Frequency Resolved Optical Gating (FROG)

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A FROG is a spectrogram.

If $E(t)$ is the waveform of interest, its spectrogram is:

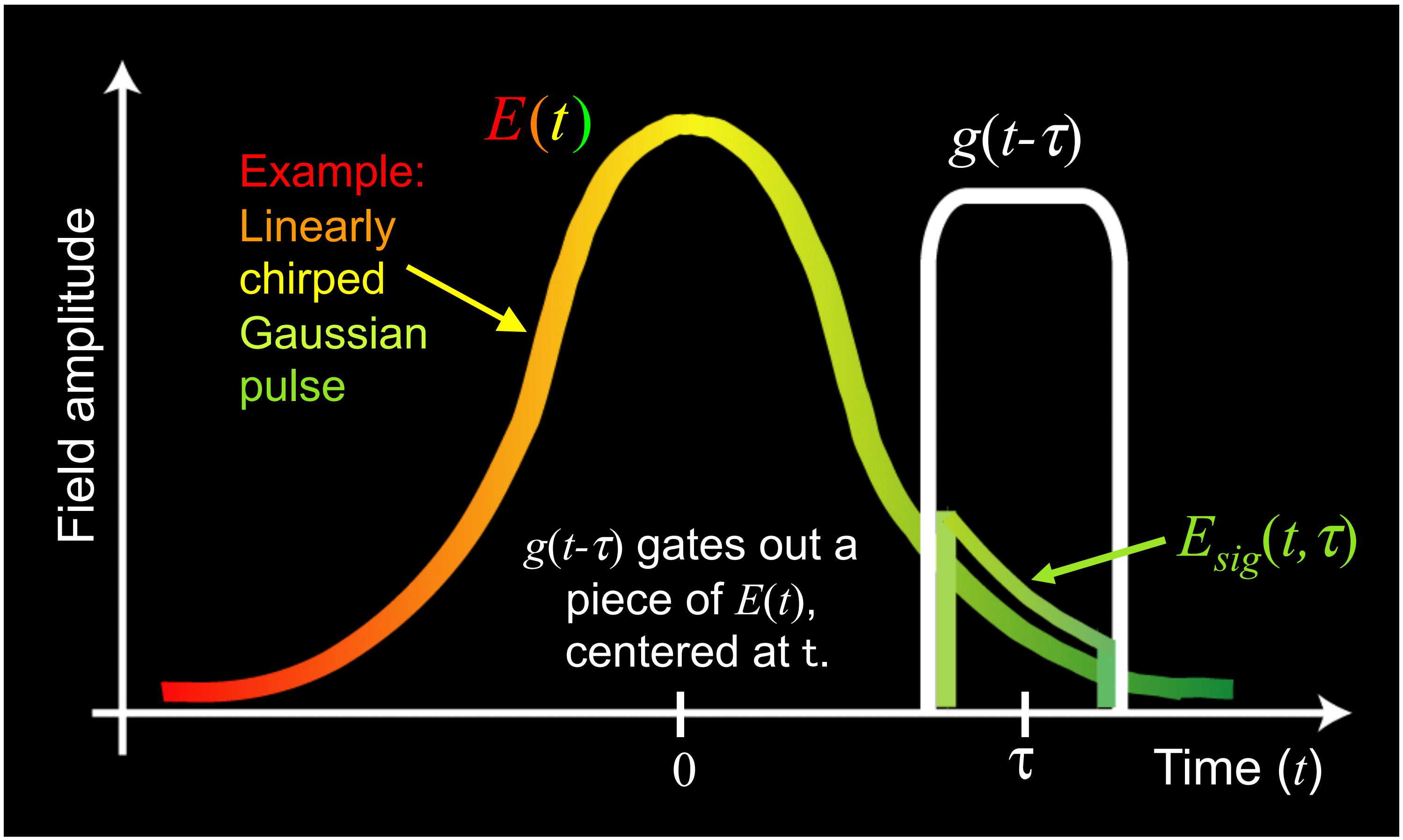
$$\Sigma_E(\omega,\tau) \equiv \left| \int_{-\infty}^{\infty} E(t)\, g(t-\tau)\, \exp(-i\omega t)\, dt \right|^2$$

where $g(t\text{-}\tau)$ is a variable-delay gate function and τ is the delay.

Without $g(t\text{-}\tau)$, $\Sigma_E(\omega,\tau)$ would simply be the spectrum.

The spectrogram is a function of ω and τ.
It is the set of spectra of all temporal slices of $E(t)$.

The Spectrogram of a waveform $E(t)$



Properties of the Spectrogram

Algorithms exist to retrieve $E(t)$ from its spectrogram.

The spectrogram essentially uniquely determines the waveform intensity, $I(t)$, and phase, $\Phi(t)$.

There are a few ambiguities, but they're "trivial."

The gate need not be—and should not be—much shorter than $E(t)$.

Suppose we use a delta-function gate pulse:

$$\left| \int_{-\infty}^{\infty} E(t)\, \delta(t-\tau) \exp(-i\omega t)\, dt \right|^2 = \left| E(\tau) \exp(-i\omega\tau) \right|^2$$

$$= \left| E(\tau) \right|^2$$ = The Intensity.

No phase information!

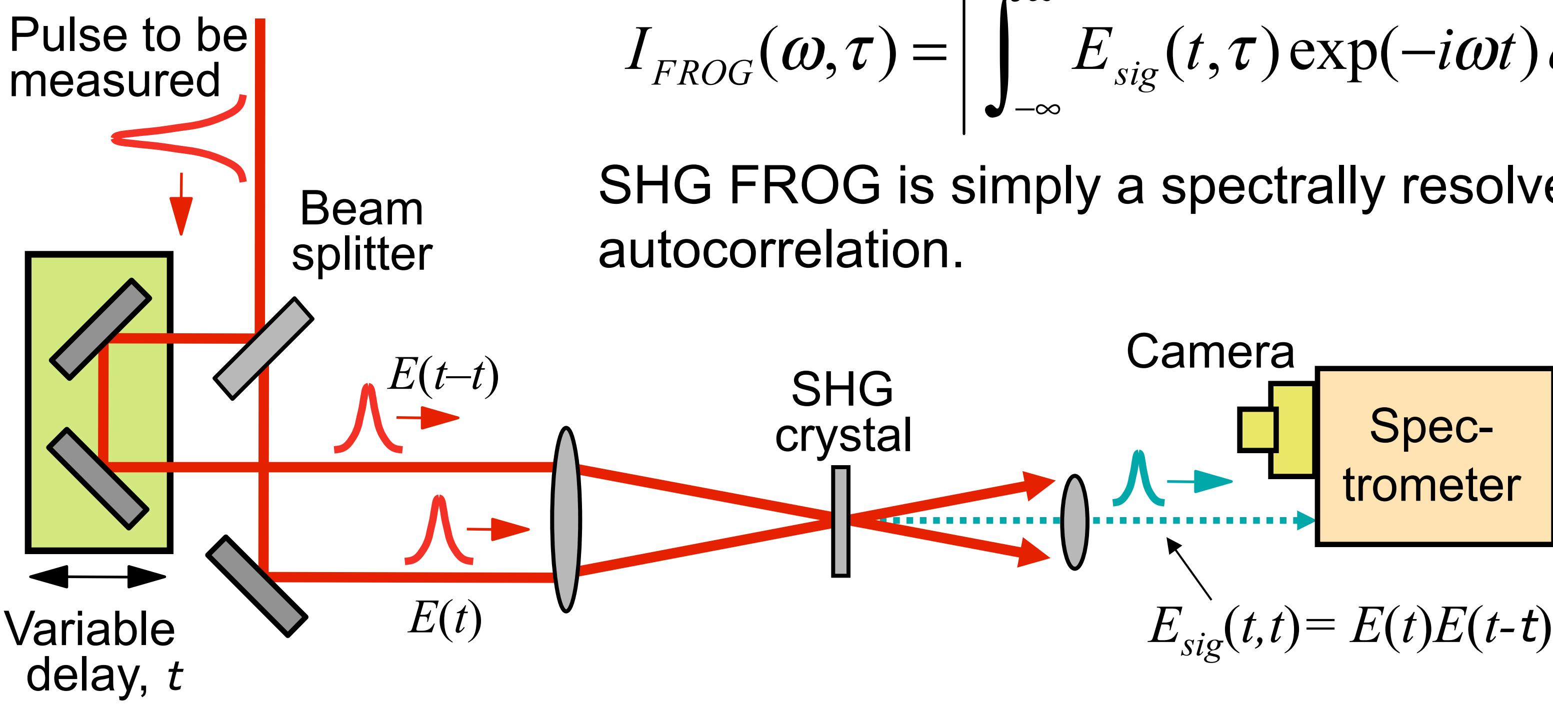
The spectrogram resolves the dilemma! It doesn't need the shorter event! It temporally resolves the slow components and spectrally resolves the fast components.

Frequency-Resolved Optical Gating (FROG)

FROG involves gating the pulse with a variably delayed replica of itself in an instantaneous nonlinear-optical medium and then spectrally resolving the gated pulse vs. delay.

$$I_{FROG}(\omega,\tau) = \left| \int_{-\infty}^{\infty} E_{sig}(t,\tau)\exp(-i\omega t)\,dt \right|^2$$

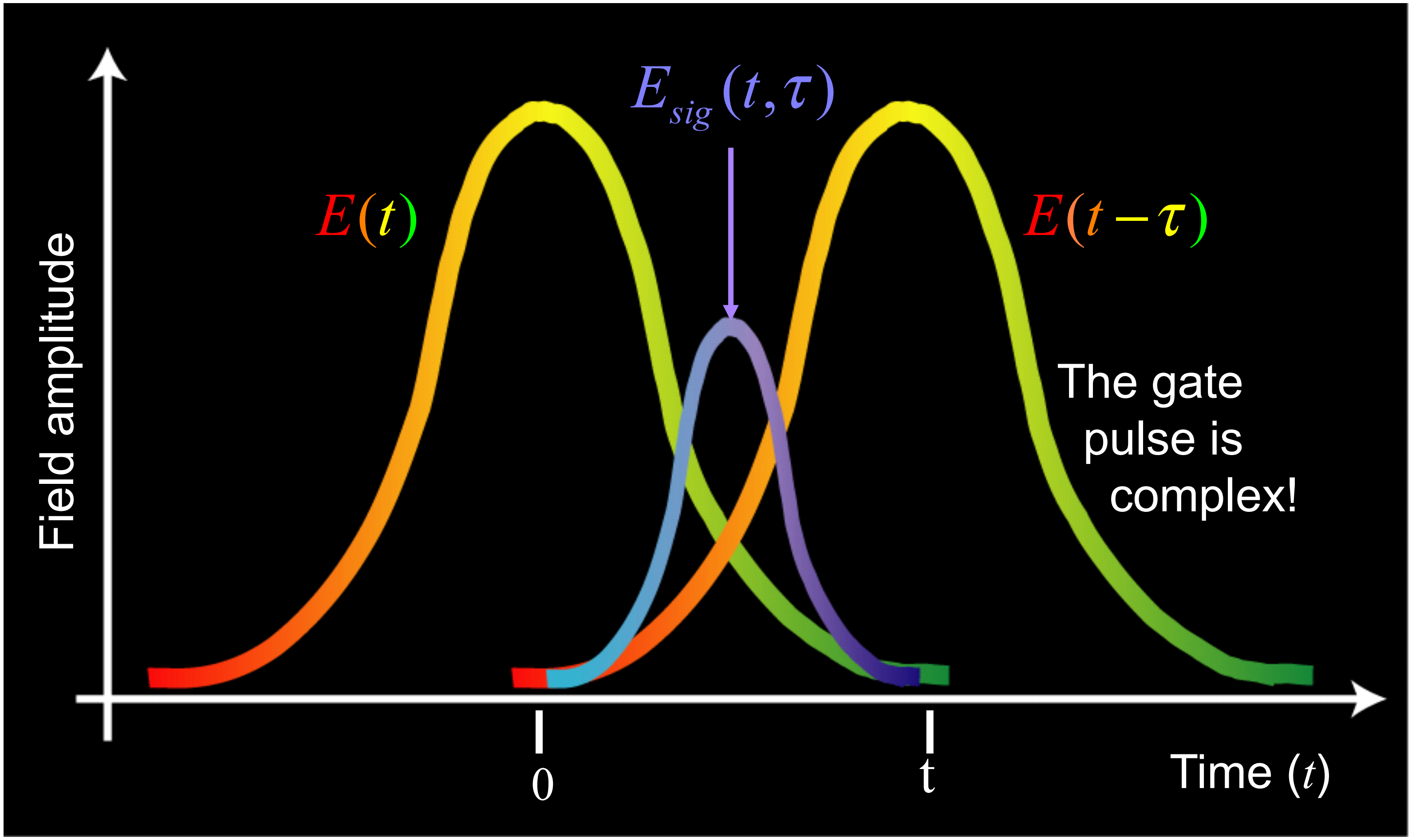
SHG FROG is simply a spectrally resolved autocorrelation.



Use any ultrafast nonlinearity: Second-harmonic generation, etc.

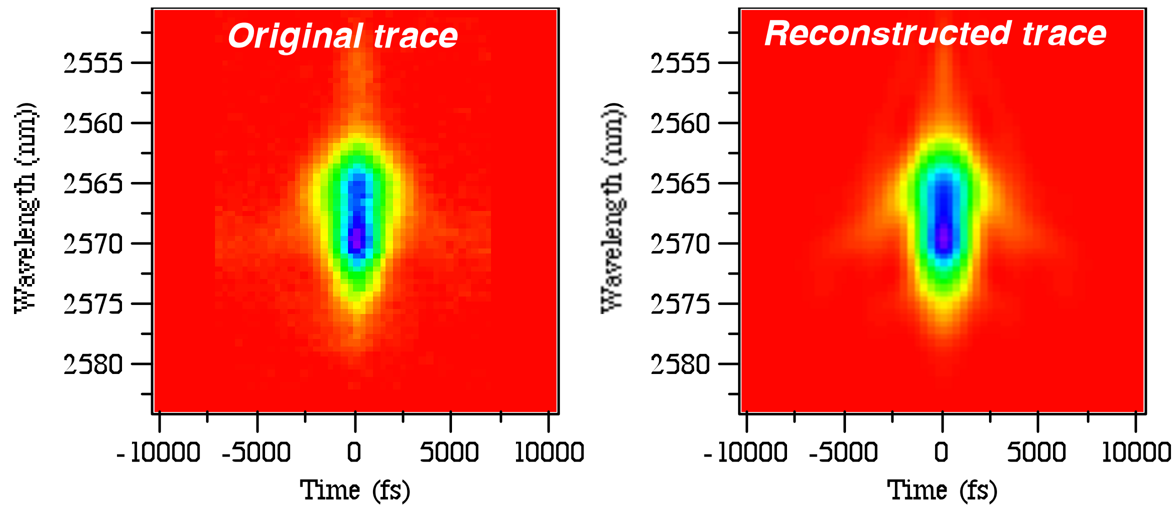
SHG FROG

$$E_{sig}(t,\tau) \propto E(t)E(t-\tau)$$

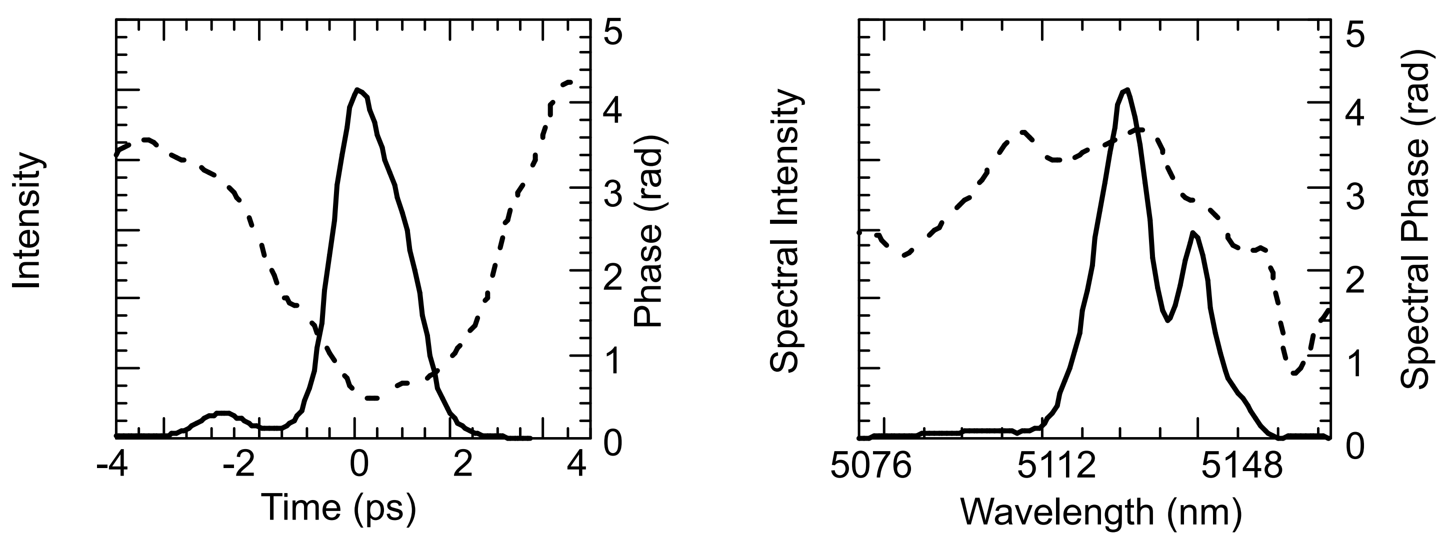


SHG FROG is also a spectrogram, but its interpretation is more complex.

SHG FROG Measurements of a Free-Electron Laser



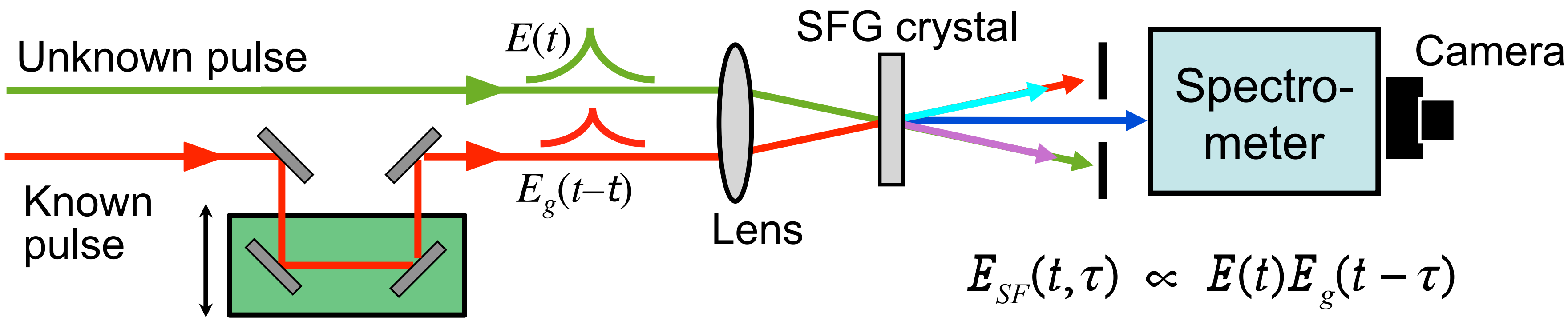
Richman, et al., Opt. Lett., 22, 721 (1997).



SHG FROG works very well, even in the mid-IR and for difficult sources.

When a known reference pulse is available: *Cross-correlation FROG (XFROG)*

If a known pulse is available (it need not be shorter), then it can be used to fully measure the unknown pulse. In this case, we perform sum-frequency generation, and measure the spectrum vs. delay.



$$E_{SF}(t,\tau) \propto E(t)E_g(t-\tau)$$

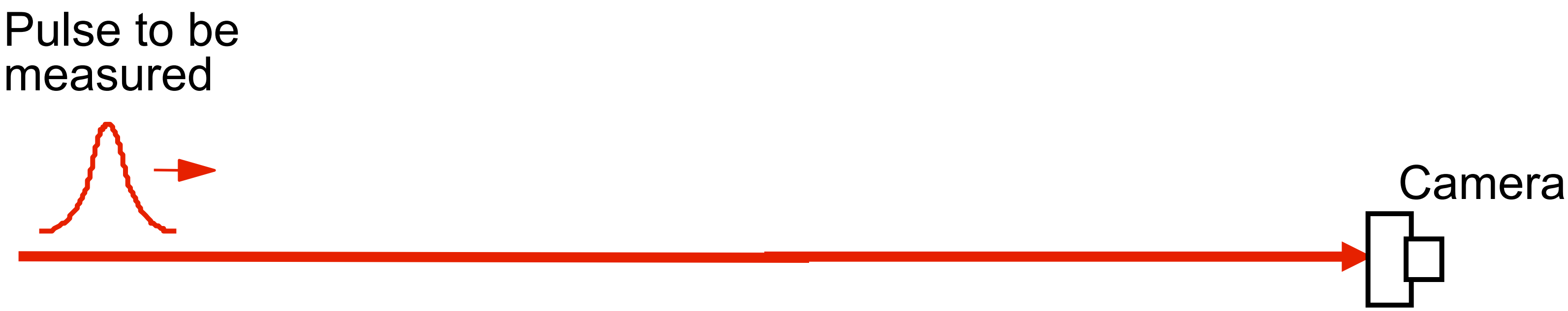
The XFROG trace (a spectrogram):

$$I_{XFROG}(\omega,\tau) \equiv \left| \int_{-\infty}^{\infty} E(t)\, E_g(t-\tau)\exp(-i\omega t)\, dt \right|^2$$

XFROG completely determines the intensity and phase of the unknown pulse, provided that the gate pulse is not too long or too short.
If a reasonable known pulse exists, use XFROG, not FROG.

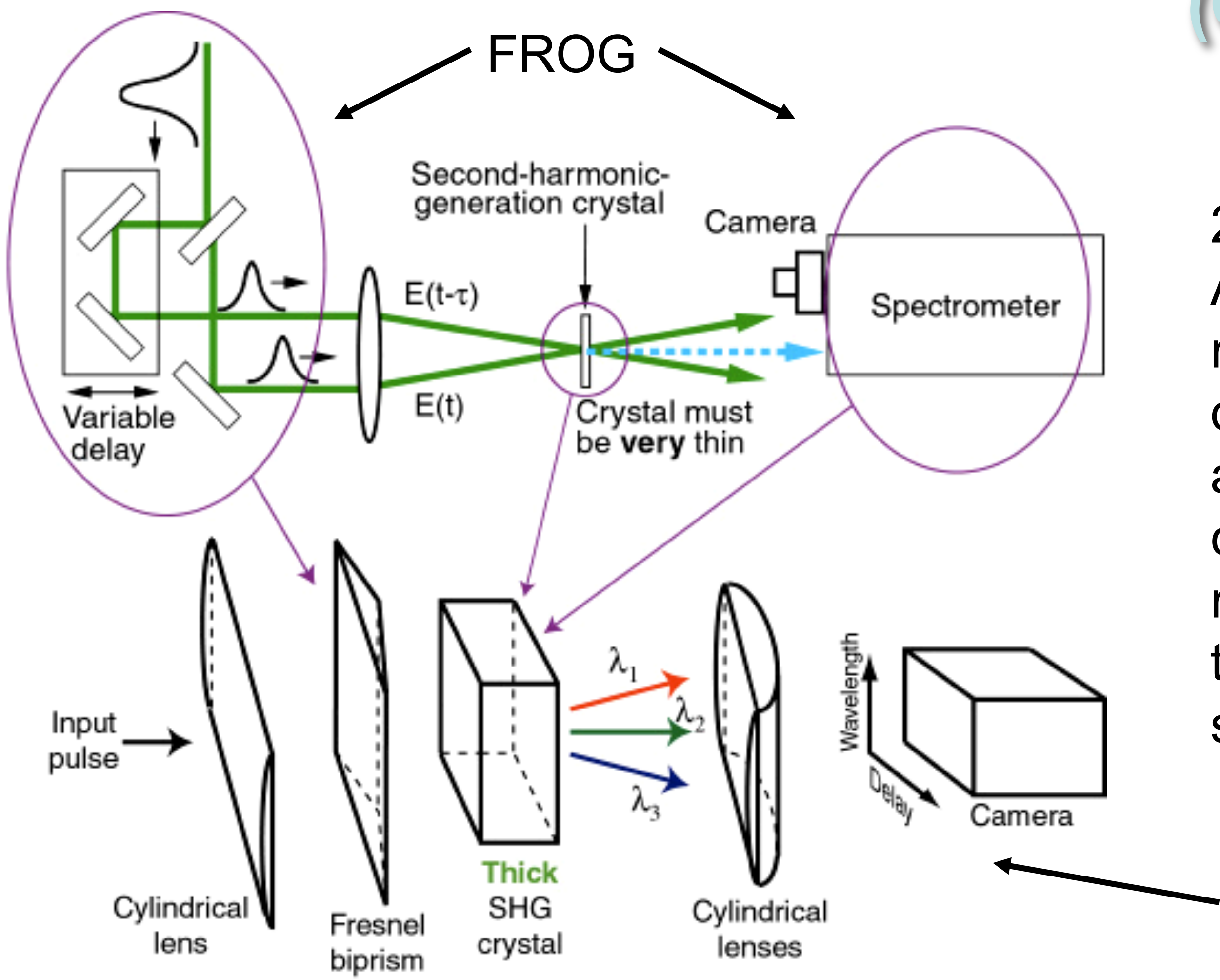
Linden, et al., Opt. Lett., 24, 569 (1999).

Can we simplify FROG?



Remarkably, we can design a FROG without these components!

GRating-Eliminated No-nonsense Observation of Ultrafast Incident Laser Light E-fields (GRENOUILLE)

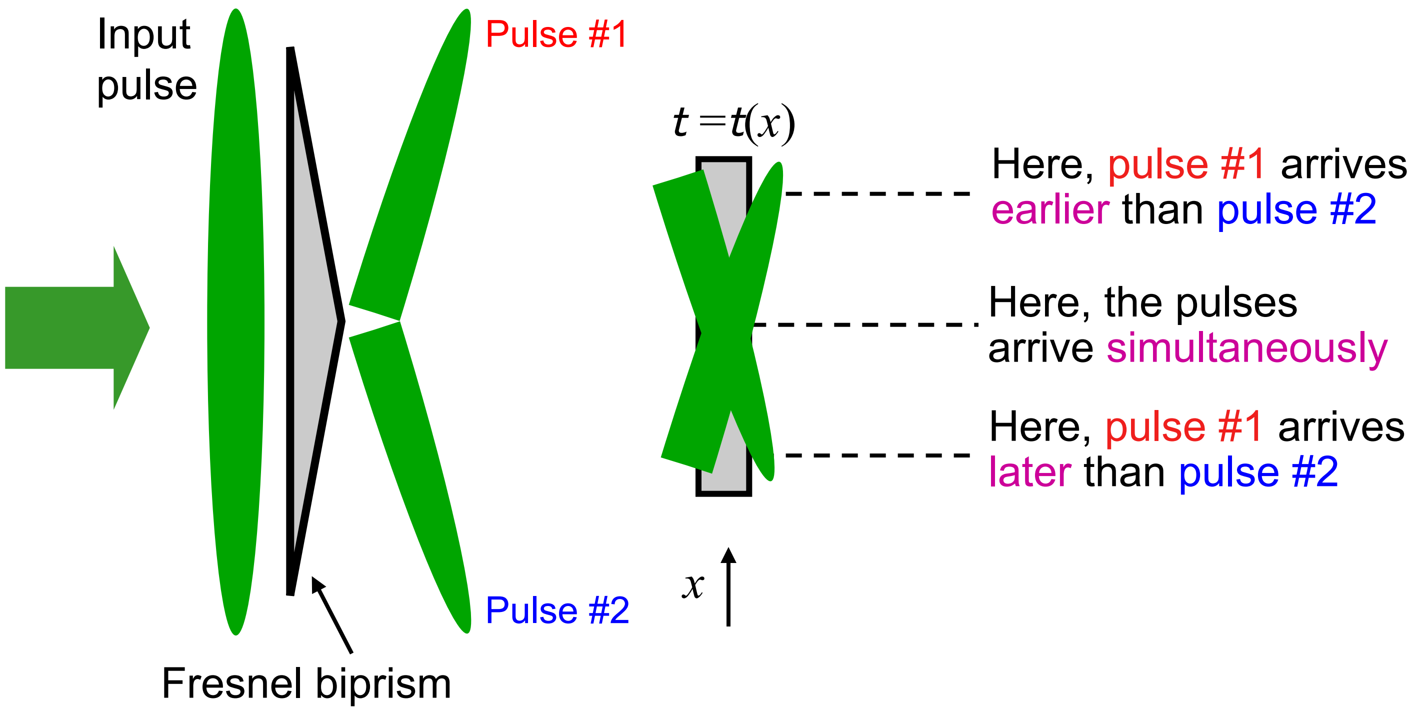


2 key innovations:
A *single* optic that replaces the *entire* delay line,
and a *thick* SHG crystal that replaces *both* the thin crystal *and* spectrometer.

C. Radzewicz, P. Wasylczyk, and J. S. Krasinski, Opt. Comm. 2000.
P. O'Shea, M. Kimmel, X. Gu and R. Trebino, Optics Letters, 2001.

The Fresnel biprism

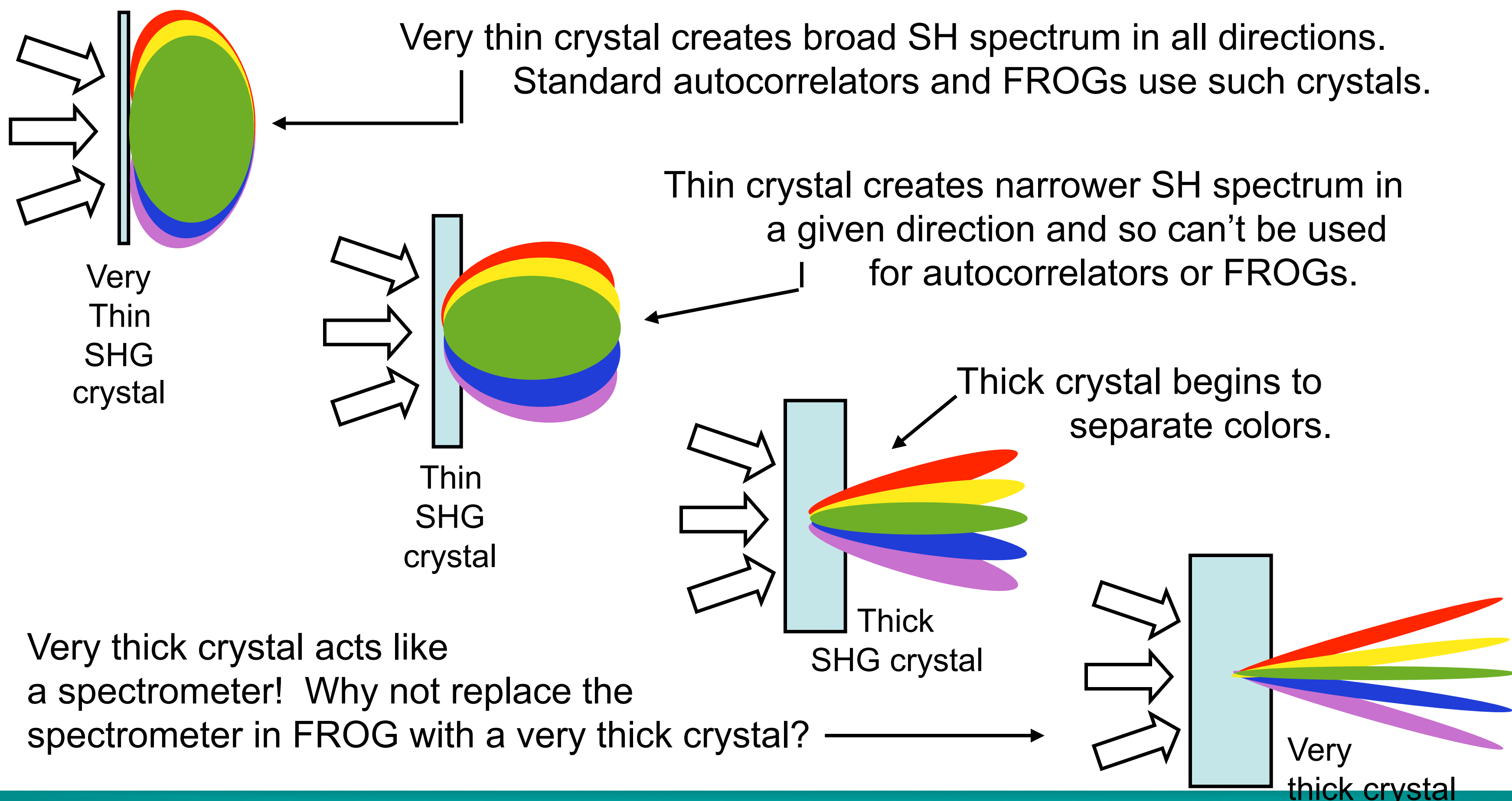
Crossing beams at a large angle maps delay onto transverse position.



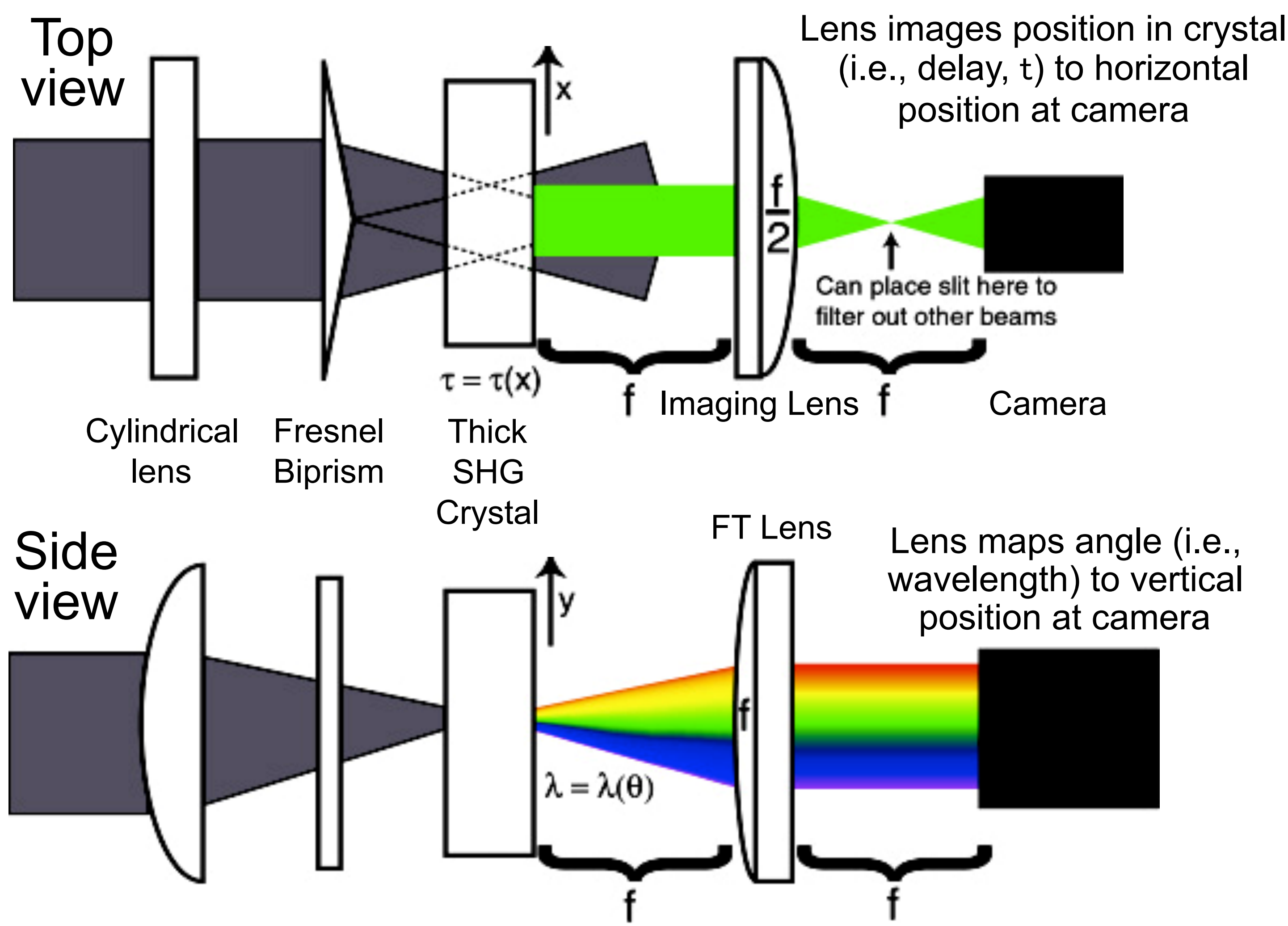
Even better, this design is amazingly compact and easy to use, and it never misaligns!

The thick crystal

Suppose white light with a large divergence angle impinges on an SHG crystal. The SH generated depends on the angle. And the angular width of the SH beam created varies inversely with the crystal thickness.



GRENOUILLE Beam Geometry

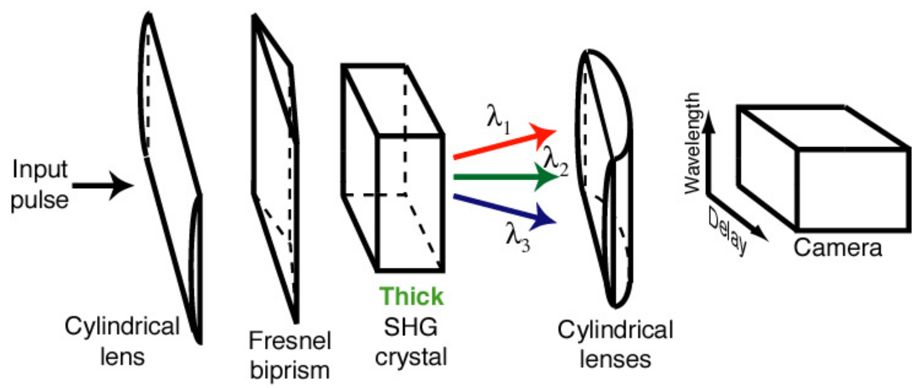


Yields a complete single-shot FROG. Uses the standard FROG algorithm. Never misaligns. Is more sensitive. Measures spatio-temporal distortions!

Disadvantages of GRENOUILLE

Its low spectral resolution limits its use to pulse lengths between ~ 20 fs and ~ 1 ps.

Like other single-shot techniques, it requires good spatial beam quality.

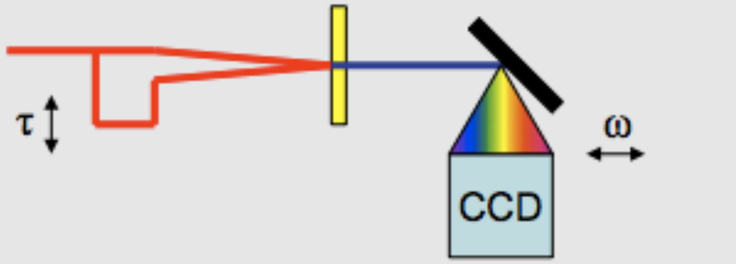


Improvements on the horizon:

Inclusion of GVD and GVM in FROG code to extend the range of operation to shorter and longer pulses.

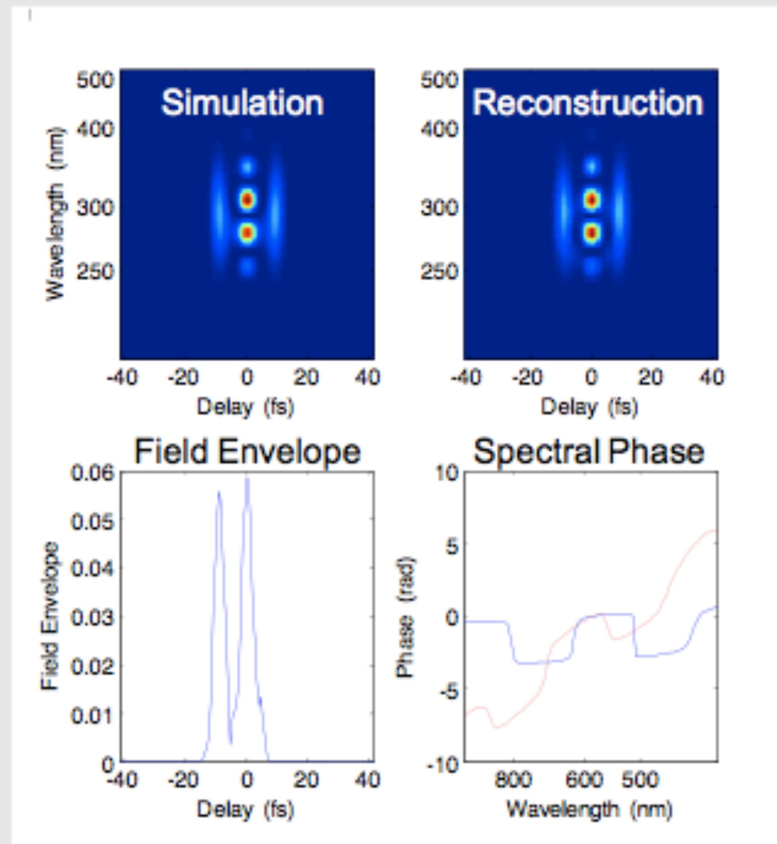
Optical techniques: FROG

- Measure spectrum of SHG in BBO vs. delay

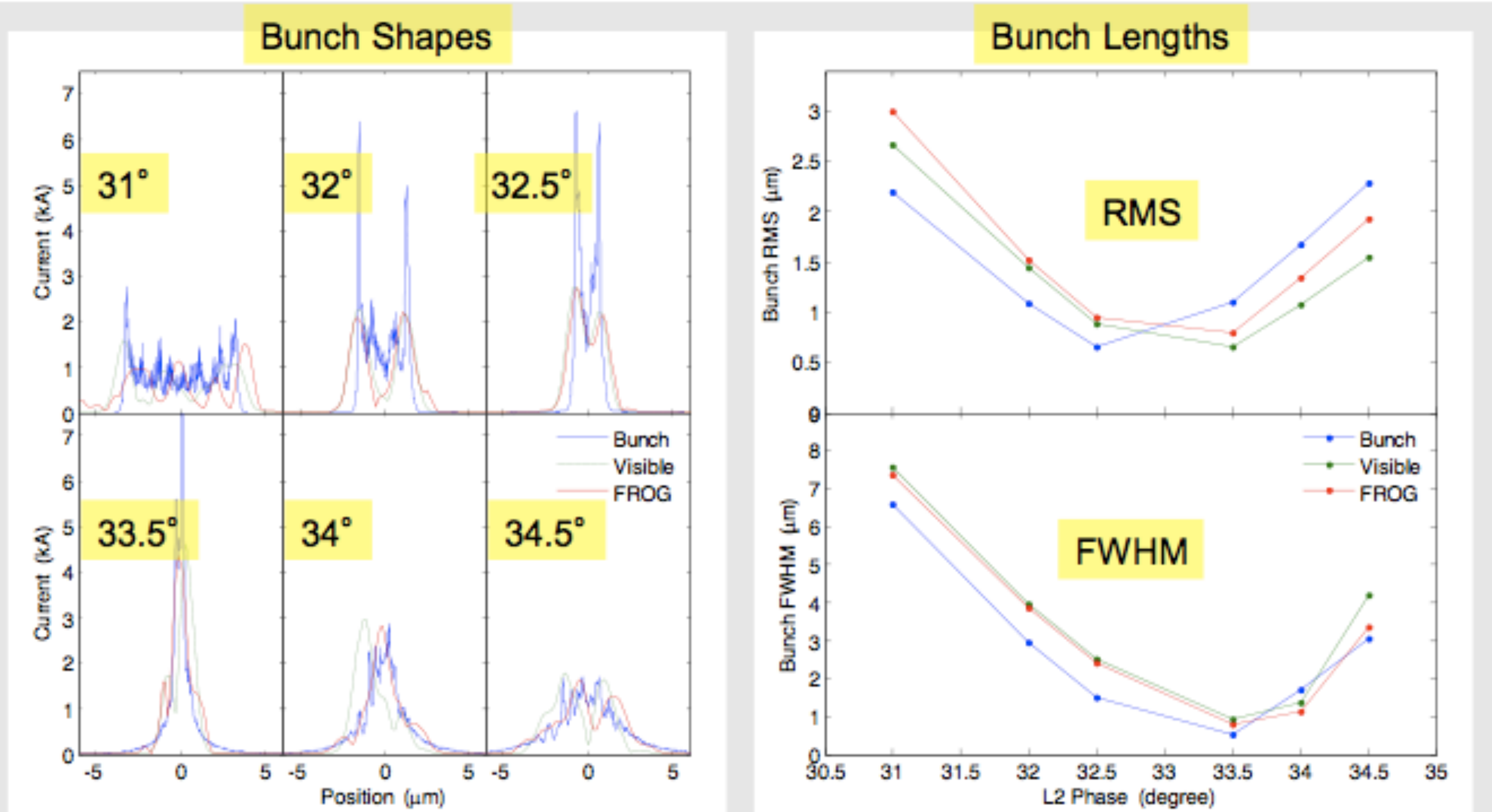


$$I(\omega,\tau) \propto \left| \int E(t)E(t-\tau)e^{-i\omega t}dt \right|^2$$

- Remove carrier frequency from reconstructed field
- Envelope is $|E(t)|^2$
- Required pulse energy is few 100 nJ
- COTR energy between 0.1 – 1 μJ
- Phase matching over 300 nm BW requires few μm crystal



Simulated FROG results for LCLS



- Reconstruction longer for undercompressed and shorter for overcompressed bunches