Unit 4 - Lecture 10

RF-accelerators: Standing wave linacs

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Converting the denominator of $Z$ to a real number we see that

$$|Z(\omega)| \sim \left[ \left(1 - \frac{\omega^2}{\omega_0^2}\right)^2 + (\omega RC)^2 \right]^{-1}$$

The width is

$$\frac{\Delta \omega}{\omega_0} = \frac{R}{\sqrt{L/C}}$$
RF-Cavity without Beam

represent transmission line, RF-coupler and cavity by a lumped circuit model

\[
\frac{1}{Z_c} = Y_c = \frac{I_L + I_c + I_{R_s}}{U_{Z_c}} = i\omega C + \frac{1}{i\omega L} + \frac{1}{R_s}
\]

\[
Z_c = \frac{R_s}{1 + i\frac{R_s}{\omega L} \left( \frac{\omega^2}{\omega_0^2} - 1 \right)} = \frac{R_s}{1 + iQ_0 \frac{\omega}{\omega_0} \left( \frac{\omega}{\omega_0^2} - 1 \right)} = \left( R_s + iQ_0 \frac{\omega}{\omega_0} \right) \left( \frac{\omega}{\omega_0} - \frac{\omega_0}{\omega} \right)
\]

\[
\omega_0^2 = \frac{1}{LC}
\]

resonance frequency of RF-structure

\[
f_0 \approx 50 \text{MHz} - 3 \text{GHz}
\]

\[
Q_0 = \frac{R_s}{\omega_0 L} = R_s \omega_0 C
\]

unloaded quality factor of cavity

Interaction between RF-System, RF-Cavity and Beam
Measuring the energy stored in the cavity allows us to measure

- We have computed the field in the fundamental mode

\[ U = \int_0^d dz \int_0^b dr 2\pi r \left( \frac{\varepsilon E_o^2}{2} \right) J_1^2 \left( \frac{2.405 r}{b} \right) \]

\[ = b^2 d \left( \frac{\varepsilon E_o^2}{2} \right) J_1^2 \left( 2.405 \right) \]

- To measure Q we excite the cavity and measure the E field as a function of time

- Energy lost per half cycle = \( U\pi Q \)

- Note: energy can be stored in the higher order modes that deflect the beam
Figure of Merit: Accelerating voltage

The voltage varies during time that bunch takes to cross gap

\[ \Gamma = \frac{\sin\left(\frac{\vartheta}{2}\right)}{\frac{\vartheta}{2}} \]

where \( \vartheta = \frac{\omega d}{\beta c} \)

For maximum acceleration

\[ T_{\text{cav}} = \frac{d}{c} = \frac{T_{\text{rf}}}{2} \quad \Rightarrow \quad \Gamma = \frac{2}{\pi} \]
Compute the voltage gain correctly

The voltage gain seen by the beam can be computed in the co-moving frame, or we can use the transit-time factor, $\Gamma$ & compute $V$ at fixed time

$$V_o^2 = \Gamma \int_{z_1}^{z_2} E(z) \, dz$$
Make the linac with a series of pillbox cavities

Power the cavities so that $E_z(z,t) = E_z(z)e^{i\omega t}$
How can we improve on an array of pillboxes?

- Return to the picture of the re-entrant cavity

- Nose cones concentrate $E_z$ near beam for fixed stored energy

- Optimize nose cone to maximize $V^2$; i.e., maximize $R_{sh}/Q$

- Make H-field region nearly spherical; raises Q & minimizes $P$ for given stored energy
In warm linacs “nose cones” optimize the voltage per cell with respect to resistive dissipation

\[ Q = \sqrt{\frac{L}{C \cdot R_{\text{surface}}}} \]

Usually cells are feed in groups not individually…. and
Linacs cells are linked to minimize cost

$$\Rightarrow$$ coupled oscillators $$\Rightarrow$$ multiple modes

Zero mode

π mode
Modes of a two-cell cavity
9-cavity TESLA cell
Example of 3 coupled cavities

\[ x_0 \left( 1 - \frac{\omega_0^2}{\Omega^2} \right) + x_1 k = 0 \quad \text{oscillator } n = 0 \]

\[ x_1 \left( 1 - \frac{\omega_0^2}{\Omega^2} \right) + (x_0 + x_2) \frac{k}{2} = 0 \quad \text{oscillator } n = 1 \]

\[ x_2 \left( 1 - \frac{\omega_0^2}{\Omega^2} \right) + x_1 k = 0 \quad \text{oscillator } n = 2 \]

\[ x_j = i_j \sqrt{2L_o} \quad \text{and} \quad \Omega = \text{normal mode frequency} \]
Write the coupled circuit equations in matrix form

$$Lx_q = \frac{1}{\Omega_q^2} x_q \quad \text{where} \quad L = \begin{pmatrix} \frac{1}{\omega_o^2} & \frac{k}{\omega_o^2} & 0 \\ \frac{k}{2\omega_o^2} & \frac{1}{\omega_o^2} & \frac{k}{2\omega_o^2} \\ 0 & \frac{k}{\omega_o^2} & \frac{1}{\omega_o^2} \end{pmatrix} \quad \text{and} \quad x_q = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

• Compute eigenvalues & eigenvectors to find the three normal modes

Mode q = 0: zero mode \[ \Omega_0 = \omega_o \sqrt{1 + k} \quad x_0 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \]

Mode q = 1: \(\pi/2\) mode \[ \Omega_1 = \omega_o \quad x_1 = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} \]

Mode q = 2: \(\pi\) mode \[ \Omega_2 = \omega_o \sqrt{1 - k} \quad x_2 = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \]
For a structure with N coupled cavities

- Set of N coupled oscillators
- N normal modes, N frequencies
- From the equivalent circuit with magnetic coupling

\[
\omega_m = \frac{\omega_o}{\left(1 - B \cos \frac{m\pi}{N}\right)^{1/2}} \approx \omega_o \left(1 + B \cos \frac{m\pi}{N}\right)
\]

where \( B \) = bandwidth (frequency difference between lowest & high frequency mode)

- Typically accelerators run in the \( \pi \)-mode
Magnetically coupled pillbox cavities
The tuners change the frequencies by perturbing wall currents $\Rightarrow$ changes the inductance
$\Rightarrow$ changes the energy stored in the magnetic field
\[
\frac{\Delta \omega_o}{\omega_o} = \frac{\Delta U}{U}
\]
Dispersion diagram for 5-cell structure

\[ E_m(\text{cell } n) = A_n \sin \left( \frac{m \pi (2n - 1)}{2N} \right) \]
Evolution of the Los Alamos structure

\[ \pi/2 \text{-mode has high } v_g \text{ & good frequency stability, BUT low } R_{sh} \]

Bi-periodic structure raises \( R_{sh} \) by shrinking the unexcited cells
Side-coupled cavity

Side-Coupled Structure shrinks the unexcited cells to zero. 
$R_{sh}$ is the same as the $\pi$-mode
Power exchange with resonant cavities

Rf power in

Beam power out

$I_{beam}$

\[ V_{rf} \]

\[ R_t \]

\[ A_l \]

\[ L \]

\[ C \]

\[ R_{sh} \]

\[ L'_{l} \]
Define “wall quality factor”, $Q_w$, & “external” quality factor, $Q_e$.

Power into the walls is $P_w = \frac{\omega U}{Q_w}$.

If $P_{in}$ is turned off, then the power flowing out $P_e = \frac{\omega U}{Q_e}$

Net rate of energy loss $= \frac{\omega U}{Q_w} + \frac{\omega U}{Q_e} = \frac{\omega U}{Q_{loaded}}$
**Loaded fill time**

\[ T_{\text{fill}} = \frac{2Q_L}{\omega} \]

**Critically coupled cavity:** \( P_{\text{in}} = P_w \implies \frac{1}{Q_e} = \frac{1}{Q_w} \)

**In general, the coupling parameter** \( \beta = \frac{Q_w}{Q_e} \)
At resonance, the rf source & the beam have the following effects

- Voltage produced by the generator is

\[
V_{gr} = \frac{2\sqrt{\beta}}{1 + \beta} \cdot \sqrt{R_{shunt} P_{gen}}
\]

- The voltage produced by the beam is

\[
V_{b,r} = \frac{i_{beam}}{Z_{tr}(1 + \beta)} \approx \frac{I_{dc} R_{shunt}}{(1 + \beta)}
\]
At resonance, the rf source & the beam have the following effects

- The accelerating voltage is the sum of these effects

\[ V_{\text{accel}} = \sqrt{R_{\text{shunt}}P_{\text{gen}}} \left[ \frac{2\sqrt{\beta}}{1 + \beta} \left( 1 - \frac{K}{\sqrt{\beta}} \right) \right] = \sqrt{R_{\text{shunt}}P_{\text{wall}}} \]

where \( K = \frac{I_{dc}}{2} \sqrt{\frac{R_{\text{shunt}}}{P_{\text{gen}}}} \) is the "loading factor".

- \( V_{\text{acc}} \) decreases linearly with increasing beam current

![Image of RF envelope and bunch train](image-url)
Power flow in standing wave linac

Equilibrium value with $I_{beam}$

Inject beam at this time

Time
Efficiency of the standing wave linac

\[ \eta = \frac{I_{dc} V_{acc}}{P_{gen}} = \frac{2\sqrt{\beta}}{1 + \beta} \left[ 2K \left( 1 - \frac{K}{\sqrt{\beta}} \right) \right] \]
Schematic of energy flow in a standing wave structure
What makes SC RF attractive?
## Comparison of SC and NC RF

<table>
<thead>
<tr>
<th>Superconducting RF</th>
<th>Normal Conductivity RF</th>
</tr>
</thead>
<tbody>
<tr>
<td>● High gradient</td>
<td>● High gradient</td>
</tr>
<tr>
<td>==&gt; 1 GHz, meticulous care</td>
<td>==&gt; high frequency (5 - 17 GHz)</td>
</tr>
<tr>
<td>● Mid-frequencies</td>
<td>● High frequency</td>
</tr>
<tr>
<td>==&gt; Large stored energy, $E_s$</td>
<td>==&gt; low stored energy</td>
</tr>
<tr>
<td>● Large $E_s$</td>
<td>● Low $E_s$</td>
</tr>
<tr>
<td>==&gt; very small $\Delta E/E$</td>
<td>==&gt; ~10x larger $\Delta E/E$</td>
</tr>
<tr>
<td>● Large Q</td>
<td>● Low Q</td>
</tr>
<tr>
<td>==&gt; high efficiency</td>
<td>==&gt; reduced efficiency</td>
</tr>
</tbody>
</table>
Recall the circuit analog

As $R_{surf} \to 0$, the $Q \to \infty$.

In practice,

$$Q_{nc} \sim 10^4 \quad \quad \quad Q_{sc} \sim 10^{11}$$
Resistive input (shunt) impedance at $\omega_o$ relates power dissipated in walls to accelerating voltage

$$R_{in} = \frac{\langle V^2(t) \rangle}{P} = \frac{V_o^2}{2P} = Q\sqrt{\frac{L}{C}}$$

Linac literature more commonly defines “shunt impedance” without the “2”

$$R_{in} = \frac{V_o^2}{P} \sim \frac{1}{R_{surf}}$$

For SC-rf $P$ is reduced by orders of magnitude

**BUT, it is deposited @ 2K**
Why do we need beams?

Collide beams

FOMs: Collision rate, energy stability, Accelerating field

Examples: LHC, ILC, RHIC
In LHC storage rings...

- Limited space & Large rf trapping of particles
  - \( V/cavity \) must be high

- Bunch length must be large (\( \leq 1 \) event/cm in luminous region)
  - RF frequency must be low

- Energy lost in walls must be small
  - \( R_{surf} \) must be small

**SC cavities were the only practical choice**
For ILC SC rf provides high power, high quality beams at high efficiency

- To deliver required luminosity (500 fb⁻¹ in 4 years) ==>  
  - powerful polarized electron & positron beams (11 MW /beam)  
  - tiny beams at collision point ==> minimizing beam-structure interaction

- To limit power consumption ==> high “wall plug” to beam power efficiency  
  - Even with SC rf, the site power is still 230 MW !
Why do we need beams?

Intense secondary beams

FOM: Secondaries/primary
Examples: spallation neutrons, neutrino beams

1 MW target at SNS
1 MW @ 1GeV (compare with ILC 11 MW at 500 GeV (upgradeable to 4 MW))

==> miniscule beam loss into accelerator

==> large aperture in cavities ==> large cavities

==> low frequency

==> high energy stability

==> large stored energy

==> high efficiency at $E_z$

==> SC RF
Synchrotron light source
(pulsed incoherent X-ray emission)

FOM: Brilliance $v. \lambda$

$$B = \frac{ph}{s/mm^2/mrad^2/0.1\%BW}$$

Pulse duration

Science with X-rays
- Imaging
- Spectroscopy

US Particle Accelerator School
Matter to energy: Energy Recovery Linacs
Hard X-rays ==> ~5 GeV

Synchrotron light source
(pulsed incoherent X-ray emission)

Pulse rates – kHz => MHz
X-ray pulse duration ≤ 1 ps
High average e-beam brilliance
& e-beam duration ≤ 1 ps
⇒ One pass through ring
⇒ Recover beam energy
⇒ High efficiency
⇒ SC RF
Even higher brightness requires coherent emission $\Rightarrow$ FEL

Free electron laser

FOM: Brightness $\nu \lambda$
Time structure
Full range of FEL-based science requires...

- Pulses rate 10 Hz to 10 MHz (NC limited to ~ 100 Hz)
  - High efficiency

- Pulse duration 10 fs - 1 ps

- High gain
  - Excellent beam emittance
    ==> Minimize wakefield effect
    ==> large aperture
    ==> low frequency
  - Stable beam energy & intensity
    ==> large stored energy in cavities
    ==> high Q

==> SC RF
End of unit
Field measurement

Nylon line

Motor

Rf generator

Rf generator

Frequency detector

V

R