5.1 Cryogenic system design

- Low temperature environment
- Source of refrigeration
- Heat exchange medium
- Thermal insulation
- Structural support
- Instrumentation and control
Thermal Insulation Systems

- Solid foam insulation
- Powder insulation
- Vacuum
  - Radiation heat transfer
  - Gas conduction/convection
- Multi-layer insulation
  - Radiation shields (active and passive)
  - MLI
Solid Foam Insulations

- Solid foam insulations are not used very often in cryogenics because they have relatively poor performance.
- Since these materials are typically gas filled, their thermal conductivity is > $k_{\text{air}} \sim 25 \text{ mW/m K}$.

Table 7.12. Apparent thermal conductivity of foam insulations for boundary temperatures of 300 K (80°F) and 77 K (−139°F)

<table>
<thead>
<tr>
<th>Foam</th>
<th>Density</th>
<th>Thermal Conductivity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>kg/m³</td>
<td>mW/m-K</td>
</tr>
<tr>
<td>Polyurethane</td>
<td>11</td>
<td>33</td>
</tr>
<tr>
<td>Polystyrene</td>
<td>39</td>
<td>33</td>
</tr>
<tr>
<td>Rubber</td>
<td>46</td>
<td>26</td>
</tr>
<tr>
<td>Silica</td>
<td>160</td>
<td>55</td>
</tr>
<tr>
<td>Glass</td>
<td>140</td>
<td>35</td>
</tr>
</tbody>
</table>

Example:

Consider a Polystyrene LN$_2$ vessel with 20 mm wall and 1 m$^2$ surface area.

Heat leak: $Q = kA \Delta T/L$

$= 33 \text{ mW/m K} \times 1 \text{ m}^2 \times (300 - 77) \text{ K} / 0.02 \text{ m} = 368 \text{ W}$

$h_{fg} (\text{LN}_2) = 200 \text{ J/g}; \rho \sim 800 \text{ g/L}$

$\frac{dm}{dt} = 1.84 \text{ g/s (8.3 L/hr)}$
Vacuum Insulation

- High performance insulation systems all involve some level of vacuum.
  - How low vacuum is needed?

- Even for perfect vacuum, thermal radiation can still contribute significantly to total heat leak
  - $Q_R \sim T^4$ so process is dominated by high temperature surfaces (usually 300 K)
Thermal Radiation

- Radiation from room temperature is one of the main heat loads in cryogenic systems.
- Black body spectrum is ideal emitted power versus wavelength of radiation.
- Integral of spectrum is total emitted power

\[ E_b = \int_0^\infty e_b(T, \lambda) d\lambda = \sigma T^4 \]

where \( \sigma = 5.67 \times 10^{-8} \text{ W/m}^2\text{K}^4 \), the Stefan-Boltzmann constant.

\[ e_b(T, \lambda) = \frac{8\pi hc}{\lambda^5} \left( \frac{1}{e^{hc/\lambda k_B T} - 1} \right) \]
Radiant Emissivity ($\varepsilon$)

- Emissivity is the property of a surface material that determines the fraction of radiant flux that is absorbed or emitted.
- $\varepsilon$ depends on material conductivity, temperature.
- $\varepsilon$ is also a function of wavelength, but engineering usually relies on average values measured for range of temperatures.
- For a real surface, $q_r \approx \varepsilon \sigma T^4$
Radiation heat transfer

- Net heat transfer for two facing black body surfaces
  \[ q_r = \sigma A [ (T + \Delta T)^4 - T^4 ] \]

- For non-black bodies, the heat exchange between surfaces depends on the emissivity of each surface:
  \[ q_r = \left( \frac{\varepsilon_1 \varepsilon_2}{\varepsilon_1 + \varepsilon_2 - \varepsilon_1 \varepsilon_2} \right) \sigma (T_1^4 - T_2^4) \]
  \[ \text{For } \varepsilon_1 \sim \varepsilon_2 = \varepsilon \text{ and } \varepsilon \ll 1, \, (\ ) \sim \varepsilon/2 \]

- Example: Radiant heat transfer between 300 K and 77 K
  \[ \varepsilon \sim 0.05, \, q = \frac{0.05}{2} \times 5.67 \times 10^{-8} \times (300^4 - 77^4) = 11.4 \, \text{W/m}^2 \]
  \[ h_{fg} (\text{LN}_2) = 200 \, \text{kJ/kg} \text{ and the density, } \rho = 800 \, \text{kg/m}^3 \]
  \[ \text{volume consumption} = \frac{11.4}{200} \, \text{g/sm}^2 = 0.06 \, \text{g/sm}^2 \]
  or about \( \frac{1}{4} \) liter/hour of LN\(_2\) (much better than foam)
  Note if the low T surface were at 4 K in Helium, the liquid consumption would be larger because \( h_{fg}(\text{LHe}) \) is about 21 J/g

Two surfaces facing each other with vacuum between

![Diagram of two surfaces facing each other with vacuum between](image-url)

Photon radiation exchange
Heat exchange with imperfect vacuum

- Residual heat leak due to gas conduction can contribute significantly to heat loading to a cryogenic system
  - At pressures near 1 Atm, the heat transfer is by natural convection
  - At lower pressure, convection is reduced, but gas conduction still can transfer considerable heat, \( k(T) \). This regime occurs for gas densities where the mean free path is less than the wall spacing.

- In addition to radiation heat transfer, gas conduction due to poor vacuum can seriously affect thermal performance
Gas conduction heat transfer

- At pressures below about 1 Pa, the mean free path of the molecule begins to exceed the distance between surfaces and heat is carried by Molecular-Kinetic processes.

\[ l \approx \frac{1}{n \sigma_{tot}} \approx \frac{k_B T}{\pi d^2 p} \]

Where \( d \) is the molecule diameter and \( p \) is the pressure.

- For helium gas at 1 Atm (100 kPa) and 300 K, \( l \sim 60 \text{ nm} \)
- For helium at 1 Pa and 300 K, \( l \sim 6 \text{ mm} \), a distance comparable to the spacing in containers.

- In the molecular kinetic regime, the heat exchange depends on:
  - Number of molecules striking the surface/unit time
  - The thermal equalization of the molecule with the surface
  - Probability that the molecule sticks to the surface
Adsorption & Accommodation Coef.

- Molecules are attracted to solid surfaces by Van der Waal’s forces just as with intermolecular interactions.

\[ U \]

\[ r \]

- \( \alpha \) is the accommodation coefficient that measures the amount that a molecule comes in thermal equilibrium with the wall.

\[
\alpha = \frac{T_i - T_e}{T_i - T_w} \leq 1
\]

- \( T_i \) is the temperature of the incident molecule
- \( T_e \) is the temperature of the emitted molecule
- \( T_w \) is the temperature of the wall

- For heat exchange between two surfaces, it is necessary to use an average accommodation coefficient,

\[
\bar{\alpha} = \frac{\alpha_1 \alpha_2}{\alpha_1 + \alpha_2 - \alpha_1 \alpha_2}
\]

Note: If the surfaces are not of equal area, geometric corrections are required for this formula.
Gas conduction heat exchange

In the molecular-kinetic regime, heat transfer between two parallel surfaces can be calculated using the expression,

\[ q = \frac{\bar{\alpha}}{4} \frac{\gamma + 1}{\gamma - 1} \left( \frac{2R}{\pi MT} \right)^{\frac{1}{2}} p(T_1 - T_2) \]

Where \( \gamma = \frac{C_p}{C_v} \)

Values for accommodation coefficients:
- \( \alpha \) decreases with cleaner surfaces
- \( \alpha \) increases with decreasing temperature to \( \sim 1 \) at \( T \sim T_{NBP} \)
- For rough calculations, \( \alpha \sim 0.5 \) is practical

<table>
<thead>
<tr>
<th>Surface condition</th>
<th>Transport gas</th>
<th>Temperature (K)</th>
<th>Accommodation coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>Very clean</td>
<td>helium</td>
<td>300</td>
<td>(&lt; 0.1)</td>
</tr>
<tr>
<td>Engineering</td>
<td>helium</td>
<td>300</td>
<td>0.3</td>
</tr>
<tr>
<td>Engineering</td>
<td>helium</td>
<td>20</td>
<td>0.6</td>
</tr>
<tr>
<td>Engineering</td>
<td>nitrogen</td>
<td>250</td>
<td>0.7</td>
</tr>
</tbody>
</table>
Example of gas conduction heat transfer

- Consider a 100 liter ($A = 1 \text{ m}^2$) cryostat for storing liquid nitrogen.
  - Calculate the consumption of LN$_2$ if the vessel is only vacuum insulated. ($h_{fg} = 198 \text{ kJ/kg}$). Radiant heat transfer between 300 K and 77 K (assume $\varepsilon \sim 0.05$)
    \[
    q = \frac{0.05}{2} \times 5.67 \times 10^{-8} \times (300^4 - 77^4) = 11.4 \text{ W (0.06 g/s)}
    \]
  - Calculate the consumption if the vessel had a poor vacuum with helium at $p \sim 0.1 \text{ Pa (10}^{-6} \text{ Atm)}$
    \[
    q = \frac{\bar{\alpha}}{4} \frac{\gamma + 1}{\gamma - 1} \left( \frac{2R}{\pi MT} \right)^{\frac{1}{2}} p(T_1 - T_2) \approx 7.6 \frac{p\Delta T}{T^{\frac{1}{2}}} \approx 12W
    \]
    Adds to the radiation heat transfer doubling the heat load

**Conclusion:** Good vacuum is highly desirable
Multi layer shielding

- Adding shielding between the radiant surfaces can significantly reduce the heat transfer. For $n$ shields with emissivity $\varepsilon$, the heat exchange is

$$ q_r = \left( \frac{\varepsilon}{(n+1)(2-\varepsilon)} \right) \sigma (T_1^4 - T_2^4) $$

which for $\varepsilon \ll 1$, reduces the $q_r$ by a factor of $1/n + 1$

- Note that the shield temperatures are not equally distributed because the heat exchange is not linear. Consider one shield and all emissivities $= \varepsilon$ in steady state:

$$ q_r(1,s) = \frac{\varepsilon}{2} \sigma (T_1^4 - T_s^4) = q_r(2,s) = \frac{\varepsilon}{2} \sigma (T_s^4 - T_2^4) $$

or

$$ T_s = \left( \frac{T_1^4 + T_2^4}{2} \right)^{1/4} \sim 252 \text{ K for } T_1 = 300 \text{ K and } T_2 = 77 \text{ K} $$
Refrigerated radiation shields

- There is significant thermodynamic advantage to actively cooling radiation shields in a cryogenic system. Examples:
  - LN$_2$ shield cooling in a cryostat
  - Vapor cooling in LHe storage vessels
  - Refrigerated shields

- Why would you want to do this?
  - Thermodynamic advantage of removing heat at higher temperature (COP)
  - Reduce boil-off of expensive fluid (LHe)
  - Can be done in conjunction with active cooling of other components (structural supports, current leads)
Multilayer Insulation (MLI)

- MLI is a material developed to approximate thermally insulated shields.
- MLI consists of aluminum (5 to 10 nm thick) on Mylar film usually with low density fibrous material between layers.
- Insulation must operate in vacuum.
- Heat transfer is by a combination of conduction and radiation.
- MLI must be carefully installed covering all surfaces with parallel layers, not wrapped since conduction along layer will produce a thermal short.
- Engineering applications must include factor of safety compared to ideal data.

Radiation heat load for different densities between 4.2 K and 77 K:

Recommended conservative values:

- \( q_r \) (77 K, 4 K) \(~ 50\) to 100 mW/m\(^2\)
- \( q_r \) (300 K, 77 K) \(~ 1\) to 1.5 W/m\(^2\)
Powder insulations (perlite, glass bubbles)

- Powder insulations were developed for ease of installation in less stringent operating conditions.
- Perlite is a commercial powder of random size and shape (cheap)
- Hollow glass micro-spheres (3M) of 50 to 200 μm in diameter
- Vacuum requirements are less critical. Good performance at $p \approx 0.1$ Torr compared to $10^{-4}$ torr for MLI
- Perlite is mostly used for less stringent cryogenic vessels such as LNG containers or LN$_2$ and LO$_2$.
- NASA is planning to build new storage containers with glass bubbles

\[ k_{\text{air}} = 26 \text{ mW/mK} \]
**Perlrite or Vacuum: which is the better insulation?**

Below $T = 77$ K and $P = 5 \times 10^{-5}$ torr, pure vacuum provides superior insulation.

$r_1 = 1.25$ m  
$r_2 = 1.35$ m  
$r_3 = 1.45$ m
Structural supports

- Simple support is appropriate for small masses where the conduction heat leak is not large.
- For large mass, an actively cooled support is preferred to reduce heat load at the lowest temperatures where the thermodynamic efficiency is low.
  - Position for the intermediate cooling station
  - Thermodynamic optimization
Optimization of mechanical supports

- **Considerations:**
  - Intermediate cooling stations (number, $T_I$, $x$)
  - Variation of thermal conductivity, $k(T)$
  - Temperature dependent mechanical properties, $\sigma(T)$
    - Only an advantage if loads occur when support is cold

- **Procedure**
  - Full optimization is based on assumptions about efficiency of refrigeration
  - Vary $T_i$ and $x_i$ to minimize total refrigeration

- **Typical practical solution (easier)**
  - Intermediate temperatures are known based on available refrigeration system (e.g. 80 K (LN$_2$), 20 K)
  - Vary position ($x_i$) to match available refrigeration
Example: 45 T Hybrid Magnet Cryostat

- Magnet loads are supported by refrigerated ss support column
  - 80 K by LN$_2$ natural circulation loop
  - 20 K by He gas forced circulation loop.
- Wall thickness decreases in low temperature sections
  - Increased strength of ss
  - Major portion of load only present when magnet is energized
- Location of refrigeration stations was optimized so that equivalent refrigeration is equal at each.
Support Tube for Hybrid

- Since load only occurs when magnet is energized, the structure takes advantage of increased material at low temperature
- Design load 6.3 MPa
- Cooling supplied by refrigerator at 20 K and LN$_2$ natural circulation loop (80 K)

<table>
<thead>
<tr>
<th>Temperature</th>
<th>$\sigma$ (MPa)</th>
<th>$k_{ave}$ (W/m K)</th>
<th>Cross section (m$^2$)</th>
<th>Length (m)</th>
<th>Q (W)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.8 K to 20 K</td>
<td>300</td>
<td>0.9</td>
<td>0.021</td>
<td>0.3</td>
<td>1.2</td>
</tr>
<tr>
<td>20 K to 80 K</td>
<td>240</td>
<td>5.6</td>
<td>0.026</td>
<td>0.3</td>
<td>30</td>
</tr>
<tr>
<td>80 K to 300 K</td>
<td>150</td>
<td>12.4</td>
<td>0.042</td>
<td>0.3</td>
<td>380</td>
</tr>
</tbody>
</table>
Instrumentation leads

- Most cryogenic systems have instrumentation
  - Monitor and control function (T, P, flow)
  - Measure performance of device (B, V, I)
- Instrumentation leads can significantly impact the thermal performance of a cryo system
  - Conduction heat leak
  - Joule heating in lead
- Proper lead design is important to ensure good measurement
  - Material selection
  - Thermal anchoring

<table>
<thead>
<tr>
<th>Gauge</th>
<th>D(mil)</th>
<th>D (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>24</td>
<td>20</td>
<td>0.51</td>
</tr>
<tr>
<td>28</td>
<td>12.6</td>
<td>0.32</td>
</tr>
<tr>
<td>32</td>
<td>8</td>
<td>0.2</td>
</tr>
<tr>
<td>36</td>
<td>5</td>
<td>0.127</td>
</tr>
<tr>
<td>40</td>
<td>3.1</td>
<td>0.08</td>
</tr>
</tbody>
</table>
Thermally optimized current lead

- Balance the heat generation with the conduction
  \[ \frac{d}{dx} \left( k(T) \frac{dT}{dx} \right) + \rho(T) \left( \frac{I}{A} \right)^2 = 0 \]

- Minimize entropy generation occurs for \( dT/dx = 0 \) at \( T_H(x=0) \)

- For Wiedemann-Franz law materials
  \[ Q = I \left[ 2 \int_{T_L}^{T_H} k(T) \rho(T) dT \right]^{1/2} \]

  \[ \frac{k(T) \rho(T)}{T} = L_0 \]

  \( Q \sim \text{constant for all materials} \)
Thermally optimized lead (continued)

- For W-F materials, exact solution:

$$\frac{Q}{I} = \sqrt{L_0 \left( T_H^2 - T_L^2 \right)}$$

\[ \sim 47 \text{ mW/A for } T_H = 300 \text{ K, } T_L = 4 \text{ K} \]

- Optimum \((L/A)\) for W-F materials

\[
\frac{L}{A} = \frac{1}{I} \int_{T_L}^{T_H} \frac{k}{\sqrt{L_0 \left( T_H^2 - T^2 \right)}} dT = \frac{k}{\sqrt{L_0 I}} \left( 1.57 - \sin^{-1} \left( \frac{T_L}{T_H} \right) \right)
\]

\[ \sim 10^4 \left( \frac{k}{I} \right), \text{ m}^{-1} \]

Example: For a lead \(I = 1 \text{ A}\) and \(L = 1 \text{ m}\), \(A = 0.25 \text{ mm}^2\) (\(d = 0.56 \text{ mm}\))

\(J = 4 \text{ A/mm}^2\)

High current leads benefit from active cooling, which allows higher current density
Conduction Cooled Lead: Sample Results

![Graph showing temperature vs. L/A (m/m^2)](image)

- I = 1000 A
- RRR = 100

Temperature (K)

L/A (m/m^2)

USPAS Short course
Boston, MA 6/14 to 6/18/2010
Conduction Cooled Lead: Conclusions

- An ‘optimized’ lead is optimized for a single (maximum) current
- $Q_{c, \text{min}} \sim I$
- $Q_{c, \text{min}}$ is a function of $T_h$, $T_c$, $I$, and (weakly) on material choice
- $JL = \text{constant dependent only on } T_h, T_c, \text{ and mtl. choice}$
- $L/A \sim 1/I$
Vapor Cooled Lead - Scaling Rules

- **Minimum heat leak:**
  - As with conduction cooled leads, $Q_{\text{min}} \sim I$
  - Dependence of $Q_{\text{min}}$ on coolant is dominated by $(C_L / C_p)$

- **Optimized aspect ratio:**
  - $L/A_{\text{opt}} \sim 1/I$ \quad smaller current $\rightarrow$ larger aspect ratio
  - $L/A_{\text{opt}}$ dependence on coolant: colder range $\rightarrow$ larger aspect ratio