LOW AND MEDIUM $\beta$
SUPERCONDUCTING CAVITIES
AND ACCELERATORS

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Introduction

- There have been increased needs for reduced-beta ($\beta<1$) SRF cavity especially in CW machine (or high duty pulsed machine; duty $>10\%$)
  - Accelerator driven system (ADS)
    Nuclear transmutation of long-lived radioactive waste
    Energy amplifier
    Intense spallation neutron source
  - Nuclear physics
    Radioactive ion acceleration
    Muon/neutrino production
  - Defense applications
  - SRF technology $\rightarrow$ Critical path !!
Introduction

- SRF cavity for CW application or long pulse application
  - efforts for expanding their application regions down to $\beta \sim 0.1$,

- Reduced beta Elliptical multi-cell SRF cavity
  - for CW, prototyping by several R&D groups have demonstrated as low as $\beta = 0.47$
  - for pulsed, SNS $\beta = 0.61, 0.81$ cavities & ESS

- Elliptical cavity has intrinsic problem as $\beta$ goes down
  - mechanical problem, multipacting, low RF efficiency

- Spoke cavity; supposed to cover ranges $\beta = 0.1 \sim 0.5 (6)$, $f = 300 \sim 900$ MHz
  - design & prototype efforts in RIA, AAA, EURISOL, XADS, ESS, etc.
  For proton $\beta = 0.12$ corresponds $\sim 7$ MeV $\rightarrow$ all the accelerating structures (except RFQ)
## Low and Medium $\beta$ Superconducting Accelerators

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High-current cw accelerators

• Beam: p, H-, d
• Technical issues and challenges
  – Beam losses (~ 1 W/m)
  – Activation
  – High cw rf power
  – Higher order modes
  – Cryogenics losses
• Implications for SRF technology
  – Cavities with high acceptance
  – Development of high cw power couplers
  – Extraction of HOM power
  – Cavities with high shunt impedance
High-current pulsed accelerators

- Beam: p, H⁻
- Technical issues and challenges
  - Beam losses (~ 1 W/m)
  - Activation
  - Higher order modes
  - High peak rf power
  - Dynamic Lorentz detuning
- Implications for SRF technology
  - Cavities with high acceptance
  - Development of high peak power couplers
  - Extraction of HOM power
  - Development of active compensation of dynamic Lorentz detuning
Medium to low current cw accelerators

- Beam: p to U
- Technical issues and challenges
  - Microphonics, frequency control
  - Cryogenic losses
  - Wide charge to mass ratio
  - Multicharged state acceleration
  - Activation
- Implications for SRF technology
  - Cavities with low sensitivity to vibration
  - Development of microphonics compensation
  - Cavities with high shunt impedance
  - Cavities with large velocity acceptance (few cells)
  - Cavities with large beam acceptance (low frequency, small frequency transitions)
Common considerations (I)

• Intermediate velocity applications usually do not require (or cannot afford) very high gradients

• Operational and practical gradients are limited by
  – Cryogenics losses (cw applications)
  – Rf power to control microphonics (low current applications)
  – Rf power couplers (high-current applications)

• High shunt impedance is often more important

• To various degrees, beam losses and activation are a consideration
Common considerations (II)

- Superconducting accelerators in the medium velocity range are mostly used for the production of secondary species
  - Neutrons (spallation sources)
  - Exotic ions (radioactive beam facilities)
- Medium power (100s kW) to high power (~MW) primary impinging on a target
- Thermal properties and dynamics of the target are important considerations in the design of the accelerator (frequency, duration, recovery from beam trips)
- Some implications:
  - Operate cavities sufficiently far from the edge
  - Provide an ample frequency control window
Design considerations

• Low cryogenics losses
  – High $QR_s \times R_{sh}/Q$
  – Low frequency

• High gradient
  – Low $E_p/E_{acc}$
  – Low $B_p/E_{acc}$

• Large velocity acceptance
  – Small number of cells
  – Low frequency

• Frequency control
  – Low sensitivity to microphonics
  – Low energy content
  – Low Lorentz coefficient

• Large beam acceptance
  – Large aperture (transverse acceptance)
  – Low frequency (longitudinal acceptance)
A Few Obvious Statements

Low and medium $\beta$

$\beta < 1$

Particle velocity will change

The lower the velocity of the particle or cavity $\beta$

The faster the velocity of the particle will change

The narrower the velocity range of a particular cavity

The smaller the number of cavities of that $\beta$

The more important it is that the particle achieve design velocity

Be conservative at lower $\beta$

Be more aggressive at higher $\beta$
Two main types of structure geometries
TEM class (QW, HW, Spoke)
TM class (elliptical)

Design criteria for elliptical cavities

Challenges and the future of reduced beta srf cavity design
Sang-ho Kim, LINAC 2002.

Low and intermediate β cavity design
Jean Delayen, SRF 2003

High-energy ion linacs based on superconducting spoke cavities
β<1 Superconducting Structures – Circa 1989

[Graph with symbols and labels representing different superconducting structures and parameters.]
β<1 Superconducting Structures – 2002..
# Basic Structure Geometries

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- Elliptical
- Reentrant
- Alvarez
- Slotted-iris
A Word on Design Tools

TEM-class cavities are essentially 3D geometries

3D electromagnetic software is available
MAFIA, Microwave Studio, HFSS, etc.

3D software is usually very good at calculating frequencies
Not quite as good at calculating surface fields
Use caution, vary mesh size
Remember Electromagnetism 101
Design Tradeoffs

Number of cells
  Voltage gain
  Velocity acceptance

Frequency
  Size
  Voltage gain
  Rf losses
  Energy content, microphonics, rf control
  Acceptance, beam quality and losses
Energy Gain
Transit Time Factor - Velocity Acceptance

\[ \Delta W = q \int_{-\infty}^{+\infty} E(z) \cos(\omega t + \phi) \, dz \]

Assumption: constant velocity

\[ \Delta W = q \cos \phi \Delta W_0 \, T(\beta) \quad \Delta W_0 = \Theta \int_{-\infty}^{+\infty} |E(z)| \, dz \]

\[ \Theta = \frac{\max \int_{-\infty}^{+\infty} E(z) \cos \left( \frac{\omega z}{\beta c} \right) \, dz}{\int_{-\infty}^{+\infty} |E(z)| \, dz} \]

Transit Time Factor

\[ T(\beta) = \frac{\int_{-\infty}^{+\infty} E(z) \cos \left( \frac{\omega z}{\beta c} \right) \, dz}{\max \int_{-\infty}^{+\infty} E(z) \cos \left( \frac{\omega z}{\beta c} \right) \, dz} \]

Velocity Acceptance
Transit Time Factor

(a)

(b)
Velocity Acceptance for 2-Gap Structures

\[ T(\beta) = \frac{\beta}{\beta_0} \frac{\sin \left( \frac{\pi \alpha \beta}{2x_0} \right)}{\sin \left( \frac{\pi}{2x_0} \right)} \frac{\sin \left( \frac{\pi \beta_0}{2x_0} \right)}{\sin \left( \frac{\pi}{2x_0} \right)} \]

\[ x_0 = \frac{\beta_0}{2l} \]
Velocity Acceptance for 3-Gap Structures

\[ T(\beta) = \frac{\beta}{\beta_0} \sin \left( \frac{\pi \alpha}{3x_0} \right) \left[ \cos \left( \frac{\pi \alpha}{3x_0} \right) - \cos \left( \frac{\pi \beta}{x_0} \right) \right] \]

\[ x_0 = \frac{\beta_0 \lambda}{2l} \]
Higher-Order Effects

$$\Delta W = q \cos \phi \Delta W_0 \ T(\beta) + \frac{(q \Delta W_0)^2}{W} \left[ T^{(2)}(\beta) + \sin 2\phi \ T_s^{(2)}(\beta) \right]$$

$$T^{(2)}(k) = -\frac{k}{4} T(k) \frac{d}{dk} T(k) \quad k = \omega / \beta c$$

$$T_s^{(2)}(k) = -\frac{k}{4\pi} \int_0^\infty \frac{T(k + k')T(k - k') - T(k)T(k)}{k'^2} \, dk'$$
A Simple Model: Loaded Quarter-wavelength Resonant Line

If characteristic length $<< \lambda$ ($\beta<0.5$), separate the problem in two parts:
- Electrostatic model of high voltage region
- Transmission line

![Diagram of loaded quarter-wavelength resonant line]

$Z = \frac{4 \xi}{\lambda}$
Basic Electrostatics

- a: concentric spheres
- b: sphere in cylinder
- c: sphere between 2 planes
- d: coaxial cylinders
- e: cylinder between 2 planes

$V_p$: Voltage on center conductor
Outer conductor at ground
$E_p$: Peak field on center conductor
Loaded Quarter-wavelength Resonant Line

Capacitance per unit length

\[ C = \frac{2\pi \varepsilon_0}{\ln \left( \frac{b}{r_0} \right)} = \frac{2\pi \varepsilon_0}{\ln \left( \frac{1}{\rho_0} \right)} \]

Inductance per unit length

\[ L = \frac{\mu_0}{2\pi} \ln \left( \frac{b}{r_0} \right) = \frac{\mu_0}{2\pi} \ln \left( \frac{1}{\rho_0} \right) \]
Loaded Quarter-wavelength Resonant Line

Center conductor voltage

\[ V(z) = V_0 \sin \left( \frac{2\pi}{\lambda} z \right) \]

Center conductor current

\[ I(z) = I_0 \cos \left( \frac{2\pi}{\lambda} z \right) \]

Line impedance

\[ Z_0 = \frac{V_0}{I_0} = \frac{\eta}{2\pi} \ln \left( \frac{1}{\rho_0} \right), \quad \eta = \sqrt{\frac{\mu_0}{\varepsilon_0}} \approx 377 \Omega \]
Loaded Quarter-wavelength Resonant Line

**Loading capacitance**

\[
\Gamma(z) = \lambda \varepsilon \frac{\cotan \left( \frac{2\pi}{\lambda} z \right)}{\ln \left( 1 / r_0 \right)} = \lambda \varepsilon \frac{\cotan \left( \frac{\pi}{2} \zeta \right)}{\ln \left( 1 / \rho_0 \right)}
\]

\[
l = \frac{\lambda}{2\pi} \arctan \left[ \frac{\lambda \varepsilon}{\Gamma \ln \left( 1 / \rho_0 \right)} \right]
\]
Loaded Quarter-wavelength Resonant Line

Peak magnetic field

\[ \frac{V_p}{b} = \left\{ \begin{array}{ll} \eta & H \\ c & B \\ 300 & B \end{array} \right\} \rho_0 \ln \left( \frac{1}{\rho_0} \right) \sin \left( \frac{\pi}{2} \zeta \right) \]

\[ \{ m, \text{A/m} \} \quad \{ m, \text{T} \} \quad \{ \text{cm}, \text{G} \} \]

\( V_p \): Voltage across loading capacitance

\( B \approx 9 \, \text{mT} \) at 1 MV/m
Loaded Quarter-wavelength Resonant Line

Power dissipation (ignore losses in the shorting plate)

\[ P = V_p^2 \frac{\pi R_s}{8 \eta^2 b} \frac{\lambda}{\ln^2 \rho_0} \left( 1 + \frac{1}{\rho_0} \right) \zeta + \frac{1}{\pi} \sin \pi \zeta \]

\[ P \propto \frac{R_s}{\eta^2} E^2 \beta \lambda^2 \]
Energy content

\[ U = V_p^2 \frac{\pi \varepsilon_0}{8} \lambda \frac{1}{\ln(1/\rho_0)} \left( \zeta + \frac{1}{\pi} \sin\pi\zeta \right) \sin^2\frac{\pi}{2\zeta} \]

\[ U \propto \varepsilon_0 E^2 \beta^2 \lambda^3 \]
Loaded Quarter-wavelength Resonant Line

Geometrical factor

\[ G = QR_s = 2\pi \eta \frac{b}{\lambda} \frac{\ln(1/\rho_0)}{1 + 1/\rho_0} \]

\[ G \propto \eta \beta \]
Loaded Quarter-wavelength Resonant Line

Shunt impedance \((4V_p^2 / P)\)

\[ R_{sh} = \frac{\eta^2}{R_s} \frac{32}{\pi} \frac{b}{\lambda} \frac{\ln^2 \rho_0}{1 + 1/\rho_0} \frac{\sin^2 \frac{\pi}{2} \zeta}{\zeta + \frac{1}{\pi} \sin \pi \zeta} \]

\[ R_{sh} \approx \eta^2 \beta \]
Loaded Quarter-wavelength Resonant Line

\[ \frac{R}{Q} = \frac{16}{\pi^2} \eta \ln \left( \frac{1}{\rho_0} \right) \frac{\sin^2 \frac{\pi}{2} \zeta}{\zeta + \frac{1}{\pi} \sin \pi \zeta} \]

\[ \frac{R_{sh}}{Q} \propto \eta \]
Loaded Quarter-wavelength Resonant Line

\[ \rho_0 = \frac{r_0}{b} \]

- loading capacitance \( \frac{\Gamma}{\lambda \varepsilon} \)
- end voltage \( 10^{-8} \frac{V}{B_b} \)
- shunt impedance \( 10^{-6} R_{sh} R_S \frac{\lambda}{b} \)

\[ \zeta = \frac{4z}{\lambda} \]
Loaded Quarter-wavelength Resonant Line

MKS units, lines of constant normalized loading capacitance $\Gamma/\lambda \varepsilon_0$
More Complicated Center Conductor Geometries

\[ \frac{d^2v}{d\zeta^2} - \frac{1}{\rho \ln \rho} \frac{d}{d\zeta} \left( \frac{dv}{d\zeta} \right) + \frac{\pi^2}{4} v = 0 \]

\[ \frac{d^2i}{d\zeta^2} + \frac{1}{\rho \ln \rho} \frac{d}{d\zeta} \left( \frac{di}{d\zeta} \right) + \frac{\pi^2}{4} i = 0 \]

\[ \Gamma(z) = -C(z) \frac{i(z)}{di / dz} \]
More Complicated Center Conductor Geometries

Constant logarithmic derivative of line capacitance

Good model for linear taper

\[
\frac{1}{C} \frac{dC}{dz} = -\frac{1}{d} \quad r(z) = b \left( \frac{r_0}{b} \right)^{\exp(z/d)}
\]

Constant surface magnetic field

\[i(z) \propto r(z)\]

\[
\frac{d^2r}{dz^2} - \frac{1}{r \ln(b / r)} \left( \frac{dr}{dz} \right)^2 + \frac{4\pi^2}{\lambda^2} r = 0
\]
Profile of Constant Surface Magnetic Field

\[ \rho = \frac{r}{b} \]

\[ \zeta = 4 \frac{z}{\lambda} \]

- center conductor radius
- loading capacitance
- end voltage
- shunt impedance

\[ \frac{\Gamma}{\lambda \epsilon} \frac{V}{Bb} \]

\[ 10^{-6} R_{sh} R_S \frac{\lambda}{b} \]
Profile of Constant Surface Magnetic Field

MKS units, lines of constant normalized loading capacitance $\Gamma/\lambda \varepsilon_0$
Another Simple Model: Coaxial Half-wave Resonator
Coaxial Half-wave Resonator

Capacitance per unit length

\[ C = \frac{2\pi \varepsilon_0}{\ln \left( \frac{b}{a} \right)} = \frac{2\pi \varepsilon_0}{\ln \left( \frac{1}{\rho_0} \right)} \]

Inductance per unit length

\[ L = \frac{\mu_0}{2\pi} \ln \left( \frac{b}{r_0} \right) = \frac{\mu_0}{2\pi} \ln \left( \frac{1}{\rho_0} \right) \]
Coaxial Half-wave Resonator

Center conductor voltage

\[ V(z) = V_0 \cos\left(\frac{2\pi}{\lambda} z \right) \]

Center conductor current

\[ I(z) = I_0 \sin\left(\frac{2\pi}{\lambda} z \right) \]

Line impedance

\[ Z_0 = \frac{V_0}{I_0} = \frac{\eta}{2\pi} \ln\left( \frac{1}{\rho_0} \right), \quad \eta = \sqrt{\frac{\mu_0}{\varepsilon_0}} \approx 377\, \Omega \]
Coaxial Half-wave Resonator

Peak Electric Field

d: coaxial cylinders

$V_p$: Voltage on center conductor
Outer conductor at ground
$E_p$: Peak field on center conductor
Coaxial Half-wave Resonator

Peak magnetic field

\[
\frac{V_p}{b} = \left\{ \begin{array}{c}
\eta \\
c
\end{array} \right\} \frac{H}{B} \left\{ \begin{array}{c}
\rho_0 \\
300
\end{array} \right\} \ln \left( \frac{1}{\rho_0} \right) \left\{ \begin{array}{c}
m, A/m \\
m, T
\end{array} \right\} \left\{ \begin{array}{c}
\text{cm, G}
\end{array} \right\}
\]

\(V_p\): Voltage across loading capacitance

\(B \approx 9\) mT at 1 MV/m
Coaxial Half-wave Resonator

Power dissipation (ignore losses in the shorting plate)

\[ P = \frac{V_p^2 \pi R_s \lambda}{4 \eta^2 b} \frac{1}{\ln^2 \rho_0} \]

\[ P \propto \frac{R_s E^2 \beta \lambda^2}{\eta^2} \]
Coaxial Half-wave Resonator

Energy content

\[ U = V^2_p \frac{\pi \varepsilon_0}{4} \lambda \frac{1}{\ln(1 / \rho_0)} \]

\[ U \propto \varepsilon_0 E^2 \beta^2 \lambda^3 \]
Coaxial Half-wave Resonator

Geometrical factor

\[ G = QR_s = 2\pi \eta \frac{b \ln (1/\rho_0)}{\lambda} \frac{1}{1+1/\rho_0} \]

\[ G \propto \eta \beta \]
Coaxial Half-wave Resonator

Shunt impedance \( \left( 4\eta_p^2 / P \right) \)

\[
R_{sh} = \frac{\eta^2}{R_s} \frac{16}{\pi} \frac{b}{\lambda} \frac{\ln^2 \rho_0}{1 + 1/\rho_0}
\]

\[R_{sh} R_s \propto \eta^2 \beta\]
Coaxial Half-wave Resonator

\[
\frac{R_{sh}}{Q} = \frac{8}{\pi^2} \eta \ln\left(\frac{1}{\rho_0}\right)
\]

\[
\frac{R_{sh}}{Q} \propto \eta
\]
Some Real Geometries ($\lambda/4$)
Some Real Geometries ($\lambda/4$)
\( \lambda/4 \) Resonant Lines
$\lambda/2$ Resonant Lines
$\lambda/2$ Resonant Lines – Single-Spoke
$\lambda/2$ Resonant Lines – Double and Triple-Spoke
$\lambda/2$ Resonant Lines – Multi-Spoke
TM Modes
Design Considerations

• Minimize the peak surface fields
  Bp; approaches to theoretical limit (190 mT)
  $\leftarrow$ high RRR, defect control, better surface treatment (~170 mT)
  Ep; fields exceed 80 MV/m $\leftarrow$ improved surface cleaning tech.
• Reasonable Inter-cell coupling between cells in Elliptical cavity
• Spoke cavity intrinsically has big coupling constant
• Provide required external Q
• In CW, higher shunt impedance (mainly determined by the cavity type)
• Reasonable mechanical stiffness
  common; reasonable tuning force, mechanical stability under vacuum pressure (test~2 atm), stable against microphonics pulsed; affordable dynamic Lorentz force detuning
• Safe from Multipacting
• Verify HOM and related issues
• Coupled field problems are common between RF, mechanical, thermal..
  $\rightarrow$ strong interfaces are needed
Elliptical cell geometry and dependencies of RF parameters on the ellipse aspect ratio $(a/b)$ at the fixed slope angle, dome radius and bore radius.
RF Geometry Optimization (Spoke Cavity)

• There have been extensive efforts for design optimization especially to reduce the ratios of Ep/Eacc and Bp/Eacc.
  • Controlling A/B (Ep/Eacc) and C/D (Bp/Eacc) \(\rightarrow\) Shape optimization
  • Flat contacting surface at spoke base will help in another minimization of Bp/Eacc
  • For these cavities:
    Calculations agree well \(\rightarrow\) Ep/Eacc \(~3\), Bp/Eacc \(~(7\sim8)\) mT/(MV/m),
    though it is tricky to obtain precise surface field information from the 3D simulation.
    Intrinsically have very strong RF coupling in multi-gap cavity.
    Have rigid nature against static and dynamic vibrations.
    Beta dependency is quite small.
    Diameter \(~\) half of elliptical cavity.
Velocity Acceptance

- Energy gain
  \[ \Delta W = q \cdot V \cdot T(x) \cdot \Phi(x) \cdot \cos \varphi \]

  \[ x = \frac{\beta \lambda}{2l} \]

  \( T(x) \) Transit time factor for single cell
  Depends on field profile in cell

  \( \Phi(x) \) Phasing factor in multicell cavities
  Depends on cell spacing and field amplitude in cells
  Does not depend on field profile in cells (assumed to be identical)
Velocity Acceptance

Velocity Acceptance for Sinusoidal Field Profile

$\beta/\beta_g$ vs. $\beta/\beta_g$
Voltage in Cells

Voltage in \( j^{\text{th}} \) cell

\[
V^M_j = \sin\left(\pi M \frac{(2j-1)}{2N}\right)
\]

**N:** Number of cells,  **M:** Mode number
Phasing Factor

For fundamental \((\pi)\) mode: 
\[
\Phi(x) = \frac{1}{\cos\left(\frac{\pi}{2x}\right)} \begin{cases} 
(-1)^{n+1} \sin\left(\frac{N\pi}{2x}\right), & N = 2n \\
(-1)^n \cos\left(\frac{N\pi}{2x}\right), & N = 2n + 1 
\end{cases}
\]

For all modes:
\[
\Phi(x) = \frac{1}{2} \left( \frac{1}{\sin\left[\frac{\pi}{2}\left(\frac{M}{N} - \frac{1}{x}\right)\right]} + (-1)^{M+1} \frac{1}{\sin\left[\frac{\pi}{2}\left(\frac{M}{N} + \frac{1}{x}\right)\right]} \right)
\]

If \(M=N\), recover previous formula

If \(x=1\) 
\[
\Phi(x) = N \delta_{MN}
\]
Phasing Factor

6 Cells, Mode 6

$\Phi(x)$

$x = \beta \lambda / 2 I$
Phasing Factor

6 Cells, Mode 5

$\Phi(x)$

$x = \beta \lambda / 2 l$

1 2 3 4 5 6
Phasing Factor

6 Cells, Mode 4

$\Phi(x)$

$x = \beta \lambda / 2 l$

1 2 3 4 5 6
Surface Electric Field

- $\text{TM}_{010}$ elliptical structures
  - $E_p/E_a \sim 2$ for $\beta = 1$
  - Increases slowly as $\beta$ decreases

- $\lambda/2$ structures:
  - Sensitive to geometrical design
  - Electrostatic model of an “shaped geometry” gives $E_p/E_a \sim 3.3$, independent of $\beta$
Surface Electric Field

- Lines: Elliptical
- Squares: Spoke
Surface Magnetic Field

• $\text{TM}_{010}$ elliptical cavities:
  – $B/E_a \sim 4 \text{ mT}/(\text{MV/m})$ for $\beta=1$
  – Increases slowly as $\beta$ decreases

• $\lambda/2$ structures:
  – Sensitive to geometrical design
  – Transmission line model gives $B/E_a \sim 8 \text{ mT}/(\text{MV/m})$, independent of $\beta$
Surface Magnetic Field

- Lines: Elliptical
- Squares: Spoke

\[ \frac{B_p}{E_a} \text{ (mT/(MV/m))} \]

\[ \text{Beta} \]
Geometrical Factor ($QR_s$)

- **TM$_{010}$ elliptical cavities:**
  - Simple scaling: $QR_s \sim 275 \beta (\Omega)$

- $\lambda/2$ structures:
  - Transmission line model: $QR_s \sim 200 \beta (\Omega)$
Geometrical Factor \((QR_s)\)

- Lines: Elliptical
- Squares: Spoke
$R_{sh}/Q$ per Cell or Loading Element

- $R_{sh} = V^2/P$

- **TM$_{010}$ elliptical cavities:**
  - Simple-minded argument, ignoring effect of beam line aperture, gives: $R_{sh}/Q \propto \beta$
  - When cavity length becomes comparable to beam line aperture: $R_{sh}/Q \propto \beta^2$
  - $R_{sh}/Q \sim 120 \beta^2$ (Ω)

- $\lambda/2$ structures:
  - Transmission line model gives: $R_{sh}/Q \sim 205$ Ω
  - Independent of $\beta$
$R_{sh}/Q$ per Cell or Loading Element

Lines: Elliptical  Squares: Spoke

Beta

$R_{sh}/Q$ per cell (Ω)

0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
Shunt Impedance $R_{sh}$
($R_{sh}/Q QR_s$ per Cell or Loading Element)

- **TM$_{010}$ elliptical cavities:**
  - $R_{sh} R_s \sim 33000 \beta^3$ ($\Omega^2$)

- **$\lambda/2$ structures:**
  - $R_{sh} R_s \sim 40000 \beta$ ($\Omega^2$)
Shunt Impedance $R_{sh}$
($R_{sh}/Q \ QR_s$ per cell or loading element)

- Lines: Elliptical
- Squares: Spoke
Energy Content per Cell or Loading Element

Proportional to $E^2 \lambda^3$

At 1 MV/m, normalized to 500 MHz:

- **TM$_{010}$ elliptical cavities:**
  - Simple-minded model gives $U / E^2 \sim 200-250$ mJ
  - In practice: $U/E^2 \sim 200-250$ mJ
  - Independent of $\beta$ (seems to increase when $\beta < 0.5 - 0.6$)

- **$\lambda/2$ structures:**
  - Sensitive to geometrical design
  - Transmission line model gives $U/E^2 \sim 200 \beta^2$ (mJ)
Energy Content per Cell or Loading Element

$U/E^2$ per cell (mJ) @ 1 MV/m, 500 MHz
Size & Cell-to-Cell Coupling

$\text{T}M_{0,10}$ Structures
Dia $\sim 0.88 - 0.92 \lambda$
Coupling $\sim 2\%$

$\frac{\lambda}{2}$ Structures
Dia $\sim 0.46 - 0.51 \lambda$
Coupling $\sim 20 - 30\%$

Example: $350 \text{ MHz, } \beta = 0.45$
Multipacting

- **$\text{TM}_{010}$ elliptical structures**
  - Can reasonably be modeled and predicted/avoided
  - Modeling tools exist

- **$\lambda/2$ Structures**
  - Much more difficult to model
  - Reliable modeling tools do not exist
  - Multipacting “always” occurs
  - “Never” a show stopper
TM Structures – Positive Features

- Geometrically simple
- Familiar
- Large knowledge base
- Good modeling tools
- Low surface fields at high $\beta$
- Small number of degrees of freedom
\( \lambda/2 \) Structures – Positive Features

- Compact, small size
- High shunt impedance
- Robust, stable field profile (high cell-to-cell coupling)
- Mechanically stable, rigid (low Lorentz coefficient, microphonics)
- Small energy content
- Low surface fields at low \( \beta \)
- Large number of degrees of freedom
How Low Can We Go with $\beta_g$ in TM Cavities?

- Static Lorentz force detuning (LFD) at EoT($\beta_g$)=10 MV/m, 805 MHz (Magnification; 50,000)
- In CW application LFD is not an issue, but static LFD coeff. provides some indication of mechanical stability of structure

<table>
<thead>
<tr>
<th>$\beta_g$</th>
<th>RF efficiency; x</th>
<th>Will work in CW</th>
<th>Suitable for all CW &amp; pulsed applications</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.35</td>
<td>Mechanical Stability; x</td>
<td>Pessimistic in Pulsed application</td>
<td>Recent test results of SNS prototype cryomodule, $\beta_g=0.61$; quite positive; piezo compensation will work</td>
</tr>
<tr>
<td>0.48</td>
<td>Multipacting; Strong possibility</td>
<td>Would be a competing Region with spoke cavity</td>
<td></td>
</tr>
<tr>
<td>0.61</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>0.81</td>
<td></td>
<td></td>
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</tbody>
</table>

- Suitable for all CW & pulsed applications
- Recent test results of SNS prototype cryomodule, $\beta_g=0.61$; quite positive; piezo compensation will work
How High Can We Go with $\beta_g$ in Spoke Cavities?

- What are their high-order modes properties?
  - Spectrum
  - Impedances
  - Beam stability issues
- Is there a place for spoke cavities in high-high-current applications?
  - FELs, ERLs
  - Higher order modes extraction
Layout of the AEBL at ANL – 200 MeV/u, 400 kW

Color code:
Black = existing facility
Blue+ green = AEBL baseline
Red = Low-cost upgrade

Courtesy P. Ostroumov and K. Shepard
Driver linac

Layout for the AEBL driver linac

HWR 172.5 MHz
Double-Spoke 345 MHz
Triple-Spoke 345 MHz

400 kW @200 MeV/u (for $^{238}\text{U}$)

Q = 77*, 78*, 79*, 80*, 81*

β = 0.26
β = 0.39
β = 0.50, 0.62

QWR 57.5, 115 MHz
RFQ 57.5 MHz

Q = 33*, 34*

Stripper

17 MeV/u

β = 0.03, 0.06, 0.15

ECR Ion Sources
(H$^+$ to $^{238}\text{U}$)

300 keV/u 14 keV/u

Crafty P. Ostroumov and K. Shepard

Advanced Exotic Beam Laboratory

Jefferson Lab
Thomas Jefferson National Accelerator Facility
### AEBL Driver Linac - SC Resonator Configuration

- Input of uranium 33+ and 34+ at beta = 0.0254

<table>
<thead>
<tr>
<th>Beta</th>
<th>Type</th>
<th>Freq</th>
<th>Length</th>
<th>Esurf</th>
<th>Eacc</th>
<th># Cav</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.031</td>
<td>FORK</td>
<td>57.5</td>
<td>25</td>
<td>22.4</td>
<td>5.60</td>
<td>3</td>
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<tr>
<td>0.061</td>
<td>QWR</td>
<td>57.5</td>
<td>20</td>
<td>27.5</td>
<td>9.29</td>
<td>21</td>
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<tr>
<td>0.151</td>
<td>QWR</td>
<td>115.0</td>
<td>25</td>
<td>27.5</td>
<td>8.68</td>
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</table>

**STRIPPER**

<table>
<thead>
<tr>
<th>Beta</th>
<th>Type</th>
<th>Freq</th>
<th>Length</th>
<th>Esurf</th>
<th>Eacc</th>
<th># Cav</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.263</td>
<td>HWR</td>
<td>172.5</td>
<td>30</td>
<td>27.5</td>
<td>9.45</td>
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<tr>
<td>0.393</td>
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<tr>
<td>0.500</td>
<td>3SPOKE</td>
<td>345.0</td>
<td>65.2</td>
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<td>9.55</td>
<td>54</td>
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<tr>
<td>0.620</td>
<td>3SPOKE</td>
<td>345.0</td>
<td>80.9</td>
<td>27.5</td>
<td>9.26</td>
<td>24</td>
</tr>
</tbody>
</table>

**Subtotal**

**Total Cavity Count = 206**

Courtesy P. Ostroumov and K. Shepard
SC cavities covering the velocity range $0.12 < \nu < 0.8$
developed for the RIA driver linac and will be used in AEBL

115 MHz $= 0.15$
Steering-corrected QWR

172.5 MHz $= 0.28$
HWR

345 MHz $= 0.5$
Triple-spoke

345 MHz $= 0.4$
double-spoke

345 MHz $= 0.62$
Triple-spoke

See publications by K.W. Shepard, et al.

Courtesy P. Ostroumov and K. Shepard
Cavity Walk – Voltage Gain per Cavity for Uranium Beam

\[ \beta = \frac{v}{c} \]

Courtesy P. Ostroumov and K. Shepard

Thomas Jefferson National Accelerator Facility
ANL extended to TEM-class SC cavities the very high-performance techniques pioneered by TESLA

Courtesy P. Ostroumov and K. Shepard
Effects of interstitial hydrogen on triple-spoke cavity performance

![Graph showing the effect of interstitial hydrogen on cavity performance](image_url)

- **Cavity Q**
- **Eacc - MV/m**
- **at 1.9K**
- **at 4.2 K**
- **at 4.2K after 600C bake**
- **at 1.9K after 600C bake**

Courtesy P. Ostroumov and K. Shepard
Parting Words

In the last 30+ years, the development of low and medium $\beta$ superconducting cavities has been one of the richest and most imaginative area of srf

The field has been in perpetual evolution and progress

New geometries are constantly being developed

The final word has not been said

The parameter, tradeoff, and option space available to the designer is large

The design process is not, and probably will never be, reduced to a few simple rules or recipes

There will always be ample opportunities for imagination, originality, and common sense