Bunch Compressors

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(based on a talk by P. Piot)
Why bunch compressors?

Light sources (single-pass FEL), linear colliders and advanced accelerator physics require high peak current compression.

Generate short bunch directly at the e-source:
- pulse DC e-source,
- X-band rf-gun,
- laser/plasma e-sources

Manipulate the bunch at a later stage during transport:
Select part of the bunch during the transport:
- collimator, beam spoiling,
- Laser slicing, etc…
Magnetic bunch compression

- Energy modulator: rf-structure, laser, wake-field
- Non-isochronous section
- In practice: multi-stage compression
To compress a bunch longitudinally, trajectory in dispersive region must be shorter for tail of the bunch than it is for the head.

\[ V = V_0 \sin(kz) \]

\[ D_z = R_{56} d \]

**RF Accelerating Voltage**

**Path-Length Energy-Dependent Beamline**
**Linear Effects**

- **Energy time correlation:**
  \[ E(z) = E_0 + eV_0 \cos(kz + \phi) \]

  \[ \delta = \frac{eV_0}{E_0 + eV_0 \cos \phi} \left[ \cos(kz + \phi) - \cos \phi \right] = \kappa z + O(z^2) \]

  - **chirp:** \( \kappa = \frac{d\delta}{dz} = \frac{-keV_0}{E_0 + eV_0 \cos \phi} \sin \phi \)

- **Bunch compressor**
  \[ z_f = z_i + R_{56} \delta_i \]

- **Final bunch length and energy spread (1st order):**
  \[ \sigma_{z,f} = \sqrt{\left(1 + \kappa R_{56}\right)^2 \sigma_{z,i}^2 + R_{56}^2 \sigma_{\delta,i}^2 \frac{E_0^2}{E^2}} \]
  \[ \sigma_{\delta,f} = \sqrt{\kappa^2 \sigma_{z,i}^2 + \sigma_{\delta,i}^2 \frac{E_0^2}{E^2}} \]

  - Min bunch length
Nonlinear effects

Energy time correlation:

$$\delta = \kappa z + \mu z^2 + O(z^3)$$

Bunch compressor

$$z_f = z_i + R_{56} \delta_i + T_{566} \delta_i^2$$

Final bunch length is minimized if

$$0 = z_i (1 + \kappa R_{56}) + z_i^2 (\mu R_{56} + \kappa^2 T_{566})$$

Limit achievable minimum Bunch length

2nd order momentum compaction
Main issues

- How short can the bunch be compressed?
- Can low emittance be maintained?
- How large are the effects of space charge and coherent synchrotron radiation in bunch compression?
Types of bunch compressors

- Chicane
  \[ R_{56} < 0, \ T_{566} > 0 \]

- S-chicane
  \[ R_{56} < 0, \ T_{566} > 0 \]

- F0D0 arc
  \[ R_{56} > 0, \ T_{566} > 0 \]

- Dog-leg
  \[ R_{56} > 0, \ T_{566} > 0 \]
Different types of bunch compressors

- **Chicane**: Simplest type with a 4-bending magnets for bunch compression

- **Double chicane**: $R_{56}$ is sum of the $R_{56}$ values for each chicane.

- **Wiggler type**: This type can be used when a large $R_{56}$ is required. It is also possible to locate quadrupole magnets between dipole magnets where dispersion passes through zero, allowing continuous focusing across these long systems.

- **Arc type**: $R_{56}$ can be conveniently adjusted by varying betatron phase advance per cell in the bend plane. The systems chromatic aberrations, introduce large beamline geometry excursions and produce many well aligned components.
When beam passes a bunch through a RF cavity on the zero crossing of the voltage (i.e. without acceleration)

\[ z_1 = z_0 \]

\[ \delta_1 = \delta_0 + \frac{eV_{RF}}{E_0} \cos\left(\frac{\pi}{2} - k_{RF}z_0\right) \]

\[ k_{rf} = 2pf_{rf}/c \]

In general, when reference particle crosses at some \( f_{rf} \) that is not be zero crossing.

Then reference energy of the beam is changed from \( E_0 \) to \( E_1 \).

\[ E_i = E_o (1 + \delta_0) \]
\[ E_f = E_1 (1 + \delta_1) = E_i + eV_{rf} \cos(\phi_{rf} - k_{rf}z_0) \]
\[ E_1 = E_o + eV_{rf} \cos(\phi_{rf}) \]

Then,

\[ \delta_1 = \frac{E_o (1 + \delta_0) + eV_{RF} \cos(\phi_{rf} - k_{rf}z_0)}{E_o + eV_{rf} \cos(\phi_{rf})} - 1 \]
Longitudinal particle motion in bunch compressor: matrix formalism

To first order in $eV_{\text{rf}}/E_0 \ll 1$,

$$z_1 = z_0$$

$$\delta_1 = \delta_0 \left( 1 - \frac{eV_{RF} \cos(\phi_{rf})}{E_0} \right) + \frac{eV_{RF}}{E_0} \left( \cos(\phi_{rf} - k_{rf} z_0) - \cos(\phi_{rf}) \right)$$

In a linear approximation for RF,

$$\begin{pmatrix} z_1 \\ \delta_1 \end{pmatrix} \approx \begin{pmatrix} 1 & 0 \\ R_{65} & R_{66} \end{pmatrix} \begin{pmatrix} z_0 \\ \delta_0 \end{pmatrix}$$

$$R_{65} = \frac{eV_{RF}}{E_0} \sin(\phi_{RF}) k_{RF}$$

$$R_{66} = 1 - \frac{eV_{RF}}{E_0} \cos(\phi_{RF})$$
In a wiggler (or chicane),
\[ z_2 = z_1 + R_{56} \delta_1 + T_{566} \delta_1^2 + U_{5666} \delta_1^3 \ldots \]
\[ \delta_2 = \delta_1 \]

In a linear approximation \( T_{566} \delta_1 \ll R_{56} \),
\[
\begin{pmatrix}
  z_2 \\
  \delta_2
\end{pmatrix}
\approx
\begin{pmatrix}
  1 & R_{56} \\
  0 & 1
\end{pmatrix}
\begin{pmatrix}
  z_1 \\
  \delta_1
\end{pmatrix}
\]

Total transformation
\[
\begin{pmatrix}
  z_2 \\
  \delta_2
\end{pmatrix}
\approx
M \cdot \begin{pmatrix}
  z_0 \\
  \delta_0
\end{pmatrix}
\]
\[
M = \begin{pmatrix}
  1 + R_{65} R_{56} & R_{56} R_{66} \\
  R_{65} & R_{66}
\end{pmatrix}
\]

For \( f_{\text{rf}} = \frac{p}{2} \) (i.e. no acceleration), \( R_{66} = 1 \), the transformation matrix is sympletic, which means that longitudinal emittance is a conserved quantity.
\[
\varepsilon = \sqrt{\sigma_z^2 \sigma_\delta^2 - s_{z\delta}^2}, \quad \sigma_z^2 = \langle z^2 \rangle >\beta\varepsilon, \sigma_\delta^2 = \langle \delta^2 \rangle >\gamma\varepsilon, \sigma_{z\delta} = \langle z\delta \rangle >\alpha\varepsilon
\]
Zeuthen Chicane

- Zeuthen Chicane: a benchmark layout used for CSR calculation comparisons at 2002 ICFA beam dynamics workshop

- Bend magnet length: \( L_B = 0.5 \text{m} \)
- Drift length B1-B2 and B3-B4 (projected): \( DL = 5 \text{m} \)
- Drift length B2-B3: \( DL_c = 1 \text{m} \)
- Bend radius: \( r = 10.3 \text{m} \)
- Effective total chicane length (\( L_T - DL_c \)): \( 12 \text{m} \)
- Bending angle: \( q_o = 2.77 \text{ deg} \)
- Bunch charge: \( q = 1 \text{nC} \)
- Momentum compaction: \( R_{56} = -25 \text{ mm} \)
- Electron energy: \( E = 5 \text{ GeV} \)
- 2\(^{nd}\) order momentum compaction: \( T_{566} = 38 \text{ mm} \)
- Initial bunch length: \( 0.2 \text{ mm} \)
- Total projected length of chicane: \( L_T = 13 \text{ m} \)
- Final bunch length: \( 0.02 \text{ mm} \)
If a particle at reference energy is bent by $q_o$, a particle with relative energy error $d$ is bent by $q = q_o/(1+d)$.

Path length from first to final dipoles is

$$s = \frac{2a}{\cos(\theta)} + b = 2a \left[ \cos\left(\frac{\theta_o}{1+\delta}\right)\right]^{-1} + b \approx 2a + a\left(\frac{\theta_o}{1+\delta}\right) + b$$

$$R_{56} = \frac{ds}{d\delta}_{\delta=0} = -2a\theta_o^2$$
Path length in a chicane is

$$\Delta s = s(\delta) - s(\delta = 0) \approx a \left( \frac{\theta_0}{1 + \delta} \right)^2 - a\theta_0^2 = \frac{1}{2} R_{56} \left( 1 - \frac{1}{(1 + \delta)^2} \right)$$

By performing a Taylor expansion about $d=0$

$$\Delta s \approx R_{56} \delta - \frac{3}{2} R_{56} \delta^2 + 2 R_{56} \delta^3 - \ldots$$

$$T_{566} \approx -\frac{3}{2} R_{56} \quad U_{5666} \approx 2 R_{56}$$

For large $d$, $d^2$ and $d^3$ terms may cause non-linear deformations of the phase space during compression.
Ballistic bunch compression

- Usually used at very low energy, typically downstream of DC-gun
- Can be viewed as thin lens limit of velocity bunching
- Buncher imparts an energy chirp large enough to yield compression in a downstream drift

Buncher cavity

Drift space

\[ R_{56} = -\frac{L}{\gamma^2} \]
SDL Experiment

(P. Piot, et al PRSTAB(6) 033503 (2003))

Short Bunches in Accelerators– USPAS, Boston, MA 21-25 June 2010
Compressed beam to 0.4 ps for 200 pC
Operating 1\textsuperscript{st} linac far off-crest spoils ε compensation process
• Calculate bunch compression for the Zeuthen Chicane
  – Calculate R56
  – Compute