Impedance

Impedance is a measure of the ratio of the voltage across a circuit element to the current flowing through the circuit element. It is a generalization of **resistance R**.

$$R = \frac{V}{I}$$

This is adequate for DC circuits, but for RF, the voltage and current may not be in phase. Impedance includes the in-phase and quadrature phase (90^0) components. Impedance is expressed as a complex quantity, a sum of *R* and *X*, with *X* representing the quadrature component.

$$Z = R + jX, \qquad j = \sqrt{-1}$$

Frequently, in electrical circuit nomenclature, *j* instead of *i* is used for the imaginary part.

R is the **resistive**, or in-phase component
X is the **reactive**, or quadrature phase component

The impedance of an inductor *L* and capacitor *C* are, with the $j = \sqrt{-1}$ explicit:

$$X_L = j\omega L, \qquad X_C = \frac{1}{j\omega C}$$

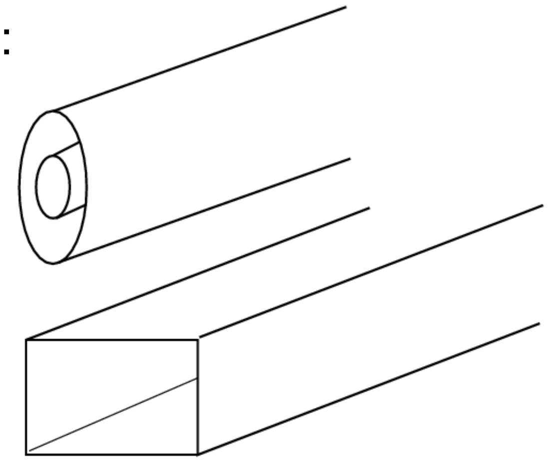
RF Transmission Lines

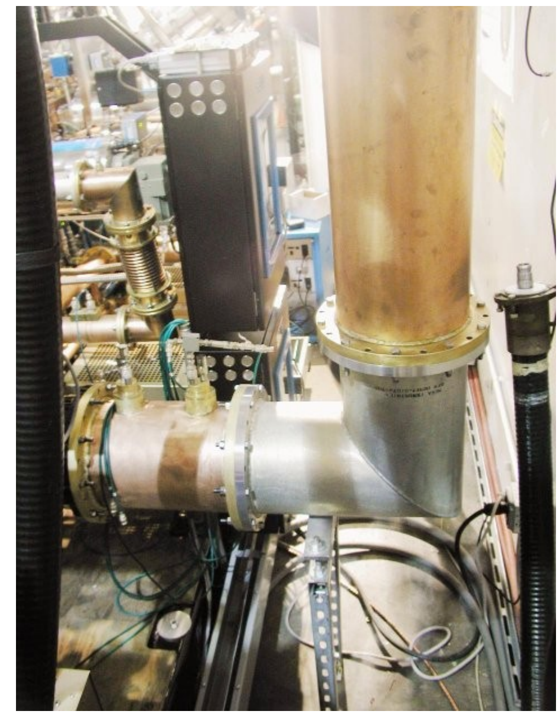
Transmission lines transmit RF power from one point to another with minimum loss and external radiation of energy.

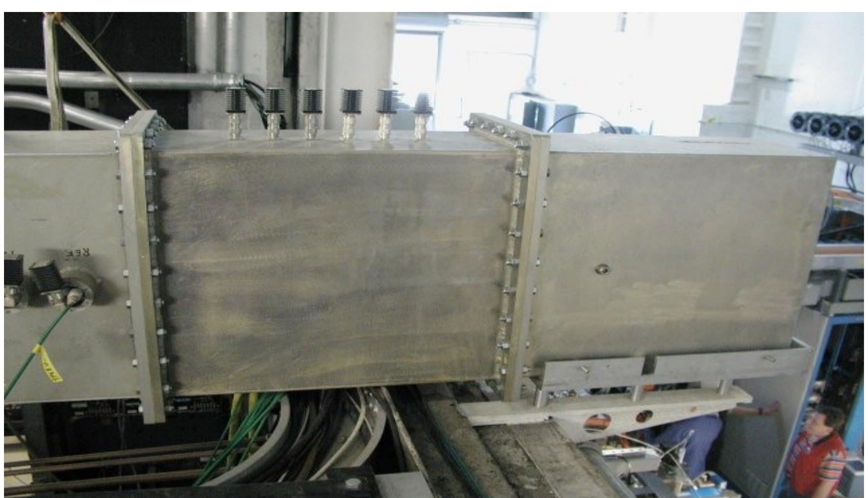
Two common types are:

Coaxial Cable

Waveguide





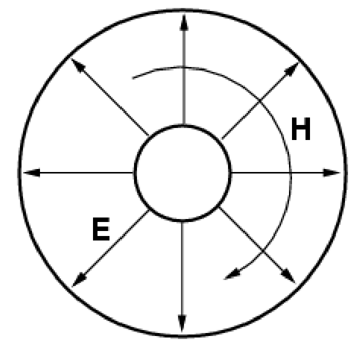


Coaxial cable (or hardline) is used for frequencies up to about 400 MHz and down to direct current, and waveguide at higher frequencies, where the loss is less than coax.

Coaxial Transmission Line

Coaxial lines carry RF in the TEM mode (usually no subscripts), where the electric and magnetic fields simultaneously have transverse components to the axis of the waveguide.

The same boundary conditions as in cavities apply:



E parallel to the surface = 0
H perpendicular to the surface = 0

At sufficiently high frequencies, the coaxial line can support other modes, which is usually undesirable.

Coaxial transmission lines have a characteristic impedance. The inner and outer conductor form a cylindrical capacitor, and the conductor also possesses an inductance. Together, they combine, resulting in a geometrically-determined impedance of

$$Z_0 = \frac{60\,ohms}{\sqrt{\epsilon}} \cdot ln\frac{r_1}{r_2}$$

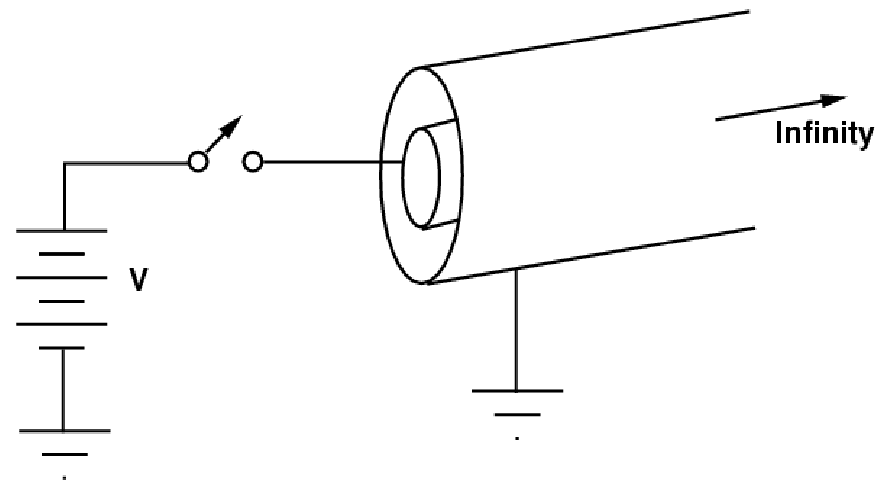
r_1 and r_2 are the outer and inner conductor radius, ϵ is the relative permeability of the dielectric between the conductors. ϵ = 1 for vacuum (and air).

The velocity of propagation is $v = \frac{c}{\sqrt{\epsilon}}$

Impedance of a Transmission Line

If the conductors of a coaxial line are lossless, how can it have an impedance?

A thought experiment:



Take an infinitely long coaxial cable and suddenly apply a voltage V to it. A current I will flow, determined by the capacitance and inductance of the cable. The ratio of the voltage V to the current I is the **characteristic (or surge) line impedance Z**.

What happens at the far end? If the line is infinitely long, it has no far end, but if the length is finite, the voltage pulse propagating along the line carrying current I will be subject to the boundary condition that no current can flow beyond the line. A wave will be reflected from the boundary unless it is terminated in a resistance equal to the line characteristic impedance.

Impedance Relationships for Coaxial Lines

The input impedance of a coaxial line depends on the impedance at the far end, as well as the characteristic impedance of the line itself. When a coaxial line transfers power to a resonant cavity, the cavity impedance that terminates the line itself varies.

For a line with no loss, the input impedance Z_i depends on the load impedance Z_{load} and the electrical length θ of the line, expressed in radians, and the line impedance Z_0.

$$Z_i = Z_0 \frac{\frac{Z_{load}}{Z_0} + j \tan\theta}{1 + j \frac{Z_{load}}{Z_0} \tan\theta}, \qquad \theta = 2\pi \sqrt{\epsilon_{relative}} \frac{L}{\lambda}$$

If $Z_{load} = Z_0$, the $Z_i = Z_0$ for any θ: the line is terminated in its characteristic impedance.

Special cases:

shorted quarter-wave line	$Z_{load} = 0$	$Z_i = \infty$,	$\theta = \pi/2$
open quarter-wave line	$Z_{load} = \infty$	$Z_i = 0$	
shorted half-wave line	$Z_{load} = 0$	$Z_i = 0$,	$\theta = \pi$
open half-wave line	$Z_{load} = \infty$	$Z_i = \infty$	