Emittance Measurements

Accelerator Beam Diagnostics
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What is “Emittance”? 

A beam is made of many, many particles, each one of these particles is moving with a given velocity. Most of the velocity vector of a single particle is parallel to the direction of the beam as a whole (s). There is however a smaller component of the particles velocity which is perpendicular to it (x or y).

\[ \vec{v}_{\text{particle}} = v_s \hat{u}_s + v_x \hat{u}_x + v_y \hat{u}_y \]
Transverse Phase Space

• The emittance describes the beam quality, assuming linear behavior due to second order differential equation.

• It is defined as the area in phase space including the particles (generally an ellipse).

$$\mathcal{E} = \frac{1}{\pi} \iint_A dx dx'$$

• The measurements are based on beam width and angular width measurements at a single location or multiple measurements with additional optics calculations.
Variation of the ellipse along the transport line

Along a beamline the orientation and aspect ratio of beam ellipse in $x, x'$ plane varies, but area $\pi \varepsilon$ remains constant.

Beam width along $z$ is described with $w(z) = \sqrt{\beta(z)} \varepsilon$
Beam ellipse and its orientation is defined by the beam matrix \( \begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{pmatrix} \) for which the emittance is \( \varepsilon = \sqrt{\sigma_{11}\sigma_{22} - \sigma_{12}^2} \),

which is related to the Twiss or Courant-Snyder parameters:

\[
\sigma = \varepsilon \begin{pmatrix} \beta & -\alpha \\ -\alpha & \gamma \end{pmatrix}
\]

The equation of the beam ellipse is:

\[
\varepsilon = \gamma x^2 + 2\alpha xx' + \beta x'^2 \quad \gamma = \frac{1 + \alpha^2}{\beta}
\]

- \( \sqrt{\beta\varepsilon} \)  beam half width
- \( \sqrt{\gamma\varepsilon} \)  beam half divergence
- \( \alpha \) correlation between \( x \) and \( x' \)

\( \alpha > 0 \) : beam is converging
\( \alpha < 0 \) : beam is diverging
\( \alpha = 0 \) : beam has minimum or maximum
6-dimensional Phase Space

• Transverse phase space:
  – x, x’ (x-position, angle in horizontal plane)
  – y, y’ (y-position, angle in vertical plane)

• Longitudinal phase space
  – E, Φ (Energy and phase or time of arrival)
Transport of a single particle along a transfer line

\[
\begin{pmatrix}
x_2 \\
x_2'
\end{pmatrix} = M \cdot \begin{pmatrix}
x_1 \\
x_1'
\end{pmatrix} = M_C \cdot M_B \cdot M_A \cdot \begin{pmatrix}
x_1 \\
x_1'
\end{pmatrix} = \begin{pmatrix} 1 & L_C \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \frac{\cos(\sqrt{k}L_B)}{\sqrt{k}} & \frac{1}{\sqrt{k}} \sin(\sqrt{k}L_B) \\ -\sqrt{k} \sin(\sqrt{k}L_B) & \frac{\cos(\sqrt{k}L_B)}{\sqrt{k}} \end{pmatrix} \cdot \begin{pmatrix} 1 & L_A \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix}
x_1 \\
x_1'
\end{pmatrix}
\]

\[
M_{\text{Drift}} = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix}, \quad M_{\text{Quadrupole}} = \begin{pmatrix} \cos(\sqrt{k}L) & \frac{1}{\sqrt{k}} \sin(\sqrt{k}L) \\ -\sqrt{k} \sin(\sqrt{k}L) & \cos(\sqrt{k}L) \end{pmatrix}, \quad M_{\text{quadrupole}} = \begin{pmatrix} 1 & 0 \\ K & 1 \end{pmatrix}
\]

Thin lens approximation:

\[
M = \begin{pmatrix} c & s \\ c' & s' \end{pmatrix}
\]

generic names of matrix elements
Adiabatic damping

\[ \varepsilon_2 = \varepsilon_1 \frac{P_1}{P_2} \quad \text{with} \quad P = m \beta_{\text{rel}} \gamma_{\text{rel}} c \]

normalised emittance \( \varepsilon_N = \beta_{\text{rel}} \gamma_{\text{rel}} \varepsilon \) preserved with acceleration

To distinguish from normalised emittance \( \varepsilon_N \), \( \varepsilon \) is quoted as “geometric emittance”!
Why measure Emittance?

The emittance tells if a beam fits in the vacuum chamber or not

\[ w(z) = \sqrt{\beta(z) \varepsilon} < a(z) \]
Why measure Emittance?

Emittance is one of key parameters for overall performance of an accelerator

- Luminosity of colliders for particle physics
- Brightness of synchrotron radiation sources
- Wavelength range of free electron lasers
- Resolution of fixed target experiments

Therefore emittance measurement is essential to guide tune-up of accelerator!
How to measure Emittance?

In a Synchrotron the lattice functions are fixed, beam width and emittance are related:

\[
\varepsilon_x = \frac{1}{\beta_x(s)} \left[ \sigma_x^2 - \left( D(s) \frac{\Delta p}{p} \right)^2 \right] \quad \text{and} \quad \varepsilon_y = \frac{\sigma_y^2}{\beta_y(s)}
\]

The \( \beta \) function and the Dispersion function \( D \) are known or measured with other means.
A thin wire is quickly moved across the beam
Secondary particle shower is detected outside the vacuum chamber
on a scintillator/photo-multiplier assembly
Position and photo-multiplier signal are recorded simultaneously
High speed needed because of heating.

Adiabatic damping

Current increases due to speed increase

Speeds of up to 20m/s $\Rightarrow$ 200g acceleration

Measure secondary particles outside the vacuum chamber or secondary emission
The Slit and Grid method

• If we place a slit into the beam we cut out a small vertical slice of phase space

• Converting the angles into position through a drift space allows to reconstruct the angular distribution at the position defined by the slit
Transforming angular distribution to profile

• When moving through a drift space the angles don’t change (horizontal move in phase space)

• When moving through a quadrupole the position does not change but the angle does (vertical move in phase space)
Secondary Emission Grids
SEMGrid electronics

[Diagram of SEMGrid electronics with connections and components labeled]
The Slit and Grid Method

3-dim plot:

Accelerator and Beam Diagnostics
Moving slit emittance measurement

- Position resolution given by slit size and displacement
- Angle resolution depends on resolution of profile measurement device and drift distance
- High position resolution → many slit positions → slow
- Shot to shot differences result in measurement errors
Single pulse emittance measurement

Every 100 ns a new profile

Kickers

slit

SEMgrid

Quadrupole

USPAS09 at UNM Accelerator and Beam Diagnostics
### Result of single pulse emittance measurement

<table>
<thead>
<tr>
<th>File</th>
<th>Control</th>
<th>View</th>
<th>Options</th>
<th>Datum</th>
</tr>
</thead>
<tbody>
<tr>
<td>LBE.SPEMgain</td>
<td>-1.0</td>
<td>LT.BH220DUMP</td>
<td>132.8 Amp.</td>
<td>Aug 15 11:24:35 2003</td>
</tr>
<tr>
<td>LTB.BH240</td>
<td>0.1 Amp.</td>
<td>LBE.QP10</td>
<td>-6.0 Amp.</td>
<td>MOPSBC</td>
</tr>
<tr>
<td>LBE.BH240</td>
<td>10.2 Amp.</td>
<td>LBE.QD20</td>
<td>399.9 V</td>
<td>PROTON</td>
</tr>
<tr>
<td>LBE.KH210</td>
<td>5.5 V</td>
<td>LBE.KVT10</td>
<td>-199.3 mV</td>
<td>LBE</td>
</tr>
<tr>
<td>LBE.DK10</td>
<td>5.5 V</td>
<td>LBE.KVT10A</td>
<td>-3220.0 mV</td>
<td></td>
</tr>
<tr>
<td>LBE.KVT10A</td>
<td>-189.3 mV</td>
<td>LBE.SML160AP</td>
<td>2.2 mm</td>
<td></td>
</tr>
<tr>
<td>LBE.SLY10AP</td>
<td>2.0 mm</td>
<td>LBE.SLY10AP</td>
<td>2.0 mm</td>
<td></td>
</tr>
</tbody>
</table>

#### Emittance Surface

- **Angle**
  - HORIZONTAL Position mm
  - MISMATCH Linac/Booster

#### Mismatch Linac/Booster

- **Reference Ellipse**
  - E(%I) 11.5 mm.mrad
  - Xmean 0.9 mm
  - Ymean 0.6 mrad
  - Xmax 8.6 mm
  - Ymax 1.5 mrad
  - α -0.5
  - β 6.4
  - γ 0.2
  - Σ 96.8
  - Misma 51.1%

**Waiting for new acquisition...**
Single Shot Emittance Measurement

**Advantage:**
- Full scan takes 20 μs
- Shot by shot comparison possible

**Disadvantage:**
- Very costly
- Needs dedicated measurement line
- Needs a fast sampling ADC + memory for each wire

**Cheaper alternative:**
- Multi-slit measurement
Multi-slit measurement

- Needs high resolution profile detector
- Must make sure that profiles don’t overlap

Scintillator + TV + frame grabber often used as profile detector

Very old idea, was used with photographic plates
Pepperpot

Uses small holes instead of slits
Measures horizontal and vertical emittance in a single shot
Photo of a Pepperpot Device
Scintillating Screens

Method already applied in cosmic ray experiments
- Very simple
- Very convincing

Needed:
- Scintillating Material
- TV camera
- In/out mechanism

Problems:
- Radiation resistance of TV camera
- Heating of screen (absorption of beam energy)
- Evacuation of electric charges
For further evaluation the video signal is digitized, read-out and treated by program.

In new cameras digitization is done within the camera and digital image information is accessed via Ethernet or USB.
Test for resistance against heat-shock

<table>
<thead>
<tr>
<th>Material</th>
<th>$\rho$ (g/cm$^3$)</th>
<th>$c_p$ at 20°C (J/gK)</th>
<th>$k$ at 100°C (W/mK)</th>
<th>$T_{\text{max}}$ (°C)</th>
<th>$R$ at 400 °C (Ω.cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{Al}_2\text{O}_3$</td>
<td>3.9</td>
<td>0.9</td>
<td>30</td>
<td>1600</td>
<td>$10^{12}$</td>
</tr>
<tr>
<td>$\text{ZrO}_2$</td>
<td>6</td>
<td>0.4</td>
<td>2</td>
<td>1200</td>
<td>$10^3$</td>
</tr>
<tr>
<td>BN</td>
<td>2</td>
<td>1.6</td>
<td>14</td>
<td>2400</td>
<td>$10^{14}$</td>
</tr>
</tbody>
</table>

Better for electrical conductivity (>400°C)
Better for thermal properties (higher conductivity, higher heat capacity)
Degradation of screen

Degradation clearly visible
However sensitivity stays essentially the same
Properties of scintillating material

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Material</th>
<th>Activator</th>
<th>max. emission</th>
<th>decay time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quartz</td>
<td>SiO₂</td>
<td>none</td>
<td>optical</td>
<td>&lt; 10 ns</td>
</tr>
<tr>
<td></td>
<td>CsI</td>
<td>Tl</td>
<td>550 nm</td>
<td>1 μs</td>
</tr>
<tr>
<td></td>
<td>ZnS</td>
<td>Ag</td>
<td>450 nm</td>
<td>0.2 μs</td>
</tr>
<tr>
<td>Chromolux</td>
<td>Al₂O₃</td>
<td>Cr</td>
<td>700 nm</td>
<td>100 ms</td>
</tr>
<tr>
<td></td>
<td>Li glass</td>
<td>Ce</td>
<td>400 nm</td>
<td>0.1 μs</td>
</tr>
<tr>
<td>P43</td>
<td>Gd₂O₂S</td>
<td>Tb</td>
<td>545 nm</td>
<td>1 ms</td>
</tr>
<tr>
<td>P46</td>
<td>Y₃Al₅O₁₂</td>
<td>Ce</td>
<td>530 nm</td>
<td>0.3 μs</td>
</tr>
<tr>
<td>P47</td>
<td>Y₂Si₅O₅</td>
<td>Ce</td>
<td>400 nm</td>
<td>50 ns</td>
</tr>
</tbody>
</table>
Screen mechanism

- Screen with graticule
To determine $\varepsilon$, $\beta$, $\alpha$ at a reference point in a beamline one needs at least three $w$ measurements with different transfer matrices between the reference point and the $w$ measurements location.

Different transfer matrices can be achieved with different profile monitor locations, different focusing magnet settings or combinations of both.

Once $\beta$, $\alpha$ at one reference point is determined the values of $\beta$, $\alpha$ at every point in the beamline can be calculated.

Three $w$ measurements are in principle enough to determine $\varepsilon$, $\beta$, $\alpha$.

In practice better results are obtained with more measurements.

However, with more than three measurements the problem is over-determined.

$\chi^2$ formalism gives the best estimate of $\varepsilon$, $\beta$, $\alpha$ for a set of $n$ measurements $w_i$, $i=1-n$ with transfer matrix elements $c_i$, $s_i$. 
3-Profile Measurement

- Quadrupole magnet
- Transverse beam envelope
- Profile measurement (e.g., SEM grid)
- Beam path $s$

Location: $S_0$, $S_1$, $S_2$, $S_3$

Phase space:
- Coordinate $x$
- Divergence $x^1$

Beam matrix:
- Twiss parameters:
  - $\sigma_{11}(0)$, $\sigma_{12}(0)$, $\sigma_{22}(0)$
- To be determined

Measurement:
- $x^2(1) = \sigma_{11}(1)$
- $x^2(2) = \sigma_{11}(2)$
- $x^2(3) = \sigma_{11}(3)$
3-Profile Measurement

- Measure 3 profiles at 3 positions around a waist
- Spot width corresponds to vertical lines
- Transform back to the first Profile
- Lines become tangents to the beam ellipse
An example from CERN PS

![Profile Measurement screenshot](image-url)
Quadrupole Scan

• Works the same way as the 3-Profile measurement
• The profile is taken at a fixed position (needs a single profile measurement system)
• Vary a quadrupole and measure the profile width for each quadrupole setting
Quadrupole Scan at CTF-3
Optical Mismatch at Injection

- Can also have an emittance blow-up through optical mismatch
- Individual particles oscillate with conserved CS invariant:

$$a_x = \gamma x^2 + 2\alpha xx' + \beta x'^2$$
Optical mismatch at injection

- Injected beam of emittance $\varepsilon$, characterised by a different ellipse ($\alpha^*, \beta^*$) to matched ellipse ($\alpha, \beta$), generates (via filamentation) a large ellipse with original shape ($\alpha, \beta$), **but larger $\varepsilon$**.
Filamentation

\[ \begin{align*}
    \bar{X}^* & = X \\
    \bar{X} & = X
\end{align*} \]
Filamentation
Filamentation
Filamentation
Filamentation
Filamentation
Filamentation
Filamentation
Ionisation Profile Monitor

Uses rest gas in the vacuum which is ionized.
Image Intensifier

CHANNEL
(\(\phi 12 \mu m\))

0.48 mm

INPUT SIDE ELECTRODE

CHANNEL WALL

OUTPUT ELECTRODE

OUTPUT ELECTRONS

U (about 1 kV)

negativ positiv
Result from IPM
Optical Transition Radiation
Synchrotron Radiation

dipole magnet
bending radius $\rho$

e–beam

cone of synch. radiation

angle $\alpha$

lens filter

intensified CCD camera
Results from Synchrotron Radiation

\[ y = 0.165 \text{ mm} \]

\[ x = 0.797 \text{ mm} \]
Longitudinal Phase Space Transformation

• Spectrometer produces image of slit on second slit
• Second slit selects energy slice
• First kicker sweep phase space over all energies
• Buncher rotates energy slice in phase space
• At second spectrometer the phase distribution is transformed into an energy distribution analyzed by the second spectrometer
• Second kicker corrects for first kick

At slit
First drift space
Buncher
Second drift space

E

φ

Transformer

Spectrometer magnet

Kicker

SEMGrid

Buncher RF

CARE workshop on emittance measurements
U. Raich CERN
Longitudinal Emittance measurement

Diagram showing the layout of the measurement setup with various components such as Kicker, Buncher, Transformer, and Spectrometer magnet. The diagram also includes plots for energy (E) vs. phase (φ) at slit, first drift space, buncher, and second drift space.
Computed Tomography (CT)

Principle of Tomography:
- Take many 2-dimensional Images at different angles
- Reconstruct a 3-dimensional picture using mathematical techniques (Algebraic Reconstruction Technique, ART)
The reconstruction

Produce many projections of the object to be reconstructed

Back project and overlay the “projection rays”

Project the back-projected object and calculate the difference

Iteratively back-project the differences to reconstruct the original object
Some CT results
Computed Tomography and Accelerators

RF voltage

Restoring force for non-synchronous particle

Longitudinal phase space

Projection onto $\Phi$ axis corresponds to bunch profile
Reconstructed Longitudinal Phase Space
Bunch Splitting