3.3 He II Heat and Mass Transfer

- Heat transfer characteristics of He II are sufficiently unique that conventional heat transfer (convection, conduction) do not apply.

- Important questions
  - What is the limit to heat transfer, $q^*$, critical energy, $\Delta E$?
  - What is the associated thermal gradient in the He II?
  - How is the surface temperature determined?

- Understanding and modeling must be based on transport properties of He II

- Examples to be considered
  - Thermal stability of He II cooled magnet
  - Design of a He II bath heat exchanger
Surface heat transfer - general form

- Surface heat transfer characteristics look similar to ordinary fluids
- Physical interpretation different
  - $h_k$ is the Kapitza conductance regime (non-boiling)
  - $q^*$ is the peak heat flux
  - $q_{mfb}$ minimum film boiling heat flux
  - $h_{fb}$ film boiling heat transfer coefficient
- All above processes are similar to boiling heat transfer in normal liquids, but the physical interpretation is different
Heat Conductivity of He II

- Anomalous heat transport
  - Effective heat conductivity comparable to that of high purity metals
  - Low flux regime $dT/dx \sim q$
  - High flux regime $dT/dx \sim q^3$
  - Transition between two regimes depends on the diameter of channel
- Heat transport in He II can be understood in terms of the motion of two interpenetrating fluids. This “Two Fluid” model effectively describes the transport properties
- High heat flux regime of greatest technical interest
Peak Heat Flux in He II Channel

- Thermal gradient in He II channel allows $T(x = 0)$ to increase above $T_b$
  - For practical channel dimensions, gradient is controlled by mutual friction interaction

\[
\frac{dT}{dx} = -f(T)q^3 \quad f(T) = \frac{A\rho_n}{\rho_s L^4 T^3}
\]

- Thermal resistivity function

\[
q = -\left[f^{-1}(T)\frac{dT}{dx}\right]^{1/3}
\]

- Non-linear conduction equation

- Heat flux is limited by maximum allowable temperature at $x = 0$ (usually $T_\lambda$)
  - Steady state gradient
  - Thermal diffusion
He II Heat Conductivity Function, $f^{-1}(T,p)$

- Correlation based on He II turbulent flow
  \[
  f^{-1}(T) \approx \frac{P^2 S_{\lambda}^4 T_{\lambda}^3}{A_{\lambda}} \left[ t^{5.6} \left( 1 - t^{5.6} \right) \right]^3; t = \frac{T}{T_{\lambda}(p)}
  \]
  Where $A_{\lambda} \approx 145$ cm s/g
- Above correlation is good to about 10%
- Results indicate that the peak heat flux, $Q^*$, should decrease with increasing $p$ and $T$.
- Improved correlation by Sato (2003)
Maximum Steady State Heat Flux ($q^*$)

Integrating the heat transport equation over the length of the channel:

$$q^* = \left[ \frac{1}{L} \int_{T_c}^{T_b} \frac{dT}{f(T)} \right]^{1/3} \equiv \frac{Z(T_b)}{L^{1/3}}$$

This expression is used to compute the maximum $q$ for a channel heated at one end and containing He II at moderate pressure.

Example: $L = 10$ cm at $T_b = 1.8$ K $\rightarrow$ $q^* = 5.9$ W/cm$^2$
Transient Heat Transfer in He II

- Heat pulse diffuses through conductor and is transferred to He II by conduction
- \( \delta = \) thermal diffusion length into LHe \( \approx \) cm rather than \( \mu \)m as in He I
- Surface temperature difference
  \( \Delta T_s = \Delta T_K + \Delta T_{He \text{ II}} \) where
  \( \Delta T_K = \frac{Q}{h_K S} \) (Kapitza Conductance)
- Take-off power is equivalent to energy to raise local He II temperature to \( T_\lambda = 2.2 \) K

\[
Q \cdot \Delta t = \int_{T_b}^{T_\lambda} \rho C \frac{dT_{He II}}{dx} dx
\]

Note: this problem has significant implications to the thermal stability of He II cooled superconducting magnets.
Thermal diffusion - solids

- Heat transport is described by the diffusion equation
  \[
  \frac{\partial T}{\partial t} = D_{th} \frac{\partial^2 T}{\partial x^2}
  \]
  where \( D_{th} = \frac{k}{\rho C_s} \left[ \frac{m^2}{s} \right] \)

- Characteristic time for diffusion over length \( L \):
  \[
  \tau_D \sim \frac{L^2}{D_{th}}
  \]
  Corresponds to a Fourier number = 1 \( (Fo \equiv \frac{D_{th}t}{L^2}) \)

- Metals at low temperature
  - Copper: \( D_{th} \sim 1 \text{ m}^2/\text{s} \); for \( L = 1 \text{ m} \) then \( \tau_D \sim 1 \text{ s} \)
  - Stainless steel: \( D_{th} \sim 3 \times 10^{-3} \text{ m}^2/\text{s} \); for \( L = 1 \text{ m} \) then \( \tau_D \sim 3000 \text{ s} \)
Thermal “diffusion” - He II

\[ \frac{dT}{dx} = f(T)q^3 \Rightarrow k_{\text{eff}} = \frac{1}{f(T)q^2} \]

- The effective thermal conductivity of He II is heat flux dependent
  - At \( T = 1.8 \) K, \( f^1(T) \sim 10000 \) kW\(^3\)/m\(^5\) K
  - For heat flux \( q = 10 \) kW/m\(^2\), \( k_{\text{eff}} \sim 10^5 \) W/m K
  - For heat flux \( q = 100 \) kW/m\(^2\), \( k_{\text{eff}} \sim 1000 \) W/m K \( \sim k_{\text{cu}} @ 2 \) K
- Volume heat capacity of He II is much larger than that of metals at low temperature (\( \rho C_{\text{He II}} \sim 1000 \) kJ/m\(^3\)K; \( \rho C_{\text{cu}} \sim 0.2 \) kJ/m\(^3\)K)
- Effective thermal diffusivity for He II
  - \( D_{\text{eff}} = k_{\text{eff}}/\rho C \sim 0.1 \) m\(^2\)/s @ 10 kW/m\(^2\)
  - Characteristic diffusion time (\( L = 1 \) m): \( \tau_D \sim L^2/D_{\text{eff}} \sim 10 \) s
- **Significance**: thermal diffusion is an important heat transport mechanism in He II (contrary to normal liquids except at very short times)
He II thermal diffusion equation

\[ \rho C \frac{\partial T}{\partial t} = \frac{\partial}{\partial x} \left( f^{-1} \frac{\partial T}{\partial x} \right)^{\frac{1}{3}} \]

where \[ \tau_d \approx \rho C f^{\frac{1}{3}} L^{\frac{4}{3}} \Delta T^{\frac{2}{3}} \]

- Non-linear partial differential equation
- Methods of solution
  - Approximate methods (similarity solution)
  - Numerical methods
- Problems of interest
  - Step function heat flux
  - Heat pulse

Time increasing
Step Function Heat Flux (Clamped flux)

Non-linear diffusion equation that can be solved by numerical methods of approximate methods (constant properties)

\[ \frac{\partial \theta}{\partial \tau} = \frac{\partial}{\partial x} \left[ \frac{\partial \theta}{\partial x} \right]^{1/3} \]

where \( \theta \equiv \frac{T - T_b}{T_\lambda - T_b} \) and \( \tau \equiv \frac{t}{\rho C (T_\lambda - T_b)^{2/3}} \)

Boundary conditions:
- Heat transfer b.c. at \( x = 0 \):
  \[ \left. \frac{\partial \theta}{\partial x} \right|_{x=0} = -\frac{q^3 f}{T_\lambda - T_b} \forall t > 0 \]
- \( T = T_b \) or \( \Theta = 0 @ x = \text{infinity} \) (approximate solution for \( x \ll L \))
- For finite length, \( L \), numerical solution is required. Solution should asymptotically approach steady state profile
Boundary conditions @ x = L

Time to onset of film boiling, $\Delta t^*$ depends on boundary condition at x = L.

Diffusion time ($\tau_D$) is indicated by point where boundary condition at x = L determines $\Delta t^*$

USPAS Short Course Boston, MA 6/14 to 6/
Recall for ideal superflow the (laminar) pressure drop is associated with the normal fluid viscous drag (fountain effect).

In turbulent flow \( \text{Re}_D > 1200 \) He II behaves more as a classical fluid.

\[
\text{Re}_D = \frac{\rho v D}{\mu_n}
\]

Since \( \mu_n \) is small, turbulent flow is common in helium.

Associated pressure drop is given by the classical expression:

\[
\frac{dp}{dx} = -4 f_d \left( \frac{1}{2} \rho v^2 \right)
\]

where the friction factor, \( f_d (\text{Re}_d) \)
Friction factor for He II

Blausius correlation

\[ f = \frac{0.079}{\text{Re}^{\frac{1}{4}}} \]

Von Karman-Nikuradse

\[ \frac{1}{\sqrt{f}} = 1.737 \times \ln(\text{Re}^{\frac{1}{2}} f) - 0.396 \]

Colebrook for \( \varepsilon = 1.4 \times 10^{-4} \)

\[ \frac{1}{\sqrt{f}} = -4 \log \left( \frac{\varepsilon}{3.7D} + \frac{1.25}{\text{Re} \sqrt{f}} \right) \]
There are three regimes of heat transfer that can occur at a heated surface in He II

1. Kapitza Conductance (non-boiling)
   Temperature difference occurs at surface, $\Delta T_s \sim 1$ K
   Due to a surface thermal impedance

2. Transition to film boiling (unstable)
   Exchange between boiling and non-boiling condition

3. Film boiling
   Vapor layer covering surface
   $\Delta T$ can be large $\sim 10$ to $100$ K
Kapitza Thermal Boundary Conductance

- discovered by Kapitza (1941) while studying heat flow around a heated Cu block in He II

- general term associated with thermal resistance at low temperatures

How measured:  \[ h_k = \lim_{\Delta T \to 0} \frac{q}{\Delta T_s} \]
Practical Significance of Kapitza Conductance

- Kapitza conductance causes largest $\Delta T$ in a non-boiling heat transfer process in He II
  - $h_k \sim 10 \text{ kW/m}^2 \text{ K}$ $\rightarrow$ $\Delta T_s(q = 10 \text{ kW/m}^2 \text{ K}) \sim 1 \text{ K}$
  - $dT/dx_{\text{He II}} \sim 100 \text{ mK/m}$ $\rightarrow$ $\Delta T_{\text{He II}} \sim (T_\lambda - T_b) \sim 400 \text{ mK over 4 m}$

- Kapitza conductance is important in the design of numerous technical devices
  - He II heat exchangers
  - Composite superconductors stability
  - Low temperature refrigerators and instrumentation
Theory of Kapitza Conductance

- Phonon Radiation Limit
  - Heat exchange occurs by phonons (quantized lattice vibrations) impinging on the interface
  - Analogue to radiation heat transfer

\[ q = \sigma (T + \Delta T)^4 - \sigma T^4 \]

where

\[ \sigma = \frac{\pi^4}{10h} \left( \frac{k_B}{\Theta_D} \right)^2 \left( \frac{3N}{4\pi V} \right)^{2/3} \]

- Expand

\[ q = 4\sigma T^3 \Delta T \left[ 1 + \frac{3}{2} \frac{\Delta T}{T} + \left( \frac{\Delta T}{T} \right)^2 + \frac{1}{4} \left( \frac{\Delta T}{T} \right)^3 \right] \]

\[ h_k \]

- Note: \( h_k \sim T^3/\Theta_D^{-2} \)

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Comparison of Highest Experimental Values for the Kapitza Conductance with the Phonon Radiation Limit

<table>
<thead>
<tr>
<th>Solid</th>
<th>( \Theta_D ) (K)</th>
<th>( h_k^e (1.9 \text{ K}) ) (kW/m²·K)</th>
<th>( h_k (1.9 \text{ K}) ) (kW/m²·K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hg</td>
<td>72</td>
<td>440</td>
<td>30</td>
</tr>
<tr>
<td>Pb</td>
<td>100</td>
<td>190</td>
<td>32</td>
</tr>
<tr>
<td>In</td>
<td>111</td>
<td>171</td>
<td>11</td>
</tr>
<tr>
<td>Au</td>
<td>162</td>
<td>155</td>
<td>8.8</td>
</tr>
<tr>
<td>Ag</td>
<td>226</td>
<td>55</td>
<td>6</td>
</tr>
<tr>
<td>Sn</td>
<td>195</td>
<td>54</td>
<td>12.5</td>
</tr>
<tr>
<td>Cu</td>
<td>343</td>
<td>30</td>
<td>7.5</td>
</tr>
<tr>
<td>Ni</td>
<td>440</td>
<td>19</td>
<td>4.0</td>
</tr>
<tr>
<td>W</td>
<td>403</td>
<td>18</td>
<td>2.5</td>
</tr>
<tr>
<td>KCl</td>
<td>230</td>
<td>22</td>
<td>6.9</td>
</tr>
<tr>
<td>SiO₂ (quartz)</td>
<td>290</td>
<td>19</td>
<td>5.7</td>
</tr>
<tr>
<td>Si</td>
<td>636</td>
<td>6.4</td>
<td>4.2</td>
</tr>
<tr>
<td>LiF</td>
<td>750</td>
<td>5.1</td>
<td>4.5</td>
</tr>
<tr>
<td>Al₂O₃</td>
<td>1000</td>
<td>1.5</td>
<td>1.6</td>
</tr>
</tbody>
</table>

* Compiled by Snyder."
Phonon radiation limit is an upper limit because it does not take into consideration boundary reflections due to dissimilarity between solid & He II

Acoustic mismatch theory
- Based on impedance mismatch between dissimilar materials
- Similar to optical transmission between media with different refractive indices

Similar expression to Phonon Radiation Limit

\[ q = \sigma \left( T + \Delta T \right)^4 - \sigma T^4 \]

Where in this case,
\[ \sigma = \frac{4\pi^5 k_B^4 \rho_L c_L}{15\hbar^3 \rho_s c_s^3} \approx \Theta_D^{-3} \quad \text{and} \quad h_k \sim \frac{T^3}{\Theta_D^3} \]

This is a lower limit to heat transfer
Experimental values for Kapitza Conductance

Example Copper:

i) Phonon radiation limit
   \( h_K^{pk} = 4.4 \ T^3 \ kW/m^2K \)

ii) Acoustic mismatch theory
    \( h_K^A = 0.021 \ T^3 \ kW/m^2K \)

iii) Experimental results
    a) clean surfaces
        \( h \sim 0.9 \ T^3 \ kW/m^2K \)
    b) dirty surfaces
        \( h \sim 0.4 \ T^3 \ kW/m^2K \)

Large variations ⇒ Kapitza conductance is an empirical quality.
Kapitza Conductance for $\Delta T/T \sim 1$

- Expand from theory
  \[ q = \sigma (T + \Delta T)^4 - \sigma T^4 \]
  \[ q = 4\sigma T^3 \Delta T \left[ 1 + \frac{3}{2} \frac{\Delta T}{T} + \left( \frac{\Delta T}{T} \right)^2 + \frac{1}{4} \left( \frac{\Delta T}{T} \right)^3 \right] \approx h_k \Delta T \]

- Alternate correlation
  \[ q = a \left( T_s^n - T_b^n \right) \text{ for finite } \Delta T \]

where $a$ and $n$ are empirically determined

$T_s$: surface temperature = $T_b + \Delta T$
$T_b$: fluid temperature near surface
Large Heat Flux Kapitza Conductance

Empirical correlation

\[ q = a(T_s^n - T_b^n) \]
Film Boiling Heat Transfer Modes

1. Near saturation \( (p < p_\lambda) \)
   Low density vapor blankets surface significantly reducing heat transfer

2. Pressurized to \( p > p_\lambda \)
   Triple phase phenomena \((\text{He II, He I, vapor})\)

3. Near \( T_\lambda \) permits nucleate boiling in He I phase w/o exceeding \( q^* \)
**Film Boiling Heat Transfer (He II)**

<table>
<thead>
<tr>
<th>Sample</th>
<th>$T_b$ (K)</th>
<th>$T_s$ (K)</th>
<th>$\Delta p$ (kPa)$^a$</th>
<th>$h$ (kW/m$^2$·K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wire ($d = 76 \mu$m)</td>
<td>1.8</td>
<td>150</td>
<td>0.42</td>
<td>1.1</td>
</tr>
<tr>
<td>Wire ($d = 25 \mu$m)</td>
<td>1.8 K</td>
<td>150</td>
<td>0.56</td>
<td>2.2</td>
</tr>
<tr>
<td>Flat rectangular plate</td>
<td>1.8</td>
<td>75</td>
<td>0.14</td>
<td>0.22</td>
</tr>
<tr>
<td>(39 mm × 11 mm)</td>
<td>1.8</td>
<td>75</td>
<td>0.28</td>
<td>0.3</td>
</tr>
<tr>
<td></td>
<td>1.8</td>
<td>75</td>
<td>0.84</td>
<td>0.55</td>
</tr>
<tr>
<td>Flat surface ($d = 13.7$ mm)</td>
<td>2.01</td>
<td>40</td>
<td>0.13</td>
<td>0.69</td>
</tr>
<tr>
<td></td>
<td>2.01</td>
<td>25</td>
<td>0.237</td>
<td>0.98</td>
</tr>
<tr>
<td>Horizontal cylinder</td>
<td>1.88</td>
<td>40</td>
<td>0.10</td>
<td>0.2</td>
</tr>
<tr>
<td>($d = 14.6$ mm)</td>
<td>2.14</td>
<td>40</td>
<td>0.10</td>
<td>0.2</td>
</tr>
<tr>
<td>Wire ($d = 200 \mu$m)</td>
<td>2.05</td>
<td>150</td>
<td>0.14</td>
<td>0.66</td>
</tr>
<tr>
<td>Cylinder ($d = 1.45$ mm)</td>
<td>1.78</td>
<td>80</td>
<td>0.06</td>
<td>0.22</td>
</tr>
</tbody>
</table>

$^a$ 1 kPa = 7.5 torr = 70.3 cm·He.

- **Typical value** $h_{fb} \sim 0.5$ kw/m$^2$K
- $h_{fb}$ (flat plates) < $h_{fb}$ (wires)
- $h_{fb}$ increases with pressure
Subcooled He II Refrigerator

Compressor

Counter-flow Heat Exchanger

Saturated Bath Heat Exchanger

He I
T - 4.5 K

He II
T - 1.8 K

He I \_\_s

He II_p
Application: He II Heat Exchanger

- Performance of He II heat exchangers are governed by several processes:
  - Surface heat transfer coefficient due to Kapitza conductance
  - Thermal gradient in He II channel allows $T(x = 0)$ to increase above $T_b$
  - Solution is similar to a conductive fin problem

\[
\frac{d}{dx} \left( f^{-1} \frac{dT}{dx} \right)^{\frac{1}{3}} + \frac{PU}{A} (T_b - T) = 0
\]

- Heat flux is limited by the temperature along the heat exchanger exceeding the local saturation temperature

$T_b \sim 1.8$ K
1. The heat exchanger must have sufficient surface area

\[ A_s \geq \frac{Q}{U (T_b - T_0)} \]

\[ U = \text{overall heat transfer coefficient} \]

2. Bulk boiling in the heat exchanger should be avoided.

\[ T < T_{Sat} \quad \text{everywhere within the heat exchanger} \]

3. Temperature gradient along heat exchanger should be minimized to not degrade performance

\[ T_L < T_b \quad \text{otherwise heat transfer at the end is poor} \]

\( \text{(low effectiveness)} \)
Summary of He II heat transfer

- Heat conductivity of He II is very high and thus models to interpret heat transfer are different from classical fluids.
- Thermal gradient \( \frac{dT}{dx} \) in He II governed by two mechanisms:
  - Normal fluid viscous drag \( \mu_n \) yielding \( \frac{dT}{dx} \sim q \)
  - Turbulent “mutual friction” \( \frac{dT}{dx} \sim q^3 \)
- Peak heat flux is determined by the He II near the heater reaching a maximum with onset of local boiling.
- Thermal diffusion-like mechanism controls heat transfer for short times and can result in significantly higher peak heat flux.
- Forced flow He II pressure drop is similar to that of classical fluids.
- Non-boiling heat transfer controlled by Kapitza conductance process (thermal impedance mismatch).
- Boiling heat transfer forms vapor film over surface.