Unit 4 - Lecture 10

RF-accelerators: Standing wave linacs

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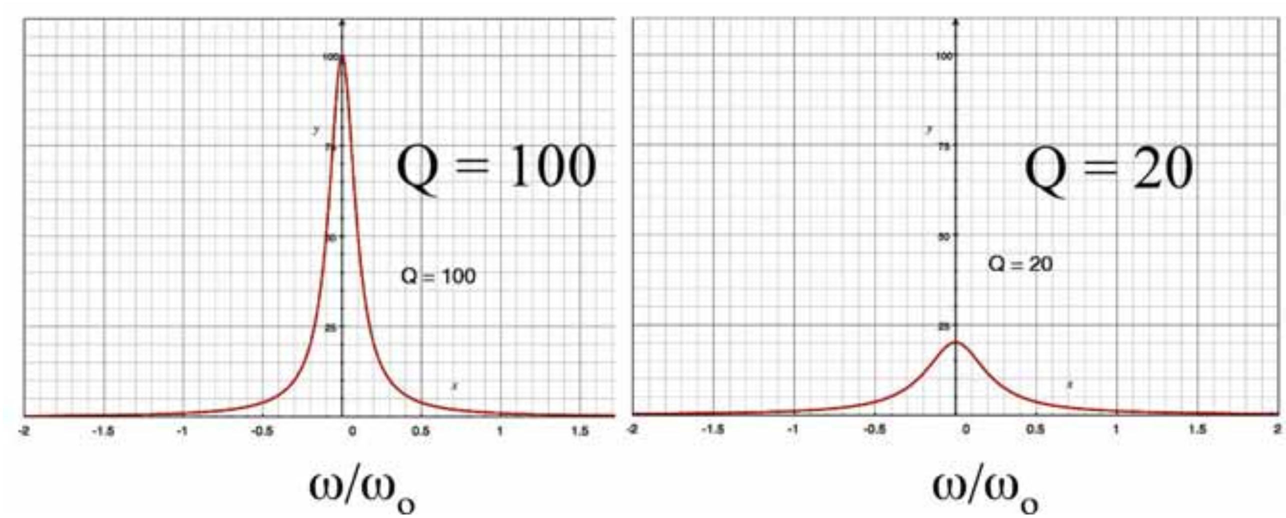
Q of the lumped circuit analogy

Converting the denominator of Z to a real number we see that

$$|Z(\omega)| \sim \left[\left(1-\frac{\omega^2}{\omega_o^2}\right)^2 + (\omega RC)^2\right]^{-1}$$

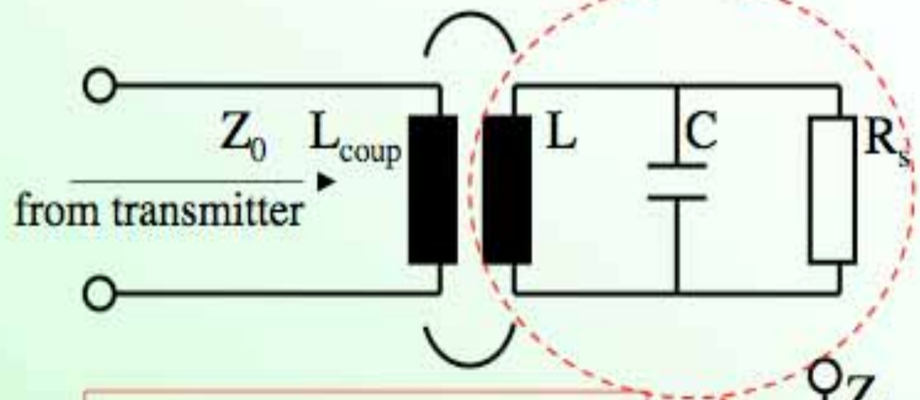


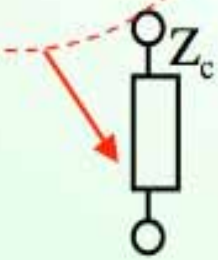
The width is $\dfrac{\Delta\omega}{\omega_0} = \dfrac{R}{\sqrt{L/.C}}$

RF-Cavity without Beam

represent transmission line, RF-coupler and cavity by a lumped circuit model

- L inductance of cavity (~ nH)
- C capacitance of cavity (~ pF)
- R_s shuntimpedance of cavity (~ MΩ)
- L_{coup} inductance of coupling
- Z_0 impedance of transmission line (~ 100 Ω)
- Z_c impedance of cavity





$$\frac{1}{Z_c} = Y_c = \frac{I_L + I_c + I_{R_s}}{U_{Z_c}} = i\omega C + \frac{1}{i\omega L} + \frac{1}{R_s}$$

$\omega_0^2 = \frac{1}{LC}$ resonance frequency of RF-structure

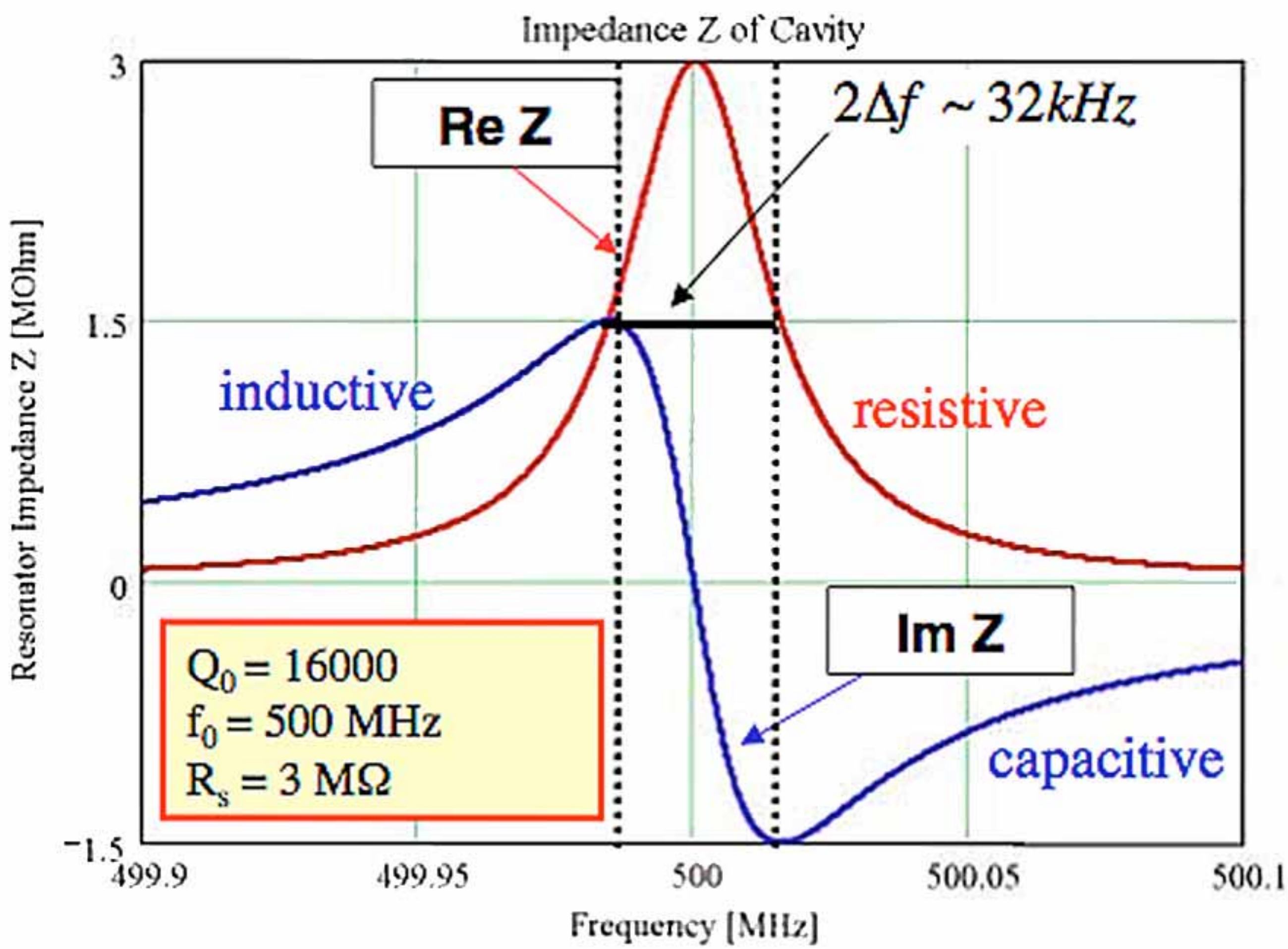
$f_0 \approx 50MHz - 3GHz$

$$Z_c = \frac{R_s}{1 + i\frac{R_s}{\omega L}\left(\frac{\omega^2}{\omega_0^2} - 1\right)} = \frac{R_s}{1 + iQ_0\frac{\omega_0}{\omega}\left(\frac{\omega^2}{\omega_0^2} - 1\right)} = \frac{R_s}{1 + iQ_0\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)}$$

norm. detuning ξ

$Q_0 = \frac{R_s}{\omega_0 L} = R_s \omega_0 C$

unloaded quality factor of cavity


Impedance Z of Cavity
Re Z
2Δf ~ 32kHz
inductive
resistive
Im Z
capacitive
Q0 = 16000
f0 = 500 MHz
Rs = 3 MΩ
Resonator Impedance Z [MOhm]
Frequency [MHz]
3
1.5
0
-1.5
499.9
499.95
500
500.05
500.1

Measuring the energy stored in the cavity allows us to measure

- We have computed the field in the fundamental mode

$$U = \int_0^d dz \int_0^b dr 2\pi r \left(\varepsilon E_o^2 / 2 \right) J_1^2(2.405 r/b)$$

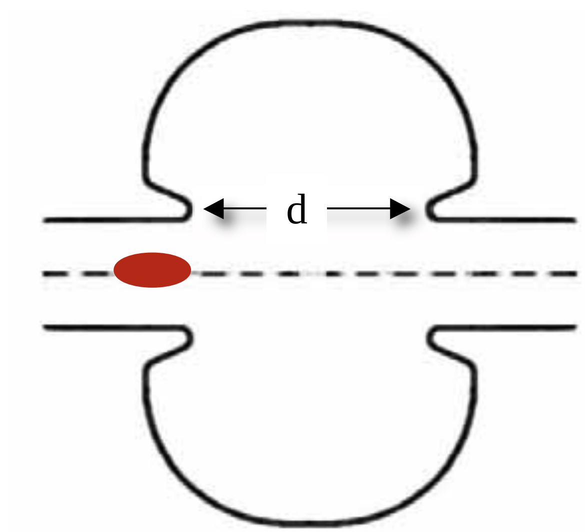
$$= b^2 d \left(\varepsilon E_o^2 /2 \right) J_1^2(2.405)$$

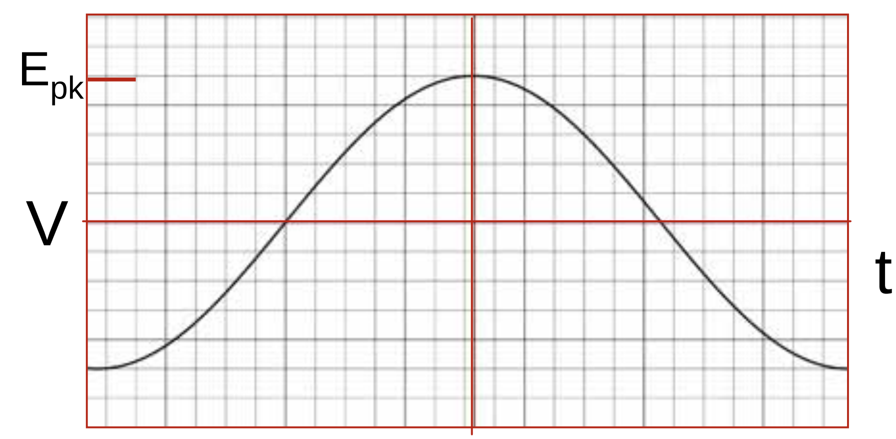
- To measure Q we excite the cavity and measure the E field as a function of time
- Energy lost per half cycle = UπQ
- Note: energy can be stored in the higher order modes that deflect the beam

Figure of Merit: Accelerating voltage

- The voltage varies during time that bunch takes to cross gap
  - ➙ reduction of the peak voltage by Γ (transt time factor)

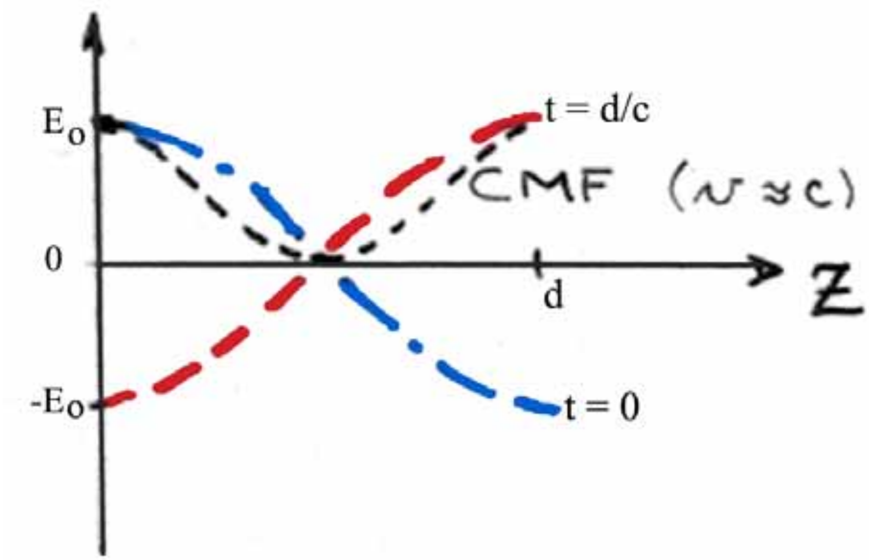
$$\Gamma = \frac{\sin\left(\vartheta/2\right)}{\vartheta/2} \quad \text{where} \quad \vartheta = \omega d/\beta c$$





For maximum acceleration $T_{\mathrm{cav}} = \frac{d}{c} = \frac{T_{\mathrm{rf}}}{2}$ ==> Γ = 2/π

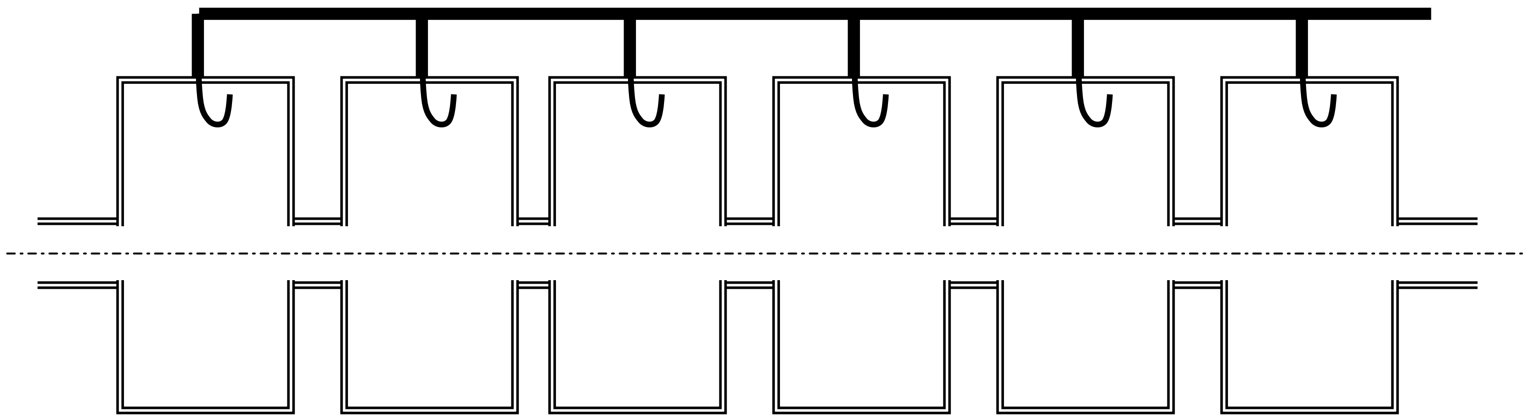
Compute the voltage gain correctly



The voltage gain seen by the beam can computed in the co-moving frame, or we can use the transit-time factor, Γ & compute V at fixed time

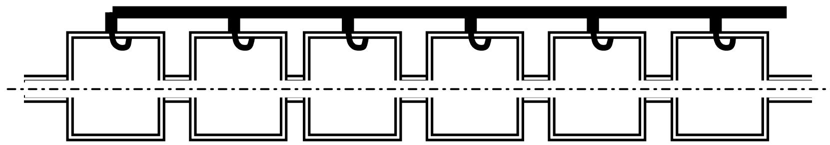
$$V_o^2 = \Gamma \int_{z_1}^{z_2} E(z) dz$$

Make the linac with a series of pillbox cavities

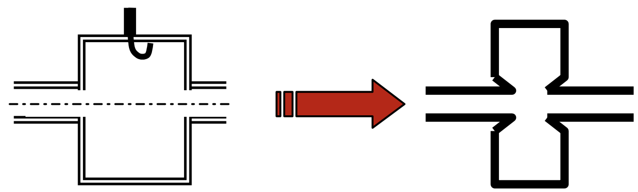


Power the cavities so that $E_z(z,t) = E_z(z)e^{i\omega t}$

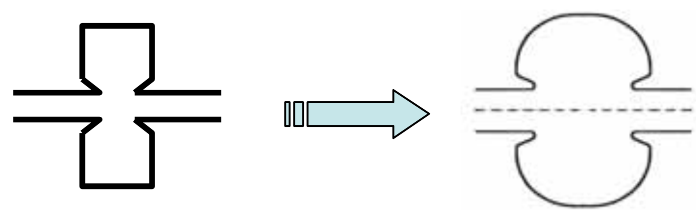
How can we improve on an array of pillboxes?



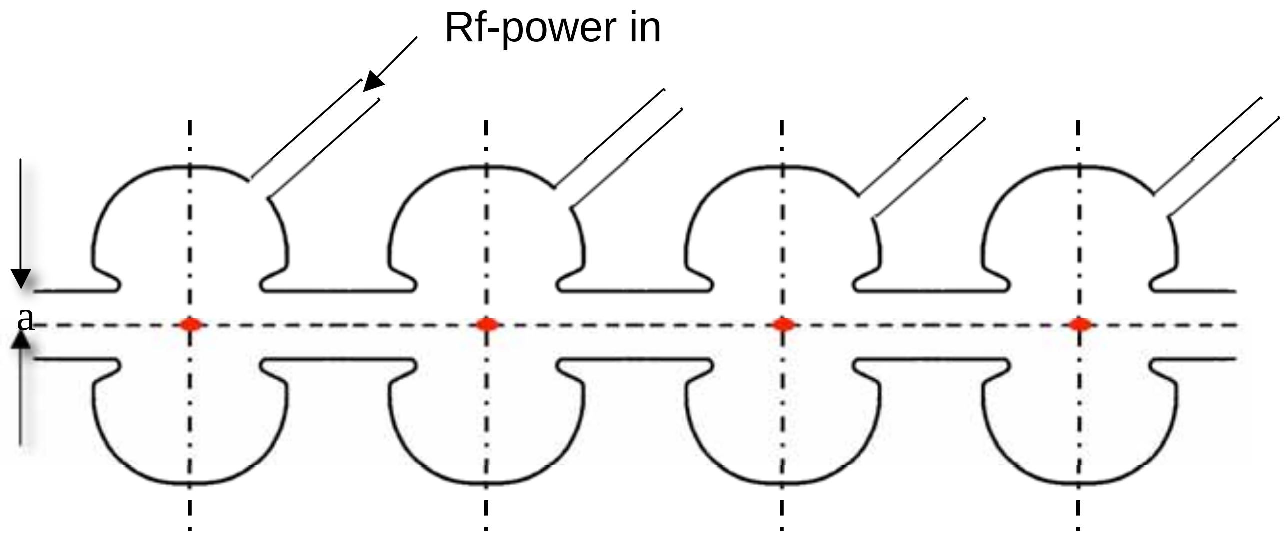
- Return to the picture of the re-entrant cavity



- Nose cones concentrate E_z near beam for fixed stored energy
- Optimize nose cone to maximize V^2; I.e., maximize R_{sh}/Q
- Make H-field region nearly spherical; raises Q & minimizes P for given stored energy



Thus, linacs can be considered to be an array of distorted pillbox cavities…

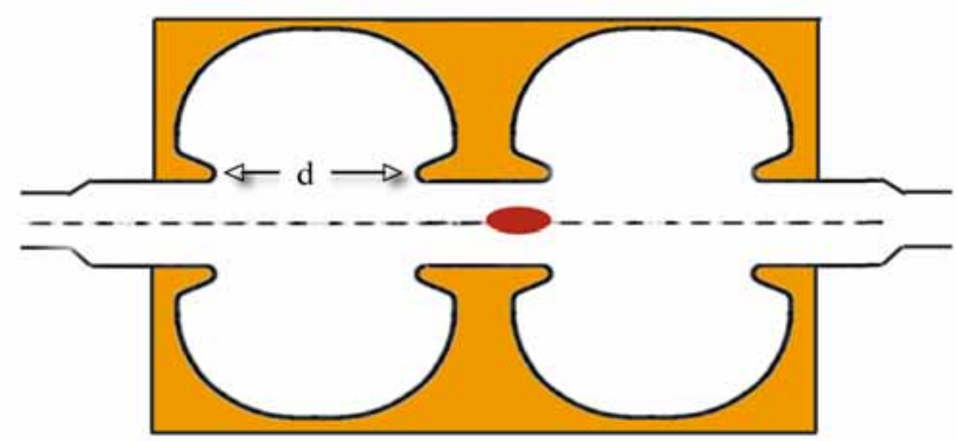


In warm linacs "nose cones" optimize the voltage per cell with respect to resistive dissipation

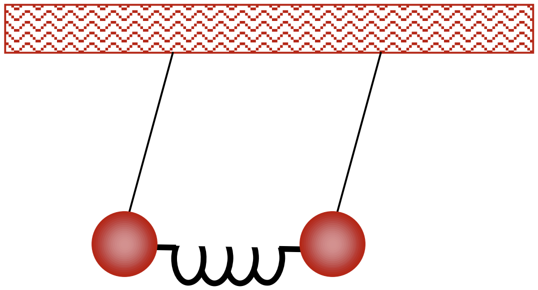
$$Q = \frac{\sqrt{L/C}}{R_{surface}}$$

Usually cells are feed in groups not individually…. and

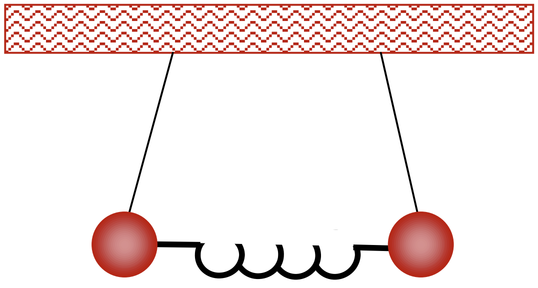
Linacs cells are linked to minimize cost



==> coupled oscillators ==>multiple modes

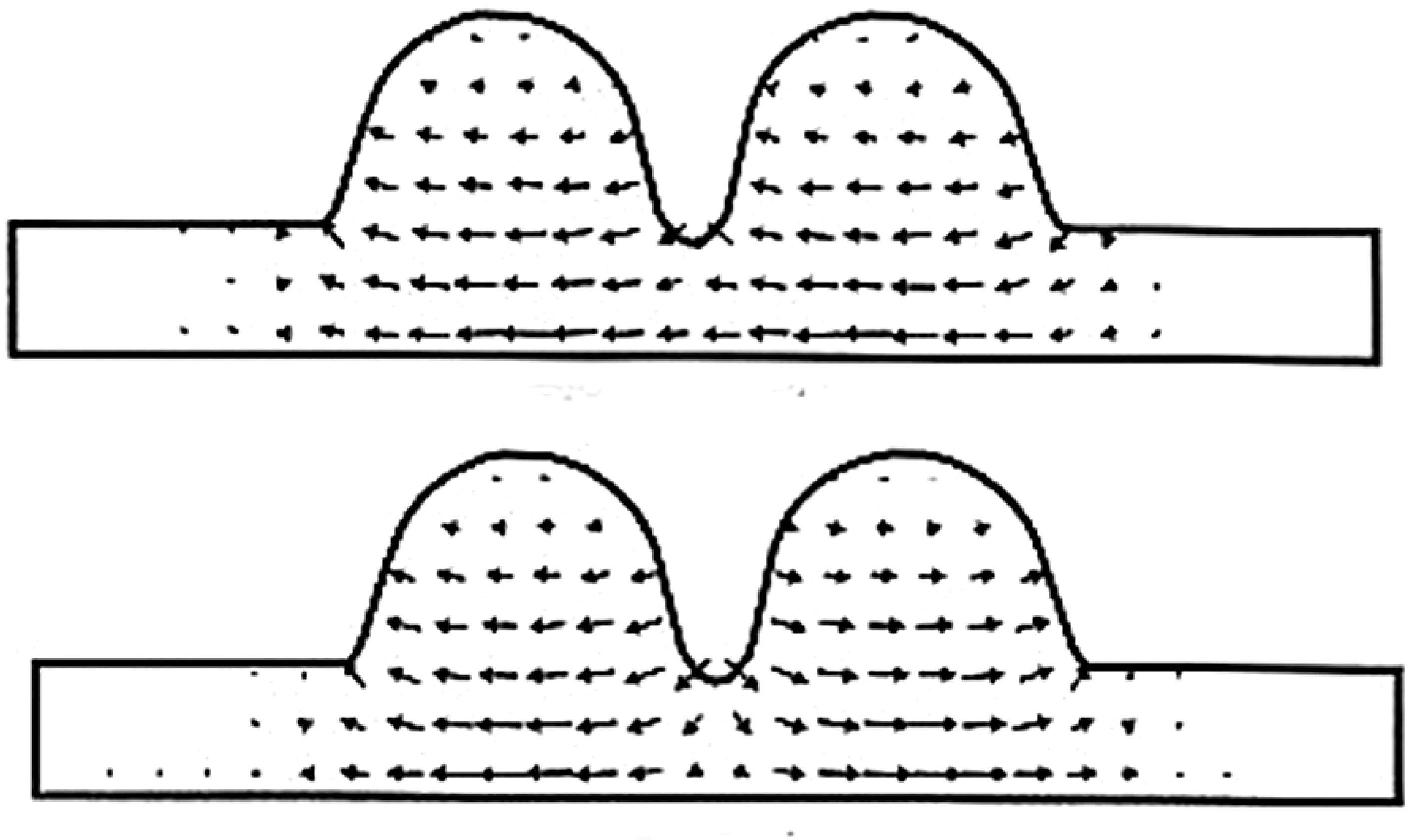


Zero mode



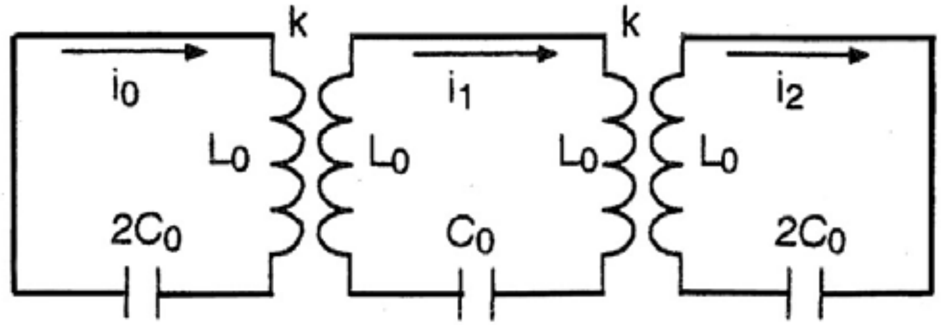
π mode

Modes of a two-cell cavity



9-cavity TESLA cell

Example of 3 coupled cavities



$$x_0\left(1-\frac{\omega_0^2}{\Omega^2}\right)+x_1 k=0 \qquad \text{oscillator } n=0$$

$$x_1\left(1-\frac{\omega_0^2}{\Omega^2}\right)+(x_0+x_2)\frac{k}{2}=0 \qquad \text{oscillator } n=1$$

$$x_2\left(1-\frac{\omega_0^2}{\Omega^2}\right)+x_1 k=0 \qquad \text{oscillator } n=2$$

$x_j = i_j\sqrt{2L_o}$ and $\Omega =$ normal mode frequency

Write the coupled circuit equations in matrix form

$$\mathbf{L}\mathbf{x}_q = \frac{1}{\Omega_q^2}\mathbf{x}_q \quad \text{where} \quad \mathbf{L} = \begin{pmatrix} 1/\omega_o^2 & k/\omega_o^2 & 0 \\ k/2\omega_o^2 & 1/\omega_o^2 & k/2\omega_o^2 \\ 0 & k/\omega_o^2 & 1/\omega_o^2 \end{pmatrix} \quad \text{and} \quad \mathbf{x_q} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

- Compute eigenvalues & eigenvectors to find the three normal modes

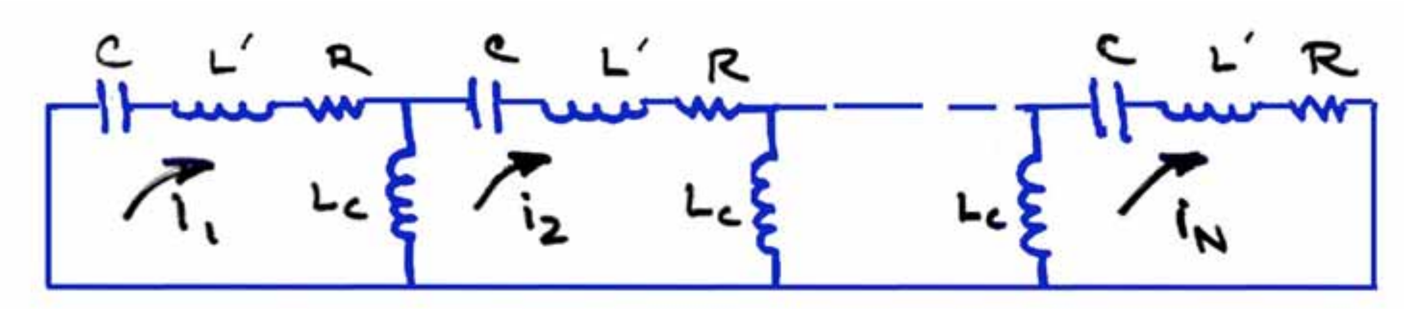
Mode q = 0: zero mode $\quad \Omega_0 = \dfrac{\omega_o}{\sqrt{1+k}} \quad \mathbf{x_0} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

Mode q = 1: $\pi/2$ mode $\quad \Omega_1 = \omega_o \quad \mathbf{x}_1 = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$

Mode q = 2: π mode $\quad \Omega_2 = \dfrac{\omega_o}{\sqrt{1-k}} \quad \mathbf{x}_2 = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$

For a structure with N coupled cavities

- ==> Set of N coupled oscillators
  - ➞ N normal modes, N frequencies
- From the equivalent circuit with magnetic coupling

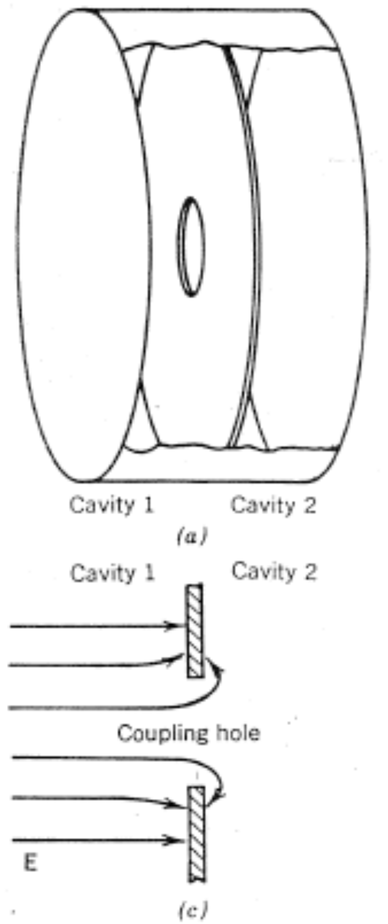


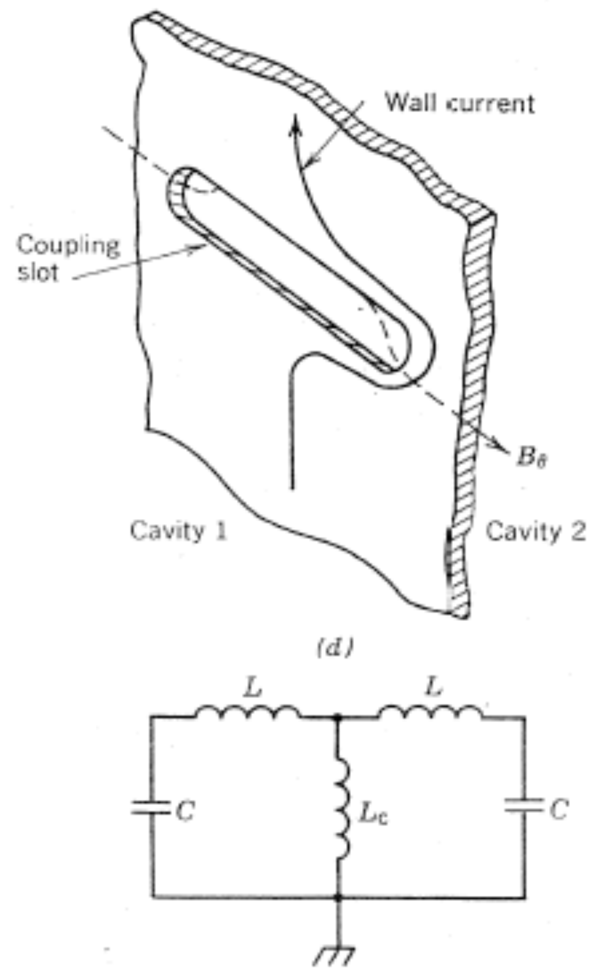
$$\omega_m = \frac{\omega_o}{\left(1 - B\cos\frac{m\pi}{N}\right)^{1/2}} \approx \omega_o\left(1 + B\cos\frac{m\pi}{N}\right)$$

where B= bandwidth (frequency difference between lowest & high frequency mode)

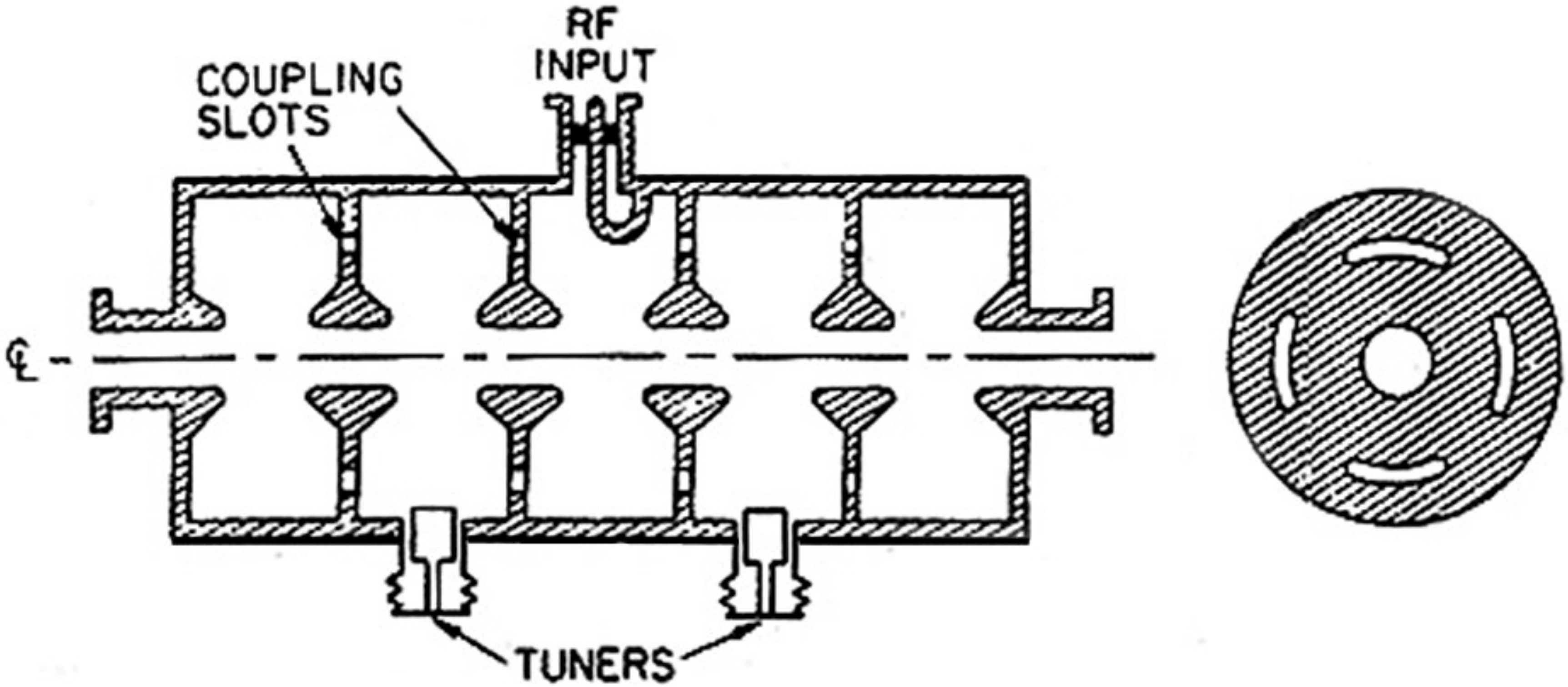
- Typically accelerators run in the π-mode

Magnetically coupled pillbox cavities





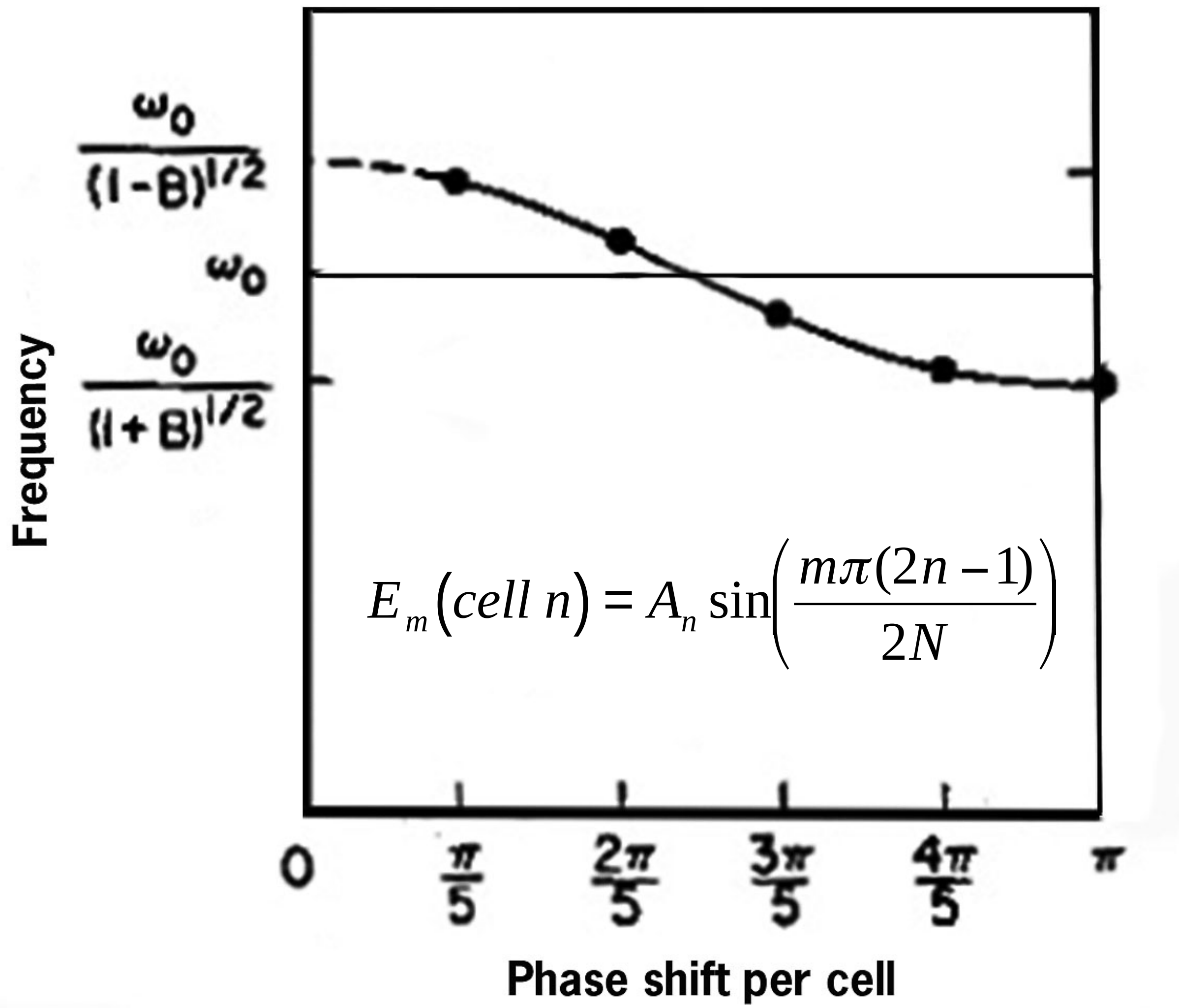
5-cell π-mode cell with magnetic coupling



The tuners change the frequencies by perturbing wall currents ==> changes the inductance ==> changes the energy stored in the magnetic field

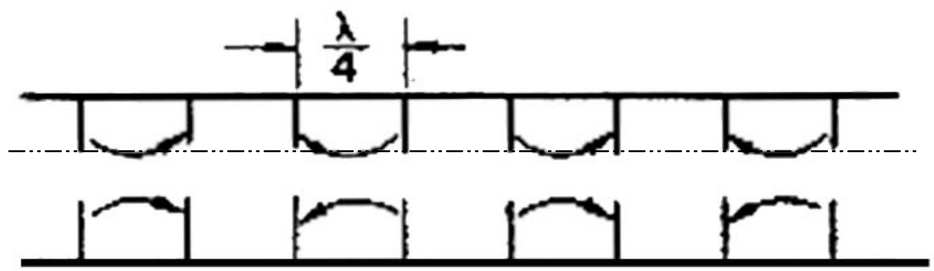
$$\frac{\Delta\omega_o}{\omega_o} = \frac{\Delta U}{U}$$

Dispersion diagram for 5-cell structure

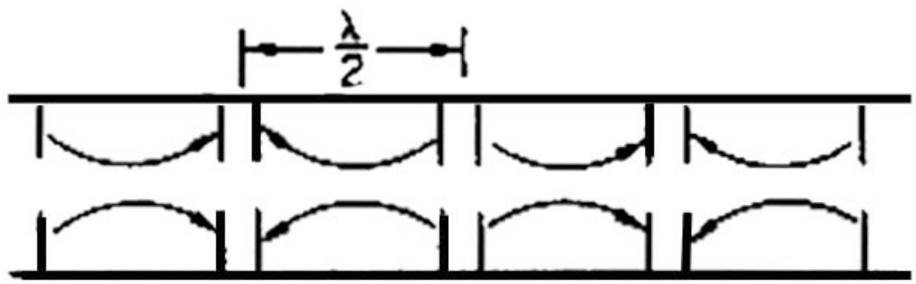


$$E_m(cell\ n) = A_n \sin\left(\frac{m\pi(2n-1)}{2N}\right)$$

Evolution of the Los Alamos structure

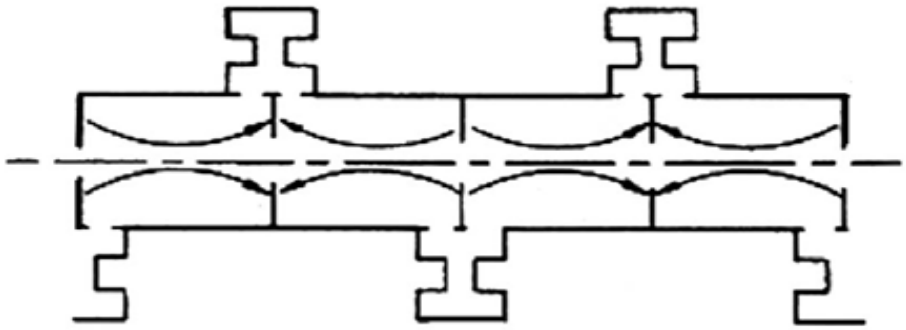


$\pi/2$-mode has high v_g & good frequency stability, BUT low R_{sh}

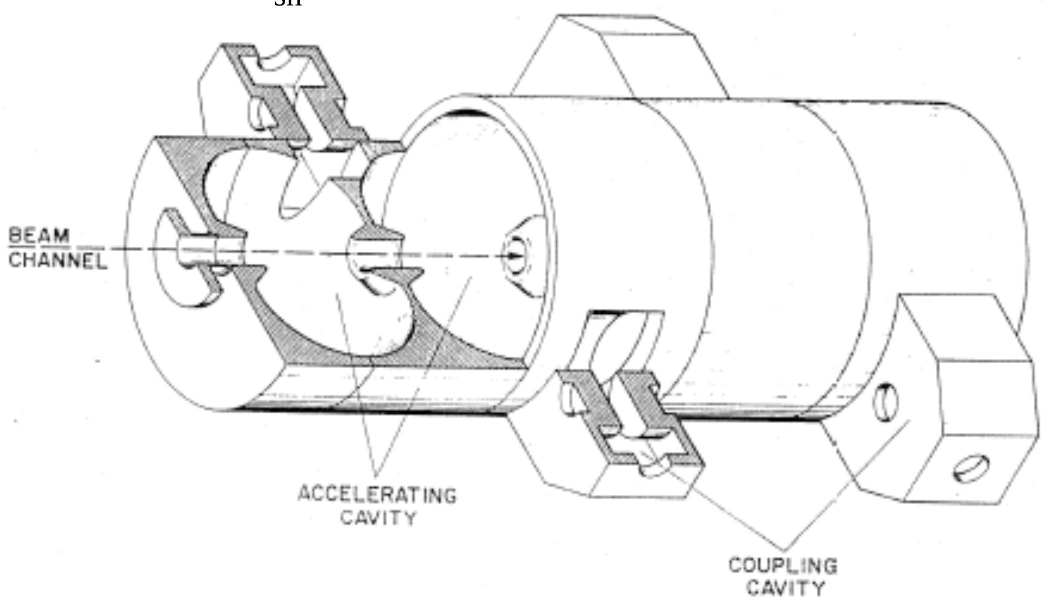


Bi-periodic structure raises R_{sh} by shrinking the unexcited cells

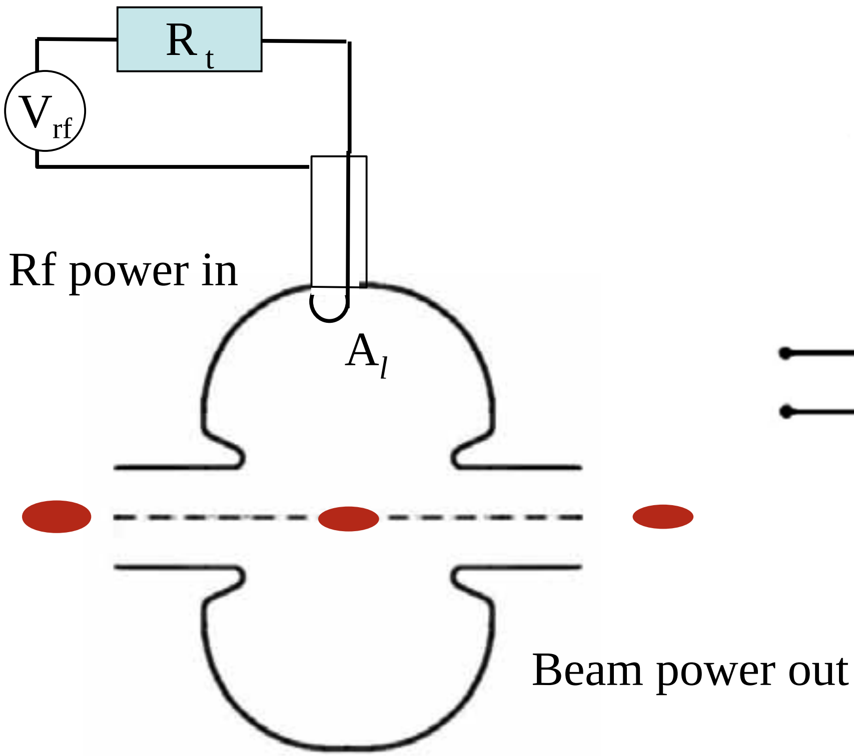
Side-coupled cavity

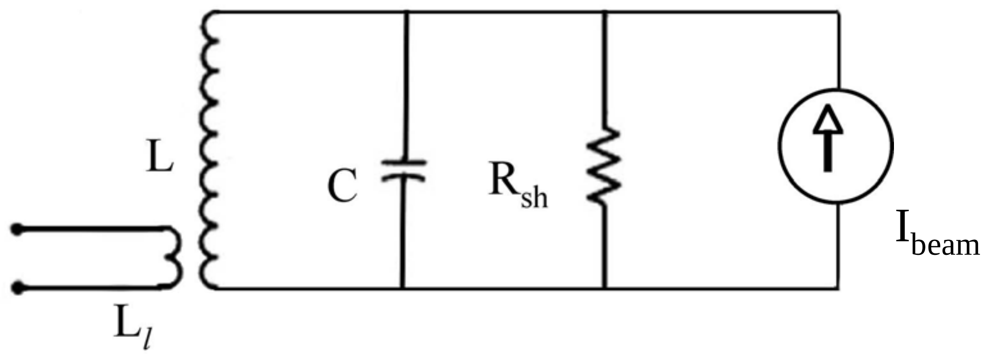


Side-Coupled Structure shrinks the unexcited cells to zero.
R_{sh} is the same as the π-mode

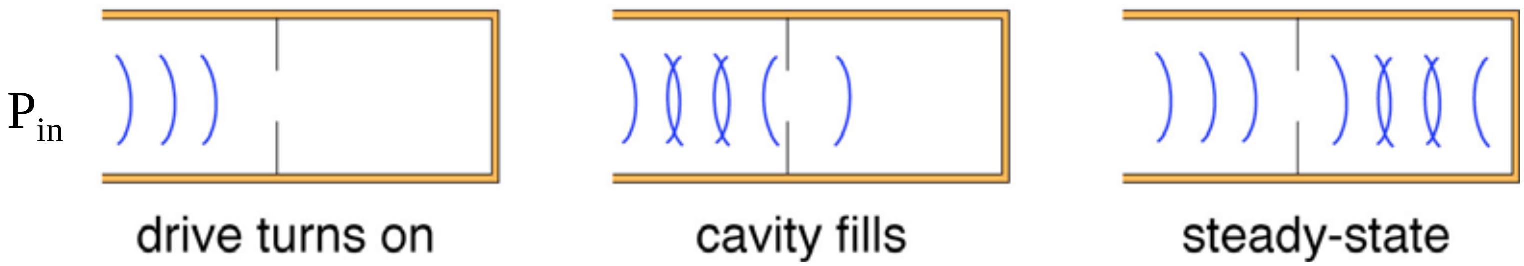


Power exchange with resonant cavities





Effects of wall-losses & external coupling on stored energy, U



- Define "wall quality factor", Q_w, & "external" quality factor, Q_e
- Power into the walls is $P_w = \omega U/ Q_w$.
- If P_{in} is turned off, then the power flowing out $P_e = \omega U/Q_e$
- Net rate of energy loss = $\omega U/ Q_w + \omega U/Q_e = \omega U/ Q_{loaded}$

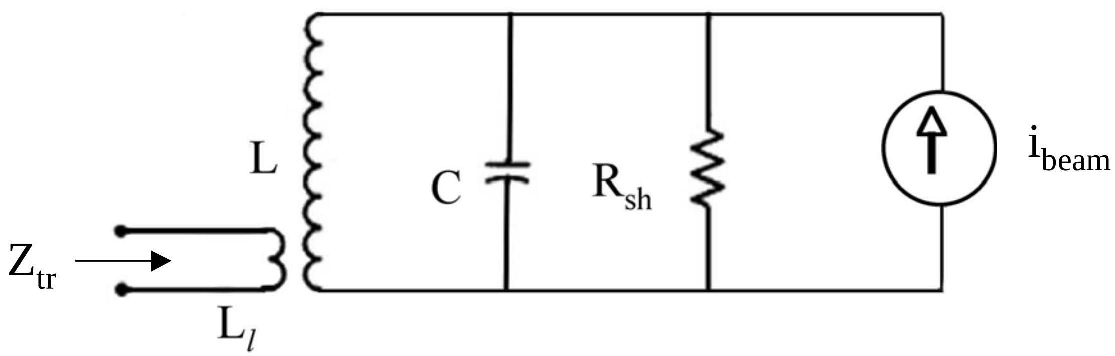
Till time & coupling

- Loaded fill time

$$T_{fill} = 2Q_L/\omega$$

- Critically coupled cavity: $P_{in} = P_w \Longrightarrow 1/Q_e = 1/Q_w$

- In general, the coupling parameter $ß = Q_w / Q_e$



At resonance, the rf source & the beam have the following effects

- Voltage produced by the generator is

$$V_{gr} = \frac{2\sqrt{\beta}}{1+\beta} \cdot \sqrt{R_{shunt} P_{gen}}$$

- The voltage produced by the beam is

$$V_{b,r} = \frac{i_{beam}}{Z_{tr}(1+\beta)} \approx \frac{I_{dc} R_{shunt}}{(1+\beta)}$$

At resonance, the rf source & the beam have the following effects

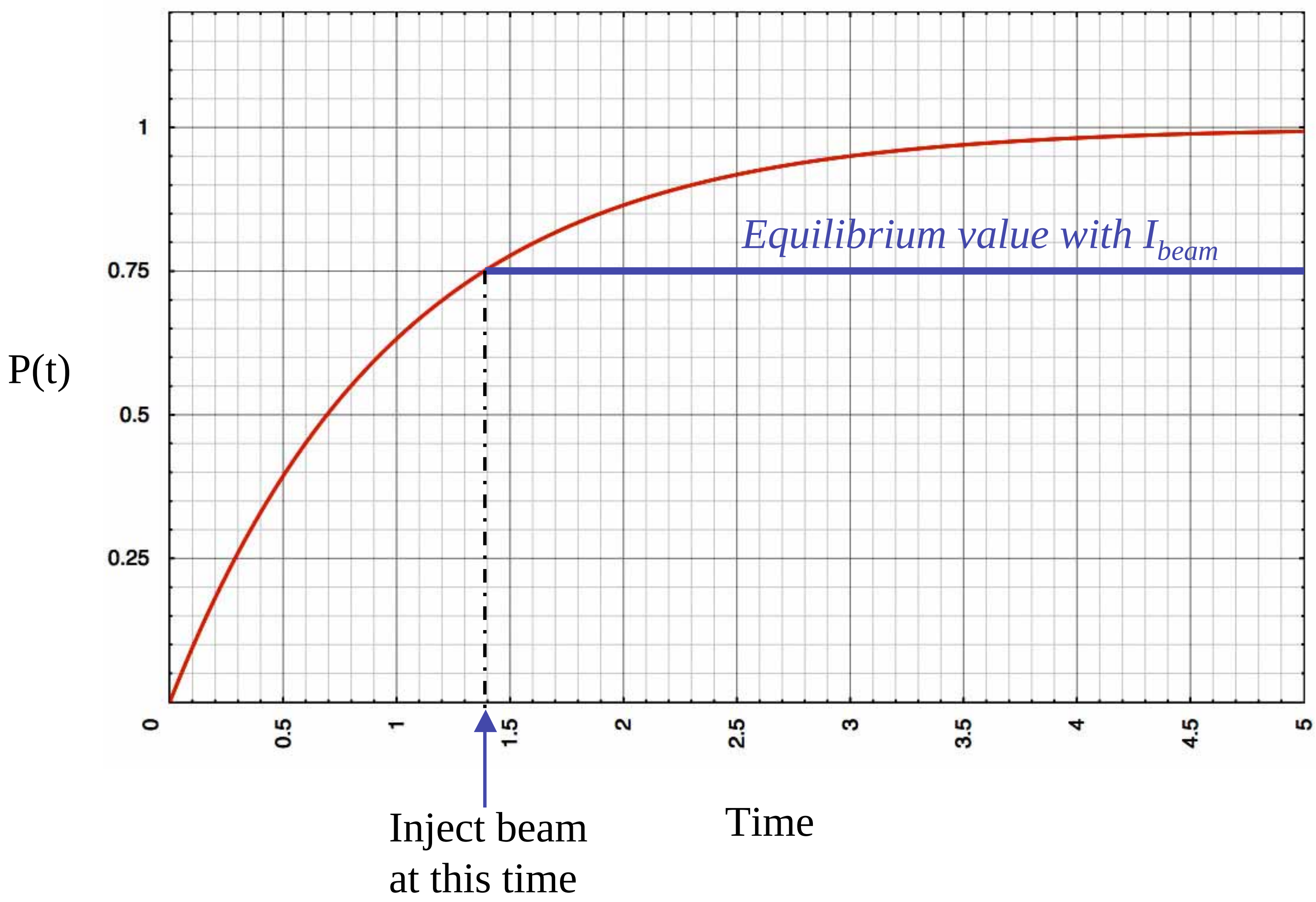
- The accelerating voltage is the sum of these effects

$$V_{accel} = \sqrt{R_{shunt} P_{gen}} \left[\frac{2\sqrt{\beta}}{1+\beta} \left(1 - \frac{K}{\sqrt{\beta}} \right) \right] = \sqrt{R_{shunt} P_{wall}}$$

where $K = \dfrac{I_{dc}}{2} \sqrt{R_{shunt} / P_{gen}}$ is the "loading factor"

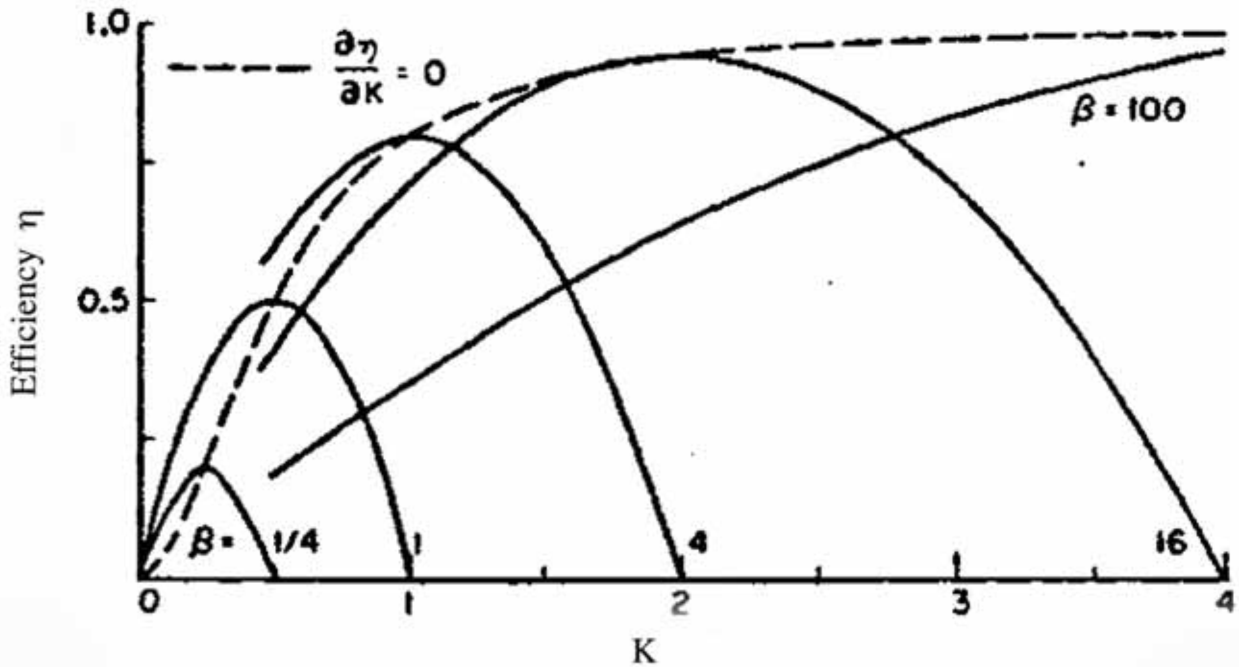
- ==> V_{acc} decreases linearly with increasing beam current

Power flow in standing wave linac

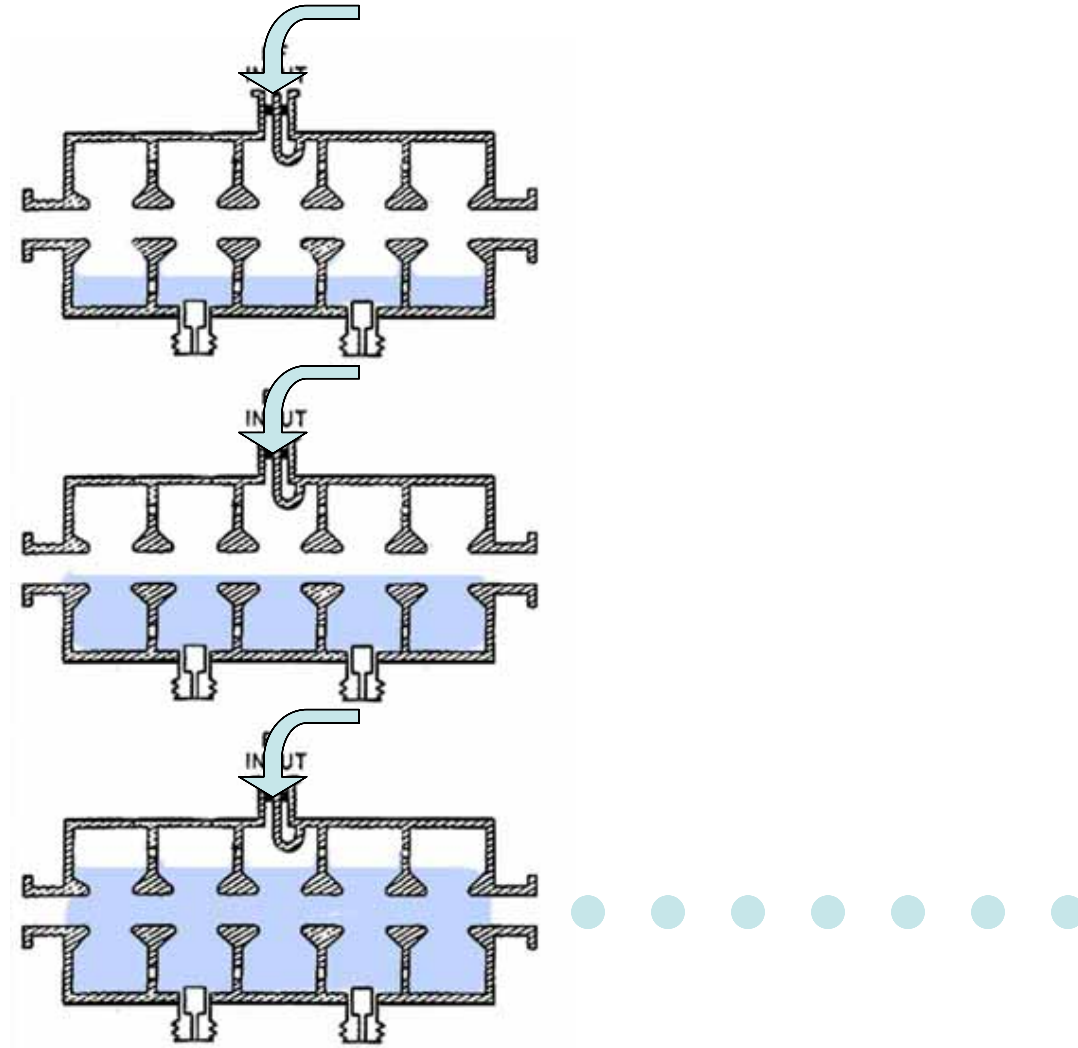


Efficiency of the standing wave linac

$$\eta = \frac{I_{dc} V_{acc}}{P_{gen}} = \frac{2\sqrt{\beta}}{1+\beta}\left[2K\left(1 - \frac{K}{\sqrt{\beta}}\right)\right]$$



Schematic of energy flow in a standing wave structure



What makes SC RF attractive?

Comparison of SC and NC RF

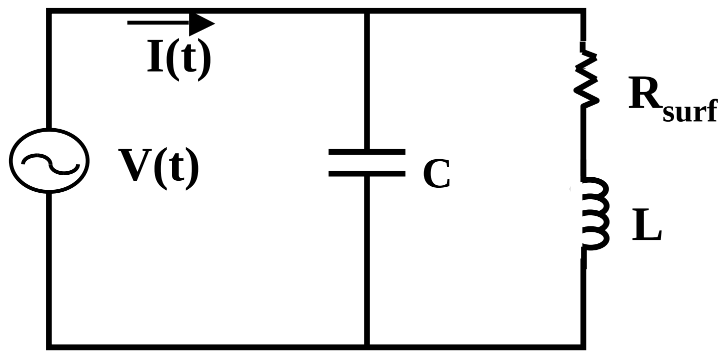
Superconducting RF

- High gradient
  ==> 1 GHz, meticulous care
- Mid-frequencies
  ==> Large stored energy, E_s
- Large E_s
  ==> very small $\Delta E/E$
- Large Q
  ==> high efficiency

Normal Conductivity RF

- High gradient
  ==> high frequency (5 - 17 GHz)
- High frequency
  ==> low stored energy
- Low E_s
  ==> ~10x larger $\Delta E/E$
- Low Q
  ==> reduced efficiency

Recall the circuit analog



As R_{surf} ==> 0, the Q ==> ∞.

In practice,

$Q_{nc} \sim 10^4$ $Q_{sc} \sim 10^{11}$

Figure of merit for accelerating cavity: power to produce the accelerating field

Resistive input (shunt) impedance at ω_o relates power dissipated in walls to accelerating voltage

$$R_{in} = \frac{\langle V^2(t) \rangle}{P} = \frac{V_o^2}{2P} = Q\sqrt{L/C}$$

Linac literature more commonly defines "shunt impedance" without the "2"

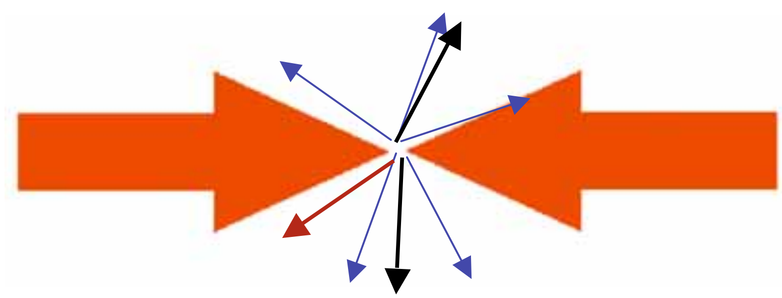
$$R_{in} = \frac{V_o^2}{P} \sim \frac{1}{R_{surf}}$$

For SC-rf P is reduced by orders of magnitude

BUT, it is deposited @ 2K

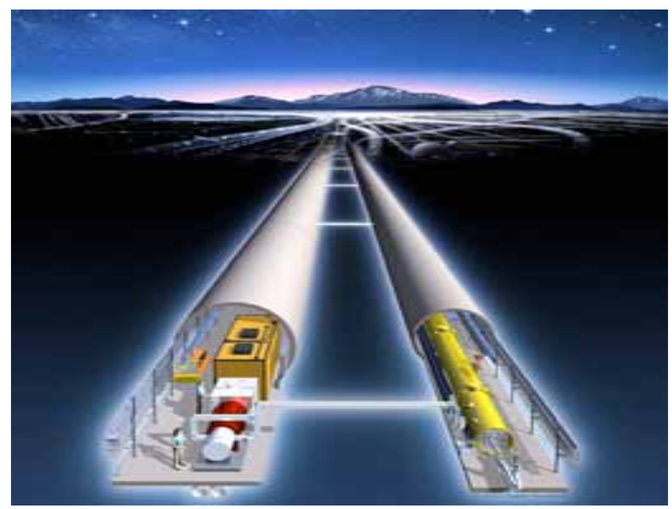
Why do we need beams?

Collide beams



FOMs: Collision rate, energy stability, Accelerating field

Examples: LHC, ILC, RHIC

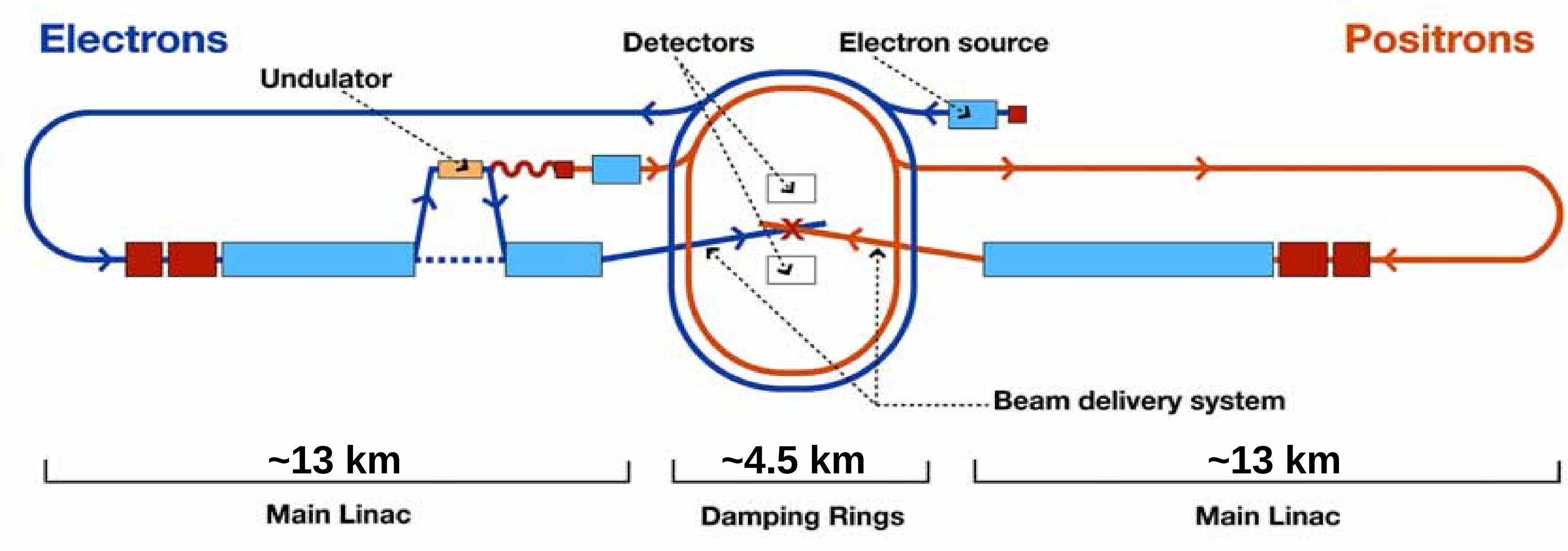


In LHC storage rings…

- Limited space & Large rf trapping of particles
  - ➙V/cavity must be high
- Bunch length must be large (≤ 1 event/cm in luminous region)
  - ➙RF frequency must be low
- Energy lost in walls must be small
  - ➙R_{surf} must be small

SC cavities were the only practical choice

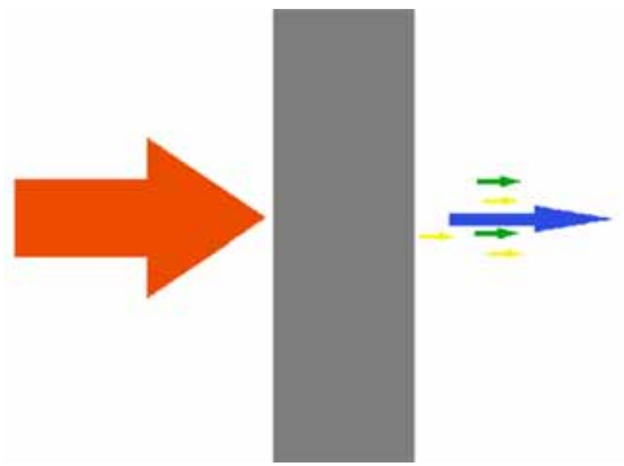
For ILC SC rf provides high power, high quality beams at high efficiency

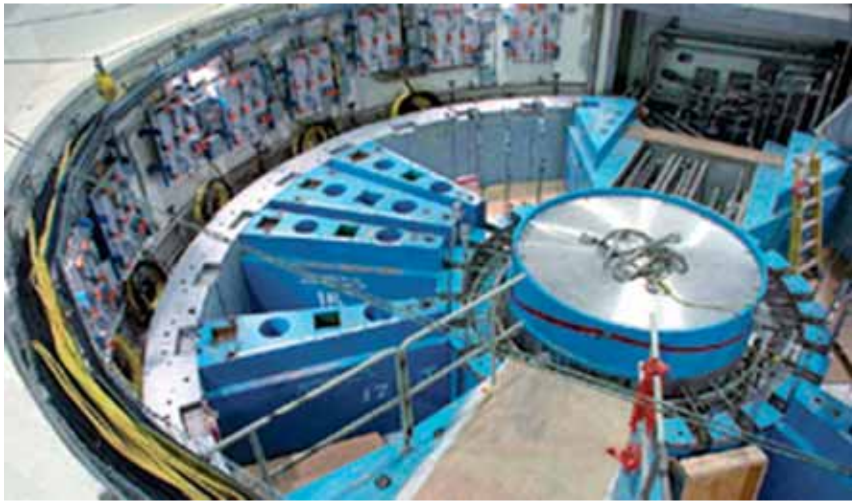


- **To deliver required luminosity (500 fb^{-1} in 4 years) ==>**
  - powerful polarized electron & positron beams (11 MW /beam)
  - tiny beams at collision point ==> minimizing beam-structure interaction
- **To limit power consumption** ==> high “wall plug” to beam power efficiency
  - Even with SC rf, the site power is still 230 MW !

Why do we need beams?

Intense secondary beams





1 MW target at SNS

FOM: Secondaries/primary
Examples: spallation neutrons,
neutrino beams

The Spallation Neutron Source

- 1 MW @ 1GeV (compare with ILC 11 MW at 500 GeV (upgradeable to 4 MW)

==> miniscule beam loss into accelerator

==> large aperture in cavities ==> large cavities

==> low frequency

==> high energy stability

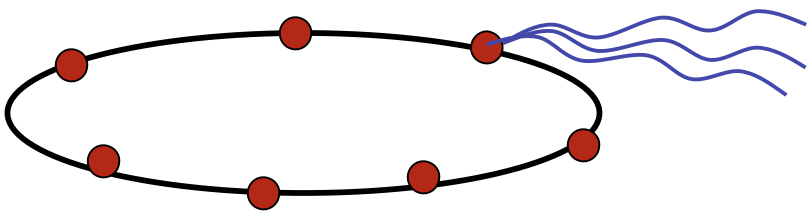
==>large stored energy

==> high efficiency at E_z

==> SC RF

Synchrotron light source

(pulsed incoherent X-ray emission)



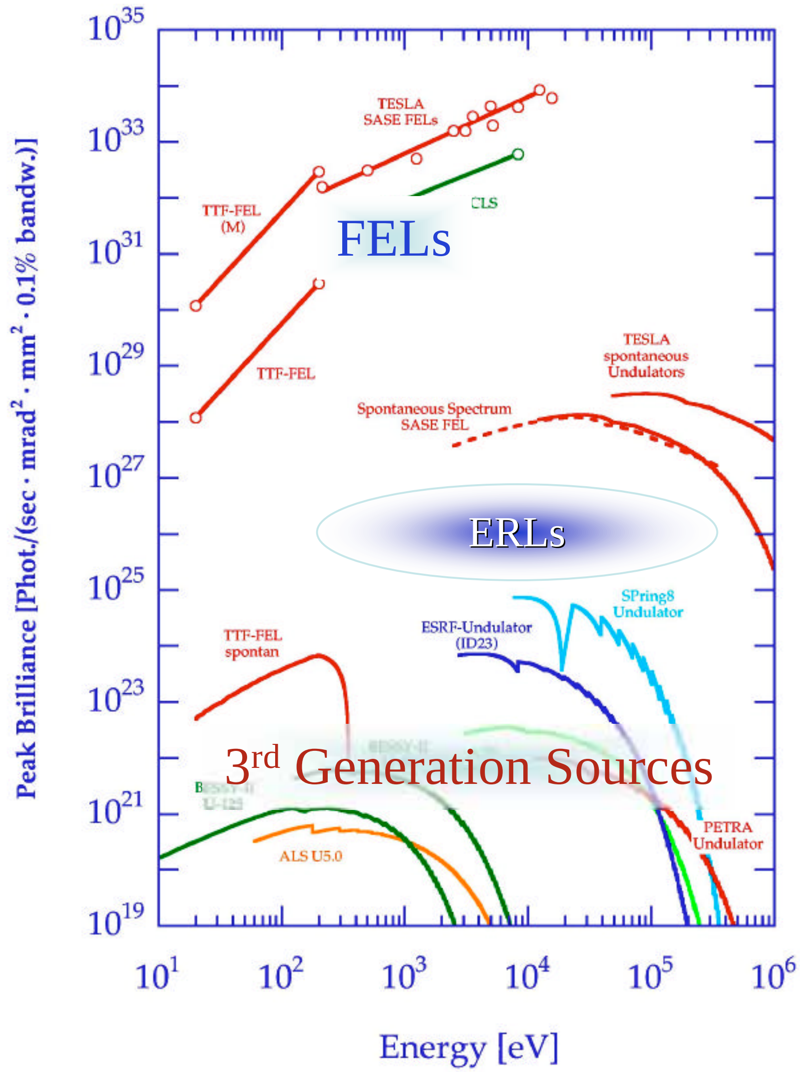
FOM: Brilliance v. λ

B = ph/s/mm^2/mrad^2/0.1%BW

Pulse duration

Science with X-rays

- Imaging
- Spectroscopy



Matter to energy: Energy Recovery Linacs
Hard X-rays ==> ~5 GeV

Synchrotron light source
(pulsed incoherent X-ray emission)

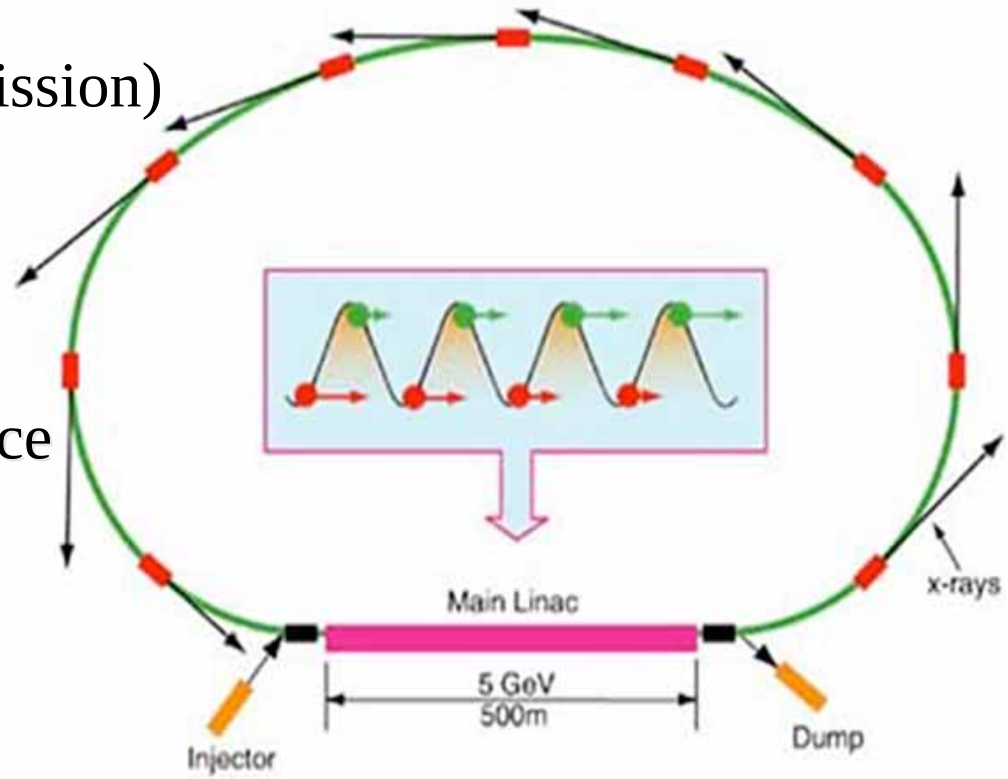
Pulse rates – kHz => MHz

X-ray pulse duration ≤ 1 ps

High average e-beam brilliance
& e-beam duration ≤ 1 ps

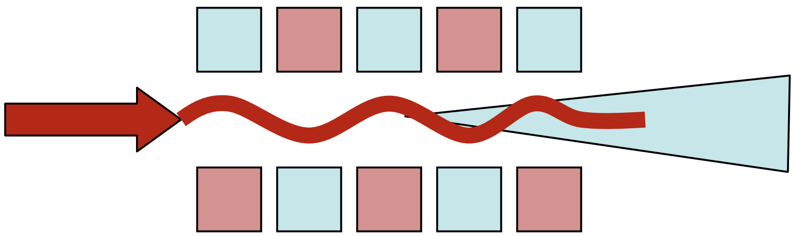
⇒ One pass through ring
⇒ Recover beam energy
⇒ High efficiency

⇒ SC RF

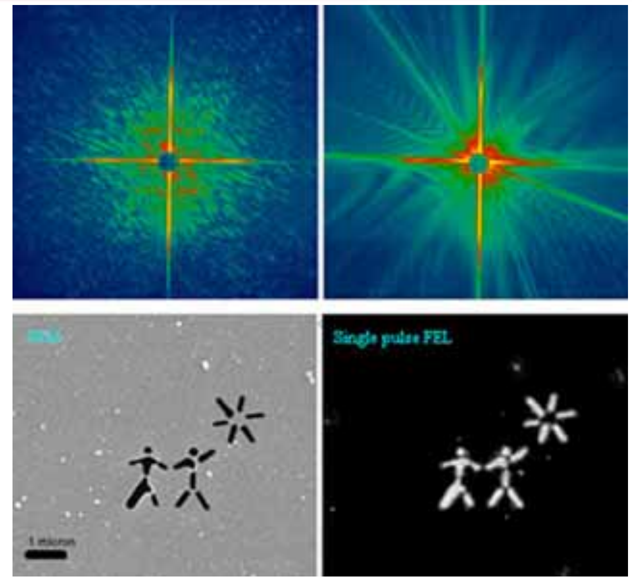


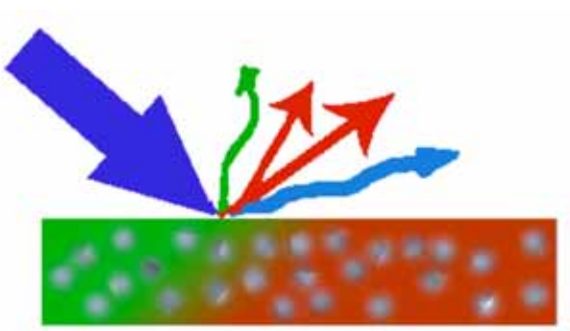
Even higher brightness requires coherent emission ==> FEL

Free electron laser



FOM: Brightness v. λ
Time structure





Full range of FEL-based science requires…

- Pulses rates 10 Hz to 10 MHz (NC limited to ~ 100 Hz)
  - ➙ High efficiency
- Pulse duration 10 fs - 1 ps
- High gain
  - ➙ Excellent beam emittance
    - ==> Minimize wakefield effect
    - ==> large aperture
    - ==> low frequency
  - ➙ Stable beam energy & intensity
    - ==> large stored energy in cavities
    - ==> high Q

===> SC RF

End of unit

Field measurement