Tuning the Longitudinal Field Profile

The z-dependence of the field distribution in both TM and TE cavities is affected by local variations in the cutoff frequency of the waveguide (cavity).

The fundamental frequency of a vibrating string is

$$f = \frac{\sqrt{T/\rho}}{2L}$$

where *T* is the string tension, ρ is the mass per unit length, and *L* is the length of the string.

What happens if ρ is non-uniform? The amplitude of the vibration will become non-uniform, compared to the uniform string.

The RFQ (or DTL) comprises a loaded waveguide with a locally varying cutoff frequency. The dependence on the local field amplitude on variation of local cutoff frequency is

$$\frac{d^2}{dz^2}\left(\frac{\delta E_0(z)}{E_0}\right) = \frac{8\pi^2}{\lambda^2}\frac{\delta f_0(z)}{f_0}$$

$\frac{\delta E_0(z)}{E_0}$ is the field variation, λ the free-space wavelength

and $\frac{\delta f_0(z)}{f_0}$ is the local frequency variation

The Field Profile Equation

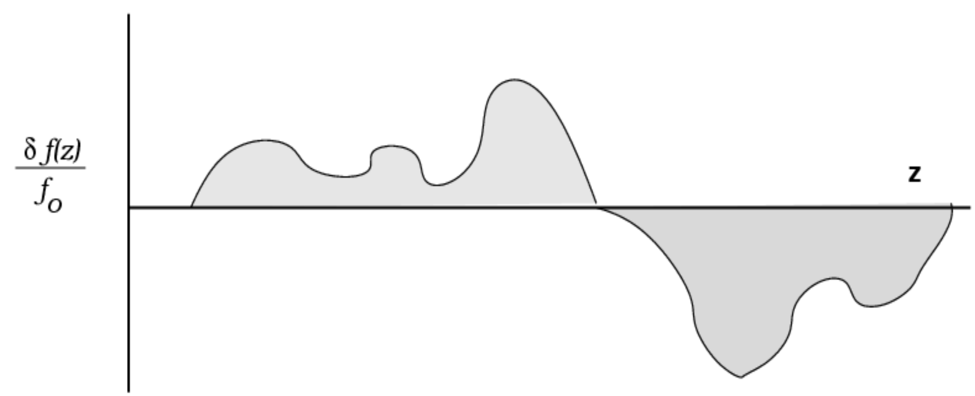
This is a second-order ordinary differential equation that can be integrated by inspection.

$$\frac{d^2}{dz^2}\left(\frac{\delta E_0(z)}{E_0}\right) = \frac{8\pi^2}{\lambda^2}\frac{\delta f(z)}{f_0}$$

The boundary condition $\frac{d}{dz}\left(\frac{\delta E_0(z=ends)}{E_0}\right) = 0$ is satisfied by requiring $\frac{\int \delta f(z)}{f_0} df = 0$

The $\frac{\int \delta f(z)}{f_0} dz = 0$

condition requires that the frequency deviation integrate to zero along the cavity. This renormalizes the frequency offset so its average is zero.



The frequency shift of the cavity due to perturbations is $\Delta f = \frac{1}{L_{cav}} \int \delta f(z) dz$

This Δf is subtracted from *df(z)* for use in the differential equation above.

Field Profile for a single frequency perturbation in the center

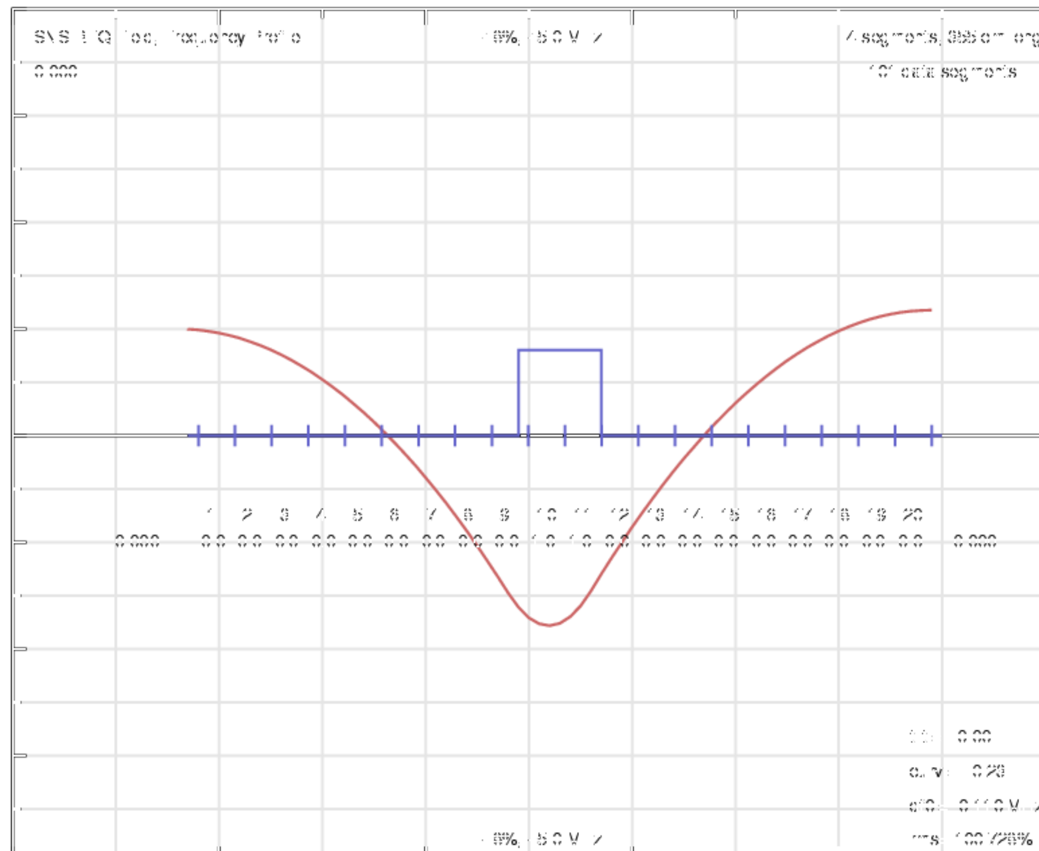
This could be the case for a single tuner in the middle of the RFQ, or a drive loop.

$$\frac{d^2}{dz^2}\left(\frac{\delta E_0(z)}{E_0}\right) = \frac{8\pi^2}{\lambda^2}\,\frac{\delta f_0(z)}{f_0}$$

Since the frequency deviation determines the curvature of the field distribution, a local positive frequency error causes a curvature upwards of the field distribution.

The boundary condition results in the first derivative of the field at the ends to be zero.

$$\frac{d}{dz}\left(\frac{\delta E_0(z=ends)}{E_0}\right) = 0$$



If the effect of the local frequency variation is to change the overall frequency by Δf, the peak-to-peak field variation is

$$\frac{\Delta E}{E_0} = \pi^2\left(\frac{L}{\lambda}\right)^2\frac{\Delta f_{cavity}}{f_0}$$

Tuner Assembly

Tuners usually operate in regions of high H-fields by removing a small volume of H-field energy.

One type is the **piston tuner**, which moves in from the sidewall, removing a volume region. Removing H-field volume **raises** the resonant frequency of the structure.

This tuner avoids moving electrical contacts by rotating a ring. When the ring is perpendicular to the direction of the H-field, a circulating current induced in the ring opposes and nulls out a volume of H-field, raising the cavity resonance. When the ring is rotated 90 degrees, it is decoupled and has negligible effect.

The vacuum seal for this tuner uses a ferrofluidic fluid in a permanent magnet.

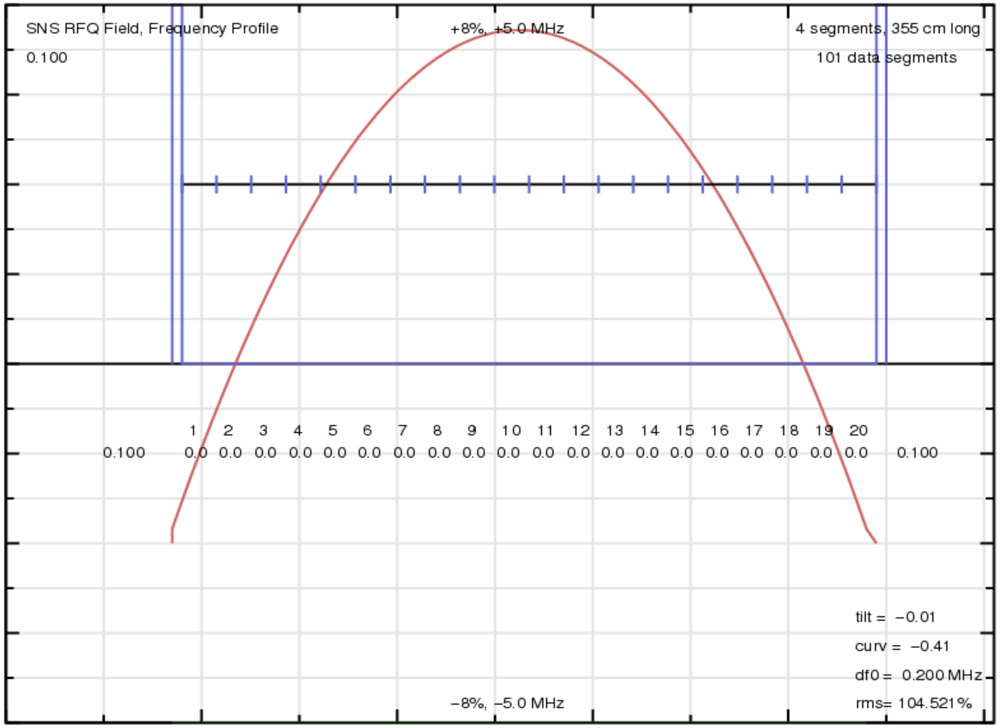
Perturbation at the Ends: End Tuners

RFQs are frequently fitted with tuners at the endwalls to adjust the field profile.

For the case where the end tuners are adjusted together to compensate a curvature in the field profile, the combined shift of the overall frequency of Δf produces a

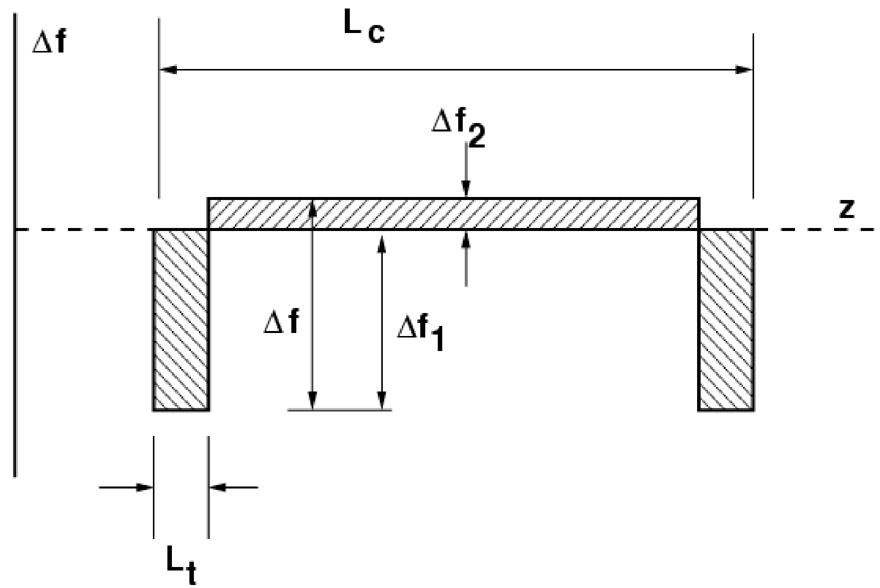
peak-to-peak variation of the field along the cavity of $\dfrac{\Delta E}{E_0} = \pi^2 \left(\dfrac{L}{\lambda}\right)^2 \dfrac{\Delta f_{cavity}}{f_0}$

Measurement of the curvature of the field indicates the difference in the cutoff frequency of the RFQ tank and the resonance of the end cut-back regions.



More Detailed Calculation, End Tuners of non-zero Length

Two tuners, each with an effective length of L_t along the cavity of length L_c, are set with a frequency offset of Δf.



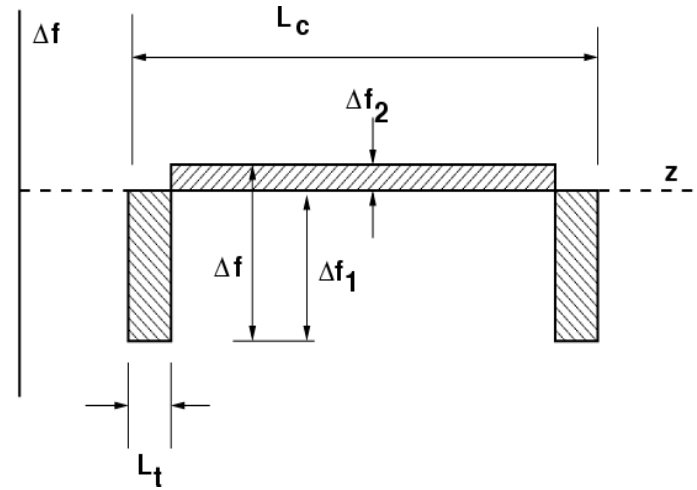
The frequency offset of the cavity, due to the tuner offset of Δf is

$$\delta f = \frac{1}{L}\int \delta f(z)\,dz = \frac{2L_t}{L_c}\Delta f$$

This is the change of overall frequency of the cavity due to the tuner offset. To satisfy the condition $\frac{\int \delta f(z)}{f_0}dz = 0$ this average value is subtracted off. To simplify the equations, the following substitutions are made:

$$\frac{d^2}{dz^2}\left(\frac{\delta E_0(z)}{E_0}\right) = \frac{8\pi^2}{\lambda^2}\frac{\delta f_0(z)}{f_0} \qquad \Delta = \frac{\delta E(z)}{E_0}, \quad k = \frac{8\pi^2}{\lambda^2}, \quad \delta(z) = \frac{\delta f}{f_0}$$

$$\frac{d^2}{dz^2}\left(\frac{\delta E_0(z)}{E_0}\right) = \frac{8\pi^2}{\lambda^2}\frac{\delta f(z)}{f_0}$$



becomes

$$\Delta'' = k\,\delta(z)$$

and

$$2L_t\,\Delta f_1 + (L_c - 2L_t)\Delta f_2 = 0, \qquad \Delta f_2 - \Delta f_1 = \Delta f, \qquad \Delta f_1 < 0, \quad \Delta f_2 > 0$$

In the center part of the cavity, the field is symmetric around the center, so we choose the origin at the center of the cavity and a parabolic solution: $-L_c/2 < z < L_c/2$

$$\Delta_c = c_2 z^2 + c_1 z + c_0, \qquad c_1 = c_0 = 0 \qquad (why?)$$

The deviation of the field in the central region of the cavity, between the end tuners is

$$\Delta_c'' = 2c_2 = k\,\Delta f_2, \qquad \Delta_c(z) = \frac{\delta E(z)}{E_0} = k\,\Delta f\,\frac{L_t}{L_c}\,z^2$$

To solve for the fields at the ends, D and its first derivative must be continuous at the interface boundary between the tuner and the rest of the cavity. The form of the field variation in the end tuners is

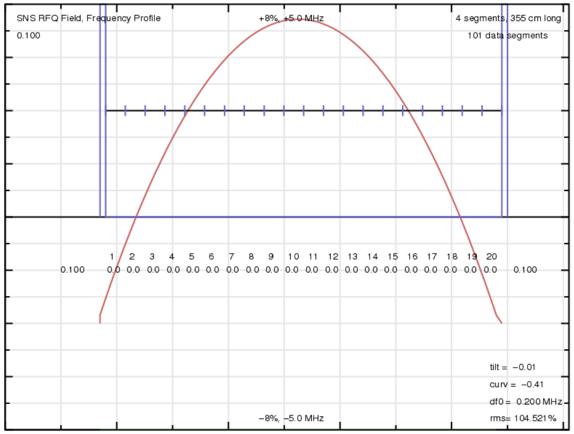
$$\Delta_e(z) \;=\; d_2 z^2 + d_1 z + d_0$$

The three conditions that are met, to solve for the three coefficients are

$$\Delta_e(L_c/2)=0 \qquad \textit{zero slope at ends}$$

$$\Delta_e(L_c/2-L_t) \;=\; \Delta_c(L_c/2-L_t)$$

$$\Delta_e{}'(L_c/2-L_t) \;=\; \Delta_c{}'(L_c/2-L_t)$$



Evaluating the coefficients, the field variation in the tuner sections are

$$\Delta_e(z) \;=\; \frac{8\pi^2}{\lambda^2}\Delta f\left(\frac{L_c}{2}-L_e\right)\left(-\frac{1}{L}z^2+z-\frac{1}{2}\left(\frac{L_c}{2}-L_e\right)\right)$$

At the center, $\qquad \Delta_c(z=0) \;=\; 0$

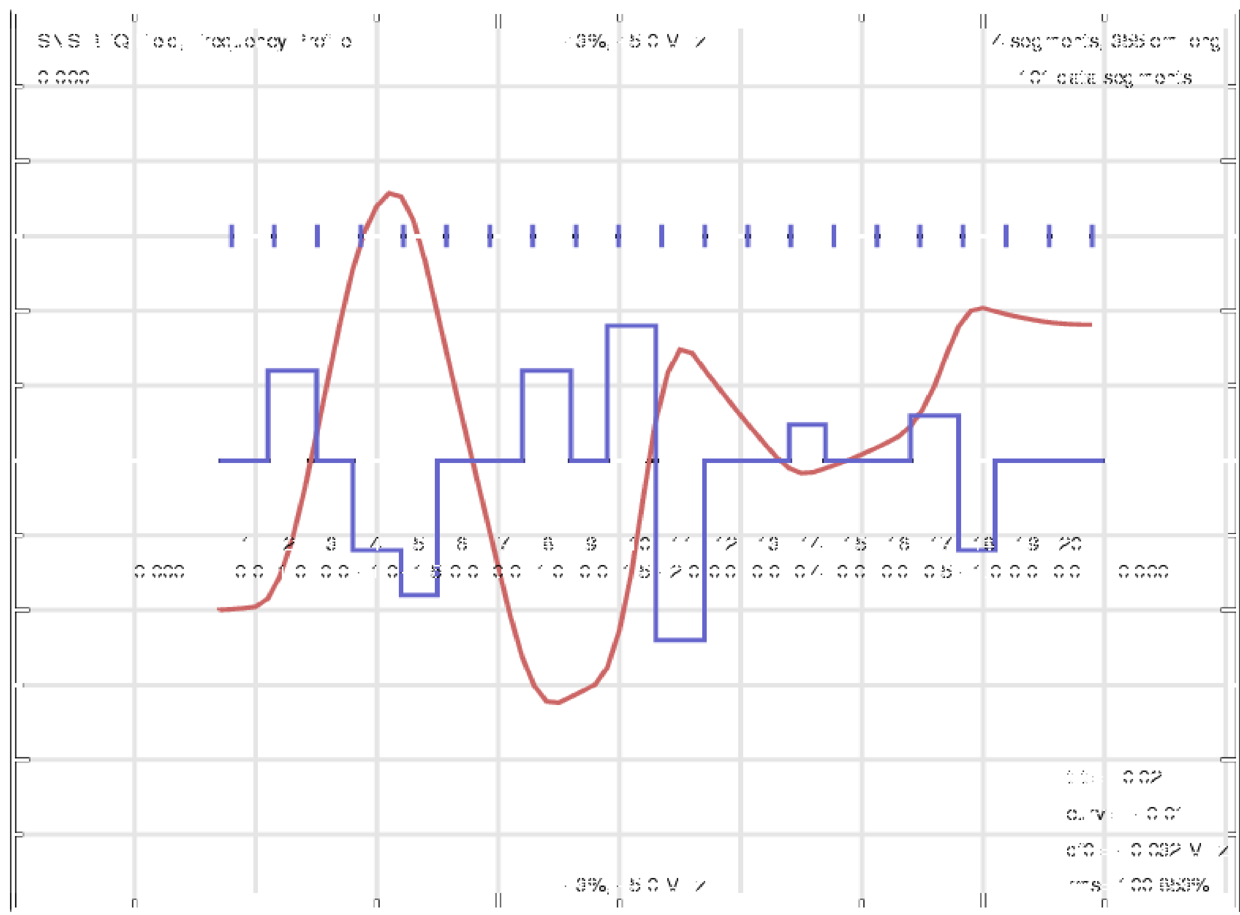
and at the ends $\qquad \Delta_c(z=L_c/2) \;=\; \dfrac{4\pi^2}{\lambda^2}\Delta f\; L_t(L_c/2-L_t)$

A More General Tuning Case

In general, a field profile determined by a bead pull will indicate a more complex variation of the field along the axis. If the cavity is supplied with tuners along the structure, the variations may be minimize by varying the local cutoff frequency with the tuners to produce the required field profile.

The program shown here, **rfqtune**, was used to tune the SNS RFQ, based on bead pull measurements, iterating the setting of temporary adjustable tuners.

The adjustable tuners were replaced by fixed tuners manufactured to the sizes determined during the tuning process.



Bead Pulling

The electric and magnetic field can be separately measured in the same location by using both a metallic bead, which removes E and H-field volume, and then retracing the path with a dielectric bead, which alters the E-field only.

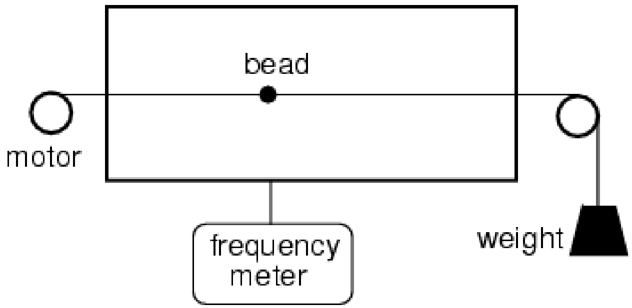
Subtracting one measurement from the other will separate the E and H fields in the path of the bead.

The constant *k* depends on the geometry of the perturber. For a sphere, *k* = 3.

$$\omega^2 = \omega_0^2 \left(1 + k \frac{\int_{bead} (\mu_0 H^2 - \epsilon_0 E^2)\, dV}{\int_{cavity} (\mu_0 H^2 + \epsilon_0 E^2)\, dV} \right)$$

For small perturbation in frequency, where τ is the volume of the bead. The frequency shift is proportional to the square of the field intensity.

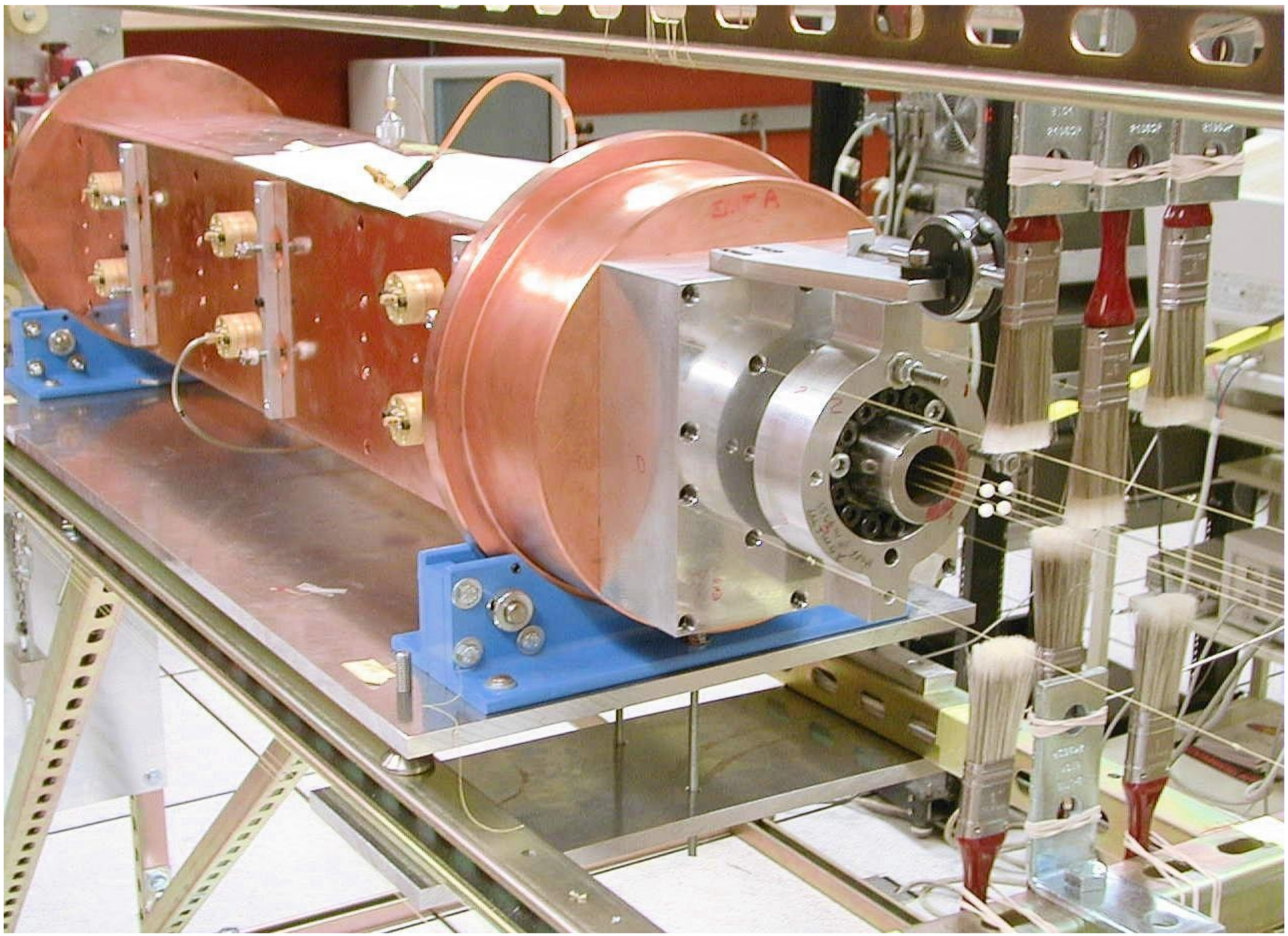
$$\frac{\Delta\omega}{\omega_0} = \frac{3\tau}{4U_{cavity}} \left(\epsilon_0 E^2 - \frac{1}{2}\mu_0 H^2 \right)$$



The beam is usually drawn through by a motor drive, and the measured frequency shift recorded on a computer.

Bead Pull Apparatus

During the development of the SNS RFQ, a cold model was constructed and the end geometry determined by the bead pulling procedure.



Here, four beads are pulled independently through the four quadrants of the RFQ near the vane tips near maximum E-field.

Also, four metallic beads are pulled near the outer wall near the location of maximum H-field.

Later, only one bead needed to be pulled, as the fields were well correlated with each other.