{\frac{volt^2}{amp^2}} = ohm$$

Ohm's Law: *E = IR*, *Easy, I Remember*. But E is reserved for electrical field strength, so we use *V = IR* (*Verily, I Remember*).

Carry units along in your calculations. It can help catch errors.

A Practical Example

I came across the following equation the other day (frequency dependence of single-point multipactoring), where *f* is frequency, *B* is magnetic field and *m* is the mass of the electron.

$$\frac{f}{N} = \frac{e\,B_0}{2\pi\,m}$$

Instead of having to look up the physical constants, and knowing that the units of magnetic field *B* are volt-seconds/meter^2, I recast the equation as:

$$\frac{f}{N} = \left(\frac{e}{mc^2}\right)\frac{c^2\,B_0}{2\pi} \qquad \left[\frac{e}{e-volt}\ \frac{m^2}{sec^2}\ \frac{volt-sec}{m^2}\right]$$

The units of the (e/mc^2) are 1/volt, and of $c^2\ B_0$ are volt/second, and I know that the value of (e/mc^2) is 1/(511000 volts), so the units come out okay (sec^{-1}) and I don't have to look up the mass of the electron or its charge.

f/N = 2.8x10^{10}. for B_0 = 1 Tesla. If you use *e* and *m*, e = 1.6x10^{-19} coul, and m_e = 9.11x10^{-31} kg, and you get the same answer, but I had to look it up.

Wave Equation in Free Space

Let's determine the relationship between the electric and magnetic field vector in free space.

$$\nabla \times E = -\dot{B}$$

$$\nabla \times (\nabla \times E) = -\frac{\partial}{\partial t}(\nabla \times B) = \frac{1}{c^2}\frac{\partial E}{\partial t}$$

For a plane wave in x

Remember: $\nabla \times H = \dot{D} \rightarrow \nabla \times B = \frac{1}{c^2}\dot{E}$

$$\frac{d^2 E_x}{d z^2} = \frac{1}{c^2}\frac{d^2 E_x}{d t^2}$$

And a similar equation for *B*. These equations can be solved by

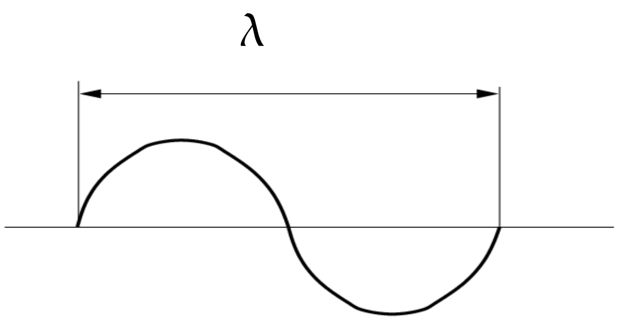
$$E_x = E_0 e^{i(kz-\omega t)} \qquad \dot{E}_x = -i\omega E_x \qquad \ddot{E}_x = -\omega^2 E_x$$

$$B_y = B_0 e^{i(kz-\omega t)} \qquad \dot{B}_y = -i\omega B_y \qquad \ddot{B}_y = -\omega^2 B_y$$

Plane Wave

Define a wavenumber *k,* increases by 2**π** for each wave of length ***λ***.

$$k = \frac{2\pi}{\lambda}, \qquad f\lambda = c, \qquad \frac{1}{\lambda} = \frac{f}{c} = \frac{\omega}{2\pi c}$$



We can rewrite the plane wave as

$$E_x = E_0 e^{i(kz-\omega t)} = E_0 e^{i(\frac{\omega}{c}z - \omega t)} = E_0 e^{2\pi i(\frac{z}{\lambda} - \frac{t}{\tau})} \qquad \tau = \frac{1}{f}$$

For a constant time t, moving along z gives one oscillation period per wavelength ***λ***.

Sitting at a particular location z, one oscillation occurs for every period t = τ.

One can ride along a particular phase at velocity c (in the lab) as time progresses.

Ratio of *E* to *H* in a plane wave

Back to Maxwell's equations $\dot{B}_y = -\nabla \times E = \frac{dE}{dz}$

$$E_x = E_0 e^{i(\frac{\omega}{c} z - \omega t)} \qquad \frac{dE_x}{dz} = i \frac{\omega}{c} E_x$$

$$B_y = B_0 e^{i(\frac{\omega}{c} z - \omega t)} \qquad \dot{B}_y = -i \omega B_y$$

$$-i \omega B_y = i \frac{\omega}{c} E_x$$

$$B = \mu_0 H$$

$$\frac{E}{H} = \mu_0 c = Z_0$$

The ratio of *E* to *H* fields in free space is Z_o, the free-space impedance.
The units are an indication:

volts/meter / amps/meter = volts/amps = ohms