3.2 Cryogenic Convection Heat Transfer

- Involves process of heat transfer between solid material and adjacent cryogenic fluid
- Classic heat transfer problem (Newton’s law)
  \[ q = h(T_s - T_f) \]
- Configurations of interest
  - Internal forced flow (single phase, \( T_f = T_{\text{mean}} \))
  - Free convection (single phase, \( T_f = T_{\infty} \))
  - Internal two phase flow
  - Pool boiling (two phase)
- Understanding is primarily empirical leading to correlations based on dimensionless numbers
- Issue is relevant to the design of:
  - Heat exchangers
  - Cryogenic fluid storage
  - Superconducting magnets
  - Low temperature instrumentation
Single phase internal flow heat transfer

Forced Convection

Classical fluid correlations

- The heat transfer coefficient in a classical fluid system is generally correlated in the form where the Nusselt number, \( \text{Nu}_D \equiv \frac{hD}{k_f} \), and \( D \) is the characteristic length.
- For laminar flow, \( \text{Nu}_D = \text{constant} \approx 4 \) (depending on b.c.).
- For turbulent flow (\( \text{Re}_D > 2000 \))

\[
\text{Nu}_D = f(\text{Re}_D, \text{Pr}) = C \text{Re}_D^n \text{Pr}^m \quad \text{and} \quad \text{Pr} \equiv \frac{\mu_f C_p}{k_f} \quad (\text{Prandtl number})
\]

- Dittus-Boelter Correlation for classical fluids (+/- 15%) \( \text{Nu}_D = 0.023 \text{Re}^{4/5} \text{Pr}^{2/5} \)

Note that fluid properties should be computed at \( T_f \) (the “film temperature”):

\[
T_f \equiv \frac{T_s + T_f}{2}
\]
Johannes Correlation (1972)

- Improved correlation specifically for helium (+/- 8.3%)

\[ Nu_D = 0.0259 \text{Re}^{4/5} \text{Pr}^{2/5} \left( \frac{T_s}{T_f} \right)^{-0.716} \]

- Last factor takes care of temperature dependent properties
- Note that one often does not know \( T_f \), so iteration may be necessary.

Example
Application: Cryogenic heat exchangers

- Common types of heat exchangers used in cryogenic systems
  - Forced flow single phase fluid-fluid
    - E.g. counterflow heat exchanger in refrigerator/liquefier
  - Forced single phase flow - boiling liquid (Tube in shell HX)
    - E.g. LN$_2$ precooler in a cooling circuit
  - Static boiling liquid-liquid
    - E.g. Liquid subcooler in a magnet system
Simple 1-D heat exchanger

- Differential equation describing the temperature of the fluid in the tube:

\[ mC \frac{dT_f}{dx} + hP(T_f - T_s) = 0 \]

- For constant \( T_s \), the solution of this equation is an exponential

\[ T_s - T_f = (T_s - T_f)_0 \exp \left( - \frac{hP}{mC} x \right) \]
Liquid nitrogen precooler

- **Assumptions & givens**
  - $T_s$ is a constant @ 77 K (NBP of LN$_2$)
  - Helium gas ($C_p = 5.2$ kJ/kg K; $\mu = 15 \times 10^{-6}$ Pa s; $\rho = 0.3$ kg/m$^3$, $k = 0.1$ W/m K)
  - Allowed pressure drop, $\Delta p = 10$ kPa
  - Helium mass flow rate = 1 g/s

- Find the length and diameter of the HX (copper tubing)
- Total heat transfer:

$$Q = \dot{m}C_p(T_{in} - T_{out}) = 1 \text{ g/s} \times 5.2 \text{ J/g K} \times 220 \text{ K} = 1144 \text{ W}$$

- Log mean $\Delta T$:

$$\Delta T_{lm} = \frac{\Delta T_f(x = 0) - \Delta T_f(x = L)}{\ln \left( \frac{\Delta T_f(x = 0)}{\Delta T_f(x = L)} \right)} = \frac{(300-77) - (80-77) \text{ K}}{\ln [(300-77)/(80-77)]} = 51 \text{ K}$$

Properties are average values between 300 K and 80 K
Liquid nitrogen precooler (continued)

\[ UA = h \pi DL = \frac{Q}{\Delta T_{lm}} = 1144 \text{ W/51 K} = 22.4 \text{ W/K} \]

- The heat transfer coefficient is a function of \( \text{Re}_D \) and \( \text{Pr} = 0.67 \)
- Assuming the flow is turbulent and fully developed, use the Dittus Boelter correlation

\[ Nu_D = \frac{hD}{k_f} = 0.023 \text{Re}_D^{0.8} \text{Pr}^{0.3} \quad \text{and} \quad \text{Re}_D = \frac{4\dot{m}}{\pi D \mu} \]

Substituting the \( \text{Re}_D \) and solving for \( h \)

\[ h = 0.023 \frac{k_f}{D} \left( \frac{4\dot{m}}{\pi D \mu} \right)^{0.8} \text{Pr}^{0.3} = \frac{0.0247 \times 0.1 \text{ W/m K} \times (10^{-3} \text{ kg/s})^{0.8}}{(15 \times 10^{-6} \text{ Pa s})^{0.8} \times D^{1.8}} \]

\[ = 0.07/D(m)^{1.8} \]

\[ h\pi DL = 0.224 \times (L/D^{0.8}) = 22.4 \text{ W/K} \]

or \( L/D^{0.8} = 100 \text{ m}^{0.2} \)

1 equation for two unknowns
Liquid nitrogen precool (continued)

- Pressure drop equation provides the other equation for L & D

\[ \Delta p = f \frac{L}{2 \rho D} \left( \frac{\dot{m}}{A_{flow}} \right)^2 ; A_{flow} = \pi D^2/4 \text{ and } f \sim 0.02 \text{ (guess)} \]

\[ \text{Re}_D = \frac{4\dot{m}}{\pi D \mu} \]

Substituting for \( \text{Re}_D \) and \( f \)

\[ \Delta p \approx 0.016 \frac{\dot{m}^2}{\rho} \frac{L}{D^5} = \frac{0.016 \times (10^{-3} \text{ kg/s})^2}{0.3 \text{ kg/m}^3} \left( \frac{L}{D^5} \right) \]

Substitute:

\[ \Delta p = 5.33 \times 10^{-8} \left( \frac{L}{D^5} \right) \]

2nd equation for two unknowns

Eq. 1: \( L = 100 D^{0.8} \quad \rightarrow \quad \Delta p = 5.33 \times 10^{-6}/D^{4.2} \quad \text{with } \Delta p = 10,000 \text{ Pa} \)

\[ D = \left[ 5.33 \times 10^{-6}/\Delta p \right]^{1/4.2} = 6.2 \text{ mm and } L = 100 \times (0.0062 \text{ m})^{0.8} = 1.7 \text{ m} \]
Single phase free convection heat transfer

- Compressible fluid effect: Heat transfer warms the fluid near the heated surface, reducing density and generating convective flow.

- Free convection heat transfer is correlated in terms of the Rayleigh number,

\[ Nu_L = f(Gr, Pr) \sim C Ra_L^n \text{ where } Ra_L \equiv Gr Pr = \frac{g \beta \Delta T L^3}{D_{th} \nu} \]

where \( g \) is the acceleration of gravity, \( \beta \) is the bulk expansivity, \( \nu \) is the kinematic viscosity (\( \mu/\rho \)) and \( D_{th} \) is the thermal diffusivity (\( k/\rho C \)). \( L \) (or \( D \)) are the scale length of the problem (in the direction of \( \vec{g} \)).
Free convection correlations

- For very low Rayleigh number, \( Nu = 1 \) corresponding to pure conduction heat transfer
- For \( Ra < 10^9 \), the boundary layer flow is laminar (conv. fluids)
  \[ Nu_L \approx 0.59Ra_L^{0.25} \]
- For \( Ra > 10^9 \), the boundary layer is turbulent
  \[ Nu_L \approx 0.1Ra_L^{0.33} \]
Pool Boiling Heat Transfer (e.g. Helium)

Factors affecting heat transfer curve

- Surface condition (roughness, insulators, oxidation)
- Orientation
- Channels (circulation)
- Time to develop steady state (transient heating)

Note: Other cryogenic fluids have basically the same behavior, although the numerical values of \( q \) and \( \Delta T \) are different.
Nucleate Boiling Heat Transfer

- Nucleate boiling is the principal heat transfer mechanism for static liquids below the peak heat flux (q* ~ 10 kW/m² for helium)

- Requirements for nucleate boiling
  - Must have a thermal boundary layer of superheated liquid near the surface
    \[ \delta_{th} = \frac{k_f \Delta T}{q} \sim 1 \text{ to } 10 \mu m \text{ for helium} \]
  - Must have surface imperfections that act as nucleation sites for formation of vapor bubbles.
Critical Radius of Vapor Bubble

Critical radius: For a given T, p, the bubble radius that determines whether the bubble grows or collapses
- \( r > r_c \) and the bubble will grow
- \( r < r_c \) and the bubble will collapse

Estimate the critical radius of a bubble using thermodynamics
- Clausius Clapeyron relation defines the slope of the vapor pressure line in terms of fundamental properties

\[
\frac{dp}{dT}_{\text{sat}} = \frac{\Delta s}{\Delta v} = \frac{h_{fg}}{T(v_v - v_L)} \approx \frac{h_{fg} p}{RT^2}
\]

If the gas can be approximated as ideal and \( v_v \gg v_L \)
Critical radius calculation

- Integrating the Clausius Clapeyron relation between $p_s$ and $p_s + 2\sigma/r$

\[ r_c = \frac{2\sigma}{p_s} \left( e^{\frac{h_{fg} \Delta T}{R T^2}} - 1 \right)^{-1} \approx \frac{2\sigma R T_s^2}{h_{fg} p_s \Delta T} \]

- Example: helium at 4.2 K (NBP)
  - Empirical evidence indicates that $\Delta T \sim 0.3$ K
  - This corresponds to $r_c \sim 17$ nm
  - Number of helium molecules in bubble $\sim 10,000$
  - Bubble has sufficient number of molecules to be treated as a thermodynamic system

- Actual nucleate boiling heat transfer involves heterogeneous nucleation of bubbles on a surface. This is more efficient than homogeneous nucleation and occurs for smaller $\Delta T$. 
Nucleate Boiling Heat Transfer He I

Note that $h_{nb}$ is not constant because $Q \sim \Delta T^{2.5}$
The mechanism for bubble formation and detachment is very complex and difficult to model.

Engineering correlations are used for analysis.

Kutateladse correlation

\[
\frac{h}{k_l} \left(\frac{\sigma}{g \rho_l}\right)^{1/2} = 3.25 \times 10^{-4} \left[\frac{q C_p \rho_l}{h_{fg} \rho_v k_l} \left(\frac{\sigma}{g \rho_l}\right)^{1/2}\right]^{0.6} \times \left[\frac{g (\rho_l/\mu_l)}{\rho \chi} \left(\frac{p}{\sigma g \rho_l}\right)\right]^{0.125} \left(\frac{p}{(\sigma g \rho_l)^{1/2}}\right)^{0.7}
\]

Rearranging into a somewhat simpler form,

\[
q = 1.90 \times 10^{-9} \left[\frac{g (\rho_l/\mu_l)^2}{\chi^3} \left(\frac{p \chi}{\sigma}\right)^{1.75} \left(\frac{\rho_l}{\rho_v}\right)^{1.5} \times \left(\frac{C_p}{h_{fg}}\right)^{1.5} \left(\frac{k_l}{\chi}\right)(T_s - T_b)^{2.5}\right]
\]

where \( \chi = \left(\frac{\sigma}{g \rho_l}\right)^{1/2} \)

\[
q(W/cm^2) = 5.8 \Delta T^{2.5} \quad \text{For helium at 4.2 K}
\]
Peak Heat Flux (theory)

- Understanding the peak nucleate boiling heat flux is based on empirical arguments due to instability in the vapor/liquid flow.

- Instability due to balance between surface energy and kinetic energy:

\[ c^2 = \frac{2\pi\sigma}{\lambda(\rho_L + \rho_v)} - \frac{\rho_L\rho_v}{(\rho_L + \rho_v)^2}(v_v - v_L)^2 \]

- Transition to unstable condition when \( c^2 = 0 \)
**Peak Heat Flux Correlations**

- **Zuber correlation:**
  
  \[ q^* \approx K h_{fg} \rho_v \left[ \frac{\sigma (\rho_l - \rho_v) g}{\rho_v^3} \right]^{1/4} \left[ \frac{\rho_l}{\rho_l + \rho_v} \right]^{1/2} \]

- **Empirical based on Zuber Correlation**

  \[ q^* \approx 0.16 h_{fg} \rho_v^{1/2} \left[ \sigma g (\rho_l - \rho_v) \right]^{1/4} \]

  \( \approx 8.5 \text{ kW/m}^2 \text{ for He I at 4.2 K} \)

- **Limits:**
  - \( T \to T_c: \) \( q^* \to 0 \) since \( h_{fg} \to 0 \) and \( \sigma \to 0 \)
  - \( T \to 0: \) \( q^* \sim \rho_v^{1/2} \) (decreases)
  - \( q^*_{\text{max}} \) near 3.6 K for LHe
Film Boiling

- Film boiling is the stable condition when the surface is blanketed by a layer of vapor
  - Film boiling heat transfer coefficient is generally much less than that in nucleate boiling
  - Minimum film boiling heat flux, $q_{mfb}$ is related to the stability of the less dense vapor film under the more dense liquid

![Diagram of film boiling with liquid and vapor layers, and a vertical arrow labeled Q.]

"Taylor Instability" governs the collapse of the vapor layer
Film Boiling Heat Transfer Correlations

- Factors affecting the process
  - Fluid properties: $C_p, h_{fg}, \sigma, \rho_l, \rho_v$
  - Fluid state: saturated or pressurized (subcooled)
  - Heater geometry (flat plate, cylinder, etc.)
- Breen-Westwater correlation

$$h_{fb} \left( \frac{\sigma}{g(\rho_l - \rho_v)} \right)^{1/8} \left( \frac{\mu_v(T_s - T_b)}{k_v \rho_v(\rho_l - \rho_v)g\lambda'} \right)^{1/4} = 0.37 + 0.28 \left( \frac{\sigma}{gD^2(\rho_l - \rho_v)} \right)^{1/2}$$

Where,

$$\lambda' = \left[ h_{fg} + 0.34C_{pv}(T_s - T_b) \right]^2$$

- Simplified form for large diameter

$$h_{fb} = 0.37 \left( \frac{g(\rho_l - \rho_v)}{\sigma} \right)^{1/8} \left( \frac{k_v \rho_v(\rho_l - \rho_v)g\lambda'}{\mu_v(T_s - T_b)} \right)^{1/4} \quad \Rightarrow \quad q \approx (T_s - T_f)^{3/4}$$
Minimum film boiling heat flux

- Minimum film boiling heat flux is less than the peak heat flux.
- Recovery to nucleate boiling state is associated with Taylor Instability.

\[ q_{mfb} = 0.16 h_{fg} \rho_v \left[ g \sigma (\rho_l - \rho_v) \right]^{1/4} \left( \rho_l + \rho_v \right)^2 \]

- Dimensionless ratio:

\[ \frac{q_{mfb}}{q^*} = \left[ \frac{\rho_v}{\rho_l + \rho_v} \right]^{1/2} \]

- \( q^* \approx 0.36 \) for LHe @ 4.2 K
- \( q^* \approx 0.1 \) @ 2.2 K
- \( q^* \approx 1 \) @ \( T_c = 5.2 \) K
- \( q^* \approx 0.09 \) for LN\(_2\) @ 80 K where \( q^* \approx 200 \) kW/m\(^2\)
Prediction of Nucleate/Film Boiling for Helium

- Breen & Westwater Correlation
- $P = 1 \, \text{atm.}$
- $D = 0.001 \, \text{cm.}$
- $D = 0.002 \, \text{cm.}$
- $D = 0.004 \, \text{cm.}$
- $D = 0.006 \, \text{cm.}$
- $D = 0.02 \, \text{cm.}$
- $D = 0.04 \, \text{cm.}$
- $D = 0.1 \, \text{cm.}$
- $D \geq 1.0 \, \text{cm.}$

- Predicted critical heat flux
- Kotasladze Correlation

- For flat plates and large diameters, use $D \geq 1.0 \, \text{cm.}$

- The points of minimum film boiling are given by either the correlation of Lienhard & Wong or Zubir, et al.
Experimental Heat Transfer (Helium)

\[ \Delta T, ^\circ K \]

- Frederking
  - \( D = 0.055 \) cm
  - \( D = 0.130 \) cm
  - \( D = 0.215 \) cm
  - \( D = 0.312 \) cm
  - \( D = 0.510 \) cm

- Breen & Westwater
  - Correlation
  - \( D = 0.04 \) cm
  - \( D = 0.1 \) cm
  - \( D \geq 1.0 \) cm

- Eastman & Dators
- Karagounis
- Kutateladze
- Lyon
- Reeber

\[ \dot{q}, \text{watts/cm}^2 \]

\( \Delta T, ^\circ K \)

(The points of minimum film boiling are given by either the correlation of Lienhard & Wong or of Zuber, et al.)
Prediction of Nucleate/Film Boiling for Nitrogen

![Graph showing the prediction of nucleate/film boiling for nitrogen with critical heat flux values plotted against temperature difference. The graph includes lines for different pressures and diameters, with annotations for Kutateladze and Breen & Westwater correlations.](image-url)
Experimental Heat Transfer (Nitrogen)

(Points of minimum film boiling are given by either the correlation of Lienhard & Wong or Zubere, et al.)
Internal two phase flow

- Heat transfer depends on various factors
  - Mass flow rate
  - Orientation w/r/t gravity
  - Flow regime
  - Quality ($\chi$)
  - Void fraction ($\alpha$)
- Total heat transfer rate
  \[ Q_T = Q_{fc} + Q_b \]
  where: $Q_{fc}$ is convective and $Q_b$ is gravity enhanced boiling.

Depending on factors above, either contribution may dominate
Horizontal flow two phase heat transfer

- Consider the case where gravitational effects are negligible
  - Horizontal flow at moderate Re so that inertial forces dominate
- Correlation based on enhanced Nusselt number
  \[
  \frac{Nu_{2\phi}}{Nu_L} = f(\chi_{tt}) \quad \text{where} \quad \chi_{tt} = \left( \frac{1-x}{x} \right)^{0.9} \left( \frac{\mu_L}{\mu_v} \right)^{0.1} \left( \frac{\rho_v}{\rho_L} \right)^{0.5}
  \]

  and \( Nu_L = 0.023 \text{Re}_L^{0.8} \text{Pr}_L^{0.4} \); \( \text{Re}_L = \frac{\dot{m}(1-x)}{\mu_L A_{\text{flow}}} \)

  Typical correlation (de la Harpe):
  \[
  \frac{Nu_{2\phi}}{Nu_L} \approx A \chi_{tt}^{-m} \quad \rightarrow \quad 1 \text{ for } \chi_{tt} \text{ large}
  \]
  with \( m \sim 0.385 \) and \( A \sim 5.4 \)
Main difference between this problem and pool boiling is that the fluid is confined within channel.

At low mass flow rate and self driven flows (natural circulation) the heat transfer is governed by buoyancy effects.

Process is correlated against classical boiling heat transfer models.

In the limit of large D the correlation is similar to pool boiling heat transfer (vertical surface).
Vertical channel maximum heat flux

Empirical observations
- $q^* \sim w$ for small $w$
- $q^* \sim z^{-1/2}$ for $w/z < 0.1$
- $q^* \sim$ constant for $w/z \gg 1$

Critical flow in an evaporator

$$q^* = \frac{w}{\sqrt{z}} \frac{h_{fg} \rho_v}{2} \left[ \frac{g \chi_c}{\beta - 1} \left( 1 - \frac{\ln[1 + \chi_c (\beta - 1)]}{\chi_c (\beta - 1)} \right) \right]^{1/2} \quad \beta = \frac{\rho_l}{\rho_v}$$

and

$$\chi_c = \frac{\dot{m}_l}{\dot{m}_v} \left( \frac{\dot{m}_v}{\dot{m}_l} \right)_{\text{max}}$$

“Critical quality” $\sim 0.3$ for helium
Example: Cryogenic Stability of Composite Superconductors (LTS in LHe @ 4.2 K)

- Used in large magnets where flux jumping and other small disturbances are possible and must be arrested
- General idea: in steady state ensure that cooling rate exceeds heat generation rate \((Q > G)\)
- Achieved by manufacturing conductor with large copper (or aluminum) fraction and cooling surface
- Lower overall current density
- Potentially high AC loss (eddy currents)

**Composite Conductor**

Copper/Aluminum stabilizer
Insulating spacer (G-10)
NbTi/Nb\(_3\)Sn
Cryogenic Stability (LHe @ 4.2 K)

Case 1: Unconditional stability, recovery to fully superconducting state occurs uniformly over length of normal zone

Case 2: Cold End recovery (Equal area criterion): Excess cooling capacity (area A) > Excess heat generation (area B)

Q/S or G/S

Q/S is the LHe boiling heat transfer curve for bath cooling normalized per surface area

G/S is the two part heat generation Curve for a composite superconductor

$T_{cs}$ is the temperature at which $T_{op} = T_c$
Transient Heat Transfer

- Heat transfer processes that occur on time scale short compared to boundary layer thermal diffusion. Why is this important in cryogenics? (D_{th} (copper) \sim 10^{-4} m^2/s @ 300 K; \sim 1 m^2/s @ 4 K)
- Normal liquid helium has a low thermal conductivity and large heat capacity

\[ D_f = \frac{k_f}{\rho C} \sim 3 \times 10^{-8} m^2/s (LHe @ 4.2 K) \]

\[ \delta_{th} = \frac{\pi}{2} (D_f t)^{1/2} \sim 3 \times 10^{-4} t^{1/2} [m] \text{ for LHe @ 4.2 K} \]
\[ \sim 1.5 t^{1/2} [m] \text{ for copper @ 4.2 K} \]

- Lumped capacitance condition: \( Bi = \frac{hL}{k} \ll 1 \sim 10L [m] \)

- Note that this subject is particularly relevant to cooling superconducting magnets, with associated transient thermal processes
- Important parameters to determine
  - \( \Delta E = q \Delta t^* \) (critical energy)
  - \( T_s \) - surface temperature during heat transfer
Transient Heat Transfer to LHe @ 4.2 K

Time evolution of the temperature difference following a step heat input:
Steady state is reached after ~ 0.1 s
**Hypothesis:** The “critical energy” is determined by the amount of energy that must be applied to vaporize a layer of liquid adjacent to the heated surface.

- Energy required
\[ \Delta E = \rho_l h_{fg} \delta_{th} \]

- Layer thickness determined by diffusion
\[ \delta_{th} = \frac{\pi}{2} \left( \frac{D_f t}{\delta} \right)^{1/2} \]

- Critical flux based on heat diffusion:
\[ q^* = \frac{\pi}{2} \rho_l h_{fg} \left( \frac{k_l}{\rho_l C \Delta t^*} \right)^{1/2} \]
\[ \sim 0.09 \Delta t^{1/2} \text{ [W/cm}^2\text{]} \text{ at } 4.2 \text{ K} \]
Surface Temperature (Transient HT)

- During transient heat transfer, the surface temperature will be higher than the surrounding fluid due to two contributions:
  - Fluid layer diffusion: transient conduction in the fluid layer will result in a finite temperature difference
  - Kapitza conductance: At low temperatures, there can be a significant temperature difference, $\Delta T_k$, due to thermal impedance mismatch (more on this subject later). This process is dominant at very low temperatures, but is small above ~ 4 K, so is normally only important for helium systems.

- Fluid layer diffusion equation:
  \[
  \frac{\partial^2 \Delta T_f}{\partial x^2} = \frac{1}{D_f} \frac{\partial \Delta T_f}{\partial t};
  D_f = \frac{k}{\rho C_p}
  \]

- Boundary conditions:
  - $\Delta T_f(x,0) = 0$; initial condition
  - $\Delta T_f(\text{infinity}, t) = 0$; isothermal bath
  - $q = -k_f \frac{d\Delta T_f}{dt}_{x=0}$; heat flux condition
  - Solid is isothermal ($Bi = hL/k \ll 1$)

Solution is a standard second order differential equation with two spatial and one time boundary condition.
Transient diffusion solution

- Integrating the diffusion equation:

\[ \Delta T_f = \frac{q}{k_f} \left[ 2 \left( \frac{D_f t}{\pi} \right)^{1/2} \exp \left( -\frac{x^2}{4D_f t} \right) - \text{erfc} \left( \frac{x}{2(D_f t)^{1/2}} \right) \right] \]

- Evaluating at \( x = 0 \) (surface of heater)

\[ \Delta T_f(x = 0) = \frac{2q}{\sqrt{\pi}} \left( \frac{t}{\rho C_p k_f} \right)^{1/2} \]

- The transient heat transfer coefficient can then be defined

\[ h = \frac{q}{\Delta T_f} = \frac{\sqrt{\pi}}{2} \left( \frac{\rho C_p k_f}{t} \right)^{1/2} \]

\[ h \approx \frac{0.1}{\sqrt{t}} \text{; kW/m}^2\text{K for helium near 4 K} \]

At \( t = 10 \mu s \); \( h \sim 30 \text{ kW/m}^2\text{K} \) and for \( q = 10 \text{ kW/m}^2 \); \( \Delta T_f \sim 0.3 \text{ K} \)

Note: this value of \( h \gg h_{nb} \) (nucleate boiling HT coefficient)
Summary Cryogenic Heat Transfer

- Single phase heat transfer correlations for classical fluids are generally suitable for cryogenic fluids
  - Free convection
  - Forced convection
- Two phase heat transfer also based on classical correlations
  - Nucleate boiling
  - Peak heat flux
  - Film boiling
- Transient heat transfer is governed by diffusive process for $\Delta T$ and onset of film boiling