Unit 6 - Lecture 14

Wakefields

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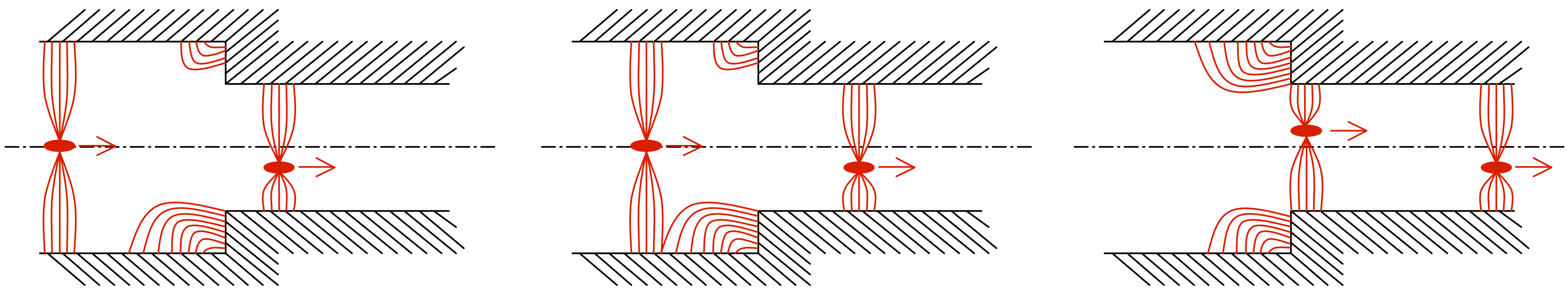
Source: Wake field slides are based on Sannibale lecture 9

Wakes are transient fields generated during the beam passage

- Duration depends on the geometry & material of the structure
- Case 1: Wake persists for the duration of a bunch passage
  - Particles in the tail can interact with wakes due to particles in the head.
  - *Single bunch instabilities* can be triggered
    - (distortion of the longitudinal distribution, bunch lengthening, transverse instabilities)

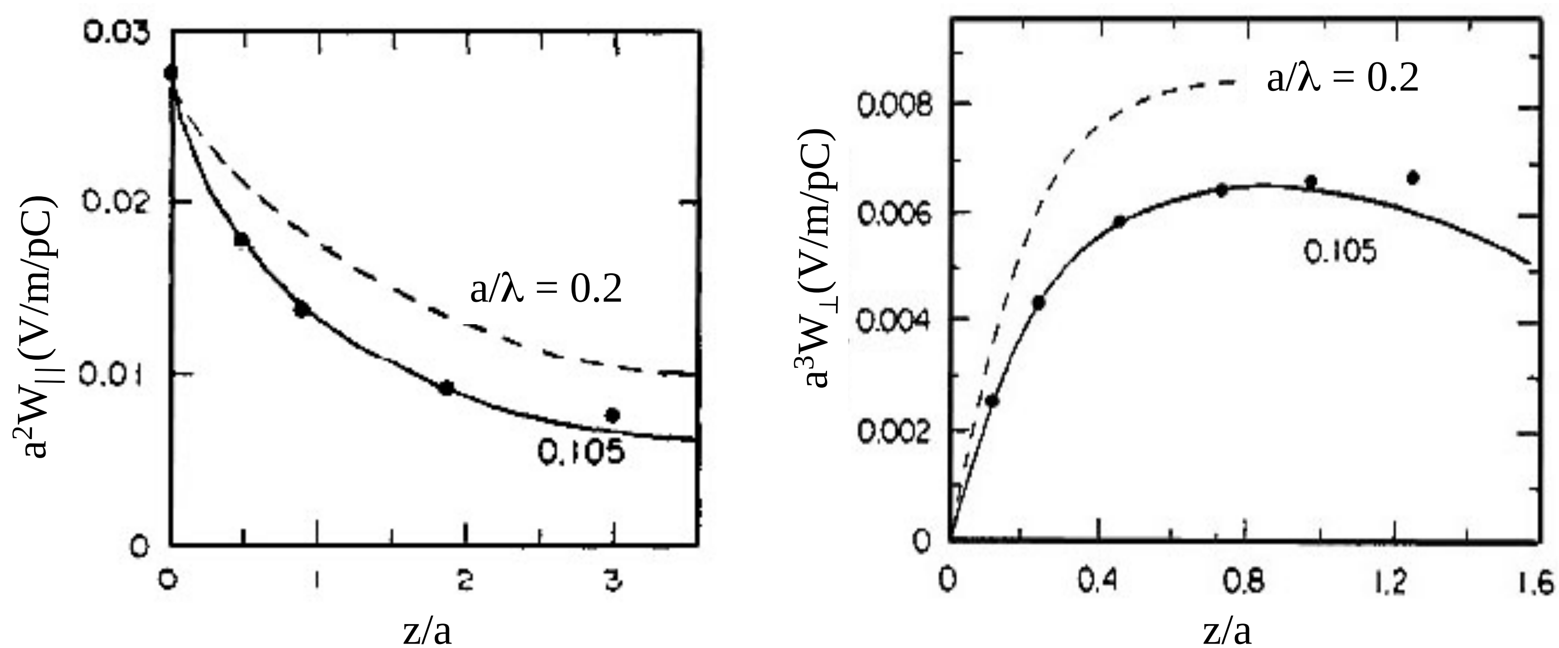


- Case 2: The wake field lasts longer than the time between bunches
  - Trailing bunches can interact with wakes from leading bunches to generate *multi-bunch bunch instabilities*

Scaling of wakefields with geometry & frequency in axisymmetric structures

For the disk-loaded waveguide structure (and typically)

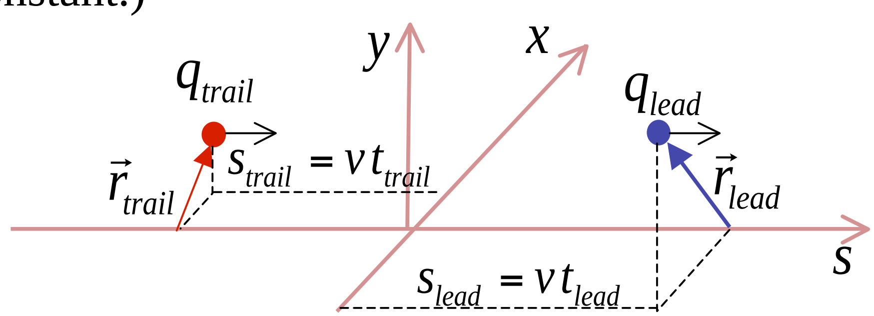
- Longitudinal wake field scales as $a^{-2} \sim \lambda_{rf}^{-2}$
- Transverse wakes scale as $a^{-3} \sim \lambda_{rf}^{-3}$



Wake Potentials

- Wake fields effects can be longitudinal or transverse.
  - ➞ Longitudinal wakes change the energy of beam particles
    - For longitudinal wakes it suffices to consider *only its electric field*
  - ➞ Transverse wakes affect beam particles' transverse momentum
- The wake potential is the energy variation induced by the wake field of the lead particle on a *unit charge* trailing particle

(Assume *v* constant.)



$$V_W\left(\vec{r}_{lead}, \vec{r}_{trail}, t_{trail} - t_{lead}\right) = \int_{-\infty}^{\infty} \vec{E}_W\left(s, \vec{r}_{lead}, \vec{r}_{trail}, t_{trail} - t_{lead}\right) \cdot d\vec{s}$$

Wake function is the wake potential induced per unit charge

$$W\left(\vec{r}_{lead},\vec{r}_{trail},t_{trail}-t_{lead}\right)=\frac{V_W\left(\vec{r}_{lead},\vec{r}_{trail},t_{trail}-t_{lead}\right)}{q_{lead}}$$

- For a bunch with charge distribution *i(**r**,t)*

$$\int i\left(\vec{r},t\right)\, d\vec{r}\,d\,t=Nq$$

the total energy variation that the trailing particle experiences due to the whole bunch is

$$V\left(\vec{r}_{trail},t_{trail}\right)=\int W\left(\vec{r}_{,}\;\vec{r}_{trail},t_{trail}-t\right)i\left(\vec{r},t\right)d\,\vec{r}\,d\,t$$

- In real accelerators, the transverse beam size << chamber aperture.
  - → It suffices to use the on-axis expression for the wakes (*monopole wake* approximation), using r and r_{trail} = 0 in the previous expressions.

Coupling Impedance

- The wake function describes the interaction of the beam with its external environment in the *time domain*

- The frequency domain “alter ego” of W is the coupling impedance (in Ohms) and defined as the *Fourier transform of the wake function*

$$Z(\vec{r},\vec{r}_{trail},\omega)=\int_{-\infty}^{\infty} W(\vec{r},\vec{r}_{trail},\tau)e^{-j\omega\tau}d\tau \qquad with \quad \tau = t_{trail}-t$$

- If *I* is the Fourier transform of the charge distribution, the Fourier transform of the total induced voltage is simply given by:

$$\tilde{V}(\vec{r},\vec{r}_{trail},\omega)=Z(\vec{r},\vec{r}_{trail},\omega)\; I(\vec{r},\omega)$$

- Then

$$V(\vec{r},\vec{r}_{trail},\tau)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\tilde{V}(\vec{r},\vec{r}_{trail},\omega)\,e^{j\omega\tau}\,d\omega$$

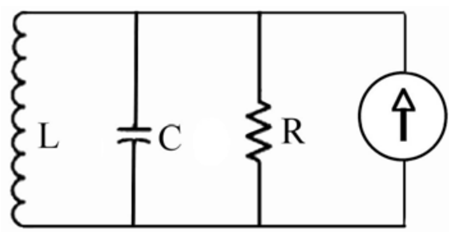
Interpretation of the coupling impedance

- The impedance is a complex quantity

$$Z(\vec{r},\vec{r}_{trail},\omega) = Z_R(\vec{r},\vec{r}_{trail},\omega) + j\, Z_j(\vec{r},\vec{r}_{trail},\omega)$$

  - ➞ Z_R is responsible for the energy losses
  - ➞ Z_j defines the phase between the beam response & exciting wake potential

- The impedance can be modeled by a parallel RLC model of the structure



$$Z(\omega) = \frac{R}{1 + jQ\left(\dfrac{\omega}{\omega_R} - \dfrac{\omega_R}{\omega}\right)}, \qquad \omega_R = \frac{1}{\sqrt{LC}}, \quad Q = R\sqrt{\frac{C}{L}}$$

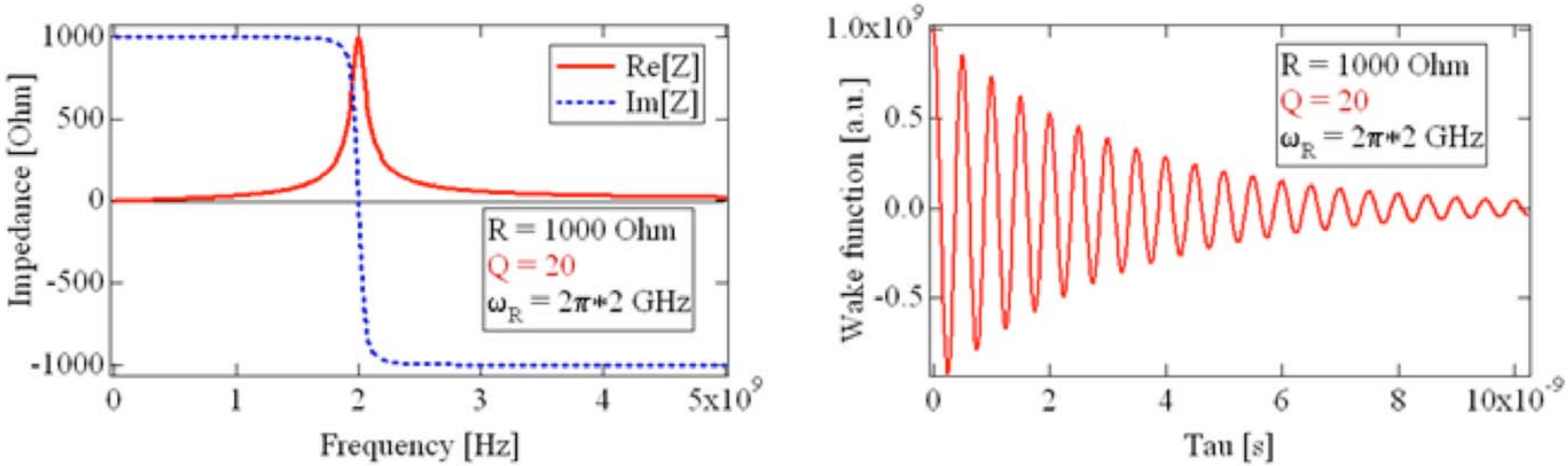
$$W(\tau) = \begin{cases} 0 & \tau < 0 \\ \dfrac{e^{-\omega_R \tau/2Q}}{C}\left[\cos\left(\omega_R\,\tau\,\sqrt{1-1/4Q^2}\right) - \dfrac{\sin\left(\omega_R\,\tau\,\sqrt{1-1/4Q^2}\right)}{\sqrt{4Q^2-1}}\right] & \tau > 0 \end{cases}$$

Narrow-band coupling impedances

- Narrow-band modes are characterized by moderate *Q* & narrow spectrum

  ==> Associated wake lasts for a relatively long time

  ==> Capable of exciting multi-bunch instabilities



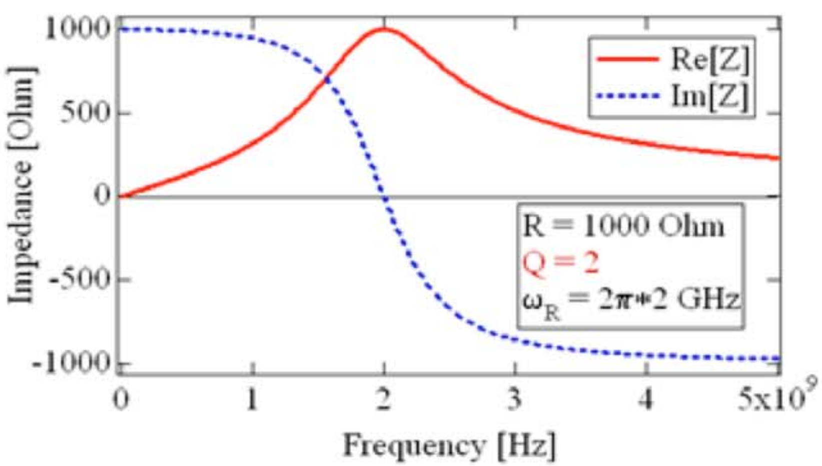
- Narrow band impedances are usually higher order modes of high Q accelerating structures

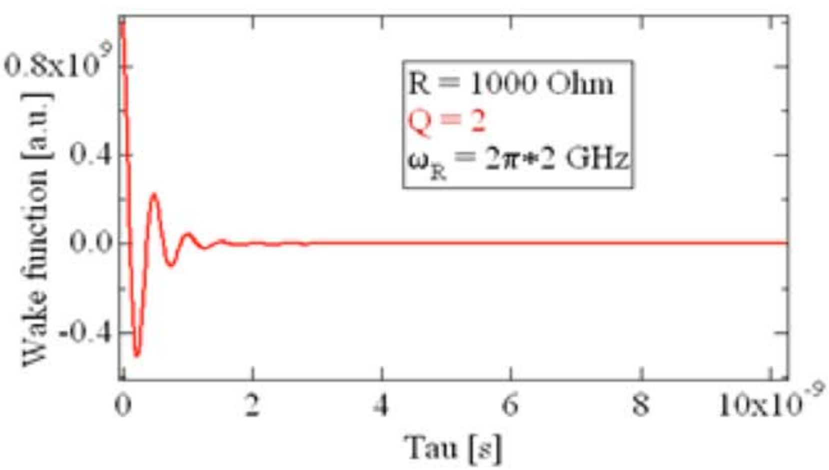
Broad band coupling impedances

- Broad-band impedance modes have a low Q and a broader spectrum.

  ==> The associated wake last for a relatively short time

  ==> Important only for single bunch instabilities





- Broad band impedances raise from irregularities or variations in the environment of the beam

Same approach applies to transverse wakes

- Transverse wake function is the transverse momentum kick per unit leading charge and unit trailing charge due to the wake fields

- Transverse wake fields are excited when the beam passes off center
  - For small displacements only the *dipole* term proportional to the displacement is important.
  - The *transverse dipole wake function* is the transverse wake function per unit displacement

- The transverse coupling impedance is defined as the Fourier transform of the transverse wake function times *j*

- Longitudinal and transverse wakes represent the same 3D wake field
  - Linked by Maxwell's equations.
  - The Panofsky-Wenzel relations allow one to calculate one wake component when the other is known.

Wakefields in real accelerators

- Accelerator vacuum chambers have complex shapes that include many components that can potentially host wake fields

- Not all wakes excited by the beam can be trapped in the chamber

- Given a chamber geometry, ∃ a cutoff frequency, f_{cutoff}
  - ➙ Modes with frequency > f_{cutoff} propagate along the chamber

$$f_{Cutoff} \approx \frac{c}{b} \qquad where\ \ b \equiv transverse\ chamber\ size$$

Categories of beam-induced wakefields

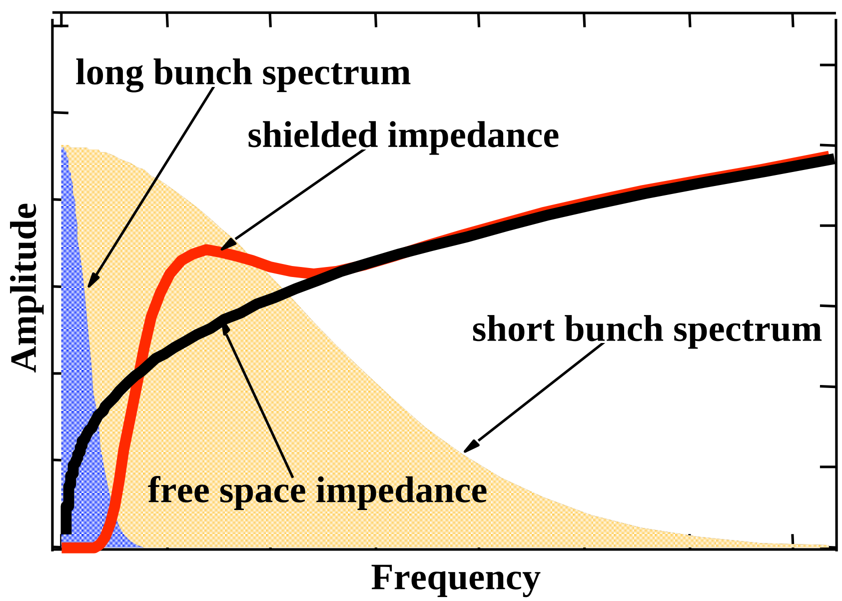
1. Wake fields that travels with the beam (e.g., the space charge)

2. Wake fields that are localized in some parts of the vacuum chamber (narrow and broad band

3. High frequency wakes > f_{cutoff} propagate inside the vacuum chamber.
   - Do not generate net interaction with the beam as long as they are not synchronous with the beam
   - A special case is synchrotron radiation which will be discussed later

When are Wakefields Dangerous?

- A wake is potentially dangerous only if it can be excited by the beam. Then, $V_{wake} \sim I_{bunch}$
  - ➙ If V_{wake} exceeds a threshold, it will trigger an instability
    - single bunch instability for broadband impedances
    - coupled bunch instability for narrowband impedances



- Impedance & beam power spectrum must overlap to allow energy transfer from beam to wake & conversely
- The larger the overlap the more dangerous is the wake
- Short bunches have a broader power spectrum than longer ones
  - ➙ bigger overlap with a wake impedance

Examples in linear accelerators

Even smooth structures can have wakes that can destabilize beams

Consider a long pulse of e⁻ moving through a smooth pipe of infinite σ.

The focusing magnets give a beam a periodic motion transversely with wave number, k_{β}.

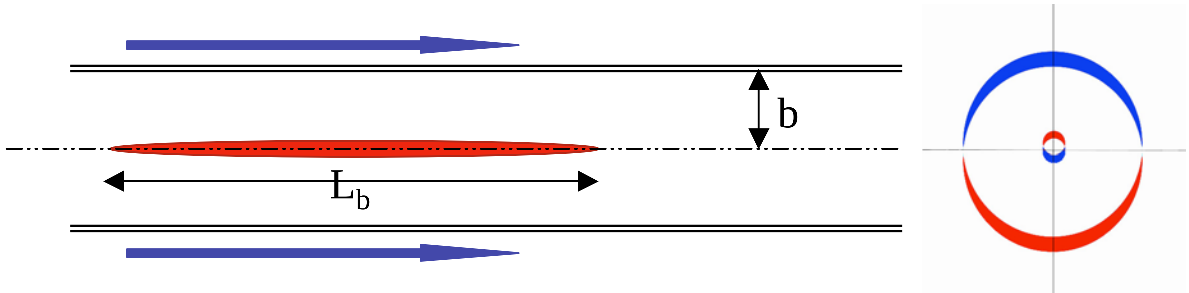


Image charges attract the beam to the wall

Image currents act to center the beam

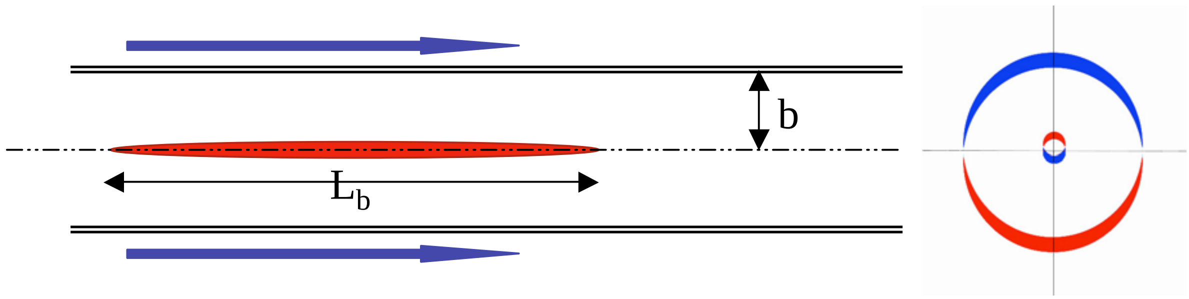
The forces cancel to a factor γ^{-2}

If the beam is off-center, focusing keeps its transverse motion bounded.

Transverse resistive wall instability

Now let the smooth pipe have finite conductivity, σ

As the pulse travels the image current diffuses into the pipe ~ $\sigma^{1/2}$



At a distance z along the pipe the initial displacement will grow as

$$\sim \exp\left[\left(z/L_{tr}\right)^{2/3}\right]$$

$$L_{tr} = \frac{2\gamma\beta I_A}{I}\sqrt{\frac{\pi\sigma_{pipe}}{\tau_{pulse}}}\frac{k_\beta b^3}{c}$$

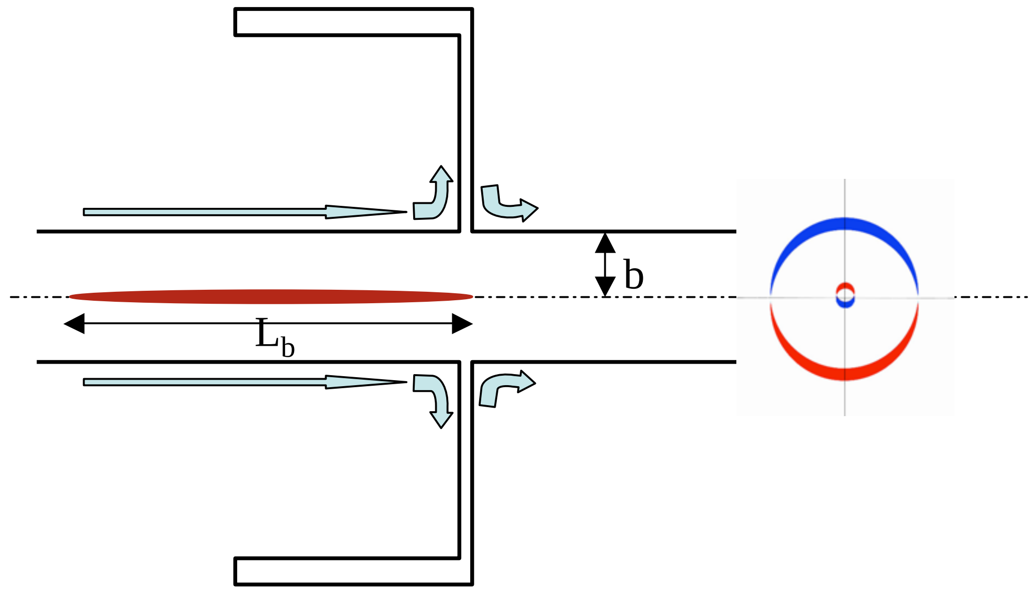
G. Caporaso, W. A. Barletta, V.K. Neil, Part. Accel., **11**, 71 (1980)

Simple example from induction linacs: Image Displacement Instability

Now add a accelerating gaps

At the gap, E_{image} is only slightly perturbed; the image current moves far away.

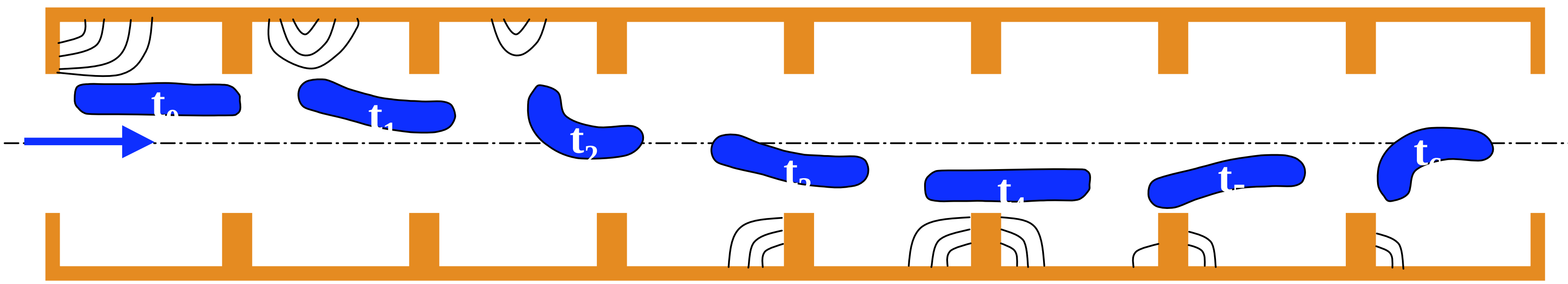
Therefore, the restoring magnetic force is absent at the gap



The displacement will grow exponentially even if σ is infinite

Beam Breakup Instability: High frequency version in rf-accelerators

- Bunch enters off-axis in a linac structure ==> transverse wakes
- Transverse wakes from the bunch head deflect the tail of the bunch
- In long linacs with high I_{bunch}, the effect amplifies distorting the bunch into a "banana" like shape. *(Single-bunch beam break up)*



Snapshots of a single bunch traversing a SLAC structure

- First observed in the 2-mile long SLAC linac

Coupled harmonic oscillator model of multi-bunch instabilities

Every mode is characterized by complex ω & by the damped oscillator equation:

$$\varphi_n(t) = \hat{\varphi}_n e^{-(\mathrm{Im}[\omega_n]+\alpha_D)\,t} \sin\left(\mathrm{Re}\left[\omega_n\right] t + \varphi_{n0}\right) \qquad \alpha_D \equiv \textit{radiation} \quad \textit{damping}$$

The oscillation becomes unstable (anti-damping) when:

$$\mathrm{Im}[\omega] + \alpha_D < 0 \qquad \left(\alpha_D > 0 \quad \textit{always}\right)$$

Wakes fields shift Im(ω):

$$\Delta\,\mathrm{Im}[\omega_n] \approx I_B \frac{e\alpha_C}{\nu_S E} Z(\omega_n)$$

Depending on the signs of momentum compaction, α_c, & the impedance Z(ω), some modes can become unstable when I per bunch is increased.

Feedback systems increase α_D ==> increase thresholds for the instabilities

The rest is Pathology