Unit 10 – Lectures 14

Cyclotron Basics

MIT 8.277/6.808 Intro to Particle Accelerators

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Outline

- Introduce an important class of circular particle accelerators: Cyclotrons and Synchrocyclotrons

- Identify the key characteristics and performance of each type of cyclotron and discuss their primary applications

- Discuss the current status of an advance in both the science and engineering of these accelerators, including operation at high magnetic field

Overall aim: reach a point where it will be possible for to work a practical exercise in which you will determine the properties of a prototype high field cyclotron design (next lecture)
Motion in a magnetic field

Shoot a charged particle into a perpendicular magnet field—what happens?

It moves in a closed circle—why?
Magnetic forces are perpendicular to the B field and the motion

\[ \vec{F} = q \vec{v} \times \vec{B} \]

It sees a sideways force - Lorentz Force

\[ F = q \vec{v} B \]
Sideways force must also be *Centripedal*

If you did not no (1) the particle had charge (2) or that there was a magnetic field

\[ F = \frac{m v^2}{r} \]

You could still infer a sideways (central force)
Governing Relation in Cyclotrons

- A charge $q$, in a uniform magnetic field $B$ at radius $r$, and having tangential velocity $v$, sees a centripetal force at right angles to the direction of motion:

$$\frac{mv^2}{r} \hat{r} = q\vec{v} \times \vec{B}$$

- The angular frequency of rotation seems to be independent of velocity:

$$\omega = \frac{qB}{m}$$
Building an accelerator using cyclotron resonance condition

- A flat pole H-magnet electromagnet is sufficient to generate require magnetic field
- Synchronized electric fields can be used to raise the ion energies as ions rotate in the magnetic field
- Higher energy ions naturally move out in radius
- Highest possible closed ion orbit in the magnet sets the highest possible ion energy
There is a difficulty—we can’t ignore relativity

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$$\frac{mv^2}{r} \hat{r} = q\vec{v} \times \vec{B}$$

- Picking an axial magnetic field $B$ and azimuthal velocity $v$ allows us to solve this relation:

$$\frac{mv^2}{r} = qvB \quad \rightarrow \quad \omega = v/r = qB/m$$

- However:

$$m = \gamma m_0$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$
Relativistic Limit on Cyclotron Acceleration

- The mass in $\omega = qB/m$ is the relativistic mass $m = \gamma m_0$
- $\omega \approx \text{constant}$ only for very low energy cyclotrons

<table>
<thead>
<tr>
<th>Proton Energy</th>
<th>% Frequency decrease</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 MeV</td>
<td>~1%</td>
</tr>
<tr>
<td>250 MeV</td>
<td>~21%</td>
</tr>
<tr>
<td>1.0 GeV</td>
<td>~52%</td>
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</table>
There are 3 kinds of Cyclotrons:

- **CLASSICAL**: (original)
  - Operate at fixed frequency \( \omega = qB/m \) and ignore the mass increase
  - Works to about 25 MeV for protons \( (\gamma \approx 1.03) \)
  - Uses slowly decreasing magnetic field ‘weak focusing’

- **SYNCHROCYCLOTRON**: let the RF frequency \( \omega \) decreases as the energy increases
  - \( \omega = \omega_0 / \gamma \) to match the increase in mass \( (m = \gamma m_0) \)
  - Uses same decreasing field with radius as classical cyclotron

- **ISOCRONOUS**: raise the magnetic field with radius such that the relativistic mass increase is just cancelled
  - Pick \( B = \gamma B_0 \) \{this also means that \( B \) increases with radius\}
  - Then \( \omega = qB/m = qB_0/m_0 \) is constant.
  - Field increases with radius– magnet structure must be different
Some Examples of Cyclotrons
1932 Cyclotron

Evacuated Beam
Chamber sits between magnet poles:

180° ‘Dee’

Vacuum Port

Internal Energy Analyzer

Ion Source is a gas feed and a wire spark gap
The Largest...

- Gatchina Synchrocyclotron at Petersburg Nuclear Physics...
  1000 MeV protons and 10,000 tons
Superconducting Isochronous Cyclotron

LHe-Supply Vessel w/4 Cryocoolers

Compressors (6x) for cryocoolers

Shield Cooler (2x)

Superconducting Coil

250 MeV Superconducting Proton Cyclotron

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The Highest Magnetic Field...

- Still River Systems 9 Tesla, 250 MeV, synchrocyclotron for Clinical Proton Beam Radiotherapy
The Newest...

- Nanotron: superconducting, cold iron, cryogen free 'portable' deuterium cyclotron
New Cyclotrons and Synchrocyclotrons are coming.

**Isotron** – for short lived PET isotope production:
- Protons or heavy ions
- 30-100 MeV
- Synchrocyclotron or isochronous cyclotron is possible

Also:
- **Gigatron**: 1 GeV, 10 mA protons for airborne active interrogation
- **Megatron**: 600 MeV muon cyclotron (requires a gigatron to produce muons and a reverse cyclotron muon cooler for capture for accel.)
Key Characteristics of the Cyclotron ‘Class’

Cyclotron utility is due to:

- Ion capture and Beam formation at low velocity, followed by acceleration to relativistic speeds in a single device
- Efficient use of low acceleration voltage makes them robust and uncritical; pulsed or CW operation allowed
- Beam characteristics are wrapped up in the design of the static magnetic guide field; ions have high orbital stability
- Ion species: H+ --> U; neg. ions (e.g. H−), molecular ions (e.g. HeH+)
- Intensities; picoamps (one ion per rf bucket) to milliamps
- γ: 0.01 --> 2.3

Have resulted in:

- 2nd largest application base historically and currently (electron linacs used in radiotherapy are 1st)
- Science (Nuclear, Atomic, Plasma, Archeology, Atmospheric, Space), Medicine, Industry, Security
- Highest energy CW accelerator in the world: K1200 heavy ion at MSU- 19.04 GeV ²³⁸U
Key Characteristics—prob. most important:

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Classical Cyclotrons

- Weak focusing
- Phase stability
- Limited by Relativistic Mass Increase
How to manage the relativistic change in mass?

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  - Field increases with radius– magnet structure must be different
The 1931 Cyclotron...
A flat pole electromagnet (3) generates a vertical magnetic field (m)

- Ions (P) rotate in the mid-plane of an evacuated split hollow conductor (1-2)
- Time varying electric fields (4) applied to the outside of this conductor raise the ion energies as ions rotate in the magnetic field and cross the split line gap- the only place where electric fields (e) appear
- Higher energy ions naturally move out in radius
- Highest allowed closed ion orbit in magnet sets the highest possible ion energy
Let’s break down the key phenomena that make cyclotrons work...

- We’ll do this in a very ‘raw’ manner- using elementary properties of ions, conductors and electromagnetic fields

- Why choose this approach?
  - To demonstrate just how utterly simple cyclotrons are
  - To get to better appreciate the key challenges in making cyclotrons work
  - To understand how the advance machines just shown are possible
Magnetic Field Generation

- A flat pole electromagnet (3) generates a vertical magnetic field (m)
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Typical large H Magnet
Magnetic field of a H Magnet

Full H-Shaped Magnet including all four quadrants
Ion Acceleration-- requires a bit more work...

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Acceleration really looks something like this...

\[ \vec{F} = q \vec{E} \quad \& \quad \Delta W = q \Delta V = q V_0 \]
Why not magnetic field only acceleration?

Why not just use a magnetic field for both acceleration and bending?

\[ \text{d}w = F \cdot ds = F \cdot \dot{r} \cdot dt \]

\[ ds = v \cdot dt \]

\[ \text{d}w = F \cdot \dot{r} \cdot dt = g(\dot{r} \times \dot{r}) \cdot \dot{r} \cdot dr 
= g \cdot \dot{r} \cdot (\dot{r} \times \dot{r}) \cdot dt 
= 0 \]

Static magnetic fields can do no work!
Ion Orbital Rotation Frequency– numerically

- Consider an arbitrary positive ion of atomic species \((A,Z)\) with \(Q\) orbital electrons removed. The ion cyclotron frequency would be:

\[
f = \frac{\omega}{2\pi} = \frac{qB}{2\pi m} = \left(\frac{Q}{A}\right) \frac{e}{2\pi m_0} \frac{B}{\gamma}
\]

- Where \(m_0\) is the rest mass of a nucleon (~940 MeV). Evaluating the constants:

\[
f = \left(\frac{Q}{A}\right) 15.23 \text{ MHz} \frac{B}{\gamma}
\]

- Some examples:
  - Low energy proton in 1 T field: 15.23 MHz
  - 250 MeV proton in 8.2T field: 98 MHz
  - 3.2GeV \(^{40}\text{Ar}^{16+}\) ion in 5.5T field: 30.8 MHz
Ion Motion in a cyclotron

- A flat pole electromagnet (3) generates a vertical magnetic field (m).
- Ions (P) rotate in the mid-plane of an evacuated split hollow conductor (1-2).
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- Highest allowed closed ion orbit in magnet sets the highest possible ion energy.
Alternative Expression in Momentum

- Again we equate the two expressions for the same force:

\[ \frac{mv^2}{r} = qvB \rightarrow p = mv = qBr \]

- The momentum at any radius is completely defined by the magnetic field there!

- Also, at the same field B,
  - If \( p_3 > p_2 > p_1 \)
  - Then \( r_3 > r_2 > r_1 \)

- Since \( \omega = \frac{d\theta}{dt} = qB/m \), even though the three orbits are different in size, the ions will make 1 complete revolution at the same angular rate (unless \( m = \gamma m_0 \) is very different for the three momenta)
Special Challenges in Cyclotrons

- **Orbit Stability**
- **Initial beam Formation**
- **RF Acceleration**
- **Getting the beam out of the machine!**
  - \[ p = e r B \quad \rightarrow \quad \frac{p}{e} = r B \]
  - We call \( R = r B \) the magnetic rigidity or magnetic stiffness
  - We will see that \( R \) shows up in the Cyclotron final energy formula— it’s in \( K_B = \frac{e^2 r^2 B^2}{2m_0} \)

*In cyclotrons, the final energy is essentially set by the radius and B field at the point of beam extraction*
Built In Orbit Stability - Weak Focusing

What happens if an ion is above or below the middle of the magnet during acceleration?

\[ F_x = -q v B_x \]

\[ \vec{F} = q \vec{v} \times \vec{B} \]

\[ \vec{B} = B_0 \vec{r} \]

This is called weak focusing - it requires this field curvature. It means that $B_0$ decreases with increasing $r$. 
The Field Index and Axial Stability

- An restoring force is required to keep ions axially centered in the gap
- We define the field index as:
  \[ n = -\frac{r}{B} \frac{dB}{dr} \]
  
  - One can show that an axial restoring force exists when \( n > 0 \) (off median plane \( B_r \) has right sign)
  - Hence \( dB/dr < 0 \) is required since \( B \) and \( r \) enter in ratios
  - This condition can be met with a flat pole H-Magnet
Field Index $n$ shows up in Equations of Motions

- Small oscillations of ions in $r$ and $z$ about equilibrium orbits:
  \[
  \ddot{x} + (1 - n)\omega^2 x = 0 \\
  \ddot{z} + n\omega^2 z = 0
  \]

- Have solutions:
  \[
  x = x_m \sin(1 - n)^{1/2} \omega t \\
  z = z_m \sin n^{1/2} \omega t
  \]

- Where $\omega$ is the cyclotron frequency

- Betatron Frequencies (Tunes):
  \[
  v_r = \omega_r / \omega = \sqrt{1 - n} \\
  v_z = \omega_z / \omega = \sqrt{n}
  \]

- Have real sinusoidal solutions for $0 < n < 1$; this condition is true in a classical cyclotron

- It’s also referred to as a weak focusing accelerator
Initial Beam Challenge

Electric Focusing & De Focusing - low velocity ions spend much time in time varying electric fields.

High Frequency Oscillator

Magnetic Field

Electric Field penetrates into cavities in the conductors

High Speed Ions

\[ r = \frac{p}{eB} \text{ so } r \propto B \]

1st Gap Problem

Ion starts from zero velocity - can it gain enough energy in first gap before voltage changes sign?

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For Example: Initial Proton trajectories at 9T
Positive Ion Source must be compact

- Straight-forward field scaling of original 5.5 T ion source of K500 cyclotron
- Chimney diameter 3 mm
- Test ion source has extra support across median plane
- Allows separated cathode geometry of Antaya thesis or Harper cyclotron
- Pulsed cathode lifetime expected to be months
RF Acceleration Challenge

What is the minimum average gap and how much voltage will such a vacuum gap hold?

magnetic fields concentrate spark energy

\[ r = \frac{p}{Be} \]

Voltage change requires

surface current & heating

\[ i \propto C V_0 \omega_{RF} \]

Stored Charge dumps into spark

\[ E = \frac{1}{2} C V_0^2 \text{ & } C \sim \frac{A_{exc}}{\text{gap}} \]
Beam Extraction Challenge

nuR
nuZ^2
rigidity

nur=2nuz @ r=0.298 m
Orbit Separation impacts Extraction

- **Turn Number**
  - Let $E_1$ be the energy gain per revolution
  - Then the total number of revolutions required to reach a final kinetic energy $T$:
    - Let the average ion phase when crossing the acceleration gap phase be $\phi$; $V_0$ is the peak voltage on the dee
    - Energy gain per gap crossing: $T_1 = V_0 \sin \phi$
    - Gaps per revolution: $n$
    - Turn number: $N = T / nT_1 = T / (nV_0 \sin \phi)$
    - 250 MeV protons; 17 KeV/turn: $N \approx 15,000$

- **Turn Spacing**:
  - $dr/dN \approx r(T_1 / T)$
  - 250 MeV protons $r=0.3$ m: $dr/dN \approx 20$ microns!
Beam Extraction: 5 micron orbit turn spacing to 1 cm in 20 orbit revolutions induced by field perturbation.
Phase Stable Acceleration aka Phase Stability

- **3 General Requirements:**
  - The required instantaneous acceleration voltage is less than the maximum available voltage.
  - A change in ion momentum results in a change in ion orbit rotation period.
  - The rate of change of the frequency is less than a limiting critical value.

- **Second Condition is the most easily accessible:**

\[
\frac{d\tau}{\tau} = \left( \frac{1}{\alpha} - \frac{1}{\gamma^2} \right) \frac{dp}{p}
\]
Acceleration in a 9T Guide Field
Cyclotrons— Final Energy Scaling with Field and Radius

(The origin of Superconducting Cyclotrons and Synchrocyclotrons)
Cyclotron Energy Scales inversely with Field

- The final energy can be written as a power series expansion in the relativistic factor $\gamma$,

- The first term in this expansion is: $T_{\text{final}} \approx K_B Q^2 / A$, for an ion of charge $Qe$ and ion mass $A m_0$.

- $K_B$ represents the equivalent proton final energy for the machine, and is related to the ion momentum a.k.a. the particle rigidity ($B\rho$):

$$K_B = (eB\rho)^2 / 2m_0$$
Almost (but not quite) spherical: Efficient cyclotron magnetic circuits include more iron laterally than axially
### Radius and Field Scaling for Fixed Energy

<table>
<thead>
<tr>
<th>$B$ (T)</th>
<th>$r_{\text{extraction}}$ (m)</th>
<th>$(r_1/r)^3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.28</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>0.76</td>
<td>1/27</td>
</tr>
<tr>
<td>5</td>
<td>0.46</td>
<td>1/125</td>
</tr>
<tr>
<td>7</td>
<td>0.33</td>
<td>1/343</td>
</tr>
<tr>
<td>9</td>
<td>0.25</td>
<td>1/729</td>
</tr>
</tbody>
</table>
Classical Cyclotrons– Energy Limit

- Historically- \( E<25 \text{ MeV} \), and high acceleration voltages were required

- WHY?
  - Relativistic mass increase lowers the ion orbital frequency:
    \[ \omega = \frac{qB}{\gamma m_0} \]
  - Ion frequency relative to the fixed RF frequency decreases (rotation time \( \tau \) increases)
  - Ions arrive increasing late with respect to the RF voltage on the dee
  - Eventually crossing the gaps at wrong phase and decelerates

- **21 MeV proton**: \( \gamma_{\text{final}} = 1.022 \) seems small, but...
  - Angular rotation slip near full energy
    \[ \frac{d\phi}{dn} = 360^\circ \Delta \omega / \omega = 360^\circ \left[ \frac{mB_0/m_0B - 1}{B - 1} \right] \approx 360\left[ \gamma - 1 \right] \rightarrow 8^\circ \]
  - An ion on peak phase is lost in 11 revolutions
  - Only solution– very high energy gain per turn - 360kV was required to reach 21 MeV in the LBL 60” Cyclotron!