1 Test-1

Determine the maximum value of the improvement factor that can be expected for the Q_0 of a 500 MHz copper cavity if it is cooled down from room temperature to liquid helium temperature. The ρl product of copper is $6.8\text{x}10^{-16}$ Ωm^2. The resistivity of copper at room temperature is $1.76\text{x}10^{-8}$ Ωm and the RRR of good-quality copper is 100.

SOLUTION

$$\text{RRR} = \frac{\rho_{300\text{K}}}{\rho_{4\text{K}}} \; \textit{3pt} \tag{1}$$

$$\rho_{4\text{K}} = \frac{\rho_{300\text{K}}}{\text{RRR}} = 1.76\text{x}10^{-10}\ \Omega\text{m} \; \textit{1pt} \tag{2}$$

$$l = \frac{\rho l}{\rho} = \frac{6.8\text{x}10^{-16}\ \Omega\text{m}^2}{1.76\text{x}10^{-10}\ \Omega\text{m}} = 3.86\ \mu\text{m} \; \textit{1pt} \tag{3}$$

$$\alpha_\text{s} = \frac{3}{4}\mu_0\omega\left(\frac{1}{\rho l}\right)l^3 \; \textit{4pt} \tag{4}$$

$$\alpha_\text{s} = \frac{3}{4}\left(1.2566\text{x}10^{-6}\ \frac{\text{m-kg}}{\text{s}^2\text{A}^2}\right)\left[\frac{2\pi(500\ \text{MHz})}{6.8\text{x}10^{-16}\ \Omega\text{m}^2}\right](3.86\ \mu\text{m})^3 \tag{5}$$

$$\alpha_\text{s} = 250 \; \textit{1pt} \tag{6}$$

$$R_\text{n}(l=\infty) = 3.789\text{x}10^{-5}\omega^{2/3}(\rho l)^{1/3} \; \textit{3pt} \tag{7}$$

$$R_\text{n}(l=\infty) = 3.789\text{x}10^{-5}[2\pi(500\ \text{MHz})]^{2/3}(6.8\text{x}10^{-16}\ \Omega\text{m}^2)^{1/3} \; \textit{1pt} \tag{8}$$

$$R_\text{n}(l=\infty) = 0.714\ \text{m}\Omega \; \textit{1pt} \tag{9}$$

$$R_{4\text{K}} = R_\infty\left(1 + 1.157\alpha_s^{-0.2757}\right) \; \textit{3pt} \tag{10}$$

$$R_{4\text{K}} = 0.714\ \text{m}\Omega\left[1 + 1.157(2.50\text{x}10^5)^{-0.2757}\right] = 0.894\ \text{m}\Omega \; \textit{1pt} \tag{11}$$

$$\sigma = \frac{1}{\rho} = 5.682\text{x}10^7\ \Omega^{-1}\text{m}^{-1} \; \textit{2pt} \tag{12}$$

$$R_{300\text{K}} = \sqrt{\frac{\omega\mu_0}{2\sigma}} \; \textit{4pt} \tag{13}$$

$$R_{300\mathrm{K}} = \sqrt{\frac{2\pi(500\ \mathrm{MHz})\left(1.2566\mathrm{x}10^{-6}\ \frac{\mathrm{m\text{-}kg}}{\mathrm{s}^2\mathrm{A}^2}\right)}{2(5.682\mathrm{x}10^7\ \Omega^{-1}\mathrm{m}^{-1})}} = 5.89\ \mathrm{m}\Omega \ \textit{4pt} \tag{14}$$

$$\frac{Q_{4\mathrm{K}}}{Q_{300\mathrm{K}}} = \frac{R_{300\mathrm{K}}}{R_{4K}} = \frac{5.89\ \mathrm{m}\Omega}{0.741\ \mathrm{m}\Omega} = 6.59 \ \textit{3pt} \tag{15}$$

2 Test-2

Derive an approximate expression for the input impedance Z_{in} of a series RLC circuit in terms of the resistance R, Q, unloaded resonant angular frequency ω_0, and the difference $\Delta\omega$ of the driven angular frequency ω and ω_0 that is valid to first order in $\Delta\omega/\omega$.
What is G/Q in terms of L and C?

SOLUTION

$$\begin{aligned}
G &= \frac{1}{R} \\
U_{\mathrm{e}} &= \frac{I^2}{4\omega^2 C} \\
U_{\mathrm{m}} &= \frac{I^2 L}{4} \\
U &= \frac{LI^2}{2} \\
P_{\mathrm{d}} &= \frac{RI^2}{2} \\
Q &= \frac{\omega_0 U}{P_{\mathrm{d}}} = \frac{\omega_0 L}{R} \ \textit{2pt} \\
\frac{G}{Q} &= \frac{1}{RQ} = \sqrt{\frac{C}{L}} \ \textit{2pt} \\
\omega_0 &= \frac{1}{\sqrt{LC}} \ \textit{2pt} \\
\omega &= \omega_0 + \Delta\omega
\end{aligned}$$

$$
\begin{aligned}
Z_{\text{in}} &= R + \mathrm{j}\,\omega L - \mathrm{j}\left(\frac{1}{\omega C}\right) \text{ \textit{3pt}} \\
&= R + \mathrm{j}\,\omega L\left(1 - \frac{1}{\omega^2 LC}\right) \\
&= R + \mathrm{j}\,\omega L\left(1 - \frac{\omega_0^2}{\omega^2}\right) \text{ \textit{1pt}} \\
&= R + \mathrm{j}\,\omega L\frac{(\omega+\omega_0)(\omega-\omega_0)}{\omega^2} \\
&= R + \mathrm{j}\,L(\omega_0+\Delta\omega)\frac{(2\omega_0+\Delta\omega)\Delta\omega}{(\omega_0+\Delta\omega)^2} \text{ \textit{2pt}} \\
&\approx R + \mathrm{j}\,\omega_0 L\left(\frac{2\omega_0\Delta\omega}{\omega_0^2}\right)\left(1 - 2\frac{\Delta\omega}{\omega_0}\right) \text{ \textit{2pt}} \\
&\approx R\left[1 + \left(\frac{\mathrm{j}\,\omega_0 L}{R}\right)\left(\frac{2\Delta\omega}{\omega_0}\right)\right] \text{ \textit{2pt}} \\
&\approx R\left(1 + \frac{2\,\mathrm{j}\,Q\Delta\omega}{\omega_0}\right) \text{ \textit{2pt}}
\end{aligned}
$$

3 Test-3

A superconducting cavity has a surface resistance of 10 nΩ and operates CW in a mode with $R/Q = 50\Omega$ at a frequency of 1.3 GHz. The geometry constant is 250Ω.
What are the values of L, R, and C for an equivalent parallel lumped-element circuit model for this cavity mode?

SOLUTION

$$
\begin{aligned}
L &= \frac{1}{\omega_0}\left(\frac{R}{Q}\right) \text{ \textit{3pt}} \\
L &= \frac{50\Omega}{2\pi(1.3\text{ GHz})} = 6.12\text{ nH} \text{ \textit{1pt}} \\
C &= \frac{1}{\omega_0\left(\frac{R}{Q}\right)} = \frac{1}{2\pi(1.3\text{ GHz})(50\ \Omega)} = 2.45\text{ pF} \text{ \textit{4pt}} \\
R &= \left(\frac{R}{Q}\right)Q = \left(\frac{R}{Q}\right)\frac{G}{R_{\text{s}}} = (50\ \Omega)\frac{250\ \Omega}{10\text{ n}\Omega} = 1.25\text{x}10^{12}\ \Omega \text{ \textit{4pt}}
\end{aligned}
$$

4 Test-4

What Q_e is required for reflectionless operation of a cavity operating in a mode with $V_\mathrm{c} = 1$ MV, $R/Q = 50\ \Omega$, $Q_0 = 10^{10}$, and a beam current of 200 mA? What is the value of Q_b, the Q value associated with the beam power? What are these values if $Q_0 = 10^4$ instead?

SOLUTION

$$
\begin{aligned}
P_\mathrm{b} &= \frac{V_\mathrm{c}^2}{2R_\mathrm{b}} \quad \textit{4pt} \\
R_\mathrm{b} &= \frac{V_\mathrm{c}^2}{2P_\mathrm{b}} = \frac{1\ \mathrm{MV}}{2(200\ \mathrm{kW})} = 2.5\mathrm{x}10^6\ \Omega \quad \textit{2pt} \\
Q_\mathrm{b} &= \frac{R_\mathrm{b}}{\frac{R}{Q}} = \frac{2.5\mathrm{x}10^6\ \Omega}{50\ \Omega} = 5\mathrm{x}10^4 \quad \textit{4pt}
\end{aligned}
$$

For reflectionless operation

$$
\begin{aligned}
\frac{1}{Q_\mathrm{e}} &= \frac{1}{Q_0} + \frac{1}{Q_\mathrm{b}} \quad \textit{4pt} \\
Q_\mathrm{e} &= \frac{Q_0 Q_\mathrm{b}}{Q_0 + Q_\mathrm{b}} = \frac{10^{10}(5\mathrm{x}10^4)}{10^{10} + 5\mathrm{x}10^4} = 5\mathrm{x}10^4. \quad \textit{2pt}
\end{aligned}
$$

If $Q_0 = 10^4$ then Q_b is the same and

$$
Q_\mathrm{e} = \frac{10^4(5\mathrm{x}10^4)}{10^4 + 5\mathrm{x}10^4} = 8.33\mathrm{x}10^3. \quad \textit{3pt}
$$