Bunch Compressors

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(based on a talk by P. Piot)

Why bunch compressors?

Light sources (single-pass FEL), linear colliders and advanced accelerator physics require high peak current

Generate short bunch directly at the e- source:
- *pulse DC e- source,*
- *X-band rf-gun,*
- *laser/plasma e- sources*

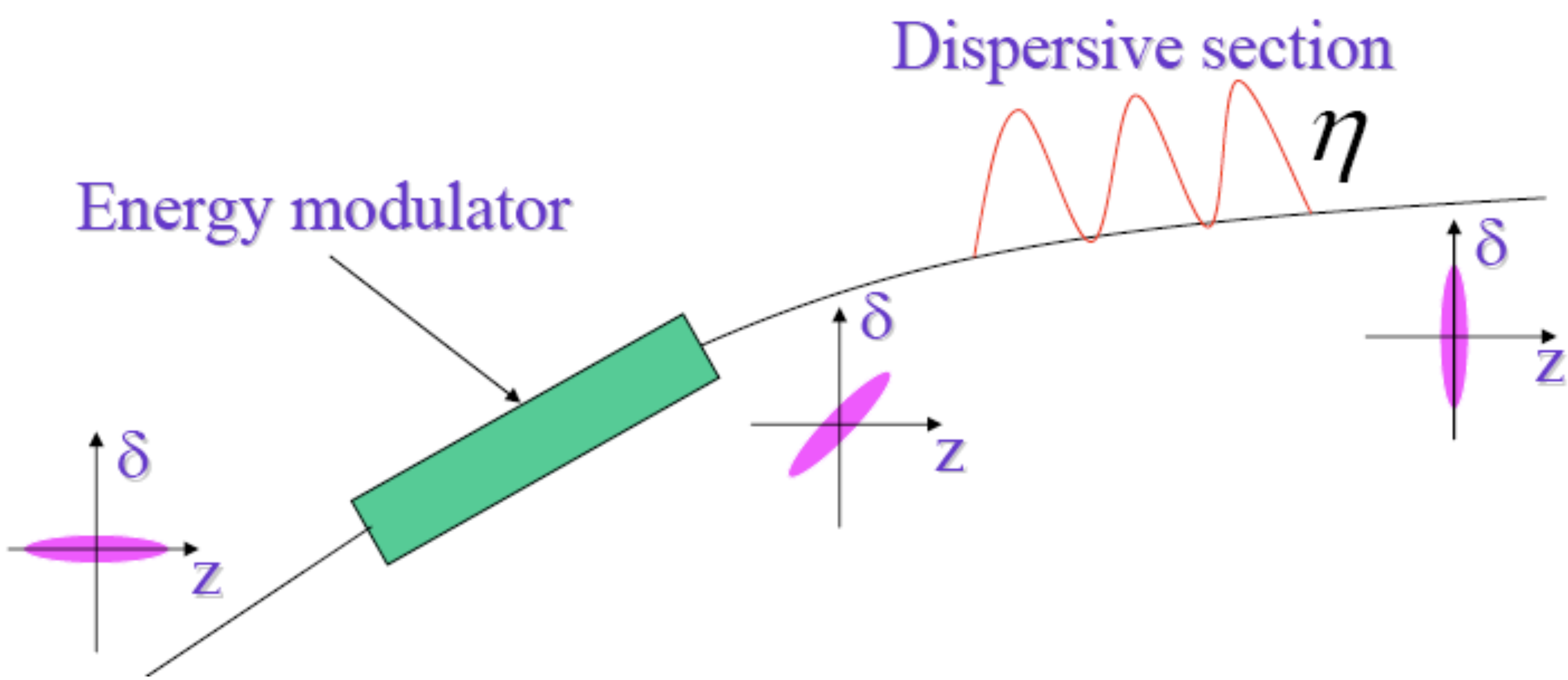
Select part of the bunch during the transport:
- *collimator, beam spoiling,*
- *Laser slicing, etc…*

compression

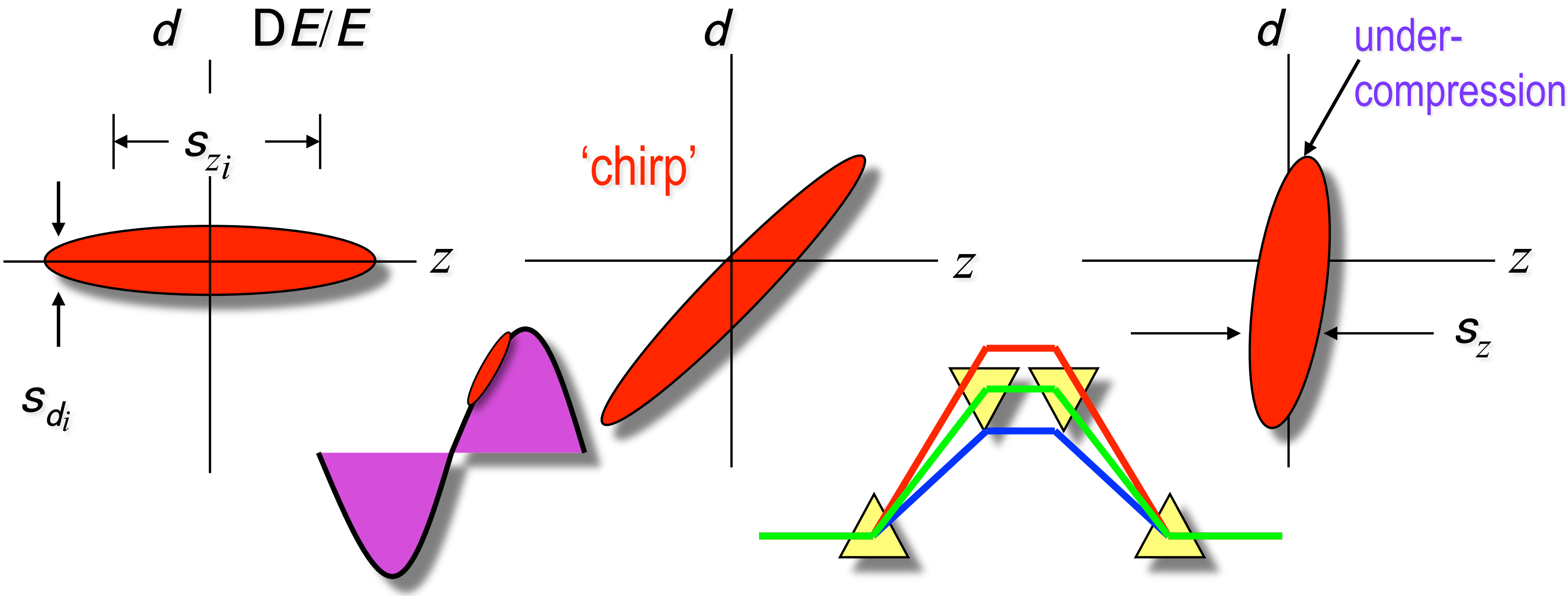
Manipulate the bunch at a later stage during transport

Magnetic bunch compression

- Energy modulator: rf-structure, laser, wake-field
- Non-isochronous section
- In practice: multi-stage compression



Chicane Bunch Compression



$$V = V_0 \sin(kz)$$

$$\mathrm{D}z = R_{56}\,d$$

RF Accelerating Voltage

Path-Length Energy-Dependent Beamline

To compress a bunch longitudinally, trajectory in dispersive region must be shorter for tail of the bunch than it is for the head.

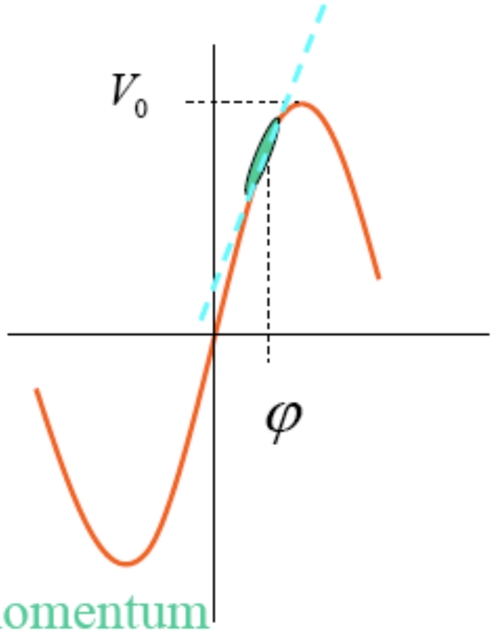
Linear Effects

- Energy time correlation:

$$E(z) = E_0 + eV_0 \cos(kz + \varphi)$$

$$\delta = \frac{eV_0}{E_0 + eV_0 \cos\varphi}\left[\cos(kz+\varphi) - \cos\varphi\right] = \kappa z + O(z^2)$$

chirp: $\kappa \equiv \dfrac{d\delta}{dz} = \dfrac{-keV_0}{E_0 + eV_0\cos\varphi}\sin\varphi$



- Bunch compressor

1st order momentum compaction

$$z_f = z_i + R_{56}\delta_i$$

- Final bunch length and energy spread (1st order):

$$\sigma_{z,f} = \sqrt{(1+\kappa R_{56})^2\sigma_{z,i}^2 + \underbrace{R_{56}^2\sigma_{\delta,i}^2 E_0^2 / E^2}_{\text{Min bunch length}}}, \quad \sigma_{\delta,f} = \sqrt{\kappa^2\sigma_{z,i}^2 + \sigma_{\delta,i}^2 E_0^2 / E^2}$$

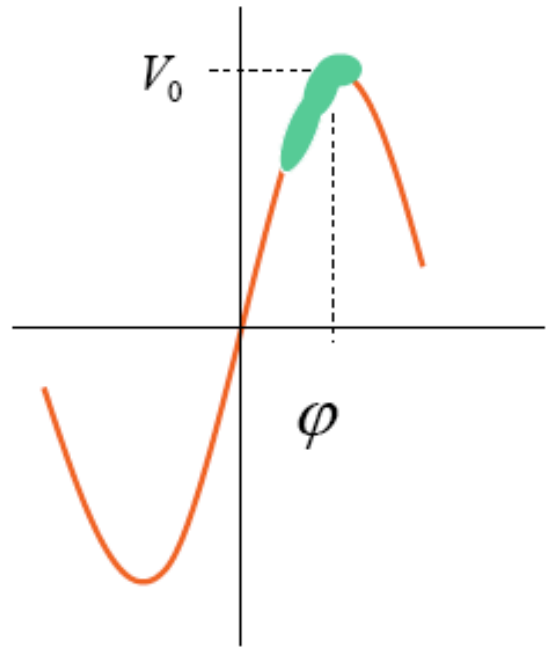
Nonlinear effects

- Energy time correlation:

$$\delta = \kappa z + \mu z^2 + O(z^3)$$

- Bunch compressor

$$z_f = z_i + R_{56}\delta_i + T_{566}\delta_i^2$$



- Final bunch length is minimized if

$$0 = z_i(1+\kappa R_{56}) + \underbrace{z_i^2(\mu R_{56} + \kappa^2 T_{566})}_{\text{Limit achievable minimum Bunch length}}$$

2nd order momentum compaction

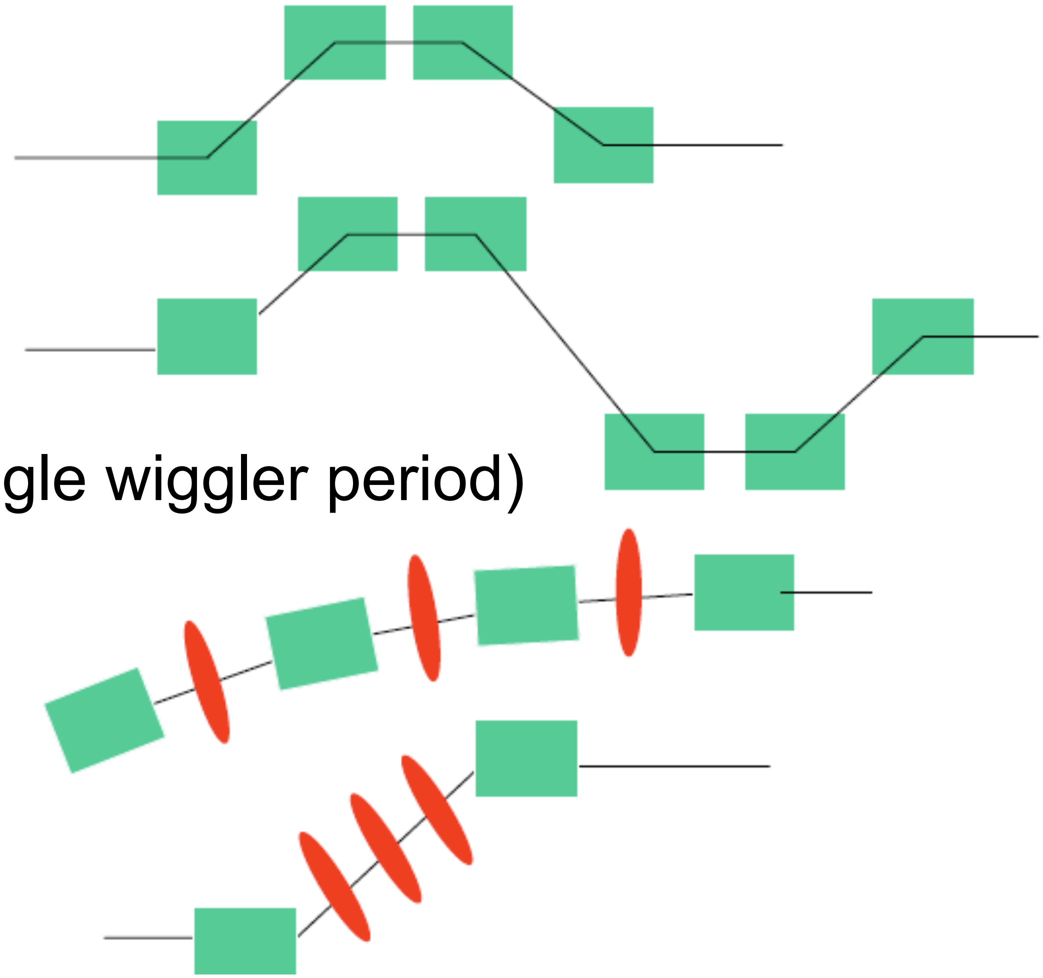
Main issues

- How short can the bunch be compressed?

- Can low emittance be maintained?

- How large are the effects of space charge and coherent synchrotron radiation in bunch compression?

Types of bunch compressors



(Single wiggler period)

- Chicane
  $R_{56}<0$, $T_{566}>0$
- S-chicane
  $R_{56}<0$, $T_{566}>0$
- F0D0 arc
  $R_{56}>0$, $T_{566}>0$
- Dog-leg
  $R_{56}>0$, $T_{566}>0$

Different types of bunch compressors

- **Chicane** : Simplest type with a 4-bending magnets for bunch compression

- **Double chicane** : R_{56} is sum of the R_{56} values for each chicane.

- **Wiggler type** : This type can be used when a large R_{56} is required, It is also possible to locate quadrupole magnets between dipole magnets where dispersion passes through zero, allowing continuous focusing across these long systems.

- **Arc type** : R_{56} can be conveniently adjusted by varying betatron phase advance per cell in the bend plane. The systems chromatic aberrations, introduce large beamline geometry excursions and produce many well aligned components.

Longitudinal particle motion in bunch compressor: matrix formalism

When beam passes a bunch through a RF cavity on the zero crossing of the voltage (i.e. without acceleration)

$$z_1 = z_0$$

$$\delta_1 = \delta_0 + \frac{eV_{RF}}{E_0}\cos\left(\frac{\pi}{2} - k_{RF} z_0\right)$$

$k_{rf} = 2pf_{rf}/c$

In general, when reference particle crosses at some f_{rf} that is not be zero crossing. Then reference energy of the beam is changed from E_o to E_1.

$$E_i = E_o(1+\delta_0)$$

$$E_f = E_1(1+\delta_1) = E_i + eV_{rf}\cos(\varphi_{rf} - k_{rf} z_o)$$

$$E_1 = E_o + eV_{rf}\cos(\varphi_{rf})$$

Then,

$$\delta_1 = \frac{E_o(1+\delta_0) + eV_{RF}\cos(\phi_{rf} - k_{rf} z_o)}{E_0 + eV_{rf}\cos(\phi_{rf})} - 1$$

Longitudinal particle motion in bunch compressor: matrix formalism

To first order in $eV_{rf}/E_o << 1$,

$$z_1 = z_0$$

$$\delta_1 = \delta_0\left(1 - \frac{eV_{RF}\cos(\phi_{rf})}{E_0}\right) + \frac{eV_{RF}}{E_0}\left(\cos(\phi_{rf} - k_{rf} z_o) - \cos(\phi_{rf})\right)$$

In a linear approximation for RF,

$$\begin{pmatrix} z_1 \\ \delta_1 \end{pmatrix} \approx \begin{pmatrix} 1 & 0 \\ R_{65} & R_{66} \end{pmatrix} \cdot \begin{pmatrix} z_0 \\ \delta_0 \end{pmatrix}$$

$$R_{65} = \frac{eV_{RF}}{E_0}\sin(\phi_{RF})k_{RF}$$

$$R_{66} = 1 - \frac{eV_{RF}}{E_0}\cos(\phi_{RF})$$

Longitudinal particle motion in bunch compressor: matrix formalism

In a wiggler (or chicane),

$$z_2 = z_1 + R_{56}\delta_1 + T_{566}\delta_1^2 + U_{5666}\delta_1^3 \ldots$$
$$\delta_2 = \delta_1$$

In a linear approximation T_{566}d$_1$ << R_{56},

$$\begin{pmatrix} z_2 \\ \delta_2 \end{pmatrix} \approx \begin{pmatrix} 1 & R_{56} \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} z_1 \\ \delta_1 \end{pmatrix}$$

Total transformation

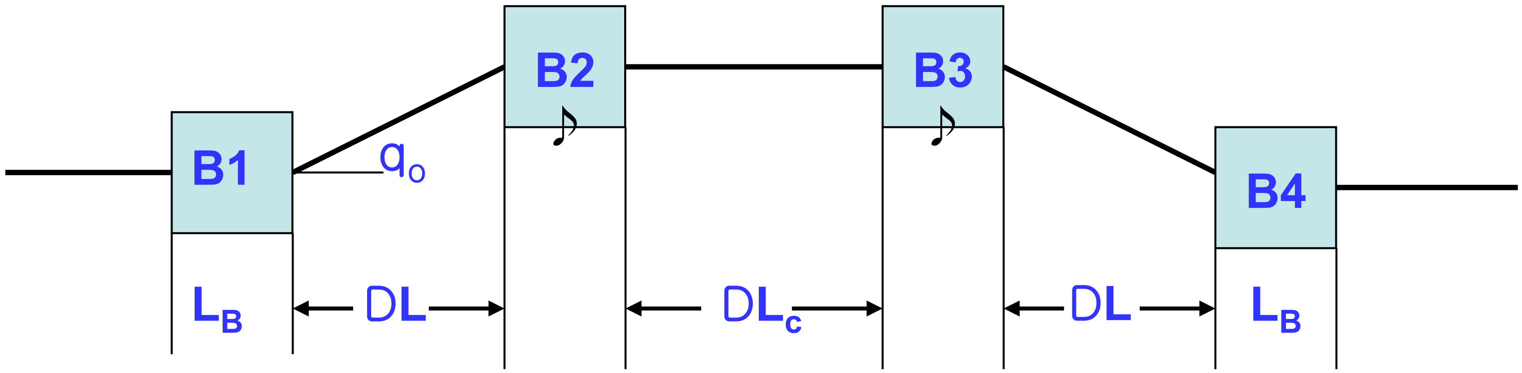
$$\begin{pmatrix} z_2 \\ \delta_2 \end{pmatrix} \approx \mathbf{M} \cdot \begin{pmatrix} z_0 \\ \delta_0 \end{pmatrix} \qquad \mathbf{M} = \begin{pmatrix} 1 + R_{65}R_{56} & R_{56}R_{66} \\ R_{65} & R_{66} \end{pmatrix}$$

For f$_{rf}$= [illegible] p/2 (i.e. no acceleration), R_{66}=1, the transformation matrix is sympletic, which means that longitudinal emittance is a conserved quantitiy.

$$\varepsilon = \sqrt{\sigma_z^2\sigma_\delta^2 - s_{z\delta}^2}, \sigma_z^2 =< z^2 >= \beta\varepsilon, \sigma_\delta^2 =< \delta^2 >= \gamma\varepsilon, \sigma_{z\delta} =< z\delta >= \alpha\varepsilon$$

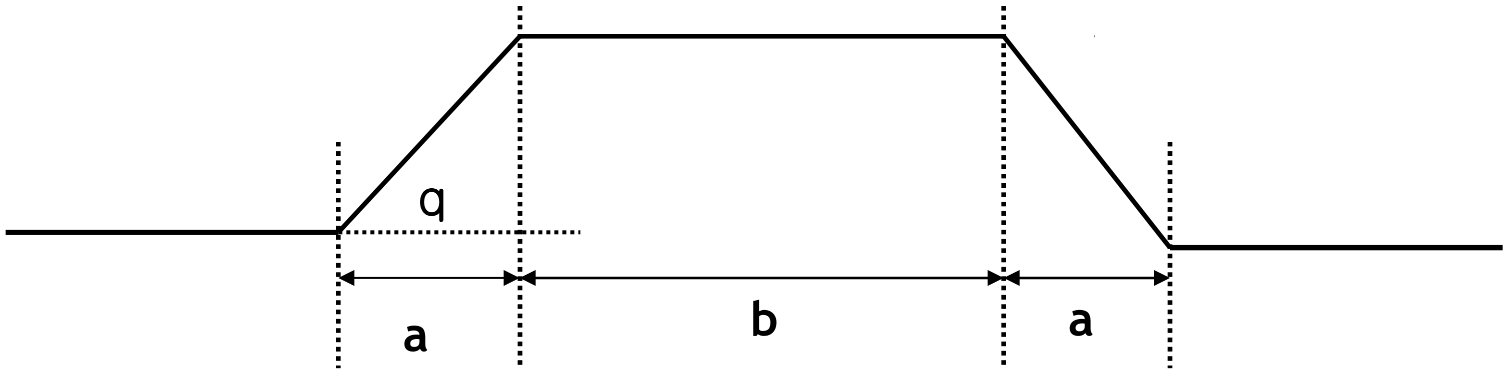
Zeuthen Chicane

- **Zeuthen Chicane : a benchmark layout used for CSR calculation comparisons at 2002 ICFA beam dynamics workshop**



- **Bend magnet length : L_B = 0.5m**
- **Drift length B1-B2 and B3-B4(projected) : DL = 5m**
- **Drift length B2-B3 : DL_c = 1m**
- **Bend radius : r = 10.3m**
- **Effective total chicane length (L_T-DL_c) = 12m**
- **Bending angle : q_o = 2.77 deg** **Bunch charge : q = 1nC**
- **Momentum compaction : R_{56} = -25 mm** **Electron energy : E = 5 GeV**
- **2nd order momentum compaction : T_{566} = 38 mm** **Initial bunch length : 0.2 mm**
- **Total projected length of chicane : L_T = 13 m** **Final bunch length : 0.02 mm**

Path length dispersion in Chicane



If a particle at reference energy is bent by q_o, a particle with relativ e energy error d is bent by $q=q_o/(1+d)$.

Path length from first to final dipoles is

$$s = \frac{2a}{\cos(\theta)} + b = 2a\left[\cos\left(\frac{\theta_o}{1+\delta}\right)\right]^{-1} + b \approx 2a + a\left(\frac{\theta_o}{1+\delta}\right) + b$$

$$R_{56} = \frac{ds}{d\delta}_{\delta=0} = -2a\theta_o^2$$

Path length dispersion in Chicane

Path length in a chicane is

$$\Delta s = s(\delta) - s(\delta = 0) \approx a\left(\frac{\theta_o}{1+\delta}\right)^2 - a\theta_o^2 = \frac{1}{2}R_{56}\left(1 - \frac{1}{(1+\delta)^2}\right)$$

By performing a Taylor expansion about d=0

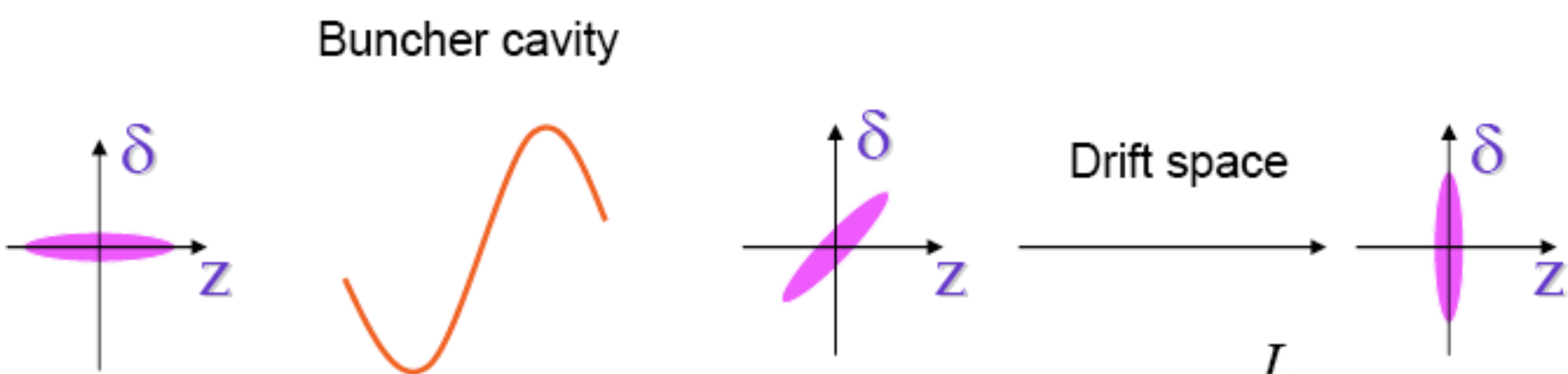
$$\Delta s \approx R_{56}\delta - \frac{3}{2}R_{56}\delta^2 + 2R_{56}\delta^3 - \ldots\ldots$$

$$T_{566} \approx -\frac{3}{2}R_{56} \qquad\qquad U_{5666} \approx 2R_{56}$$

For large d, d^2 and d^3 terms may cause non-linear deformations of the phase space during compression.

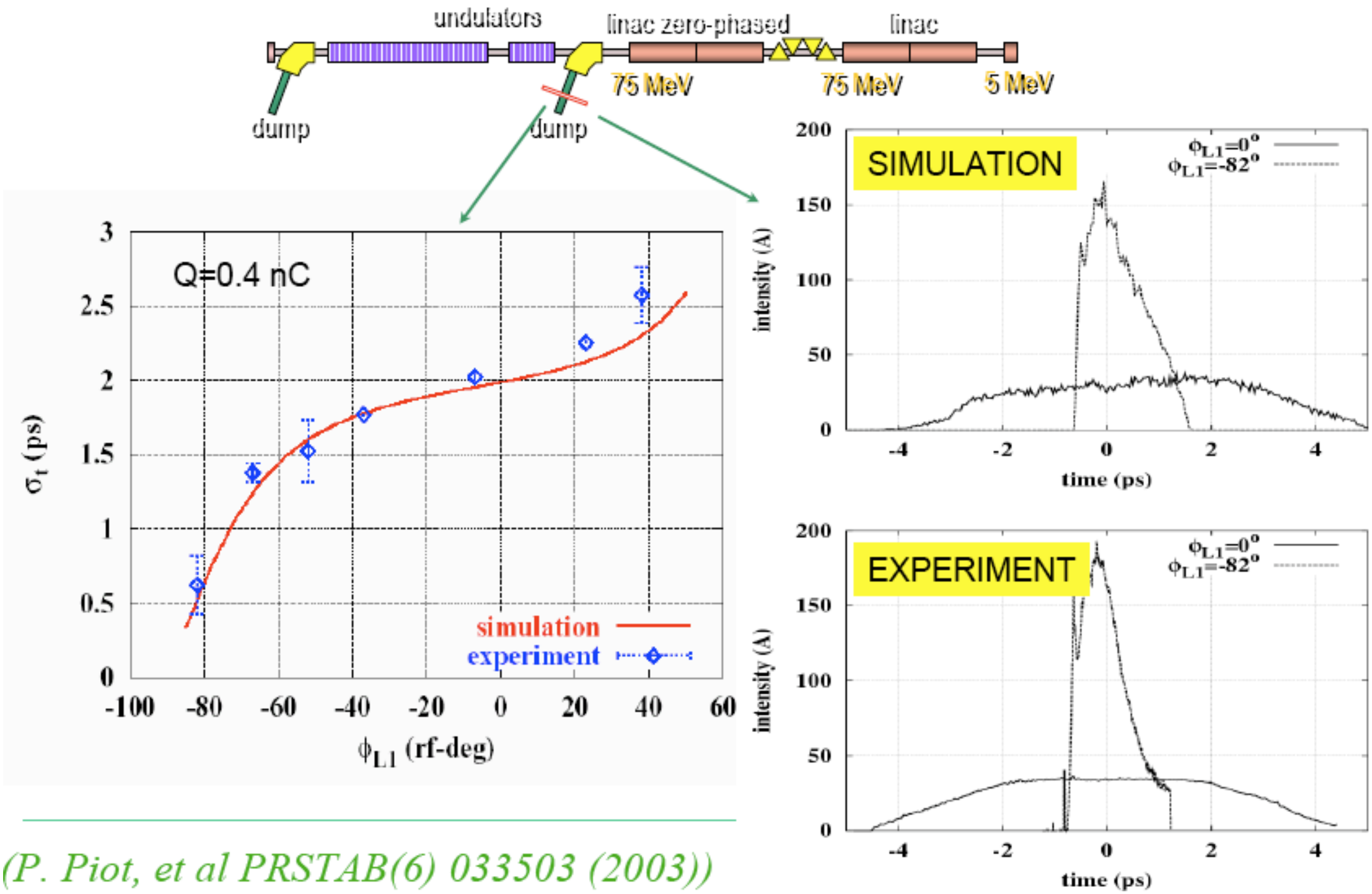
Ballistic bunch compression

- Usually used at very low energy, typically downstream of DC-gun
- Can be viewed as thin lens limit of velocity bunching
- Buncher imparts an energy chirp large enough to yield compression in a downstream drift



$$R_{56} = -\frac{L}{\gamma^2}$$

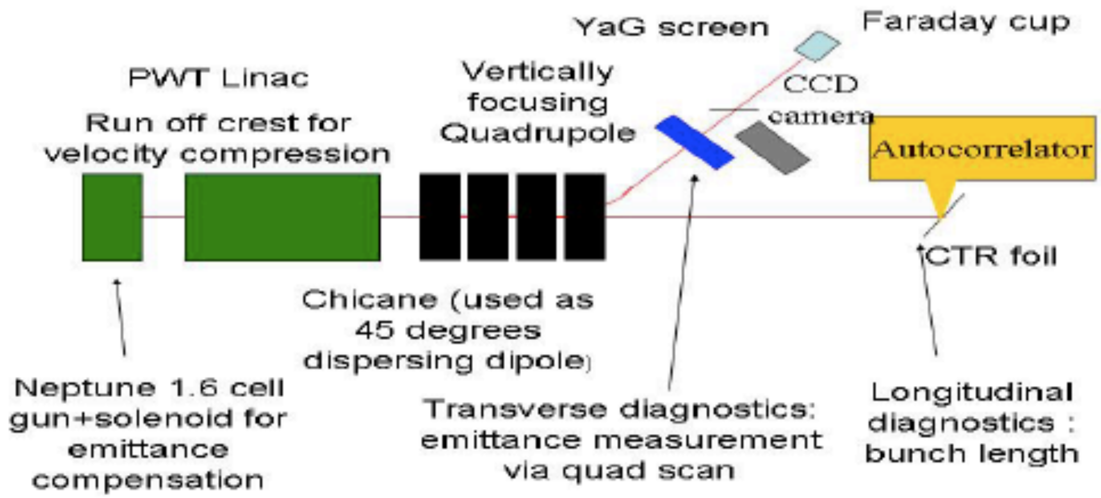
SDL Experiment

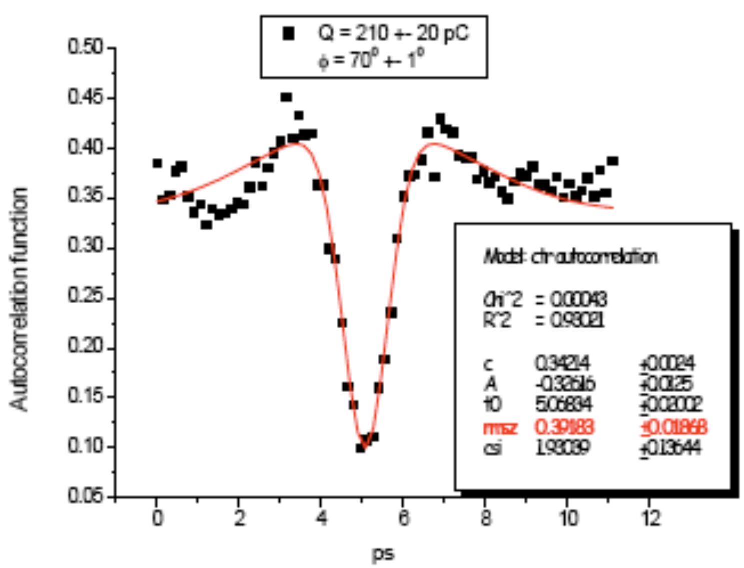


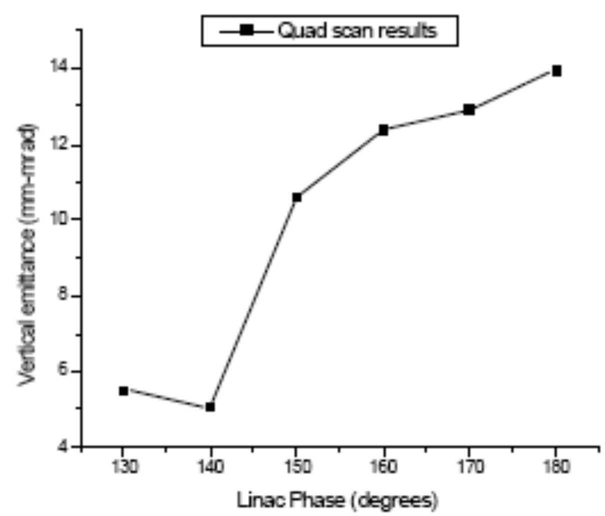
(P. Piot, et al PRSTAB(6) 033503 (2003))

UCLA Experiment

- Compressed beam to 0.4 ps for 200 pC
- Operating 1st linac far off-crest spoils ε compensation process







Homework

- Calculate bunch compression for the Zeuthen Chicane
  - Calculate R56
  - Compute