RF Systems for Accelerators

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LINAC SYSTEMS

- High Voltage
- Klystron
- Waveguides
- Accelerating Structures
  - Structure types
  - Modes
- Longitudinal Dynamics
  - Beam Loading
  - Wakefields

High Voltage Supply – The function of the high voltage supply is to produce the high voltages required for proper modulator operation. The high voltage should be regulated , filtered and have some type of feedback of both the voltage and current. It must be protected against over-voltage and over-current conditions and be capable of withstanding high stress during normal operation as well as of a failure.

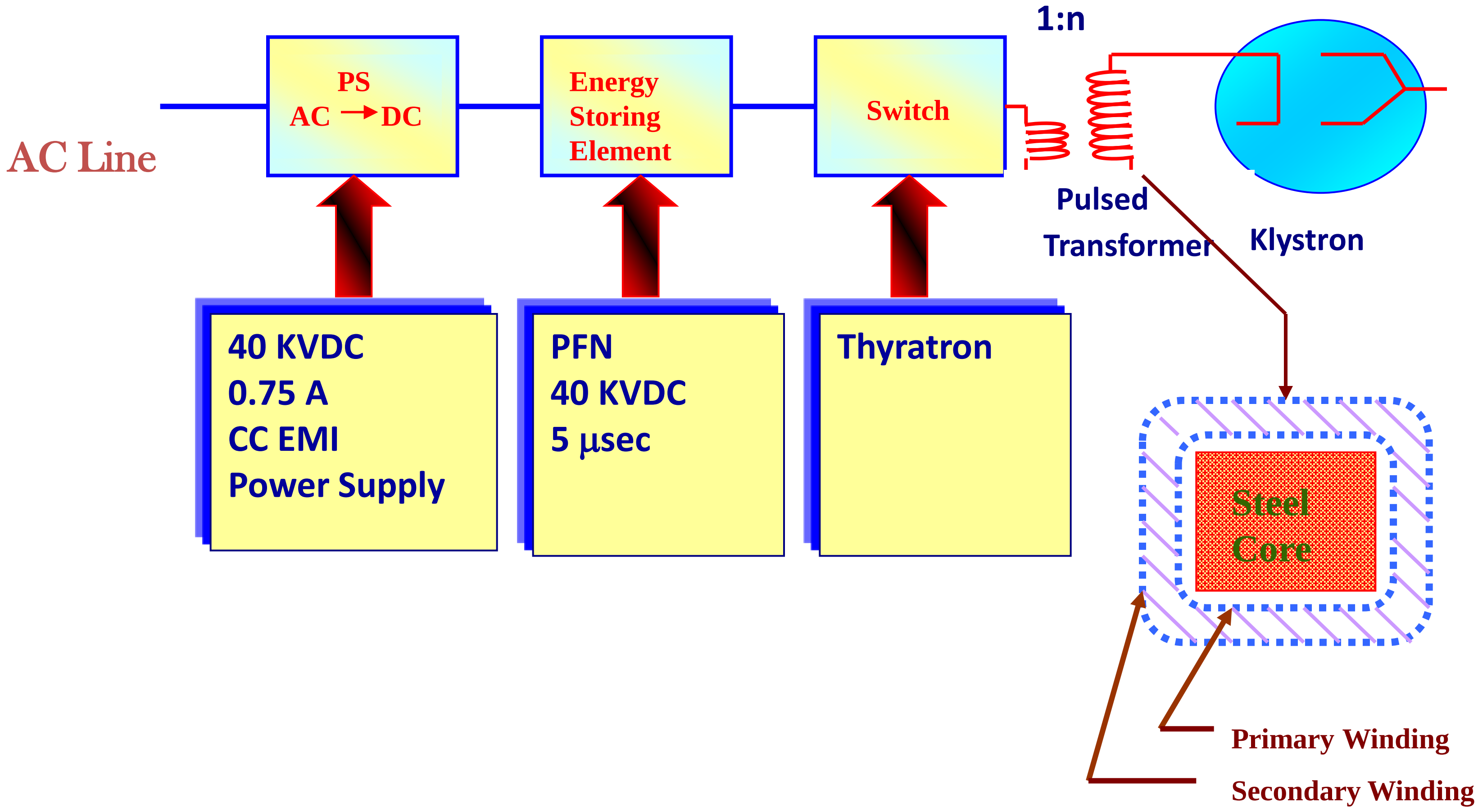
High Voltage Supply – There are two basic approaches in the design of the high voltage supply. The traditional approach is what may be called the "brute force" method. In this approach, a large high voltage transformer is used with some type of rectification and filtering.

The second method is by using high frequency switching supplies. The rend is the power supply industry has been away from linear brute force and toward switching supplies.

<u>High Voltage Supply</u> – Regulation of the high voltage is important, as changes in high voltage result in changes in RF output and ultimately causes changes in the output of the linac. There are two basic ways of providing regulation. Direct regulation of the high voltage supply and post regulation on a pulse-to-pulse.

Modulator – The function of the modulator is to provide high voltage pulses to the microwave transmitter (klystron). Almost every RF linac today uses some variation of the line type modulator. This design was used extensively during WWII for radar applications. The are called line modulators because the width of the output pulse is determined by an actual transmission line. Modern modulators use an artificial transmission line called a pulse-forming network (PFN).

Linac Modulator System



Modulator Operation:

1. Charging cycle – The charging inductor and capacitor of the PFN forma resonant circuit. This resonance causes the PFN to charge up to twice the voltage supplied by the high voltage supply. The charging diode keeps the PFN voltage at full until the discharge cycle.

2. The discharge cycle is initiated by conduction of the power switch (hydrogen thyratron). The discharge cycle results in a pulse to appear across the input of the pulse transformer. Typical ratio is 1:15 for the pulse transformer.

Radio Frequency System

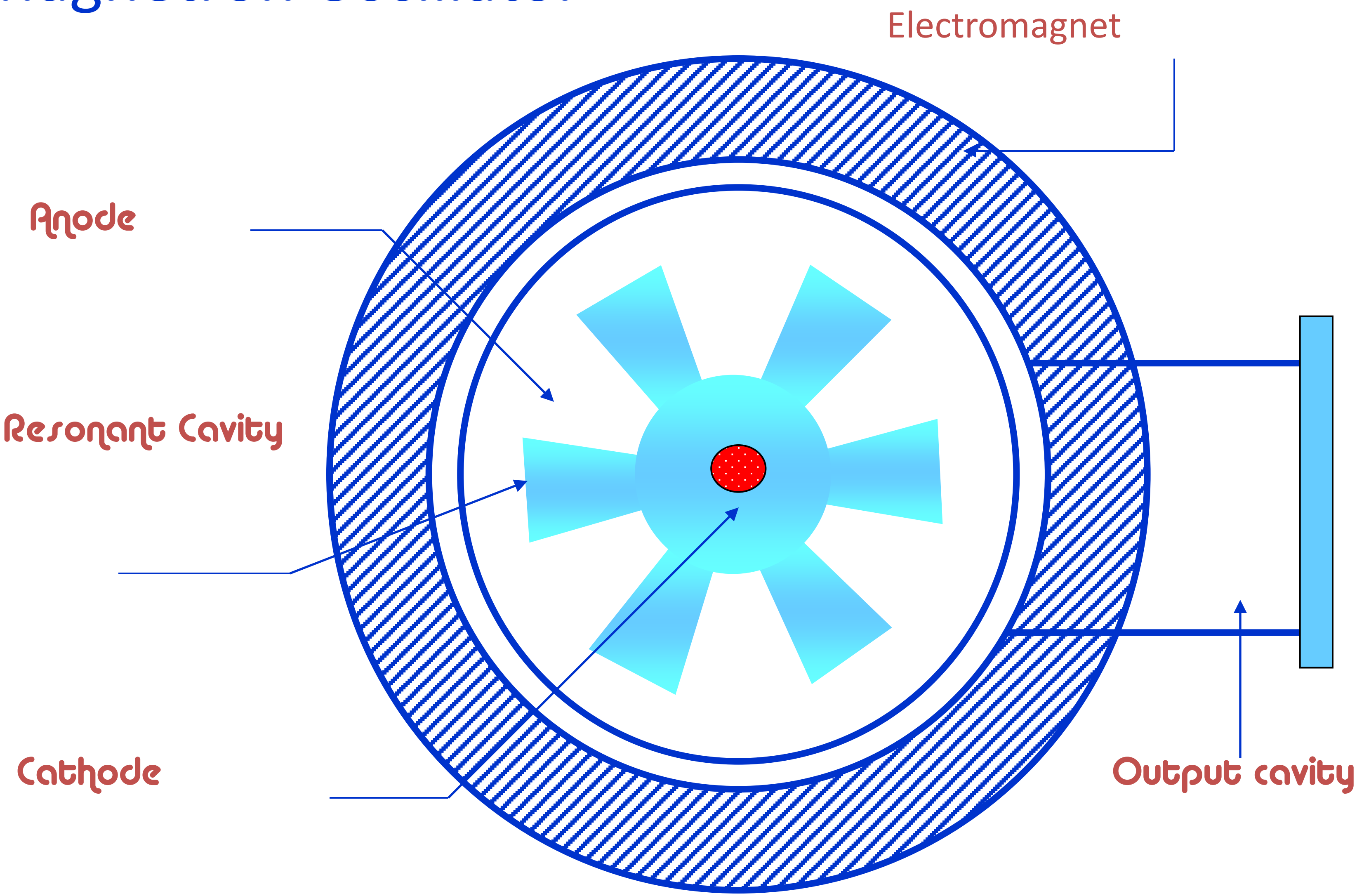
The RF system converts the high voltage pulses from the modulator into pulsed radio frequency energy. The RF pulses are sent to the accelerating structure to setup an electric field which is used for charged particle acceleration.

The main component of a RF system is the microwave source. There is a variety of microwave tubes for generating and amplifying microwave signals. The two most common ones used in linacs are magnetrons and klystrons.

Radio Frequency System

A magnetron is a microwave power oscillator which belongs to the family of electron tubes called crossed field devices. This is because it has an electric field and a magnetic field which are perpendicular to each other. The magnetron consists of a circular cathode inside a circular anode block. There are resonant cavities machined into the anode block. These cavities will resonant at microwave frequencies when excited by electrons interacting with the *E* and *H* fields.

Magnetron Oscillator

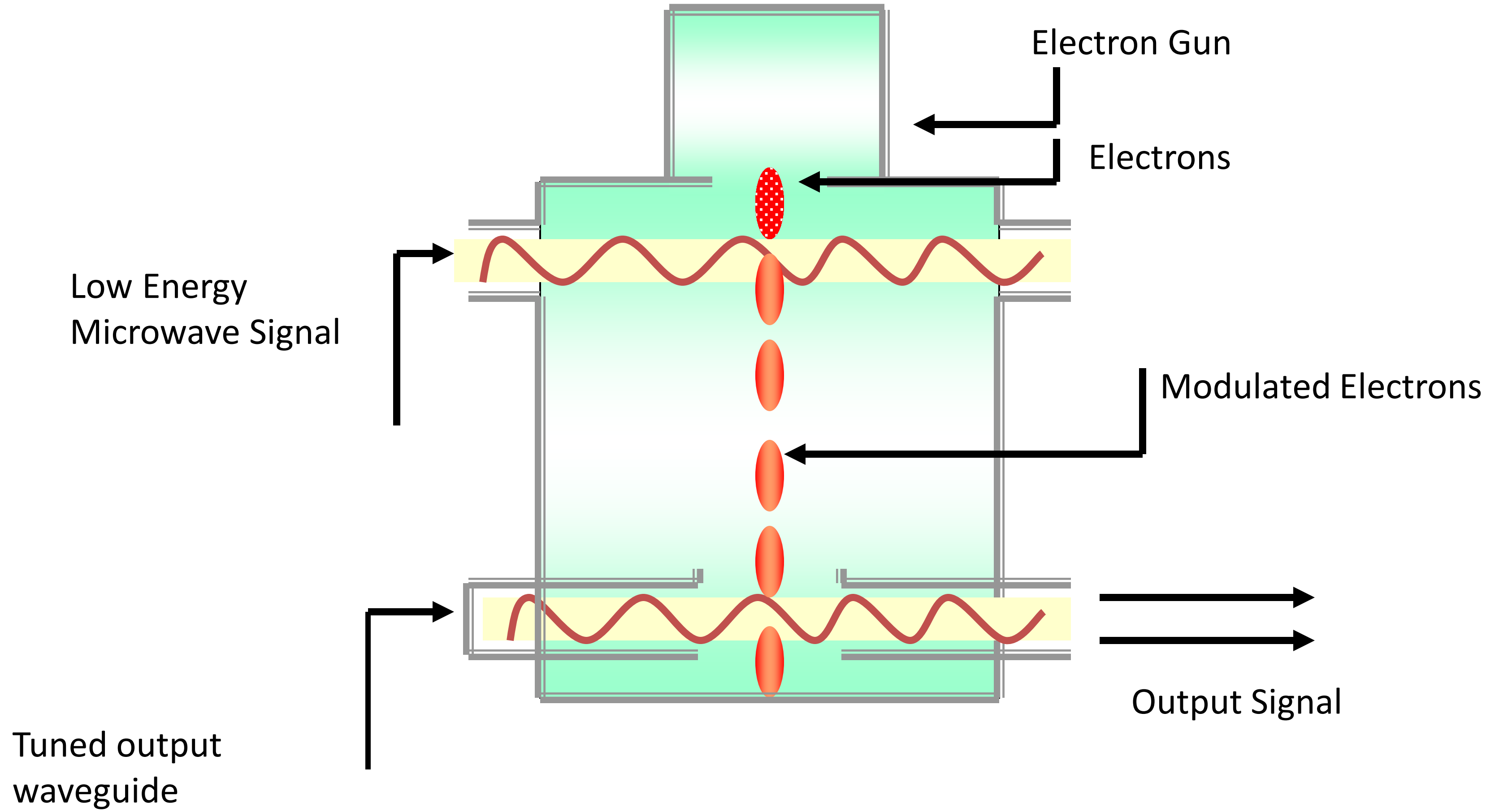


Radio Frequency System

Klystrons belong to the class of tubes called linear beam tubes. In most linac applications, the klystron is used as an amplifier, so an input signal is required. This is provided by a low power oscillator typically called an RF driver.

The choice of which type RF generator tube is used is based partially on the design requirements and partially on historical preference.

Klystron



Electron Gun
Electrons
Low Energy
Microwave Signal
Modulated Electrons
Tuned output
waveguide
Output Signal

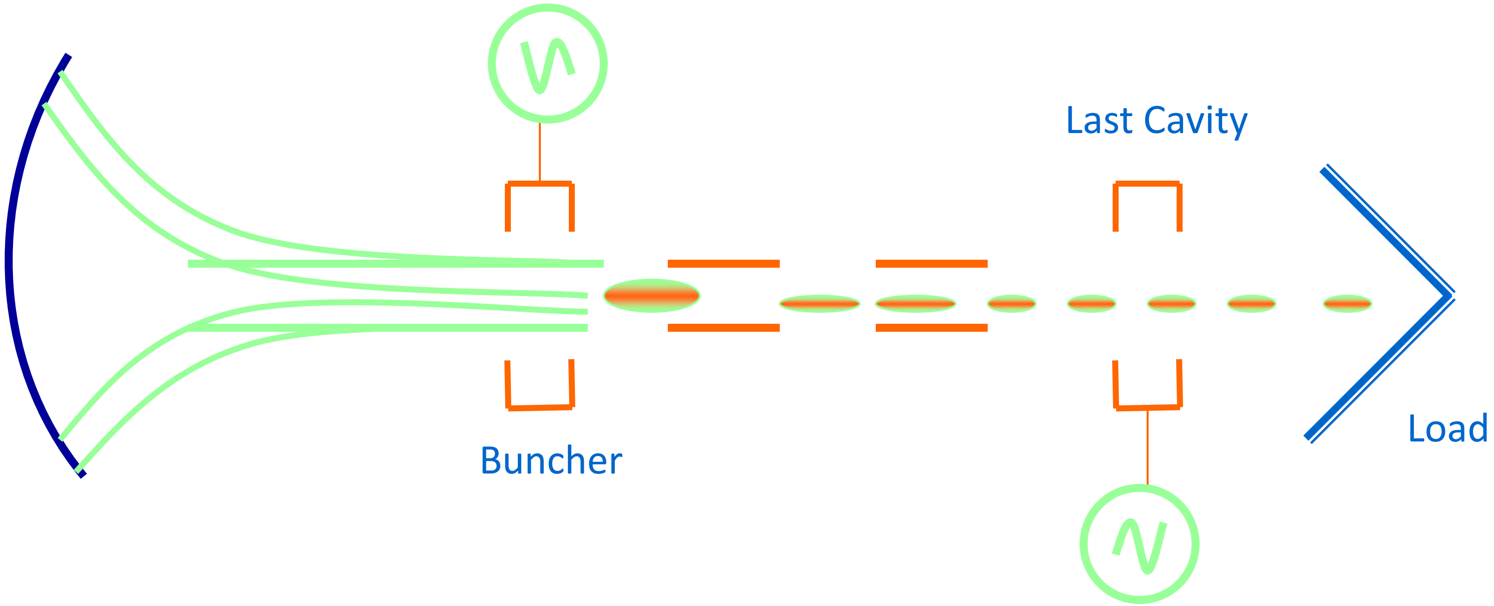
Klystrons

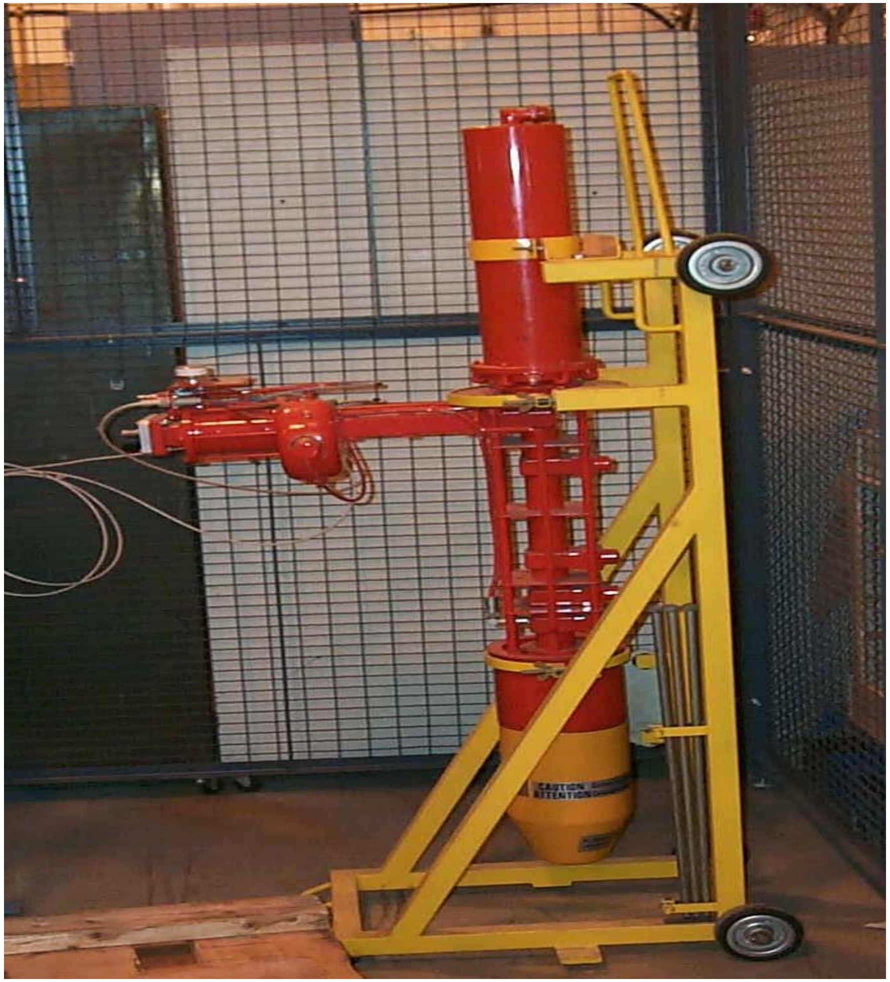
Klystron was invented at Stanford in 1937. The klystron severed as an oscillator in radar receivers during WWII. After the war, however, very high-power klystrons were built at Stanford for use in the first linear accelerators. This opened the way for the use of klystron not only in accelerators and radar, but also in UHF-TV, satellite communications, and industrial heating.

Klystrons

Klystrons are high-vacuum devices based on the interaction of well-focused pencil-like electron beam with a number of microwave cavities that it traverses, which are tuned at or near the operating frequency of the tube. The principle is conversion of the kinetic energy in the beam, imparted by high accelerating voltage, to microwave energy. Conversion takes place as a result of the amplified RF input signal, casing the beam to form “bunches.” These bunches give up their energy to the high level induced RF fields at the output cavity. The amplified signal id extracted from the output cavity through a vacuum window.

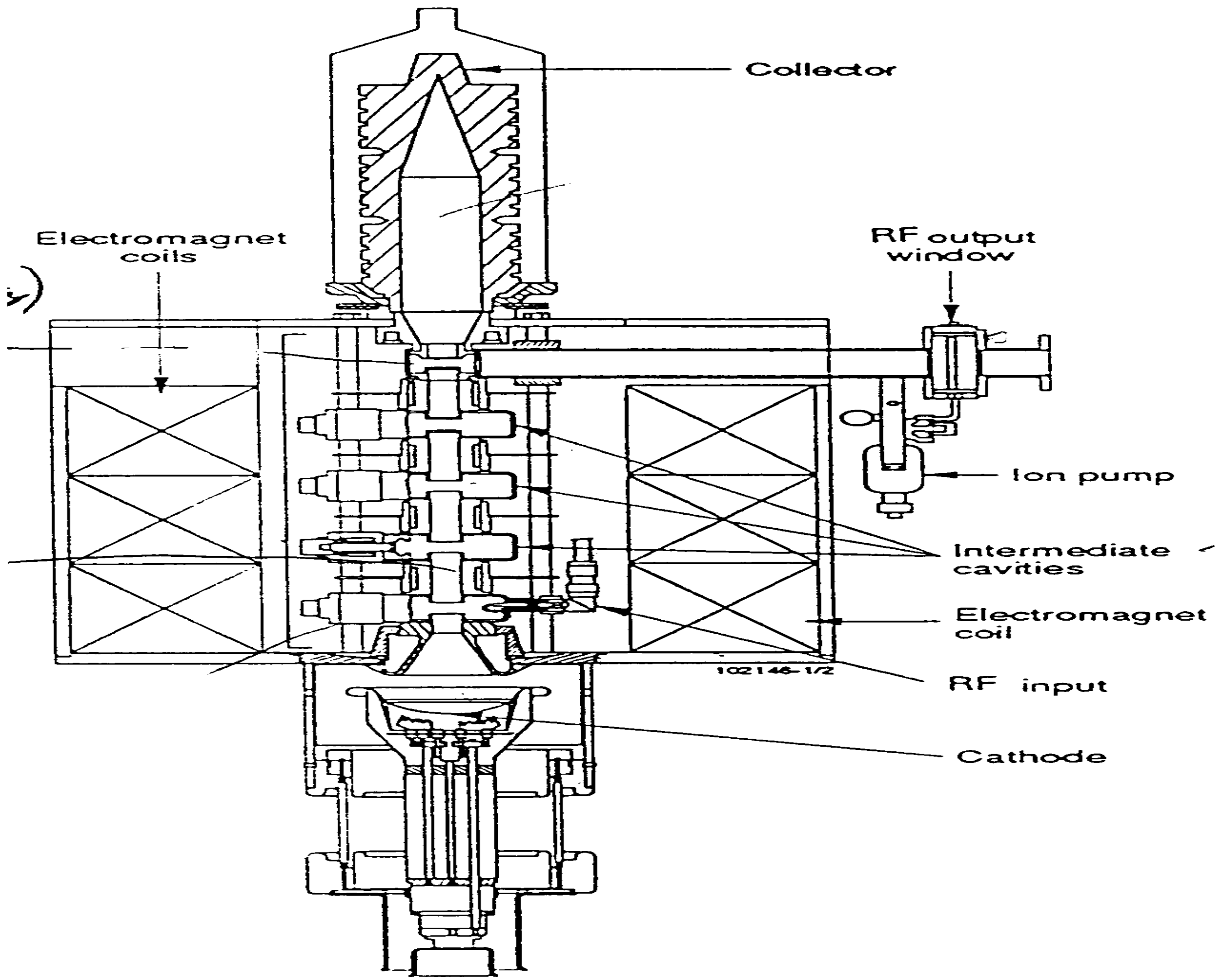
Klystron





S-Band 35 MW Klystron (TH2128)

General Description of Thomson Klystron TH2128:



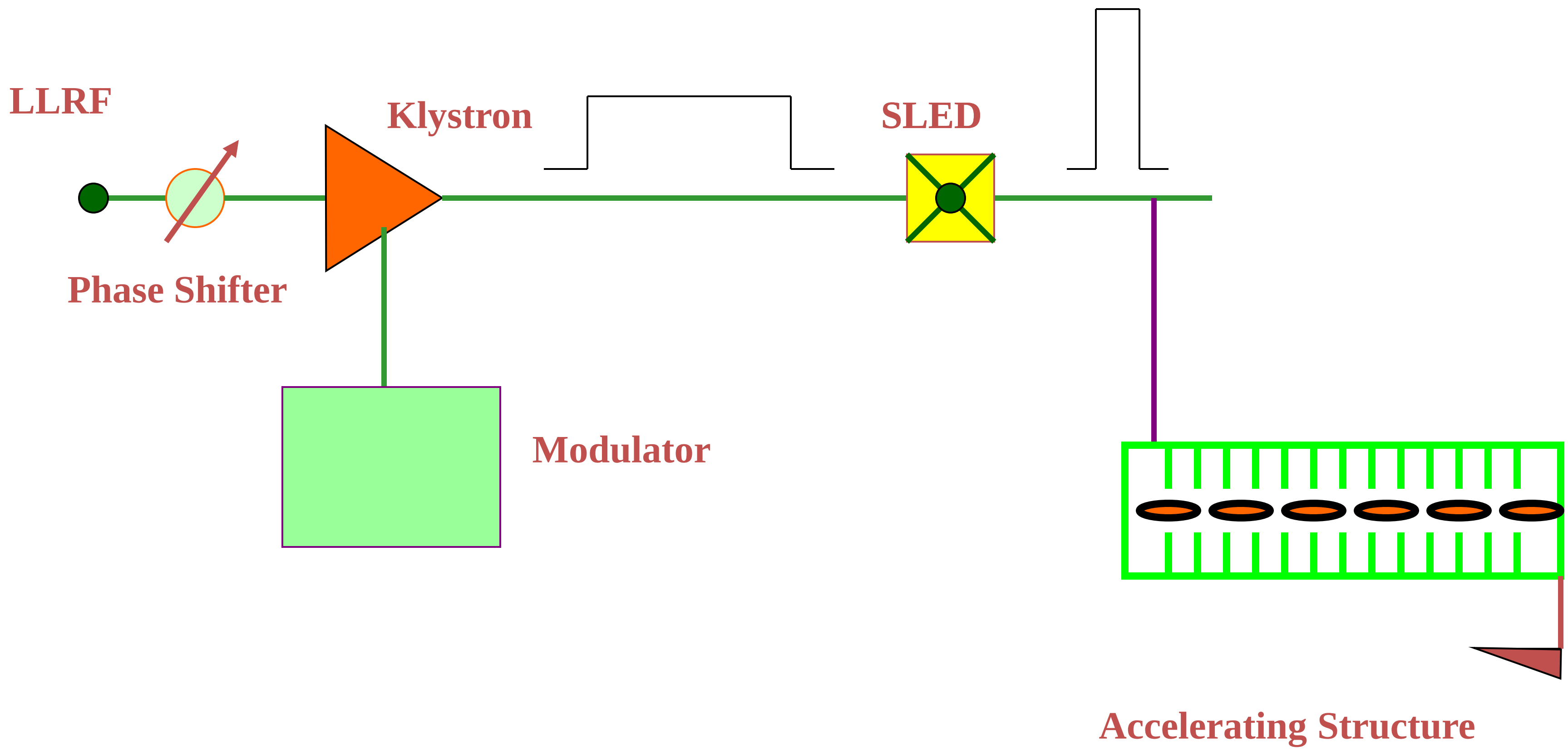
Main Parameters of TH2128 Klystron:

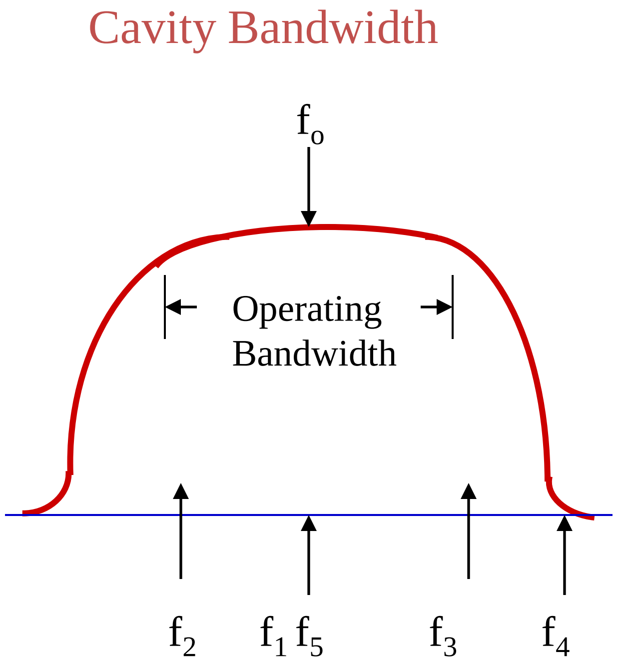
Frequency	**2856 MHz**
Peak Output Power	**35 MW**
Average Power	**11kW**
RF Pulse Duration5μsec	
Peak Beam Voltage,Max	**300 kV**
Peak beam Current,Max	**300 A**
Peak RF Drive Power, Max	**200 W**
Efficiency	**42%**
Perveance	**1.9 to 2.15 μA . $V^{-3/2}$**
Filament Voltage	**20 to 30 V**
Hot Filament Resistance	**1.1 Ω**
Cold Filament Resistance	**0.1 Ω**

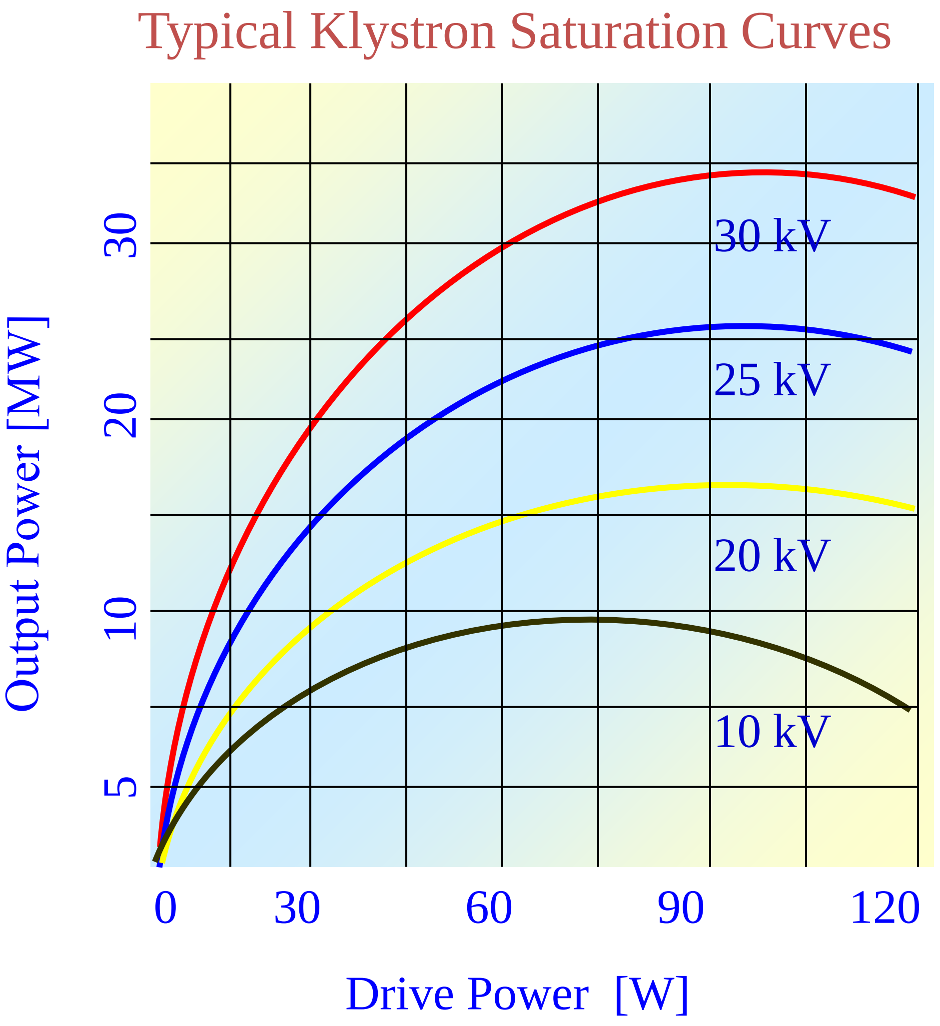
Typical Operation:

Frequency	**2856 MHz**
VSWR, Max.	**1.1:1**
Peak Beam Voltage	**280 kV**
Peak Beam Current	**297 A**
Peak RF Power	**35 MW**
Average Output Power	**10.5 KW**
RF Pulse (at -3 dB)	**5 μsec**
Power Dissipated on the Body	**800 W**

RF Power Distribution to the Accelerating Structure




Cavity Bandwidth
f_o
Operating
Bandwidth
f_2
f_1 f_5
f_3
f_4


Typical Klystron Saturation Curves
30 kV
25 kV
20 kV
10 kV
Output Power [MW]
5
10
20
30
0
30
60
90
120
Drive Power [W]

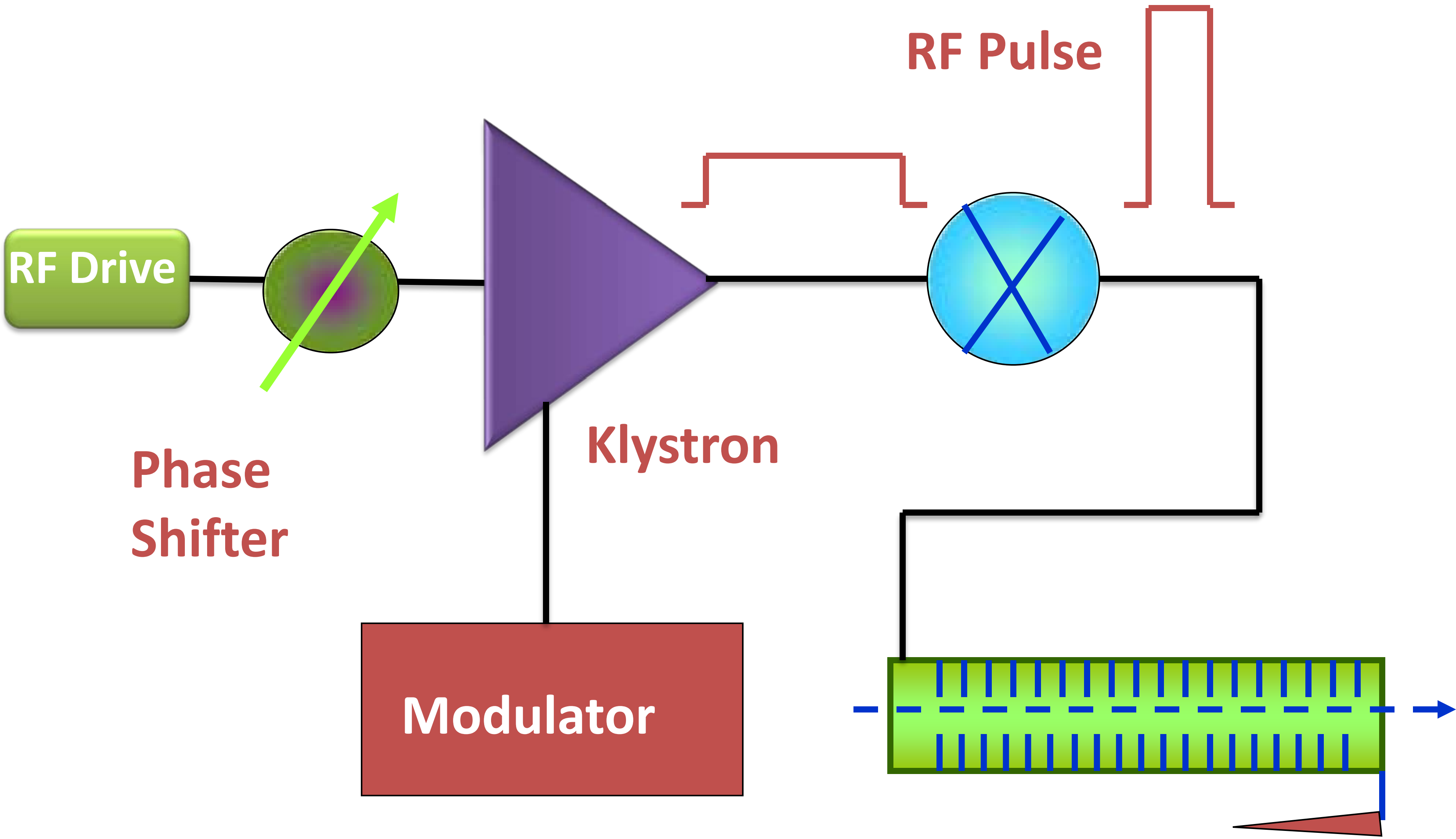
RF Components:

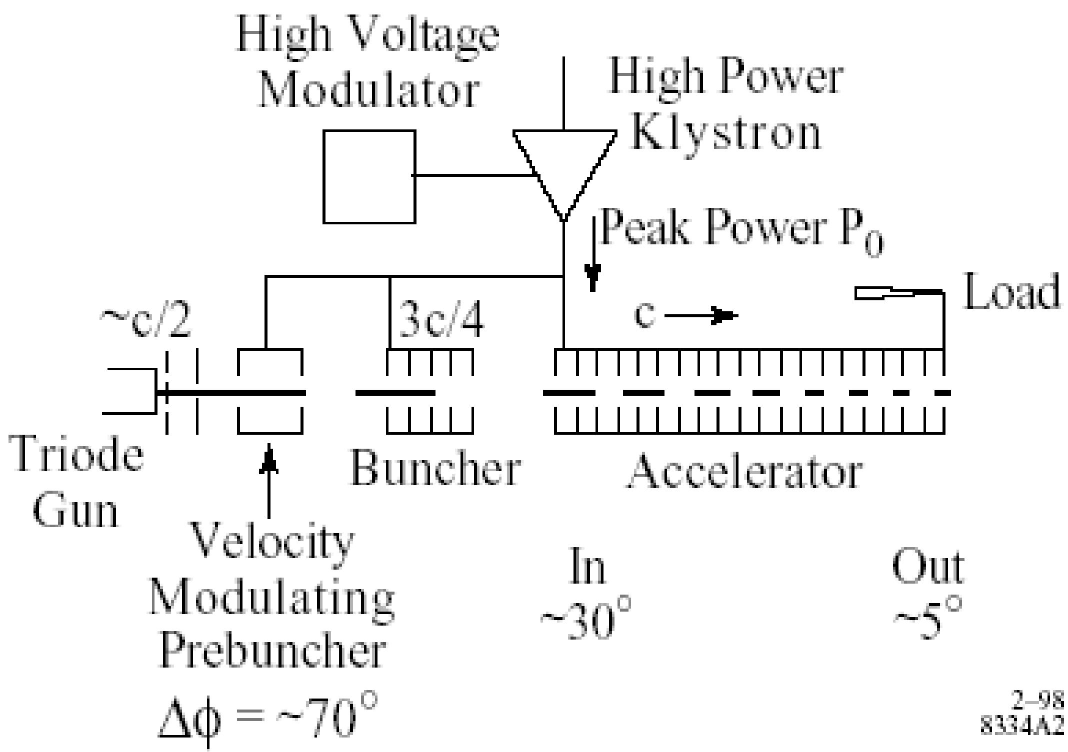
- Driver amplifier to power klystron
- Klystron is used to generate high peak power (A small accelerator)
- Need to transport power to the accelerating structure
- Waveguide is used (under vacuum) to propagate and guide electromagnetic fields
- Windows (dielectric material, low loss ceramic) are used to isolate sections of the waveguide
- Termination loads (water loads) are used to provide proper rf match and to absorb wasted power
- Power splitters are used to divide power in different branches of the waveguide run

Accelerating Structure Requirements

- High accelerating gradient to optimize length and cost (LC, NLC)
- Control of short and long range wakefields
- Preservation of low emittance for multi-bunch beams
- Minimize HOM effects
- Beam Breakup

RF Distribution





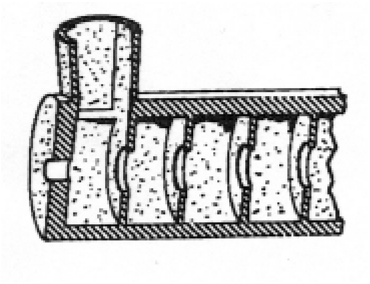
RF Control System

Vacuum System

Water Cooling System

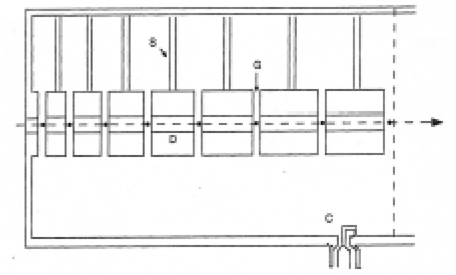
Electric Power System

- **1947 W. Hansen (Stanford) Disk-loaded waveguide linac**



DLWG

- **1955 Luis Alvarez (UC Berkeley, DTL)**

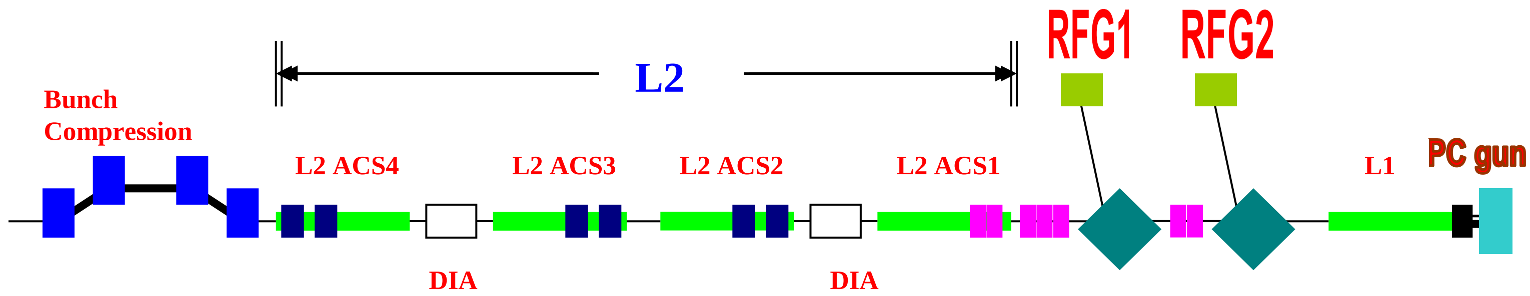


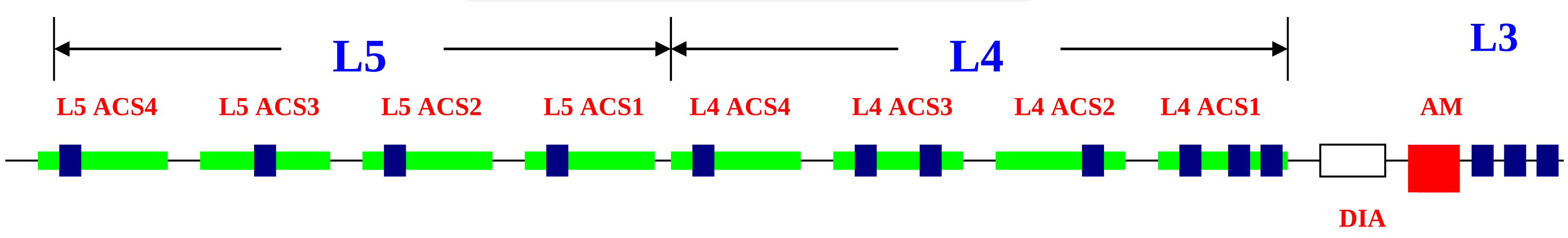
Alvarez 200MHz, 32MeV

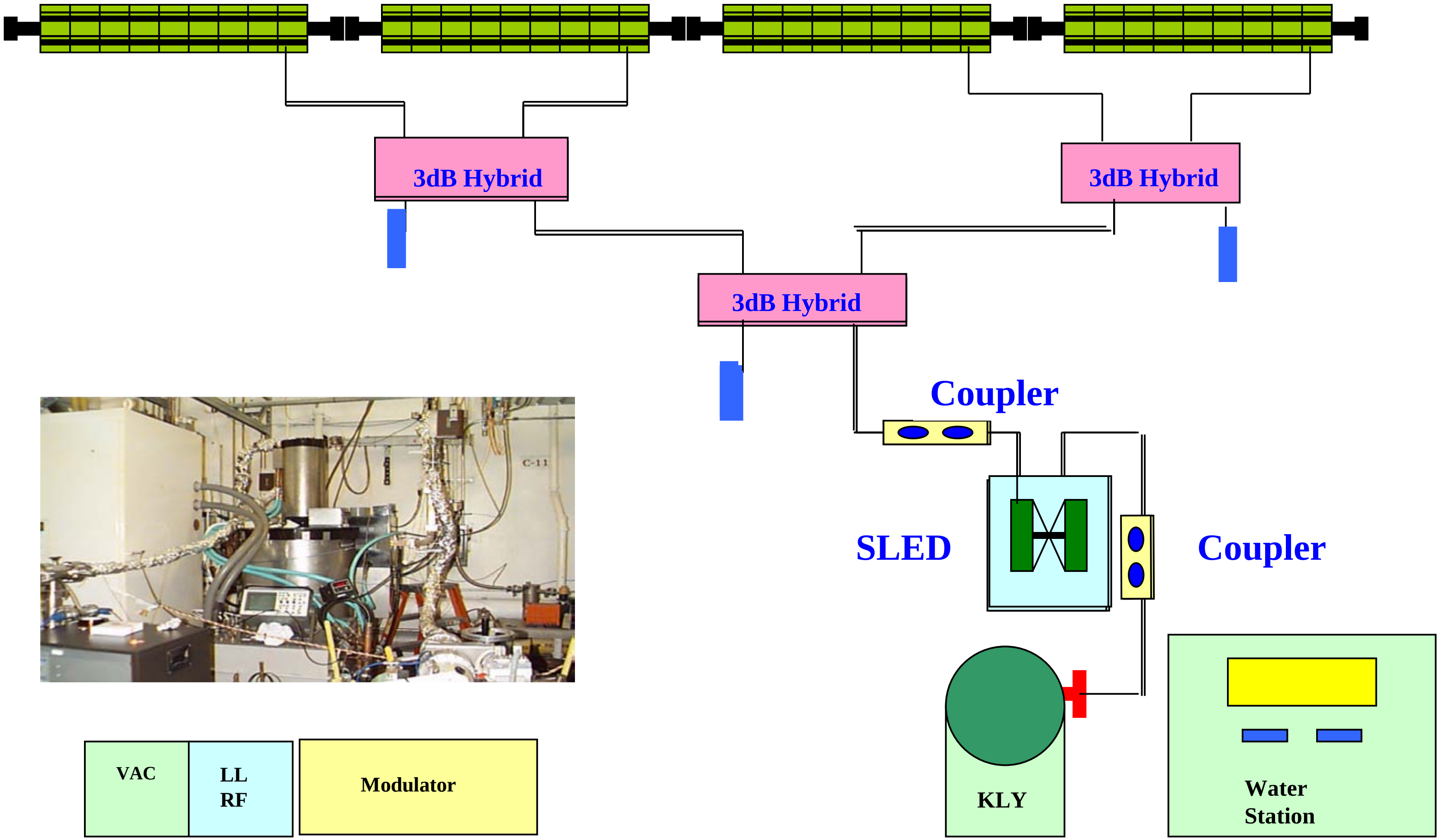
- **1970 Radio Frequency Quadruple (RFQ)**

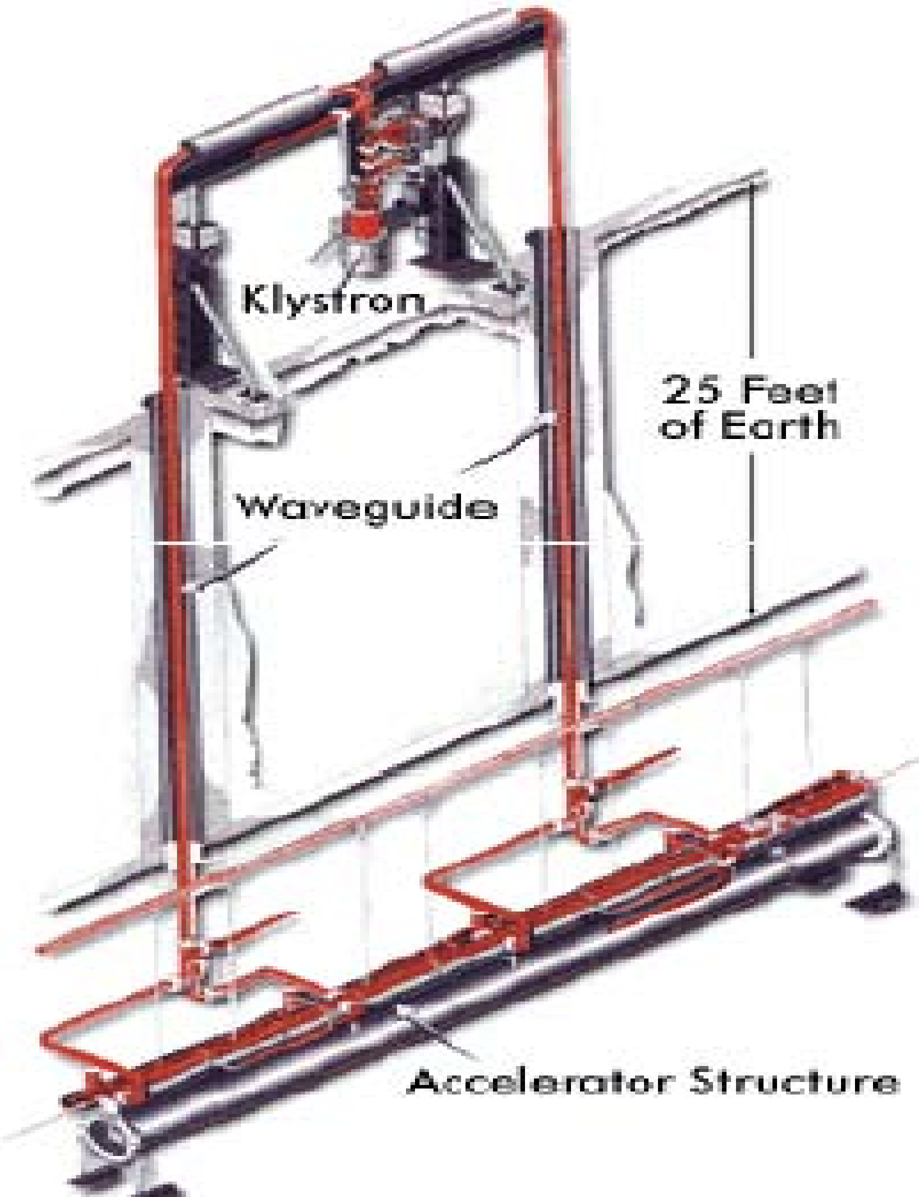
RFQ 6 - 400 MHz 0.01-0.06C

Linac RF Layout

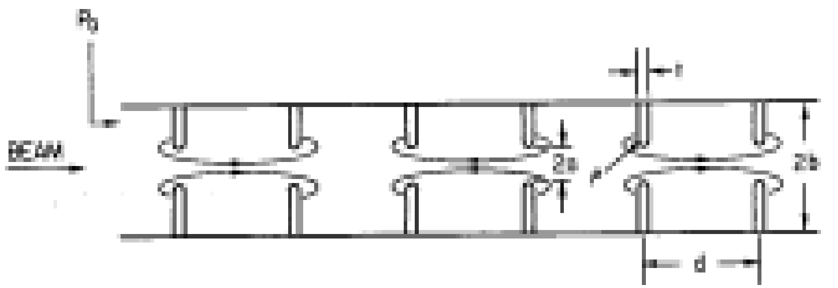


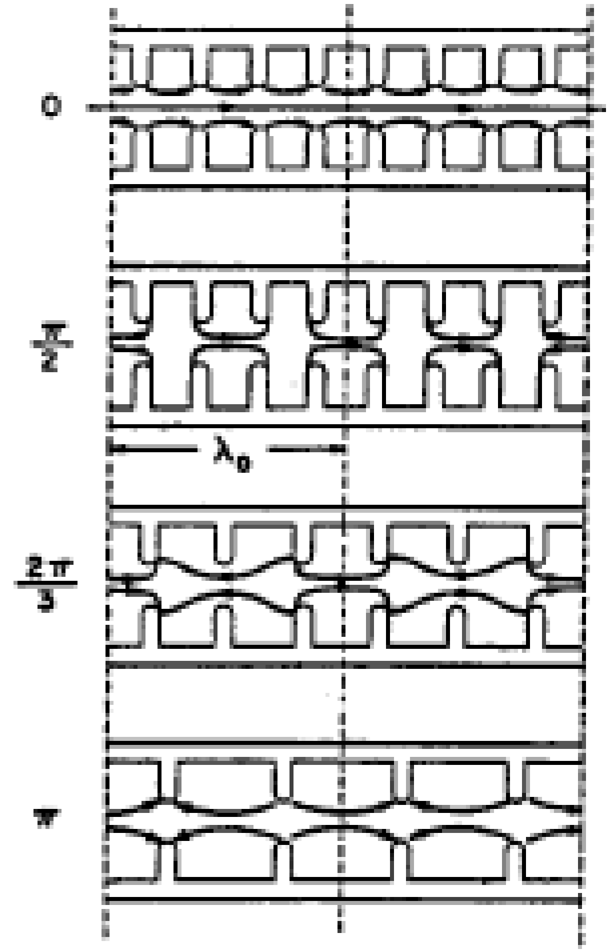



3dB Hybrid
3dB Hybrid
3dB Hybrid
Coupler
SLED
Coupler
VAC
LL
RF
Modulator
KLY
Water
Station


Klystron
25 Feet
of Earth
Waveguide
Accelerator Structure

Snapshots of e-field configuration for DL structures with various phase shift per period.

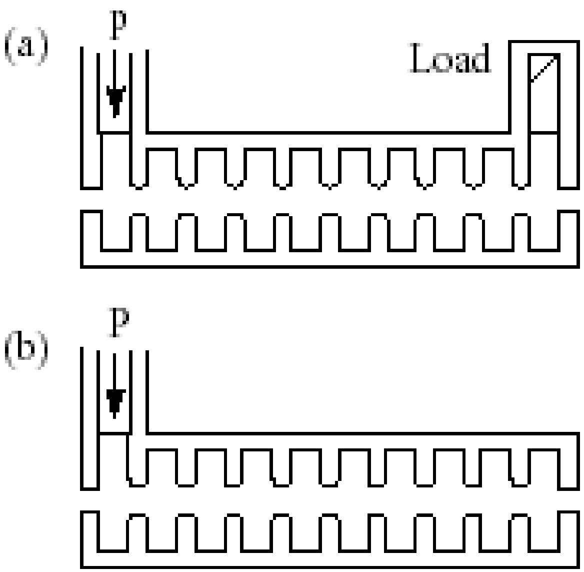




π/2 mode

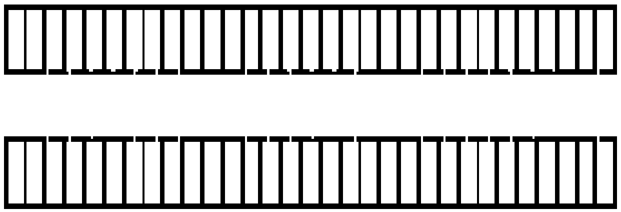
Electric field amplitude along z-axis for π/2 mode

Structure Types

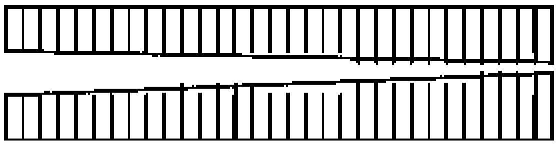


(a) Traveling Wave (TW) Structure

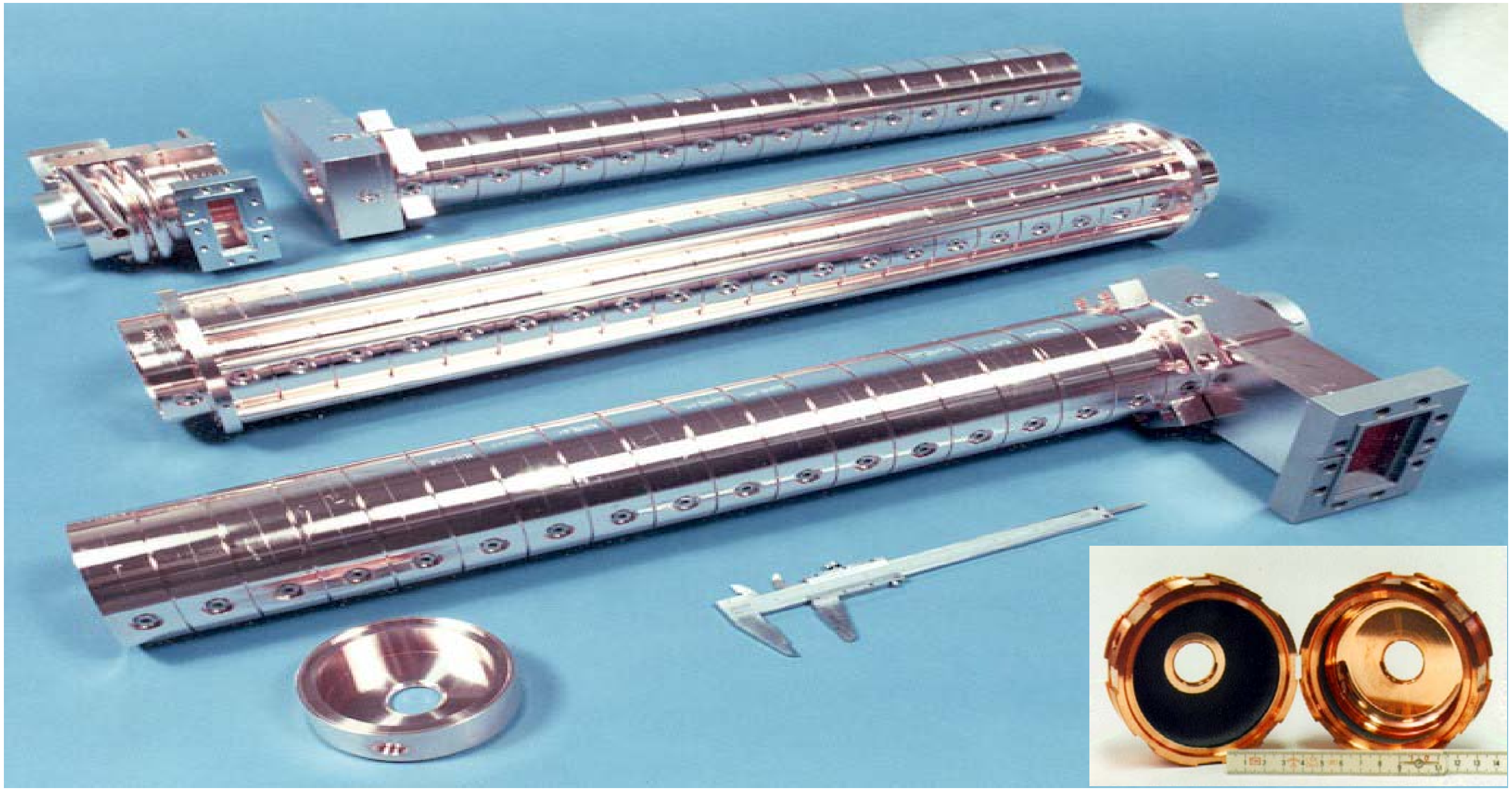
(b) Standing Wave (SW) Structure



Constant Impedance Structure (CI)

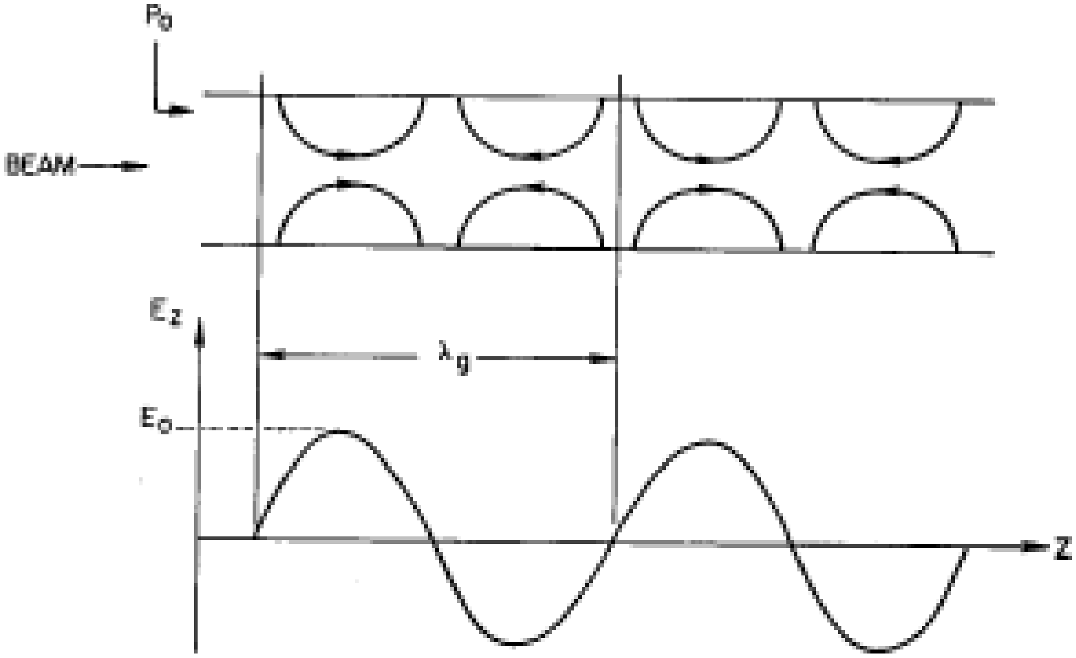


Constant Gradient Structure (CG)



Disk-Loaded Constant Gradient S-Band Structure

Circular Mode



TM_{01} mode pattern and traveling wave axial electric field in uniform cylindrical waveguide

Circular Mode

Wave equation for propagation characteristics:

$$\nabla^2 \overline{E} + k^2 \overline{E} = 0$$

$$k = \omega / c$$

K is the propagation wave number and ω is the angular frequency.

- Circular Mode

For TM_{01} mode (transverse magnetic field without θ dependence – most simple accelerating mode) in cylindrical symmetric waveguide,

$$\frac{\partial^2 E_z}{\partial r^2} + \frac{1}{r}\frac{\partial E_z}{\partial r} + \left[\left(\frac{\omega}{c}\right)^2 - \beta^2\right] E_z = 0$$

Circular Mode

The solution for TM_{01} mode is:

$$E_z = E_0 J_0(k_c r) e^{j(\omega t - \beta z)}$$

$$E_r = jE_0 \sqrt{1 - (\omega_c / \omega)^2}\, J_1(k_c r)$$

$$H_\theta = j\eta J_1(k_c r) e^{j(\omega t - \beta z)}$$

β Is propagation constant and η is the intrinsic impedance of the medium, We can consider that k_c and β to be the r and z components of k of the plane wave in free space.

- Circular Mode

$$k_c^2 = \left(\frac{\omega}{c}\right)^2 - \beta^2 = \left(\frac{\omega_c}{c}\right)^2$$

For a perfect metal boundary condition at wall, E_z=0 (the lowest frequency mode):

$$E_z(b) = 0 \Rightarrow J_0(k_c b) = 0$$

$$k_c b = 2.405$$

$$\omega_c = k_c c = {2.405c}/{b}$$

For any propagating wave, its frequency f must be greater than f_c, the field is in the form of with $\beta>0$. $$e^{j(\omega t-\beta z)}$$

Example: **An S-band (2856 MHz) structure has a diameter of 2b=8 cm, the cut-off frequency is f_c=1.9 GHz. So a 2.856GHz can propagate as TM_{01} mode.**

Phase Velocity and Group Velocity

The phase velocity V_p is the speed of RF field phase along the accelerator, it is given by

$$v_p = \frac{\omega}{\beta}$$

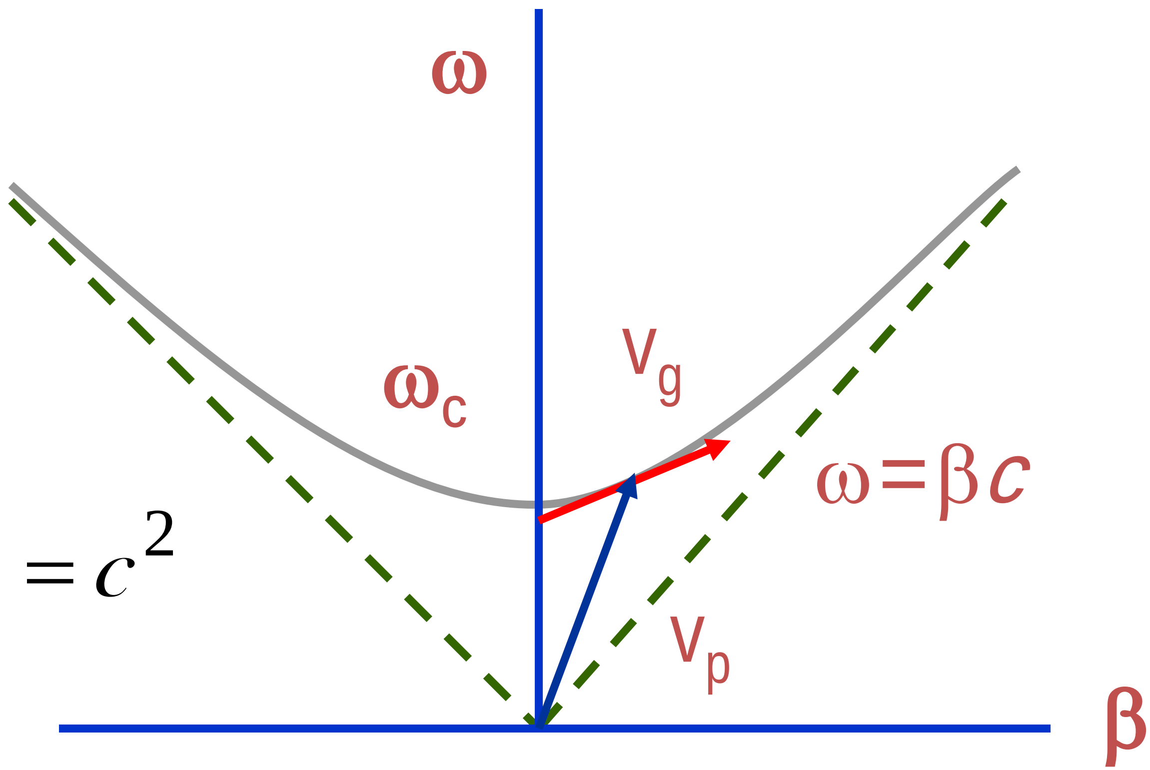
Group velocity is defined as energy propagation velocity. For waves composed of two components with different frequency ω_1 and ω_2, wave number β_1 and β_2, the wave packet travels with velocity:

$$v_g = \frac{\omega_1 - \omega_2}{\beta_1 - \beta_2} \to \frac{d\omega}{d\beta}$$

In order to use RF wave to accelerate particle beam, it is necessary to make simple cylinder “loaded” to obtain

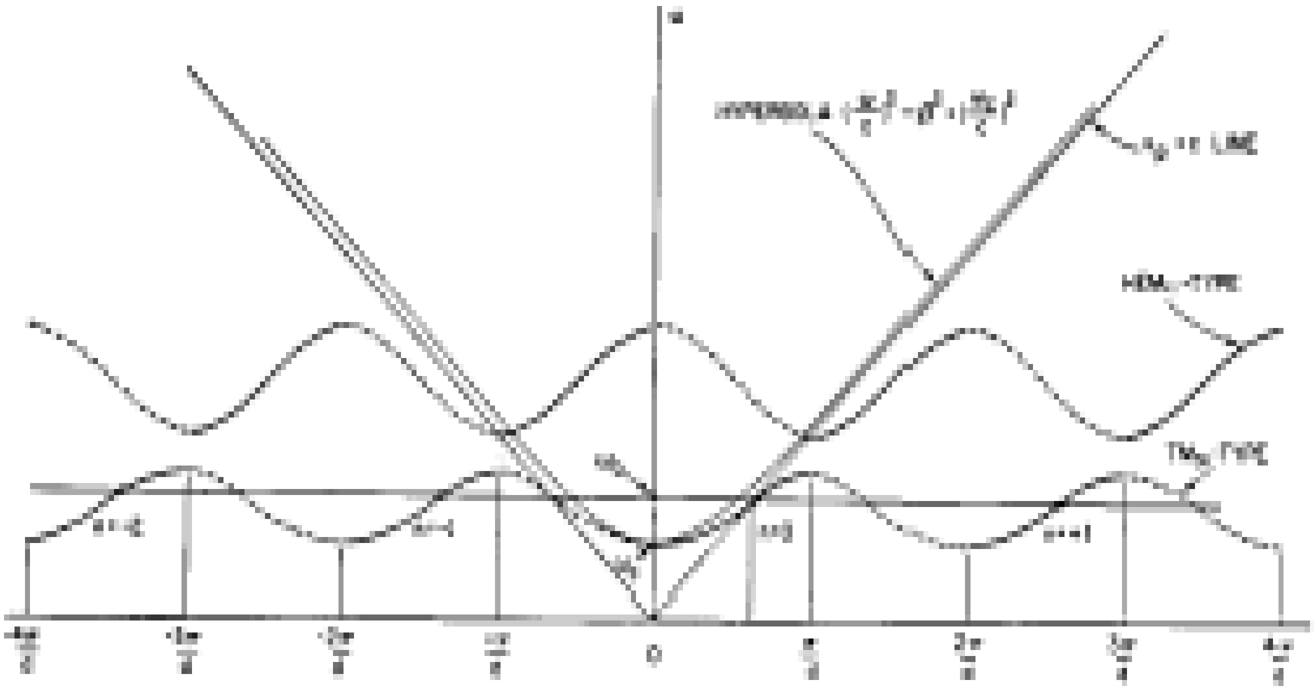
$$v_p \le c$$

For uniform waveguide, it is easy to find:



$$v_p v_g = c^2$$

Dispersion diagram for guided wave in a uniform (unloaded) waveguide.



Brillouin diagram showing propagation characteristics for uniform and periodically loaded structures with load period d.

Floquet Theorem: When a structure of infinite length is displaced along its axis by one period, it can not be distinguished from original self. For a mode with eigenfrequency ω:

$$\overline{E}\left(\bar{r}, z+d\right) = e^{-j\beta d}\,\overline{E}\left(\bar{r}, z\right) \qquad \bar{r} = x\hat{x} + y\hat{y}$$

Where βd is called phase advance per period.

Make Fourier expansion for most common accelerating TM_{01} mode:

$$E_z = \sum_{-\infty}^{\infty} a_n J_0(k_n r) e^{j(\omega t - \beta_n t)}$$

Each term is called space harmonics.

The propagation constant is

$$\beta_n = \beta_0 + \frac{2\pi n}{d} = \frac{\omega}{v_{p0}} + \frac{2\pi n}{d}$$

$$k_n^2 = k^2 - \beta_n^2$$

❑ Observations

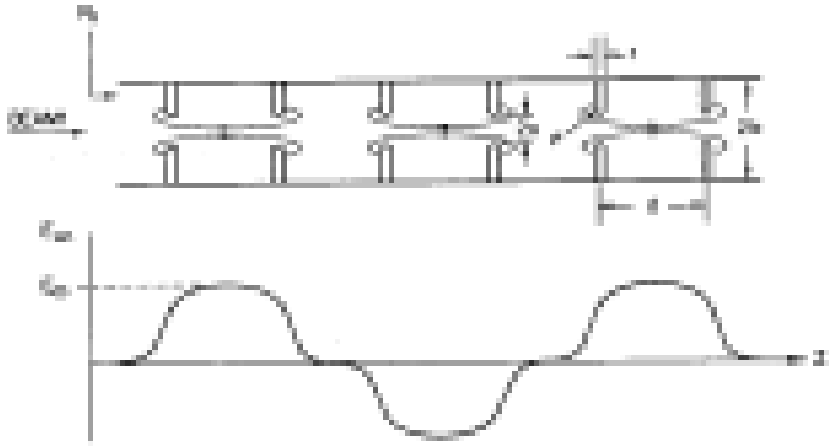
1. When the fundamental harmonic n=0 travels with v_p=c, then k_0= 0, β_0= k and $J_0(0)$=1, the acceleration is independent of the radial position for the synchronized particles.

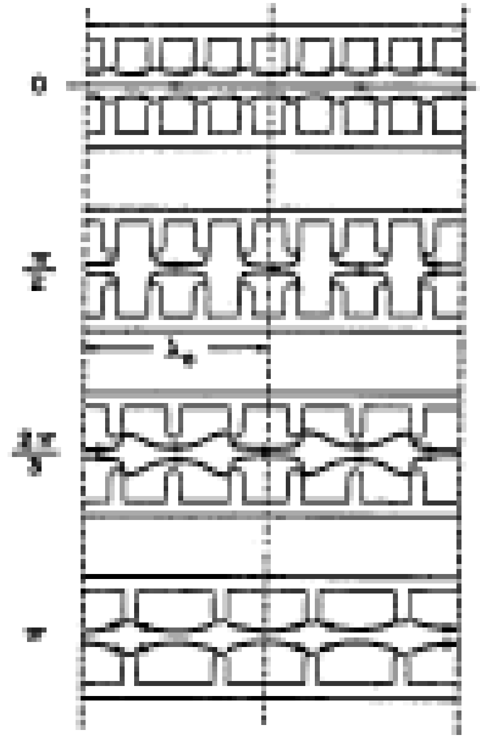
2. Each mode with specific eigenfrequency has unique group velocity.

3. Higher order space harmonics do not contribute to acceleration, but take take RF power.

RF parameters for accelerating modes

Mode which is defined as the phase shift per structure period: **$\phi=2\pi/m$** where **m** is the cavity number per wavelength.





Snapshots of electric field configurations for disk-loaded structures with various phase shift per period (left up for $\pi/2$ mode and right for 0, $\pi/2$, $2\pi/3$, π mode). Traveling wave axial electric field amplitude along z-axis for $\pi/2$ mode (left lower).

Shunt impedance per unit length r: is a measure of the accelerating quality of a structure

$$r = -\frac{E_a^2}{dp/dz}$$

Unit of MΩ/m or Ω/m

Where E_a is the synchronous accelerating field amplitude and dP/dz is the RF power dissipated per unit length.

$$R = \frac{V^2}{P_d}$$

Unit of MΩ or Ω

Factor of merit Q, which measures the quality of an EF structure as a resonator.

For a traveling wave structure $Q = -\frac{\omega W}{dP/dz}$ where W is the rf

energy stored per unit length and ω is the angular frequency and dP/dz is the power dissipated per unit length.

For standing wave structure,

$$Q = \frac{\omega W}{P_d}$$

Group velocity V_g which is the speed of RF energy flow along the accelerator is given by

$$V_g = \frac{P}{W} = \frac{-\omega P}{Q\,{dP}/{dz}} = \frac{d\omega}{d\beta}$$

Attenuation factor τ of a constant-impedance or constant-gradient is

$$\frac{dE}{dz} = -\alpha E \qquad \frac{dP}{dz} = -2\alpha P$$

α Is the attenuation constant in nepers per unit length.

Attenuation factor τ for a traveling wave section is defined as

$$\frac{P_{out}}{P_{in}} = e^{-2\tau}$$

For a constant-impedance section, the attenuation is uniform,

$$\alpha = \frac{-\,dP/dz}{2P} = \frac{\omega}{2v_g Q} \qquad \tau = \alpha L = \frac{\omega L}{2v_g Q}$$

For non-uniform structures,

$$\tau = \int_0^L \alpha(z)dz$$

For a constant-gradient section, the attenuation constant α is a function of z: $\alpha = \alpha(z) = \omega / 2V_g(z)Q$

We have the following expression

$$\frac{dP}{dz} = -2\alpha(z)P = const = \frac{P_{in}\left(1 - e^{-2\tau}\right)}{L}$$

r/Q ratio is a measure of accelerating field for a certain stored energy. It only depends on the geometry and independent of material and machining quality.

$$\frac{r}{Q} = \frac{E^2}{\omega W}$$

Filling time t_F- For a traveling wave structure, the field builds up "in space". The filling time is the time which is needed to fill the whole section of either constant impedance or constant gradient. It is given by:

$$t_F = \int_0^L \frac{dz}{V_g} = \frac{Q}{\omega} \int_0^L \frac{-dp/dz}{P} dz = \frac{2Q}{\omega} \tau$$

The field in SW structures builds uo "in time". The filling time is the the time needed to build up the field to (1-1/e)=0.632 times the steady-state field:

$$t_F = \frac{2Q_L}{\omega} = \frac{2Q_0}{(1+\beta_c)\omega} \quad \beta_c = \frac{Q_0}{Q_{ext}}$$

Q_0 is the unloaded Q value

$$Q_0 = \frac{\omega W}{P_d}$$

Q_{ext} is the external Q value

$$Q_{ext} = \frac{\omega W}{P_{ext}} = \frac{Q_0}{\beta}$$

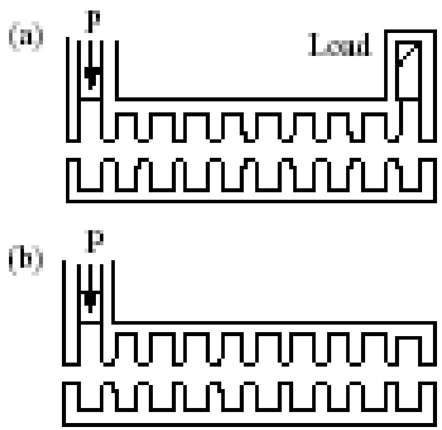
Q_L is the loaded Q value

$$Q_L = \frac{\omega W}{(P_d + P_{ext})} = \frac{Q_0}{(1+\beta)}$$

The choice of the operating frequency is of fundamental importance since almost all the basic RF parameters are frequency dependent.

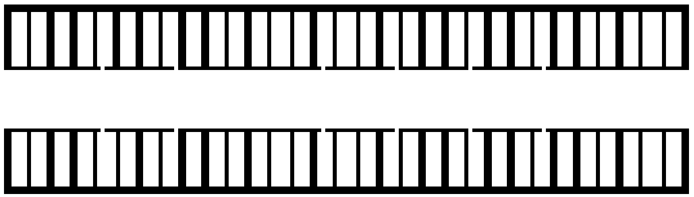
$$r \propto \sqrt{f} \quad size \propto \frac{1}{f} \quad Q \propto 1/\sqrt{f} \quad r/Q \propto f$$

Structure Types

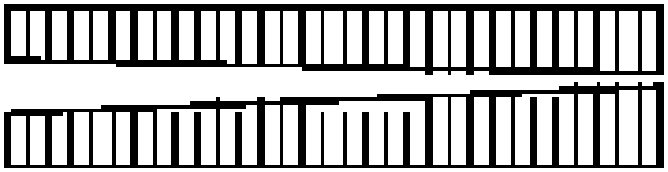


(a)Traveling Wave Structure (TW)

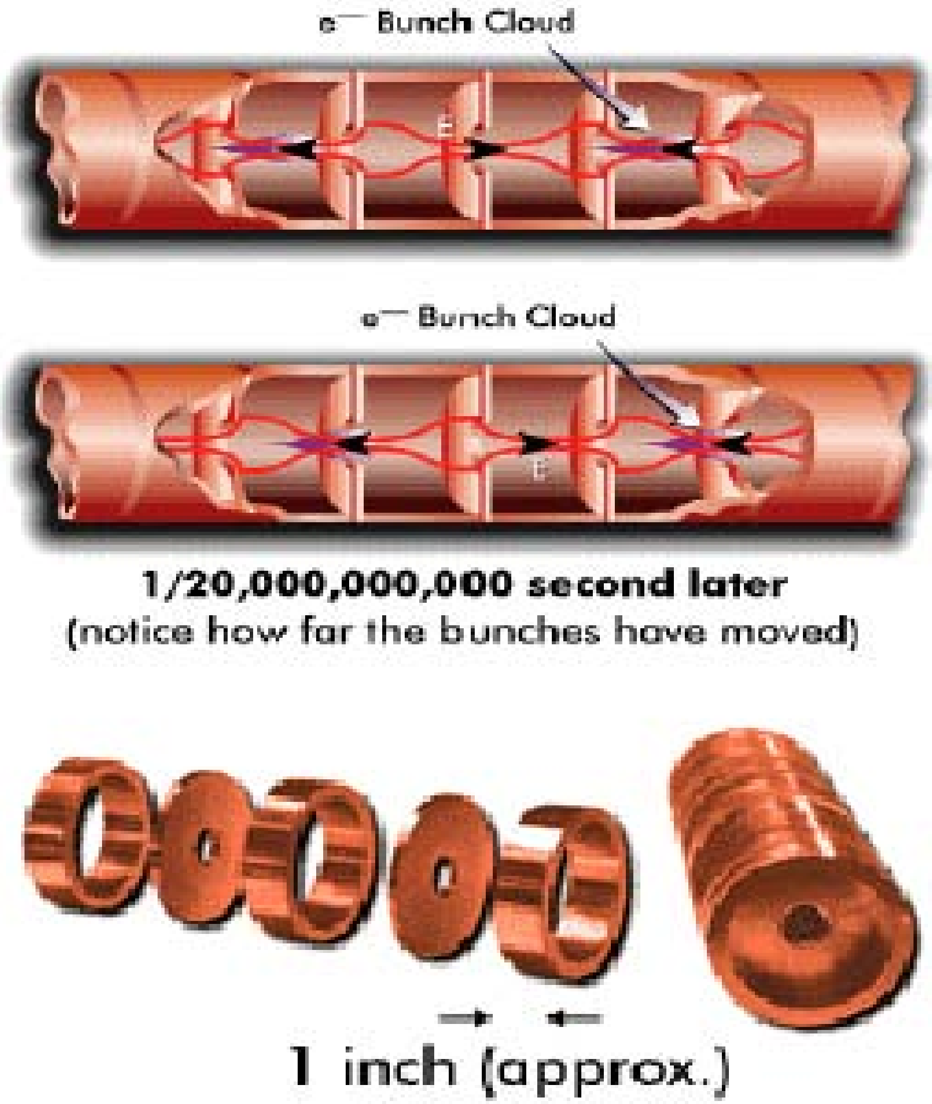
(b) Standing Wave Structure (SW)



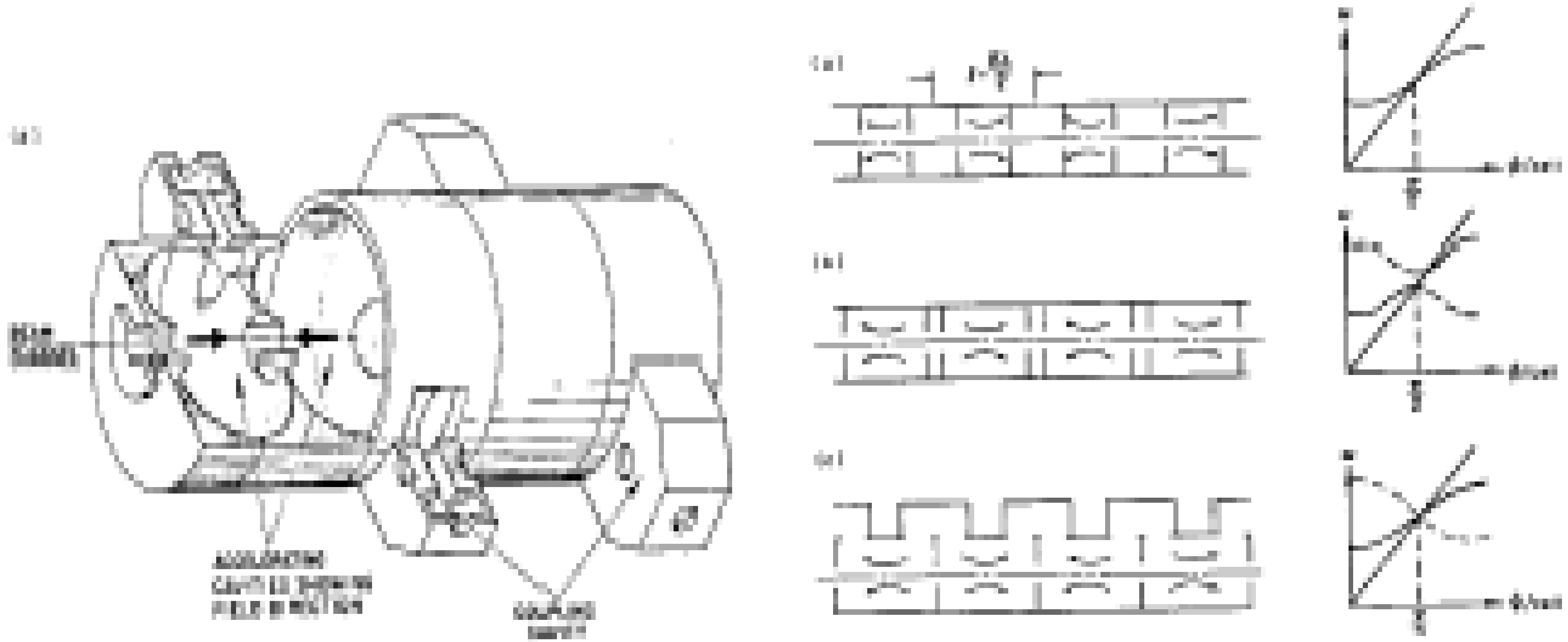
Constant Impedance Structure (CI)



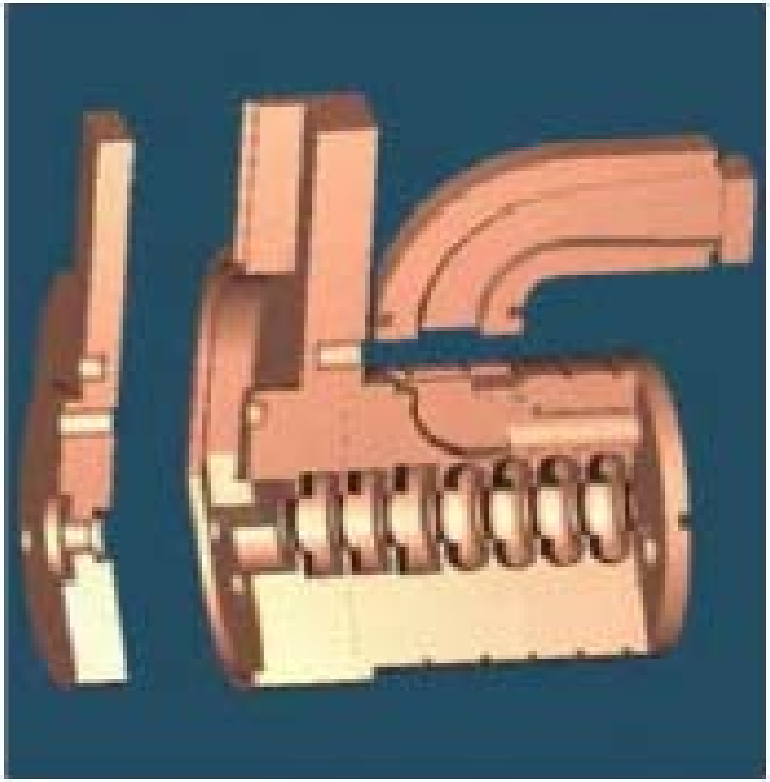
Constant Gradient Structure (CG)


e⁻ Bunch Cloud
e⁻ Bunch Cloud
1/20,000,000,000 second later
(notice how far the bunches have moved)
1 inch (approx.)

π Mode SCC



Evolution of π/2 mode disk-loaded structure to πmode side coupled structure.



Round Damped Detuned Structure (RDDS – 206 cells

Longitudinal Dynamics

Energy of particle:

$$u = \frac{m_\circ c^2}{\sqrt{1-\beta_e^2}}$$

Energy change with time: $\frac{du}{dt} = -eE_z \frac{dz}{dt} \sin\theta$

$$\theta = \frac{\omega z}{v_p} - \omega t$$

Where t is the time it takes particle to reach z

$$\theta = \omega \int \left(\frac{1}{v_p} - \frac{1}{v_e} \right) dz$$

Longitudinal Dynamics

$$d\theta = \omega\left(\frac{1}{v_p} - \frac{1}{v_e}\right)dz$$

When a particle is faster ($v_e > v_p$), $d\theta > 0$ and vise versa.

It is convenient to use z as a variable and $\frac{d}{dt} = \frac{dz}{dt}\frac{d}{dz}$

The longitudinal motion is described by the following two equations:

$$\frac{du}{dz} = -eE_z \sin\theta$$

$$\frac{d\theta}{dz} = \frac{2\pi}{\lambda}\left(\frac{1}{\beta_p} - \frac{u}{\sqrt{u^2 - u_\circ^2}}\right)$$

Longitudinal Dynamics

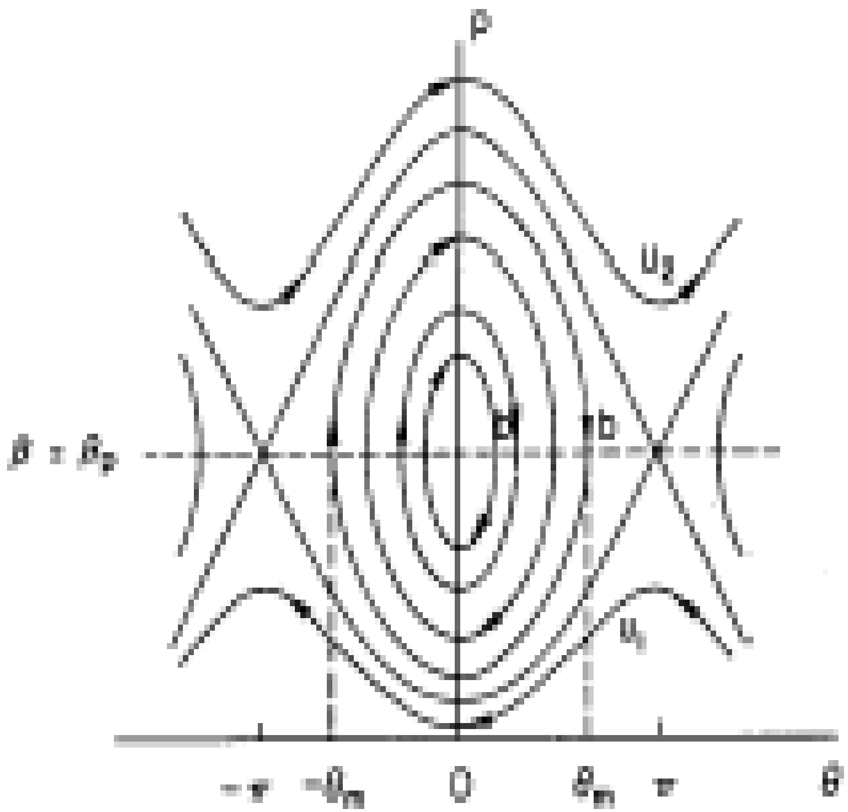
Using $p = \frac{mv_e}{m_\circ c} = \gamma\beta_e = \sqrt{\gamma^2 - 1}$ (normalized momentum)

Integration using variable substitution of $\gamma^2 = p^2 + 1,$

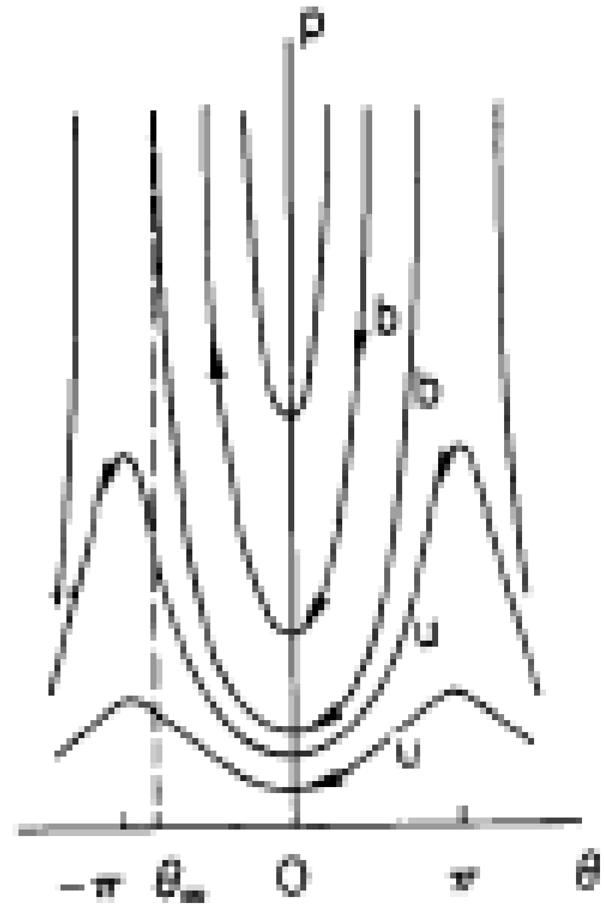
The equation for the orbit in phase space is

$$cos\,\theta - cos\,\theta_m = \frac{2\pi m_\circ c^2}{eE\lambda}\left[\sqrt{p^2+1} - \sqrt{1-\beta_p^2} - \beta_p p\right]$$

Longitudinal Dynamics



Longitudinal phase space for $\beta_p<1$



Longitudinal phase space for $\beta_p=1$

Longitudinal Dynamics

Phase velocity less than c (β_p<1)

When 1>cosθ>-1, the particles oscillate in p and θ plane with elliptical orbits centered around ($\beta = \beta_p$, θ=0). If an assembly of particles with a relative large phase extent and small momentum extent enters such a structure, then after traversing ¼ of a phase oscillation it will have a small phase extent and large momentum extent, bunching.

Longitudinal Dynamics

Phase velocity equals c (β_p=1)

When β_p=1, $d\theta/dz$ is always negative, and the orbits become open-ended. The orbit equation becomes

$$\cos\theta - \cos\theta_m = \frac{2\pi m_\circ c^2}{eE\lambda}\left[\sqrt{p^2+1} - p\right] = \frac{2\pi m_\circ c^2}{eE\lambda}\sqrt{\frac{1-\beta_e}{1+\beta_e}}$$

Where θ_m has been renamed to θ_∞ to emphasize that it corresponds to $p=\infty$. The threshold accelerating gradient for capture is $\cos\theta - \cos\theta_\infty = 2$, or

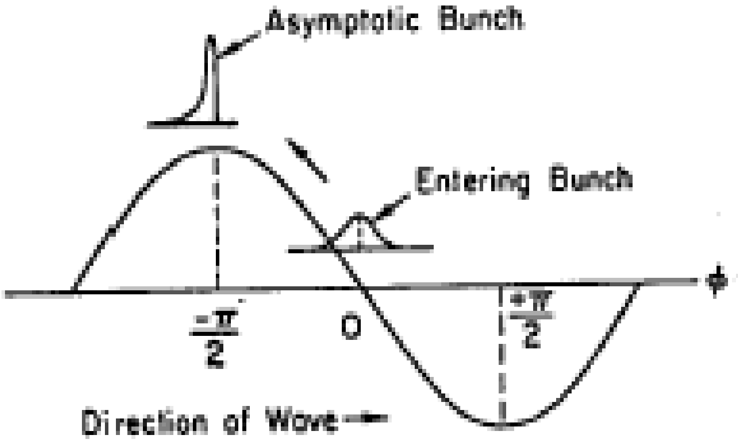
$$E_0(threshold) = \frac{\pi m_0 c^2}{e\lambda}\left[\sqrt{p^2+1} - p_0\right]$$

Longitudinal Dynamics

Lets consider a particle entering the structure with a phase θ_0=0, has an asymptotic phase $\theta_\infty=-\pi/2$, thus a assembly of particles will get maximum acceleration and minimum phase compression. For small phase extents $\pm\Delta\theta_0$ around θ_0=0,

$$\theta_\infty = -\frac{\pi}{2} - \frac{(\Delta\theta_\circ)^2}{2}$$

Example:



Traveling Wave Structures

Energy gain V of a charged particle is given by

$$CI: \; V = \sqrt{2\tau}\left[\left(1-e^{-\tau}\right)/\tau\right]\sqrt{P_{in}rL}$$

$$CG: \; V = \sqrt{1-e^{-2\tau}}\sqrt{P_{in}rL} = \sqrt{P_{disp}rL}$$

RF energy supplied during time t_F can be derived from above:

$$CI: P_{in}t_F = \left(\frac{\tau}{1-e^{-\tau}}\right)^2 \frac{V^2}{\omega \frac{r}{Q} L}$$

$$CG: P_{in}t_F = \left(\frac{2\tau}{1-e^{-2\tau}}\right)^2 \frac{V^2}{\omega \frac{r}{Q} L}$$

Traveling Wave Structures

Energy W stored in the entire section at the end of filling time is

$$CI : W = \frac{Q}{\omega}\int_0^L \frac{dP}{dz}dz = P_{in}\frac{Q}{\omega}\left(1-e^{-2\tau}\right)$$

$$CG : W = \int_0^L \frac{P}{v_g}dz = P_{in}\frac{Q}{\omega}\left(1-e^{-2\tau}\right)$$

Standing Wave Structures

Due to multi reflections, the equivalent input power is increased:

$$P = P_s + P_s e^{-2\alpha L} + P_s e^{-4\alpha L} + \ldots = \frac{P_s}{1 - e^{-4\alpha L}}$$

The slightly higher energy gain for SW is paid by field building up time (choice of length of SW structures).

For resonant cavity, power feed is related to the RF coupling:

$$\frac{1}{Q_L} = \frac{1}{Q_0} + \frac{1}{Q_{ext}} \quad \beta_c = \frac{Q_0}{Q_{ext}}$$

Standing Wave Structures

As defined before, Q_L, $Q_0=\omega W/Pd$, $Q_{ext}= \omega W/P_{ext}$ are the loaded Q, cavity Q, and external Q values. β_c is the coupling coefficient between the waveguide and the structure.

The energy gain of a charged particle is given:

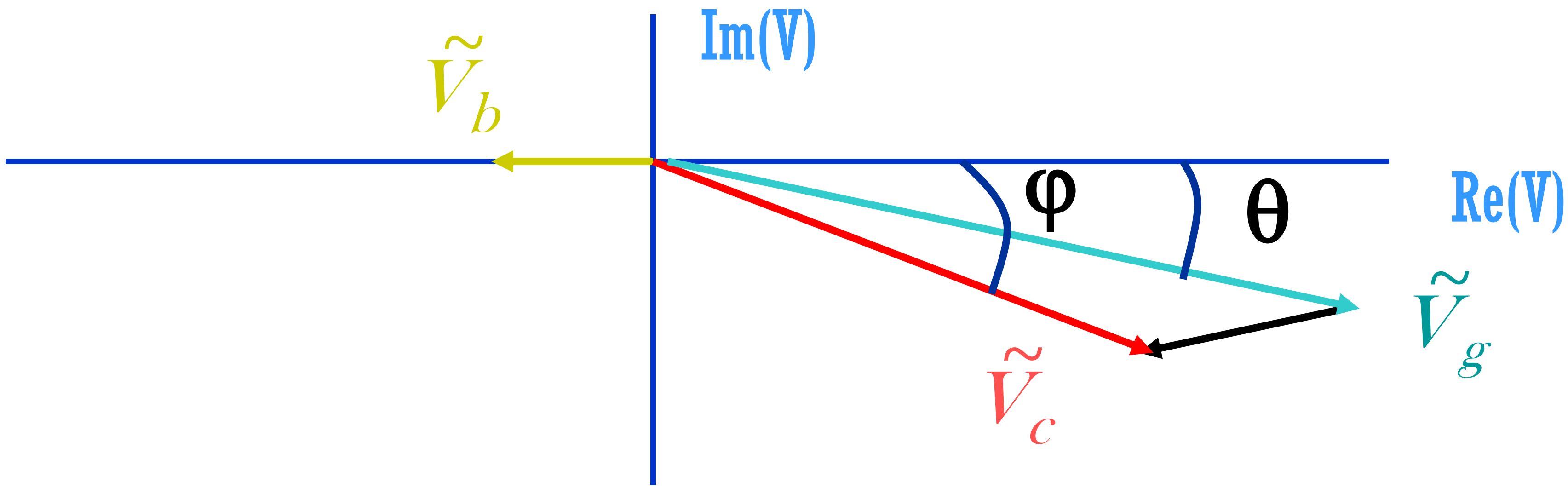
$$V = \left(1 - e^{-t/t_F}\right)\frac{2\sqrt{\beta_c}}{1+\beta_c}\sqrt{P_{in} r L} = \left(1 - e^{-t/t_F}\right)\sqrt{P_{disp} r L}$$

Beam Loading

- **Long train of bunches**
- **Bunches in from extract energy from linac**
  - **Lower gradient**
  - **Increase phase**
- **Effect on later bunches**
  - **Bunch placed directly ignoring beam loading**
  - **Bunch doesn't gain enough energy**
  - **If it gained enough energy, it would arrive at the same RF phase**
  - **Non-isochronous arc: bunch arrives in next linac late, sees higher gradient**
  - **Gaines excess energy**

Beam Loading

The effect of the beam on the accelerating field is called BEAM LOADING. The superposition of the accelerating field established by external generator and the beam-induced field needs to be studied carefully in order to obtain the net **P**hase and **A**mplitude of acceleration.



Beam Loading

In order to obtain a basic physics picture, we will assume that the synchronized bunches in a bunch train stay in the peak of RF field for both TW and SW analysis.

The RF power loss per unit length is given by:

$$\frac{dP}{dz} = \left(\frac{dP}{dz}\right)_{wall} + \left(\frac{dP}{dz}\right)_{beam}$$

$$E^2 = 2\alpha r P$$

$$E\frac{dE}{dz} = rP\frac{d\alpha}{dz} + \alpha r\frac{dP}{dz} = r\frac{E^2}{2\alpha r}\frac{d\alpha}{dz} - \alpha r\left(\frac{E^2}{r} + EI\right)$$

$$\frac{dE}{dz} = -\alpha E\left(1 - \frac{1}{2\alpha^2}\frac{d\alpha}{dz}\right) - \alpha r I$$

Beam Loading

For constant impedance structure: $\frac{dE}{dz} = -\alpha E - \alpha r I$

$$E(z) = E(0)e^{-\alpha z} - Ir\left(1 - e^{-\alpha z}\right)$$

$$E(0) = \sqrt{2\alpha r P_{in}}$$

The total energy gain through a length L is

$$V = \int_0^L E(z)dz = \sqrt{2rP_0L}\,\frac{1-e^{-\tau}}{\sqrt{\tau}} - IrL\left(1 - \frac{1-e^{-\tau}}{e^{-\tau}}\right)$$

P_0 is input rf power in MW, r is the shunt impedance per unit length in MΩ/m, L is the structure length in meters, I is the average beam current in Ampere, and V is the total energy gain in MV

Beam Loading

For constant gradient structures:

$$\frac{dE}{dz} = -\alpha r I$$

$$E = E_0 + \frac{rI}{2} ln\left(1 - \frac{z}{L}\left(1 - e^{-2z}\right)\right)$$

The attenuation coefficient is

$$\alpha(z) = \frac{\left(1 - e^{-2\tau}\right)/2L}{1 - \left(1 - e^{-2\tau}\right)(z/L)}$$

Beam Loading

The complete solution including transient can be expressed as

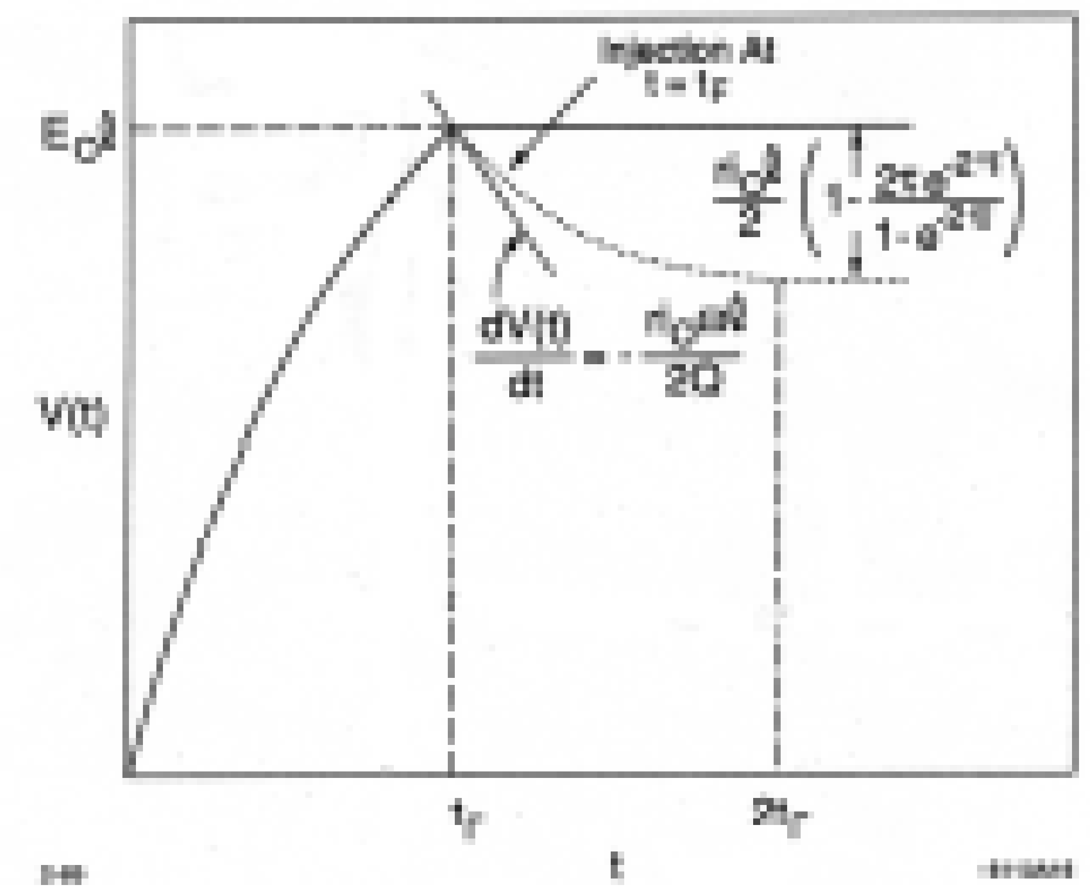
$t_F \le t \le 2t_F$:

$$V(t) = E_0 L + \frac{rI}{2}\left[\frac{\omega L e^{-2\tau}}{Q\left(1-e^{-2\tau}\right)} t - \frac{L}{1-e^{-2\tau}}\left(1-e^{-\frac{\omega}{Q}t}\right)\right]$$

$t \ge 2t_F$:

$$V(t) = E_0 L - \frac{rIL}{2}\left[1 - \frac{2\tau e^{-2\tau}}{1-e^{-2\tau}}\right]$$

Beam Loading



Transient beam loading in a TW constant gradient structure.

Beam Loading

For a standing wave structure with a coupling coefficient β_c, the energy gain V(t) is

$$V = \left(1 - e^{-t/t_F}\right)\frac{2\sqrt{\beta_c}}{1+\beta_c}\sqrt{P_{in} rL} = \left(1 - e^{-t/t_F}\right)\sqrt{P_{disp} rL}$$

$$V = \left(1 - e^{-t/t_F}\right)\sqrt{P_{disp} rL} - \frac{IrL}{1+\beta}\left(1 - e^{t-t_b/t_F}\right)$$

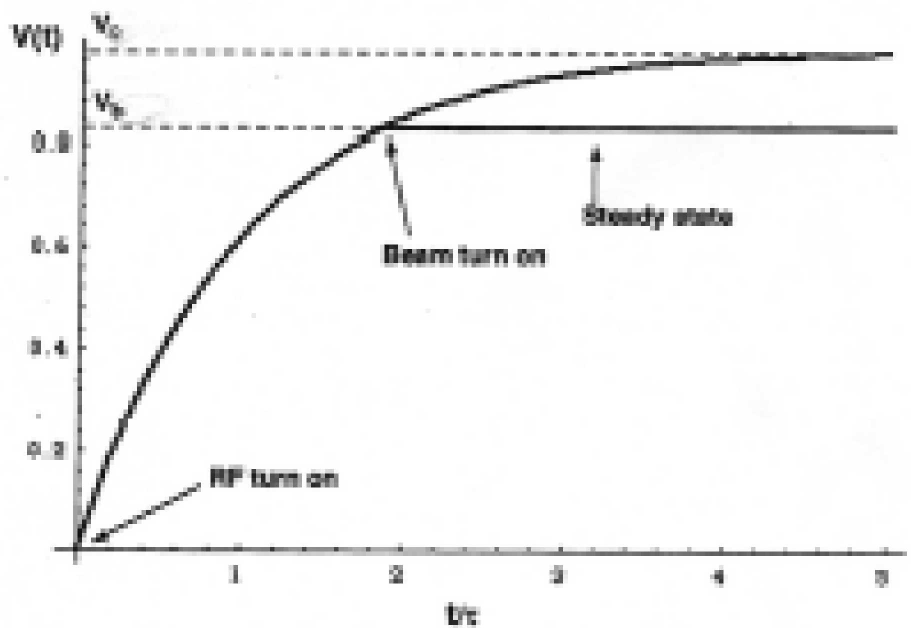
If the beam is injected at time t_b and the coupling coefficient meets the following condition:

$$\beta_c = 1 + \frac{P_0}{P_{disp}} \quad \Rightarrow \quad P_{in} = P_{disp} + P_b$$

Beam Loading

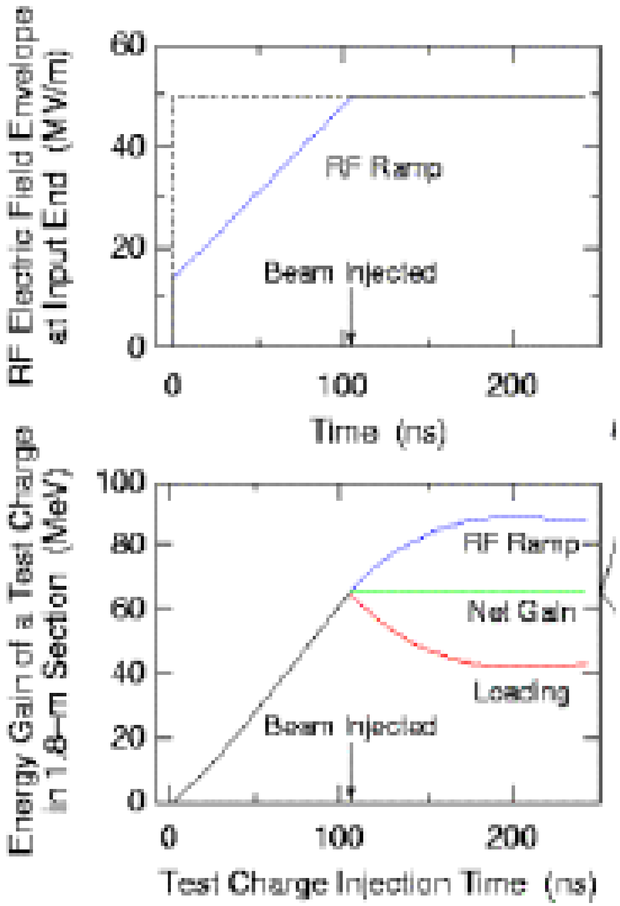
There is no reflection from the structure to power source with beam. The beam injection time can be obtained:

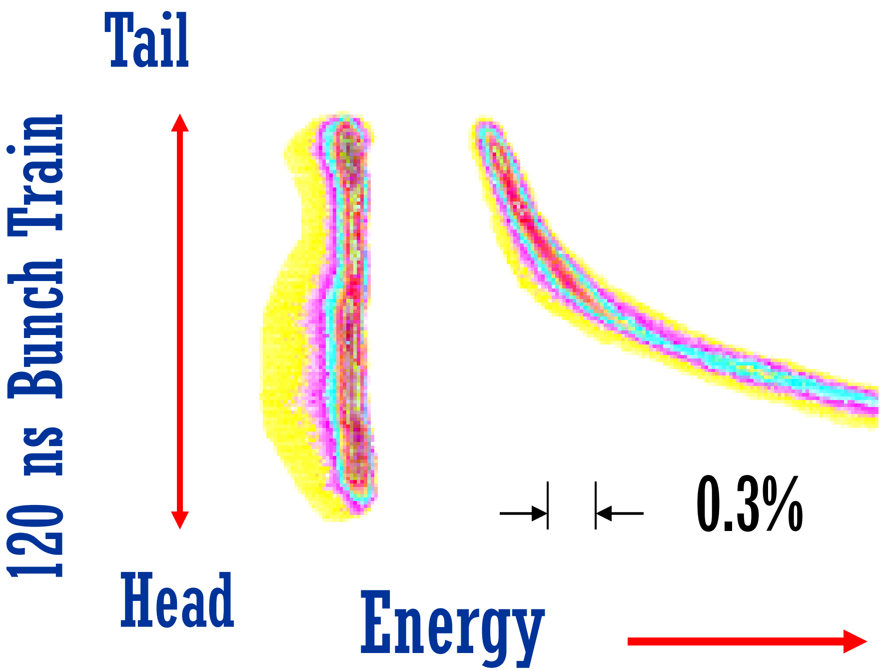
$$t_b = -t_F \, ln\left(1 - \frac{V_b}{V_0}\right) = t_F \, ln \frac{2\beta_c}{\beta_c - 1}$$



Beam Loading Compensation

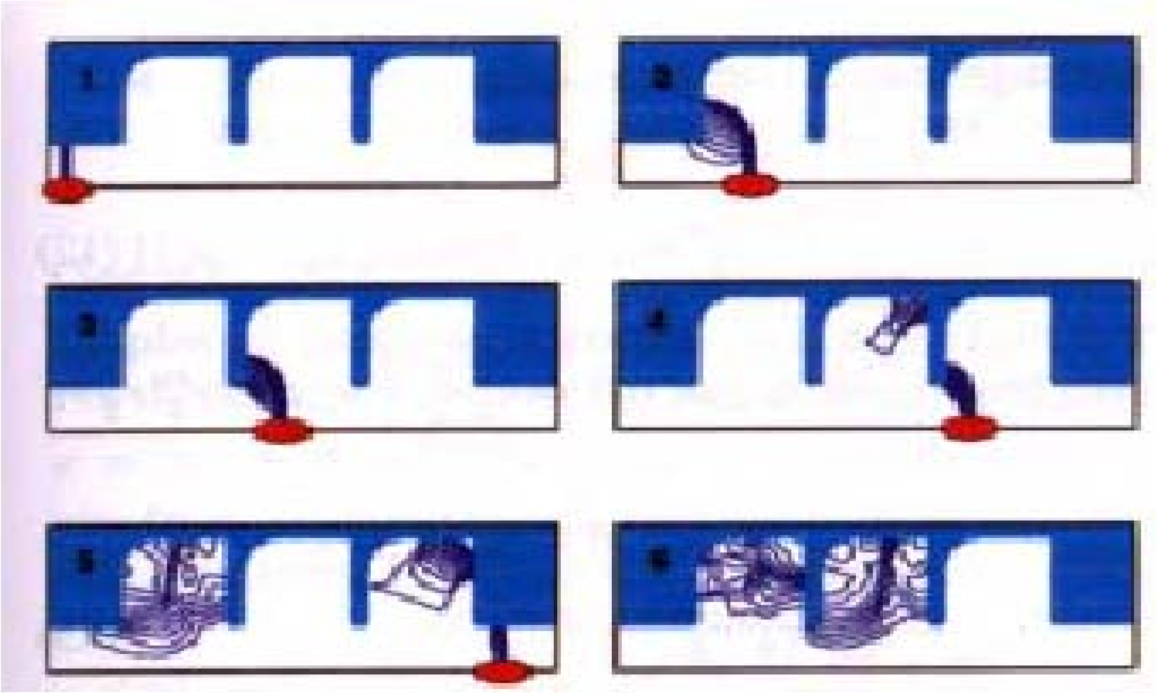
Using RF Amplitude Ramp during fill





Wakefields

The wakefield is the scattered electromagnetic radiation created by relativistic moving charged particles in RF cavities, vacuum bellows, and other beamline components.



Electric field lines of a bunch traversing through a three-cell disk-loaded structure

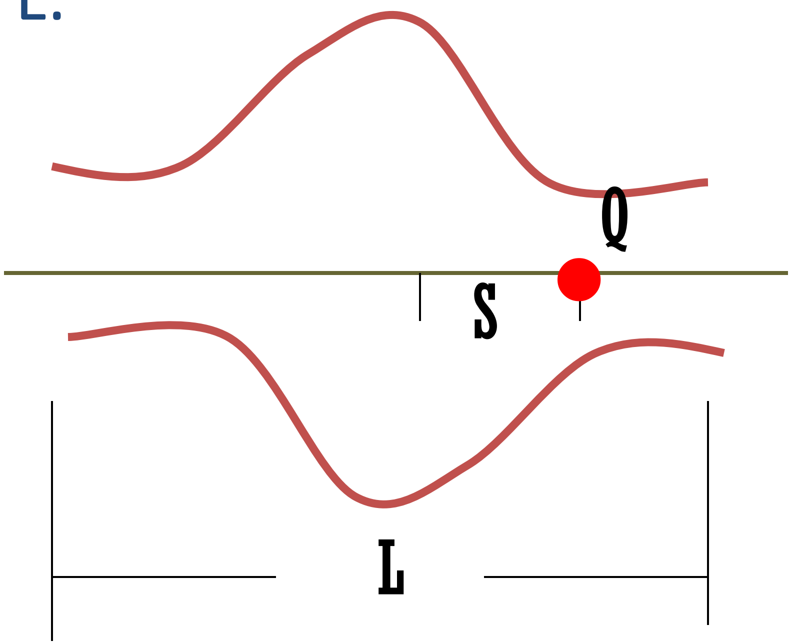
J. Wang, SLAC

Wakefields

- No disturbance ahead of moving charge – CAUSALTY.
- Wakefields behind the moving charge vary in a complex way, in space and time.
- These fields can be decomposed into MODES.
- Each mode has its particular FIELD PATTERN and will oscillate with its own frequency.
- For simplified analysis, the modes are orthogonal, i.e., the energy contained in a particular mode does not have energy exchange with the other modes.

Wakefields

Lets consider a point charge with charge Q moving at the speed of light along a path in z direction through a discontinuity L:



Longitudinal Wakefields

For practical purposes, all the bunches (driven and test bunch) are near the structure axis.

We define the longitudinal delta-function potential $W_z(s)$ as the potential (Volt/Coulomb) experienced by the test particle following along the same path at time τ (distance $s = \tau c$)behind the unit driving charge.

$$W_z(s) = \frac{1}{Q}\int_0^L dz E_z\left(z, \frac{z+s}{c}\right)$$

Longitudinal Wakefields

The longitudinal wakefields are dominated by the m=0 modes, for example TM_{01}, TM_{02}, …

$$W_z(s) = \sum_n k_n \cos\left(\frac{\omega_n s}{c}\right) \times \begin{matrix} 0(s<0) \\ 1(s=0) \\ 2(s>0) \end{matrix}$$

The loss factor k_n is:

$$k_n = \frac{|V_n|^2}{4U_n} = \frac{\omega_n}{4}\left(\frac{R_n}{Q_n}\right)$$

Where U_n is the stored energy in the n^{th} mode. V_n is the maximum voltage gain from the n^{th} mode for a unit test charge particle with speed of light.

Longitudinal Wakefields

The total amount of energy deposited in all the modes by the driving charge is:

$$U = Q^2 \sum_n k_n$$

Longitudinal wakefields are approximately independent of the transverse position of both the driving charges and the test charges.

Short range longitudinal wakefield – Energy spread within a bunch.

Long range longitudinal wakefield – Beam loading effect.

Transverse Wakefields

Transverse wakefield potential is defined as the transverse momentum kick experienced by a unit test charge following at a distance s behind the same path with a speed of light.

$$W_{\perp} = \frac{1}{Q} \int_{0}^{L} dz \left[\vec{E}_{\perp} + \left(\vec{v} \times \vec{B} \right)_{\perp} \right]_{\frac{z+s}{c}}$$

The transverse wakefields are dominated by the dipole mode (m>1), for example, HEM_{11}, HEH_{21},…

Transverse Wakefields

An expression of the transverse wakefield is approximately:

$$W_{\perp} \cong \left(\frac{r'}{a}\right)\hat{x}\sum_{n}\frac{2k_{1n}}{\omega_{1n}\,a/c}\,sin\left(\frac{\omega_{1n}s}{c}\right) \quad (s \geq 0)$$

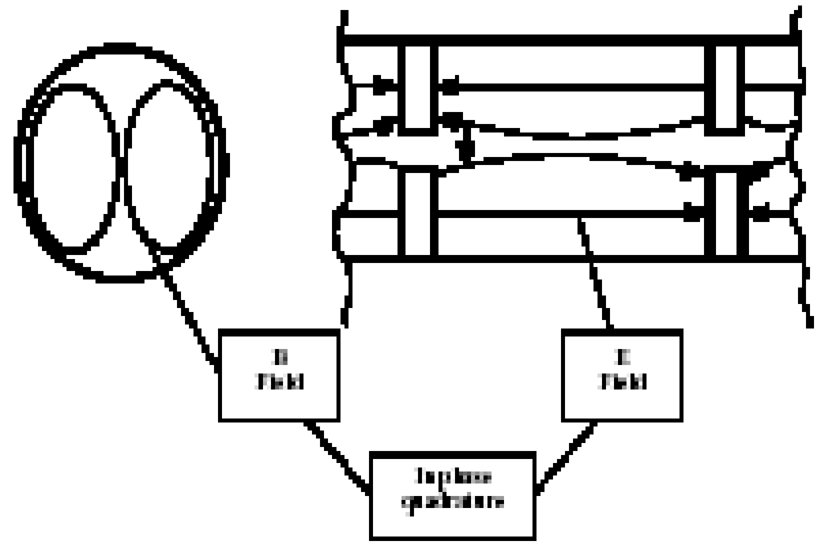
Where r′ is the transverse offset of the driving charge, a is the tube radius of the structure, and k_{1n} for m=1 nth dipole mode has a similar definition as m=0 case. The unit of transverse potential is V/Coulomb.mm.

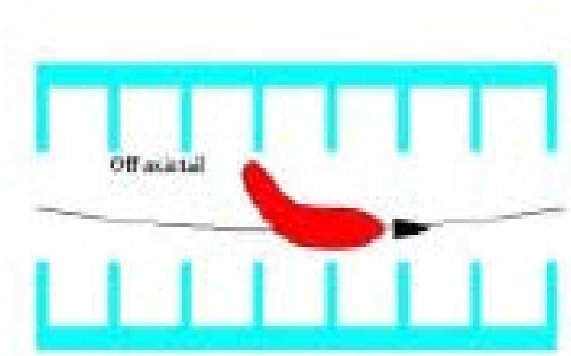
The transverse wakefields depend on the driving charge as the first power of its offset r′, the direction of the transverse wake potential vector is decided by the position of the driving charge.

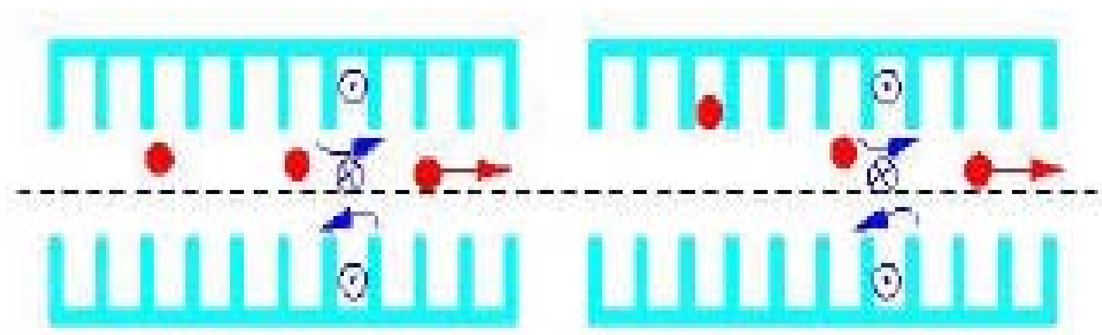
Transverse Wakefields

Example:

Schematic of field Pattern for the lowest frequency mode – HEM_{11} Mode.

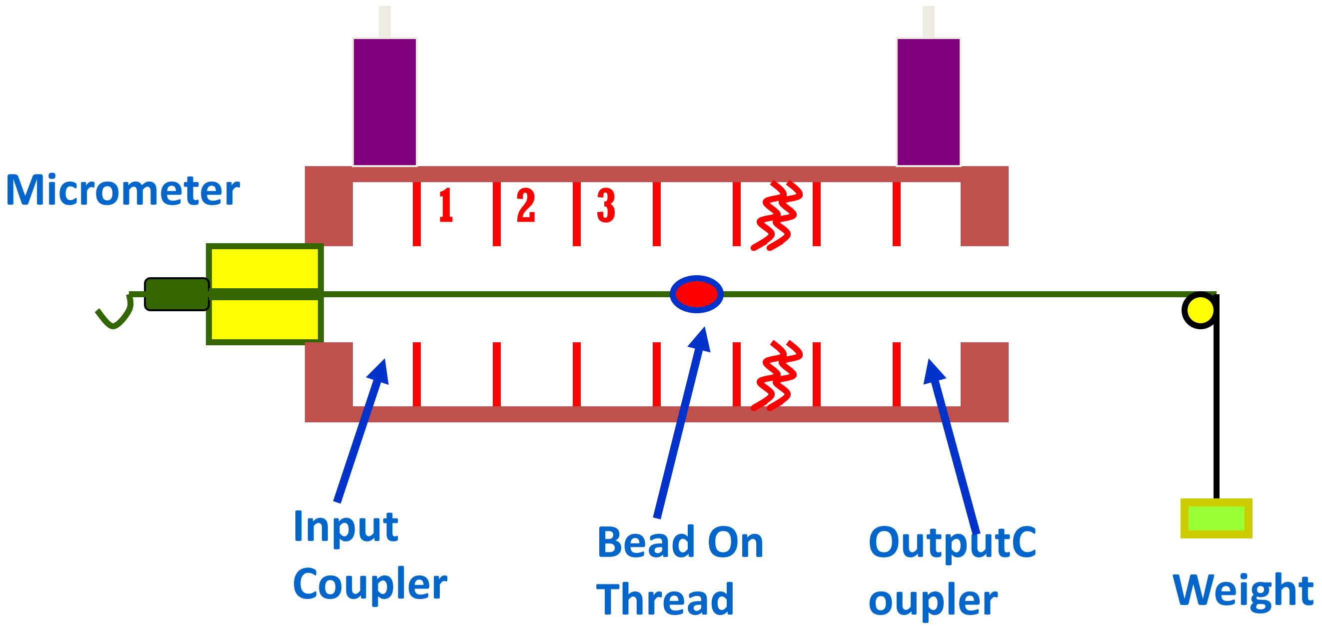






H-T Instability short range   Multi-Bunch Beam Breakup Long range.

Non-Resonance Perturbation



Non-Resonance Perturbation

Reflected wave amplitude is

$$E_r(z) = K\frac{E^2(x,y,z)}{P(z)}E_i(z)$$

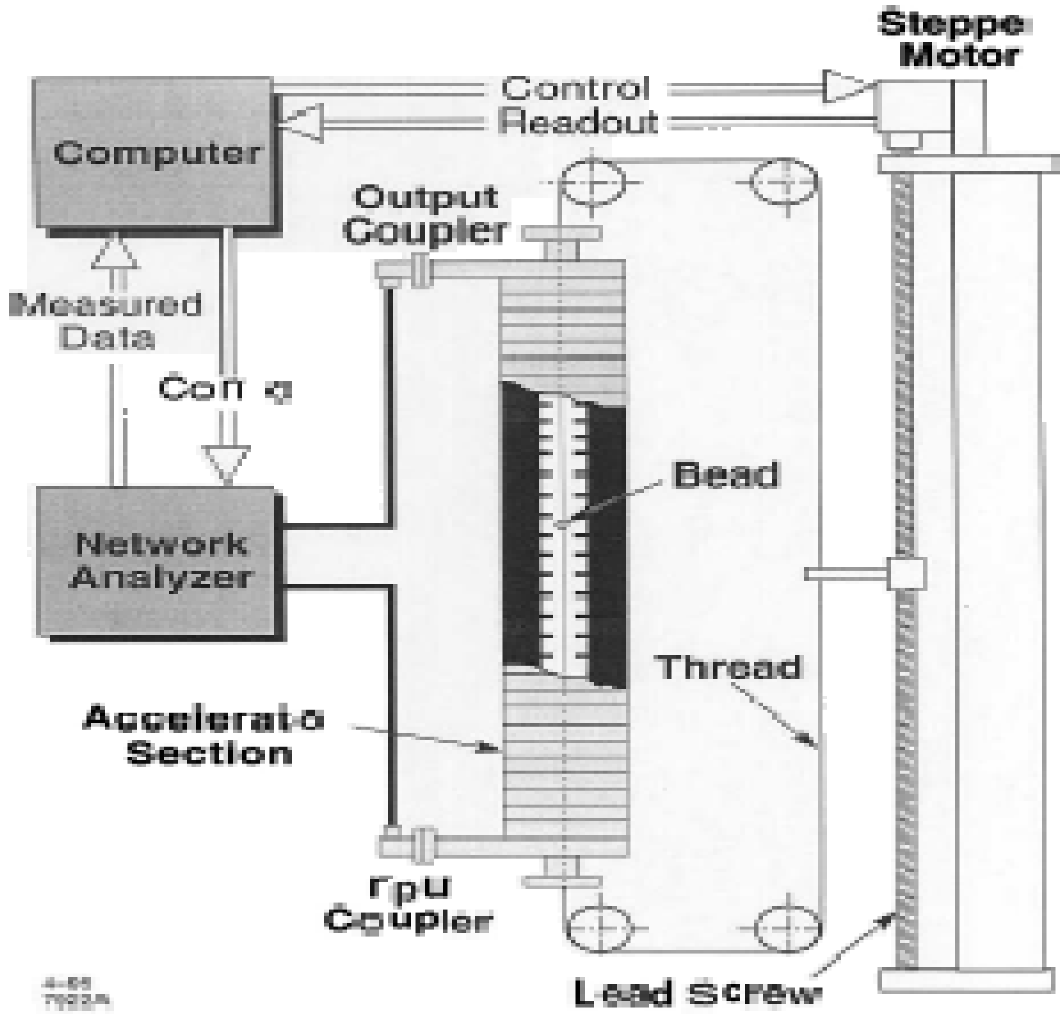
Where K is a constant which depends on the bead, E(x,y,z) is the forward power flowing across the structure at z, E_i is the incident wave amplitude.

The reflection coefficient is defined as:

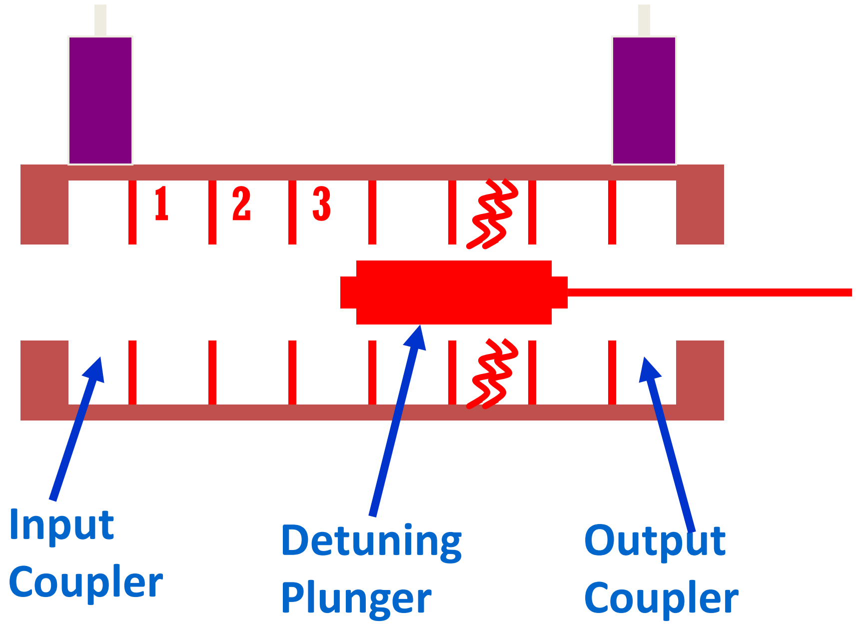
$$\rho(z) = \frac{E_r(z)}{E_i(z)}$$

For a constant gradient structure: $\rho(0) = \frac{E_r(0)}{E_i(0)} = K\frac{E^2(x,y,z)}{P(0)}$

Bead Pull Setup



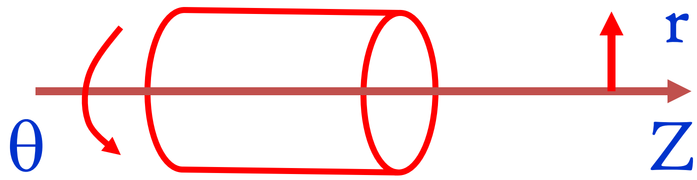
Nodal Shift Technique



"Coupled Cavity" Structures

Linacs are coupled cavities in which many connected cavities are powered by one RF source.

Single cavity modes: ω_{mnp}:



TM_{mnp} (no H_z)

TE_{mnp} (no E_z)

M = # of full period azimuthal variations 0,1,2,…

N = # of half period radial variations 1,2,…

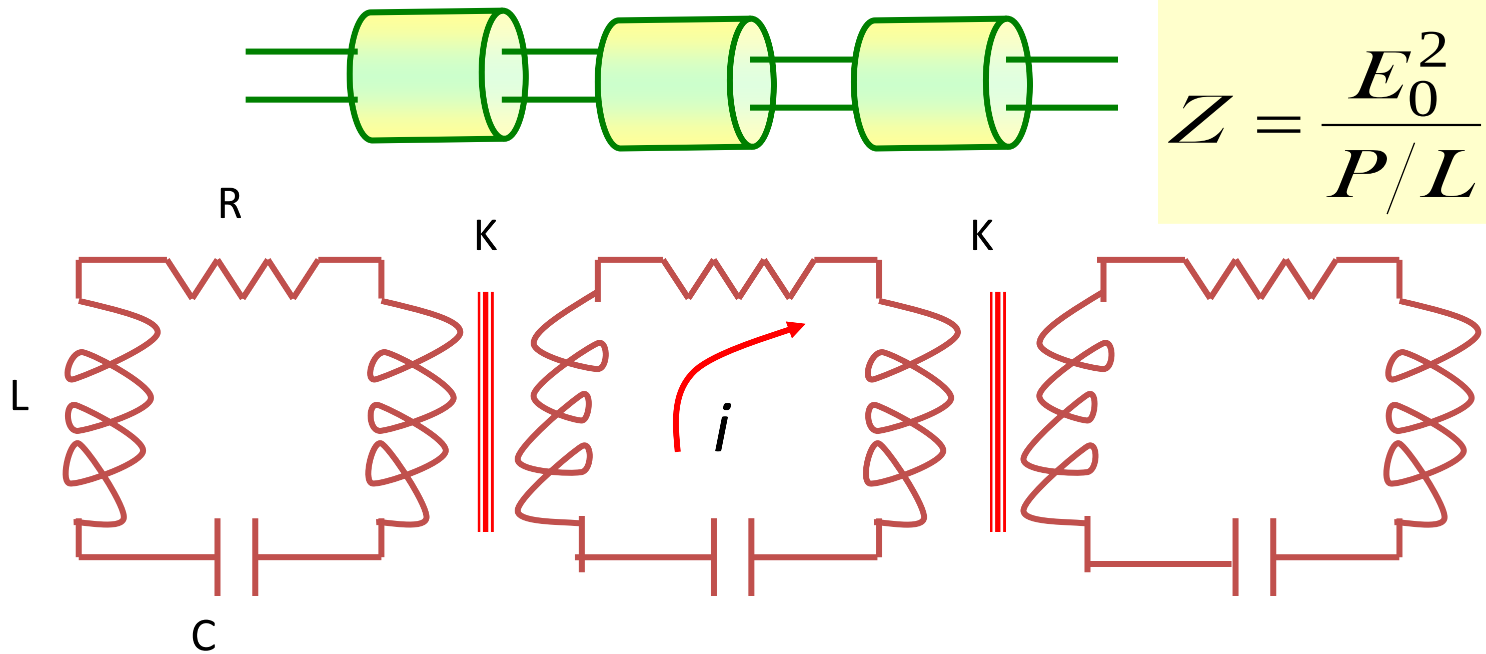
P = # of full period axial variations 0,1,2,…

“Coupled Cavity” Structures

We can describe the properties of coupled chain using separate modes of single cavity as they develop into “bands” of coupled systems.

$$Q = \frac{\omega U}{P} = \frac{\omega}{\Delta\omega}$$

$$Z = \frac{E_0^2}{P/L}$$



“Coupled Cavity” Structures

For a chain of N+1 cavities, each single cavity mode yields N+1 normal modes (Q→∞):

$$A_n^{(q)} = A_0 \cos\left(\frac{\pi q^n}{N}\right) e^{i\omega_q t}$$

Mode: q= 0,1,…N

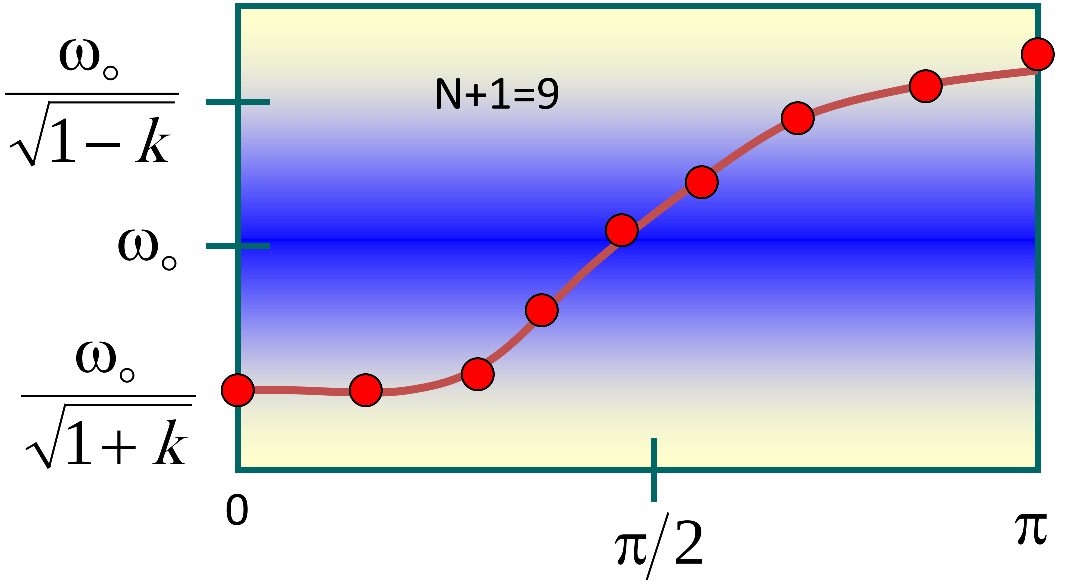
$$\omega_q^2 = \frac{\omega_0^2}{1 + k\cos\frac{\pi q}{N}}$$

Cell: n= 0,1,…N

Mode spacing:

k/N^2 near 0, π

k/N near π/2

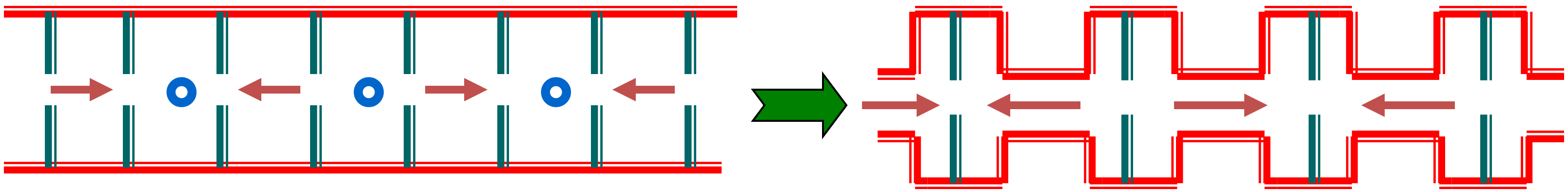


“Coupled Cavity” Structures

1. $\pi/2$ mode has special properties useful for long accelerating structures. This mode is generally insensitive to perturbations.

$\pi/2$ mode has field in cell n ($Q \to \infty$)

$$A_n^{(N/2)} = A_0 \cos\frac{n\pi}{2} e^{i\omega_0 t}$$



Optimize “accelerating Cells” for high shunt impedance. Q_c of the “coupling cells” not very important.

“Coupled Cavity” Structures

2. Losses due to finite Q excite small amplitudes ($1/kQ_0$)in the coupling cells and introduce amplitude droop [$1/(k^2 Q_0 Q_c)$] in accelerating cells. Losses do not introduce phase shift between accelerating cells up to order $1/(kQ)^2$ if all cells tuned to same frequency.

“Phase Stabilized Structure”

3. In $\pi/2$ mode, frequency errors $\Delta\omega_n$ in cells of the chain produces only second order ($\Delta\omega_n \, \Delta\omega_m$) amplitude variations in accelerating cells.

“Amplitude Stabilized Structure”