TM-CLASS CAVITY DESIGN

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Outline

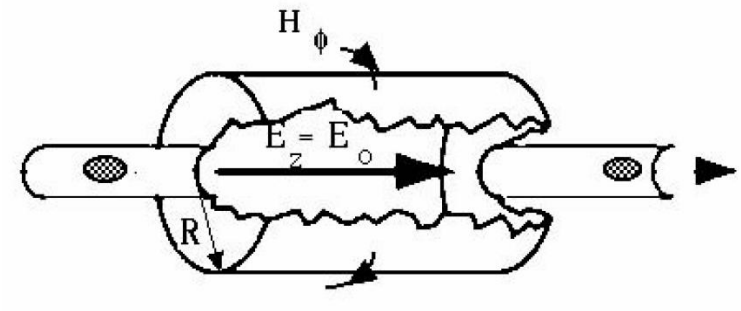
- "Pill-box" cavity
- Cell shape design
  - EM design
  - Multipacting analysis
- Multi-cell cavity design
  - HOM analysis
  - Mechanical analysis

Simplest case: “Pill-box” cavity

Hollow right cylindrical enclosure

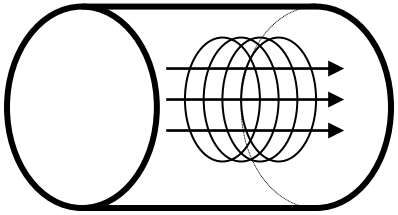
Operated in the TM_{010} mode

TM_{010} mode



$$E_z(r,z,t) = E_0 J_0\left(2.405\frac{r}{R}\right)e^{-i\omega_0 t}$$

$$H_\varphi(r,z,t) = -i\frac{E_0}{\mu_0 c}J_1\left(2.405\frac{r}{R}\right)e^{-i\omega_0 t}$$



$$\omega_0 = \frac{2.405c}{R}$$

Modes in Pill Box Cavity

- TM_{010}
  - Electric field is purely longitudinal
  - Electric and magnetic fields have no angular dependence
  - Frequency depends only on radius, independent on length
- TM_{0mn}
  - Monopoles modes that can couple to the beam and exchange energy
- TM_{1mn}
  - Dipole modes that can deflect the beam
- TE modes
  - No longitudinal E field
  - Cannot couple to the beam

TM Modes in a Pill Box Cavity

$$\frac{E_r}{E_0} = -\frac{n\pi}{x_{lm}}\frac{R}{L}J_l'\left(x_{lm}\frac{r}{R}\right)\sin\left(n\pi\frac{z}{L}\right)\cos l\varphi$$

$$\frac{E_\varphi}{E_0} = \frac{ln\pi}{x_{lm}^2}\frac{R^2}{rL}J_l\left(x_{lm}\frac{r}{R}\right)\sin\left(n\pi\frac{z}{L}\right)\sin l\varphi$$

$$\frac{E_z}{E_0} = J_l\left(x_{lm}\frac{r}{R}\right)\sin\left(n\pi\frac{z}{L}\right)\cos l\varphi$$

$$\omega_{lmn} = c\sqrt{\left(\frac{x_{lm}}{R}\right)^2 + \left(\frac{\pi n}{L}\right)^2}$$

$$\frac{H_r}{E_0} = -i\omega\varepsilon\frac{l}{x_{lm}^2}\frac{R^2}{r}J_l\left(x_{lm}\frac{r}{R}\right)\cos\left(n\pi\frac{z}{L}\right)\sin l\varphi$$

x_{lm} is the mth root of $J_l(x)$

$$\frac{H_\varphi}{E_0} = -i\omega\varepsilon\frac{R}{x_{lm}}J_l'\left(x_{lm}\frac{r}{R}\right)\cos\left(n\pi\frac{z}{L}\right)\cos l\varphi$$

$$\frac{H_z}{E_0} = 0$$

TM_{010} Mode in a Pill Box Cavity

$$E_r = E_\varphi = 0 \qquad E_z = E_0 J_0\left(x_{01}\frac{r}{R}\right)$$

$$H_r = H_z = 0 \qquad H_\varphi = -i\omega\varepsilon E_0 \frac{R}{x_{01}} J_1\left(x_{01}\frac{r}{R}\right)$$

$$\omega = x_{01}\frac{c}{R} \qquad x_{01} = 2.405$$

$$R = \frac{x_{01}}{2\pi}\lambda = 0.383\lambda$$

TM_{010} Mode in a Pill Box Cavity

Energy content

$$U = \varepsilon_0 E_0^2 \frac{\pi}{2} J_1^2(x_{01}) L R^2$$

Power dissipation

$$P = E_0^2 \frac{R_s}{\eta^2} \pi J_1^2(x_{01})(R + L) R$$

$$x_{01} = 2.40483$$

$$J_1(x_{01}) = 0.51915$$

Geometrical factor

$$G = \eta \frac{x_{01}}{2} \frac{L}{(R + L)}$$

TM_{010} Mode in a Pill Box Cavity

Energy Gain

$$\Delta W = E_0 \frac{\lambda}{\pi} \sin \frac{\pi L}{\lambda}$$

Gradient

$$E_{acc} = \frac{\Delta W}{\lambda / 2} = E_0 \frac{2}{\pi} \sin \frac{\pi L}{\lambda}$$

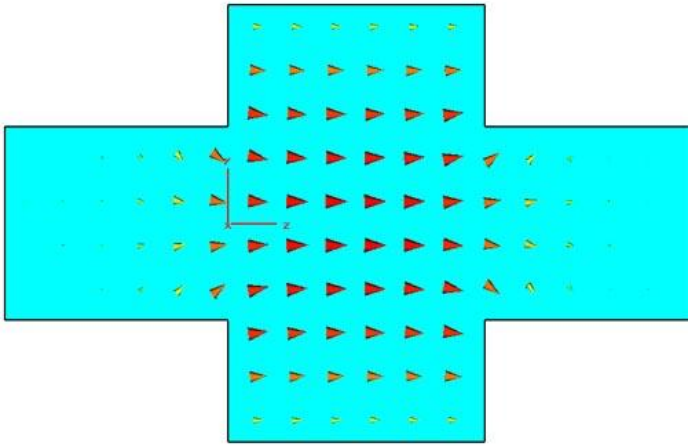
Shunt impedance

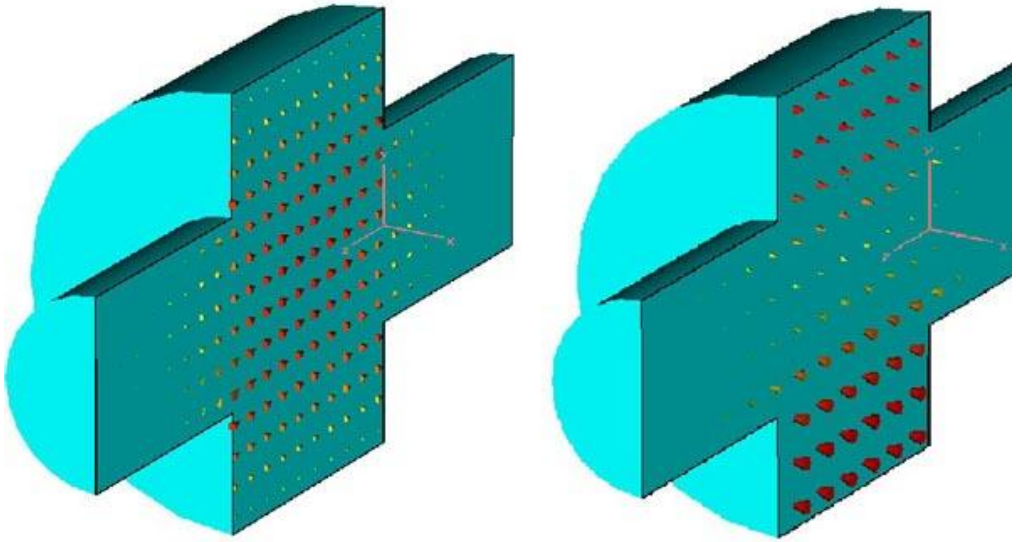
$$R_{sh} = \frac{\eta^2}{R_s} \frac{1}{\pi^3 J_1^2(x_{01})} \frac{\lambda^2}{R(R+L)} \sin^2\left(\frac{\pi L}{\lambda}\right)$$

Real Cavities

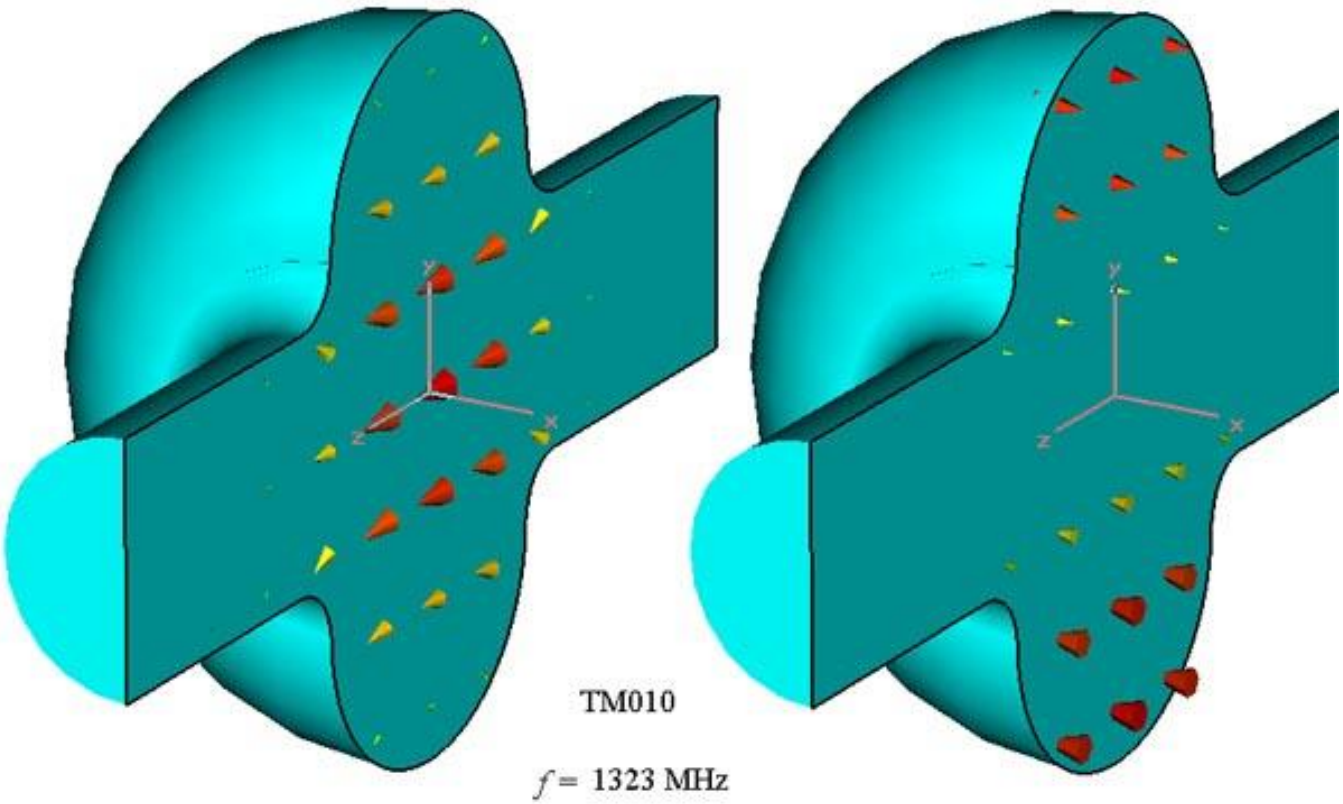
Beam tubes reduce the electric field on axis

- Gradient decreases
- Peak fields increase
- R/Q decreases



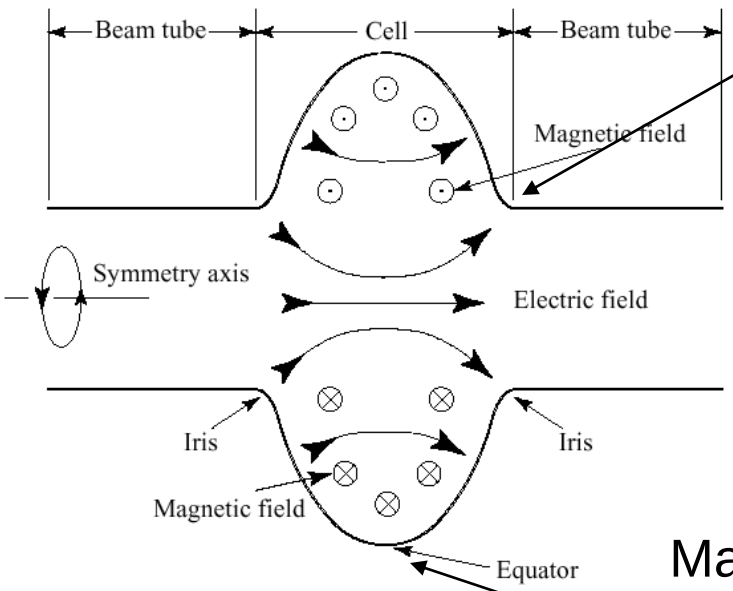


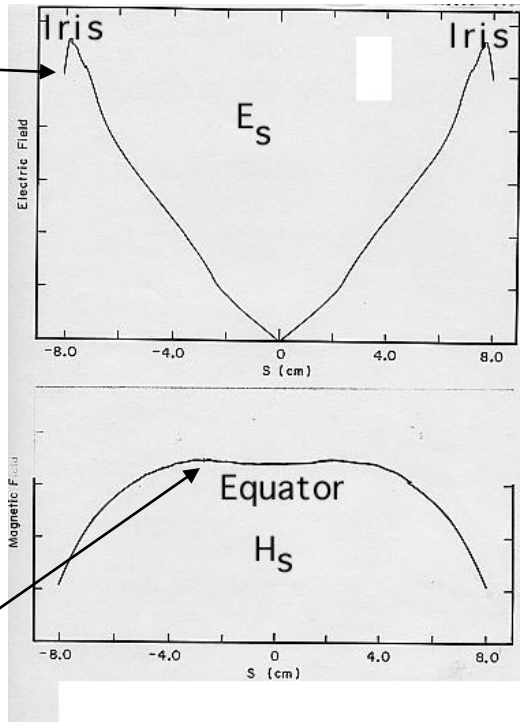
Real Cavities



Single Cell Cavities

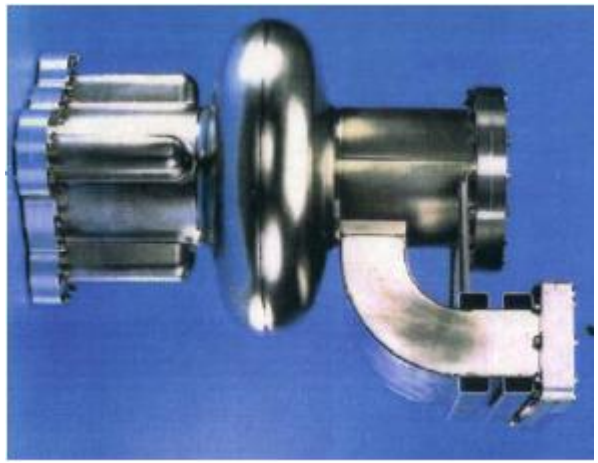
Electric field high at iris

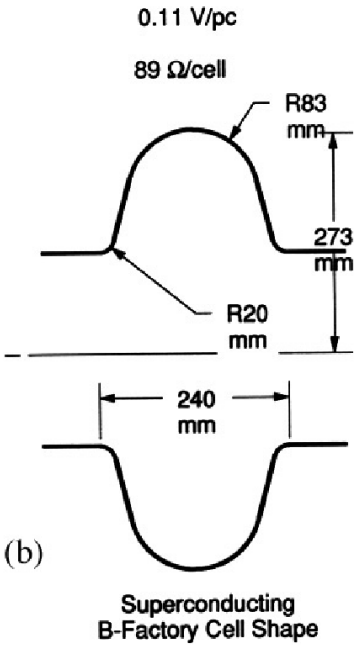




Magnetic field high at equator

Single Cell Cavities





Quantity	Cornell SC 500 MHz	Pillbox
G	270 Ω	257 Ω
R_a/Q_0	88 ohm/cell	196 Ω/cell
E_{pk}/E_{acc}	2.5	1.6
H_{pk}/E_{acc}	52 Oe/MV/m	30.5 Oe/(MV/m)

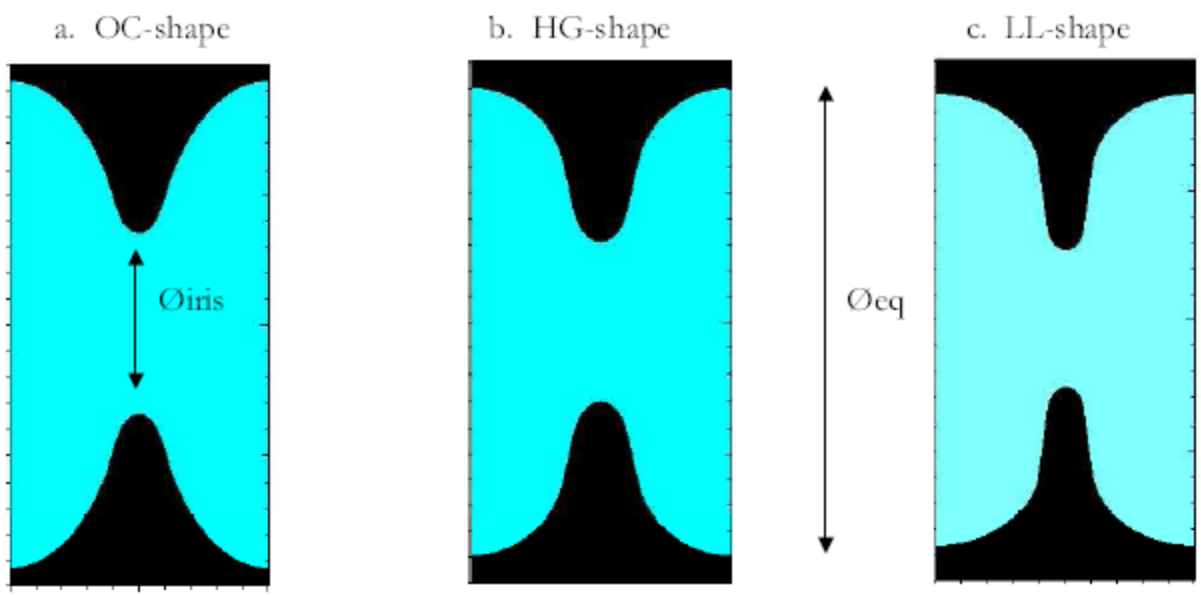
Cell Shape Design

- What is the purpose of the cavity?
- What EM parameters should be optimized to meet the design specs?

The "perfect" shape does not exist, it all depends on your application

Example: CEBAF Upgrade

- "High Gradient" shape: lowest E_p/E_{acc}
- "Low Loss" shape: lowest cryogenic losses G(R/Q)



CEBAF Upgrade Shape Comparison

Table 1. Parameters of inner dumbbells

Parameters	Unit	OC-shape	HG-Shape	LL-Shape
Øeq	[mm]	187.03	180.50	174.00
Øiris	[mm]	70.00	61.40	53.00
k_{cc}	[%]	3.29	1.72	1.49
E_{peak}/E_{acc}	-	2.56	1.89	2.17
B_{peak}/E_{acc}	$[mT\cdot(MV/m)^{-2}]$	4.56	4.26	3.74
Lorentz factor*) k_L	$[Hz\cdot(MV/m)^{-2}]$	-1.35	-1.1	-1.2
R/Q	[Ω]	96.5	111.9	128.8
r/q = (R/Q)/length	[Ω/m]	965	1119	1288
G	[Ω]	273.8	265.5	280.3
R/Q*G	[Ω*Ω]	26421	29709	36102

CEBAF Upgrade: cryo-budget limit of 30W/cavity. Higher energy gain can be obtained using LL-shape.

New Trend in TM-Cavity Design

- The **field emission is not a hard limit** in the performance of sc cavities if the surface preparation is done in the right way.

- Unlikely this, **magnetic flux on the wall** limits performance of a sc cavity (Q_0 decreases or/and quench). Hard limit ~**180 mT** for Nb.

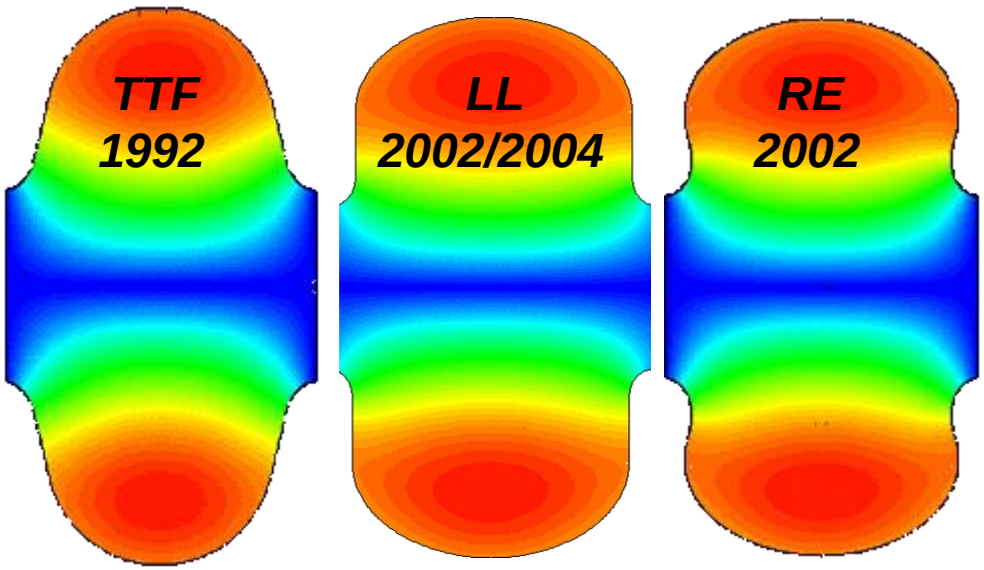
B_{peak} / E_{acc} should be low

1. Cavities may operate at higher gradients.

2. Cavities may operate at lower cryogenic load.

$$\frac{P_{dissipated}}{V_{acc}^2} \equiv \frac{R_s}{G \cdot (R/Q)}$$

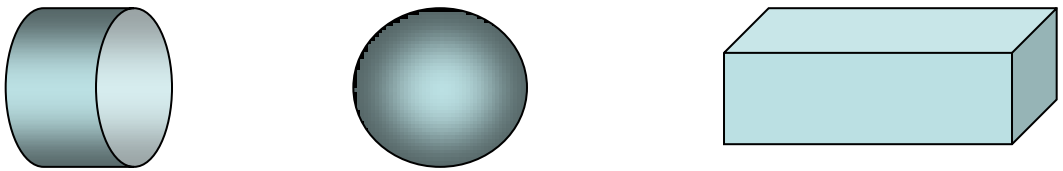
New Shapes for ILC



r_{iris}	[mm]	35	30	33
k_{cc}	[%]	1.9	1.52	1.8
E_{peak}/E_{acc}	-	1.98	2.36	2.21
B_{peak}/E_{acc}	[mT/(MV/m)]	4.15	3.61	3.76
R/Q	[Ω]	113.8	133.7	126.8
G	[Ω]	271	284	277
R/Q*G	[Ω*Ω]	30840	37970	35123

RF Simulation Codes for Cavity Design

The solution to 2D (or 3D) Helmholtz equation can be analytically find only for very few geometries (pillbox, spherical resonators or rectangular resonator).



We need numerical methods:

$$(\nabla^2 + \omega^2 \varepsilon\mu)A = 0$$

Approximating operator (Finite Difference Methods)

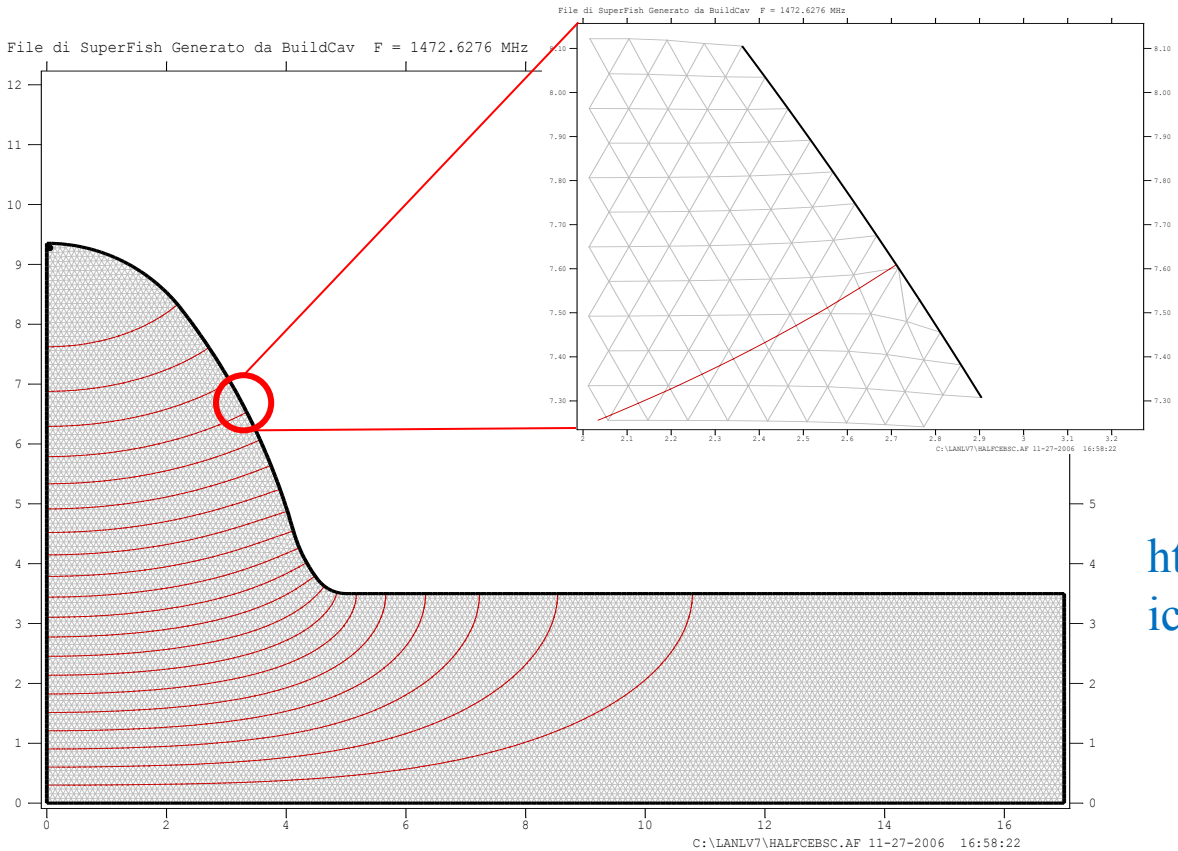
Approximating function (Finite Element Methods)

• 2D is fast and allows to define geometry of a cylindrical symmetric body (inner and end-cells) of the cavity.

• 3D is much more time consuming but necessary for modeling of full equipped cavity with FPC and HOM couplers and if needed to model fabrication errors. Also coupling strength for FPC and damping of HOMs can be modeled only 3D.

SUPERFISH

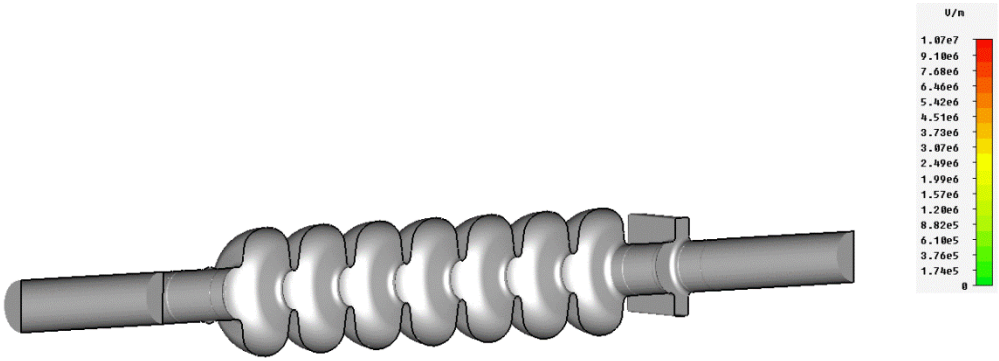
- Free, 2D finite-difference code to design cylindrically symmetric structures (monopole modes only)
- Use symmetry planes to reduce number of mesh points



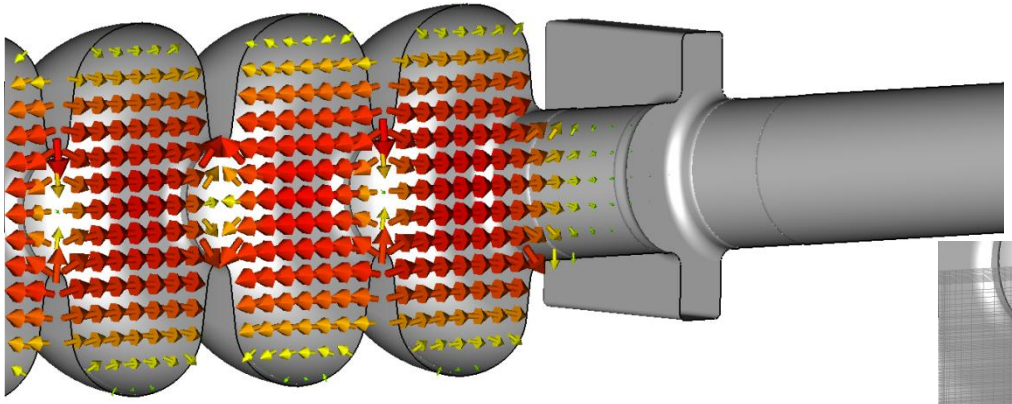
http://laacg1.lanl.gov/laacg/services/download_sf.phtml

CST Microwave Studio

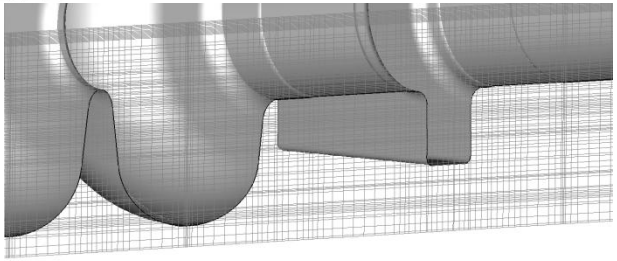
- Expensive, 3D finite-element code, used to design complex RF structure. http://www.cst.com/Content/Products/MWS/Overview.aspx



- Runs on PC
- Perfect Boundary Approximation

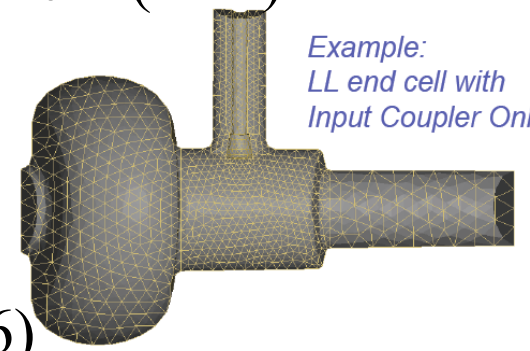


Hexahedral mesh

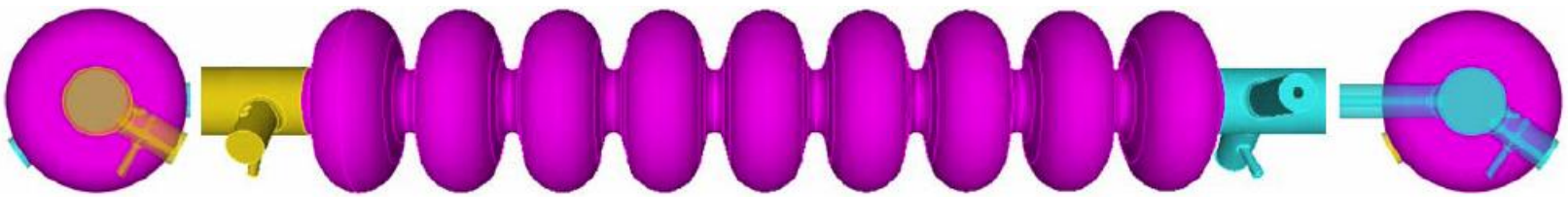


Omega3P

- SLAC, 3D code, high-order Parallel Finite Element (PFE) method
- Runs on Linux
- Tetrahedral conformal mesh
- High order finite elements (basis order p =1 – 6)
- Separate software for user interface (CuBit)

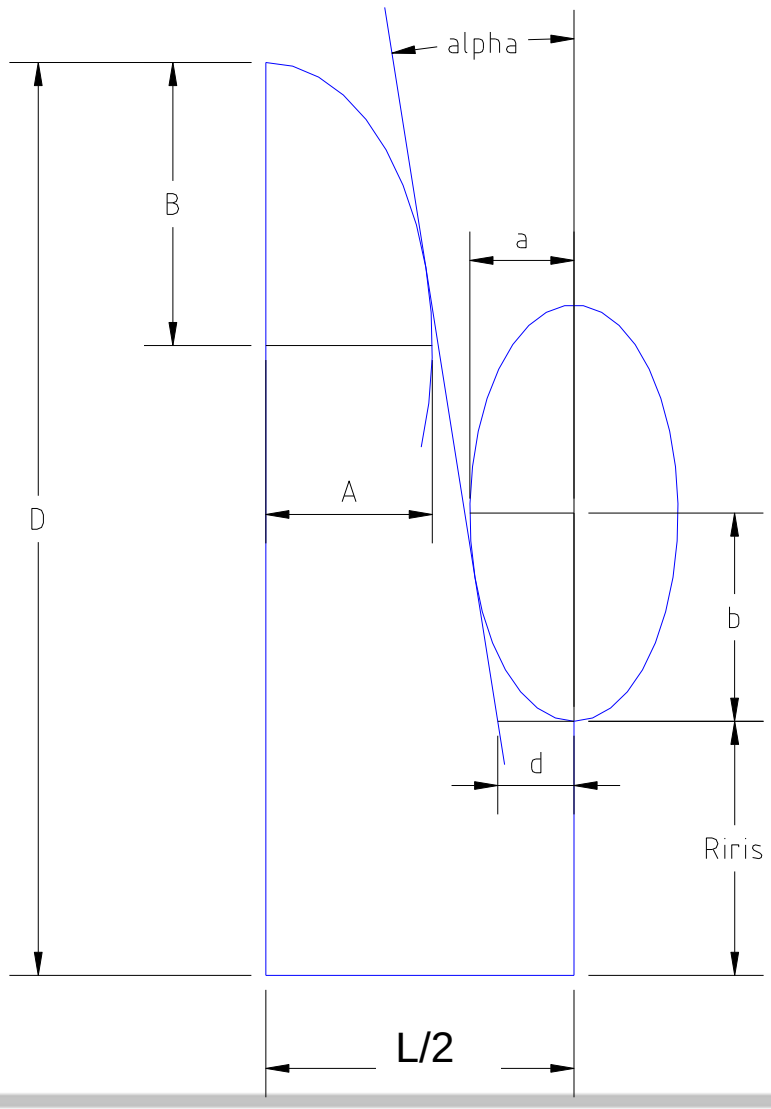


Example:
LL end cell with
Input Coupler Only



Cell Shape Parametrization

- Full parametric model of the cavity in terms of 7 meaningful geometrical parameters:
  - ✓ Ellipse ratio at the equator (R=B/A)
    ruled by mechanics
  - ✓ Ellipse ratio at the iris (r=b/a)
    E_{peak}
  - ✓ Side wall inclination (α) and position (d)
    E_{peak} vs. B_{peak} tradeoff and coupling k_{cc}
  - ✓ Cavity iris radius R_{iris}
    coupling k_{cc}
  - ✓ Half-cell Length $L/2=\lambda\beta/4$
    β
  - ✓ Cavity radius D
    used for frequency tuning
- Behavior of all e.m. and mechanical properties has been found as a function of the above parameters

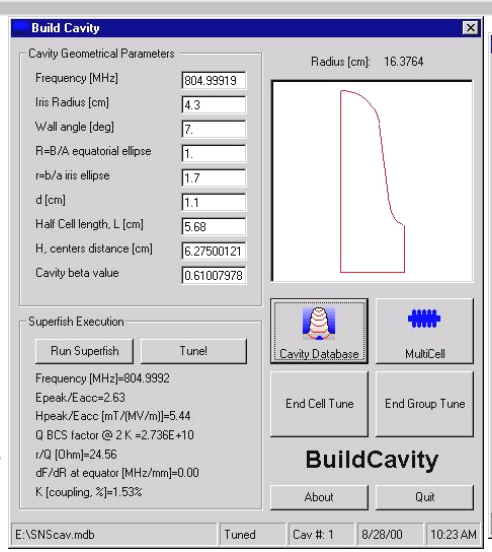


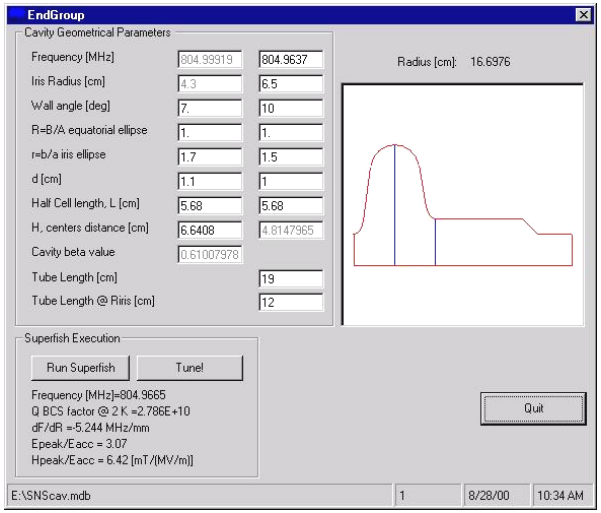
Tools used for the parametrization

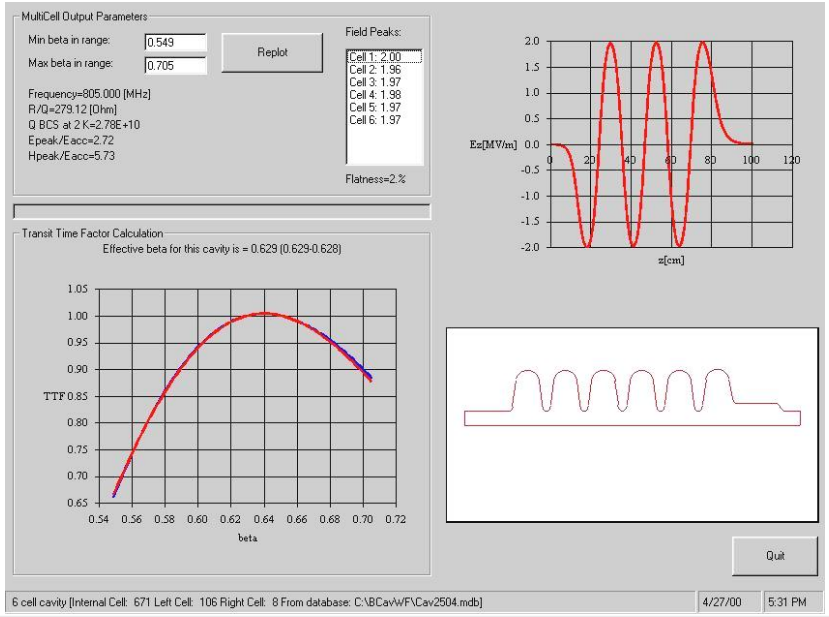
BuildCavity: parametric tool for the analysis of the cavity shape on the EM parameters:

- All RF computations are handled by SUPERFISH
- Inner cell tuning is performed through the cell diameter, all the characteristic cell parameters stay constant: R, r, α, d, L, R_{iris}
- End cell tuning is performed through the wall angle inclination, α, or distance, d.

  R, L and R_{iris} are independently settable.
- Multicell cavity is then built to minimize the field unflatness, compute the effective β and the final cavity performances.
- A proper file to transfer the cavity geometry to ANSYS is then generated







Inner cell data

L = 56.8 mm
R = 1
r = 1.7
α = 7°
d = 11 mm
R_{iris} = 43 mm

Parameter Choices

- Choose the cavity frequency ⇨ Equator diameter **D**

- Accelerate electrons (β=1) or protons (several designs with $\beta < 1$)?

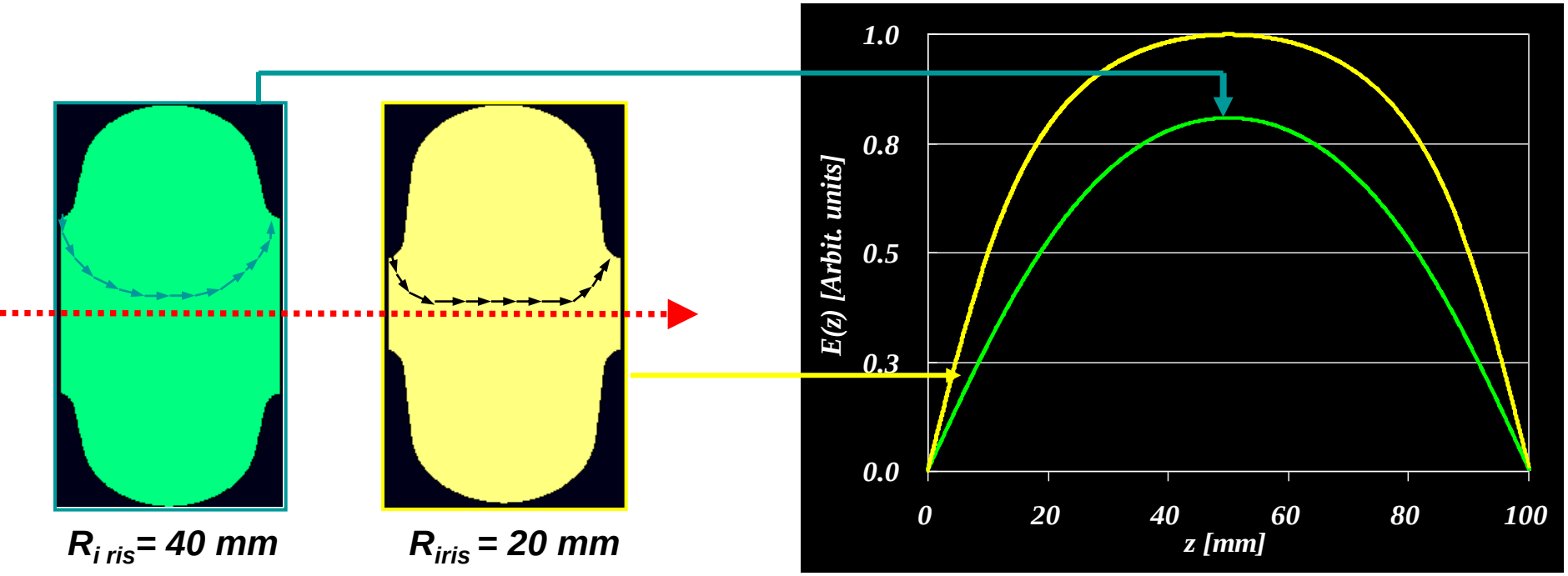
  Cell length, $\mathbf{L = \lambda\beta/2}$

One Big "Knob": R_{iris}

Why for a smaller aperture (R_{iris})?

- (R/Q) is bigger
- E_{peak}/E_{acc} , B_{peak}/E_{acc} is lower

E_{acc} is higher at the same stored energy in the cell



$R_{i\ ris}$= 40 mm ***R_{iris} = 20 mm***

E_z (z) for small and big iris radius

More on R_{iris}

We know that a smaller aperture makes:

- *(R/Q) higher*
- B_{peak}/E_{acc} , E_{peak}/E_{acc} *lower*

} **(+)**

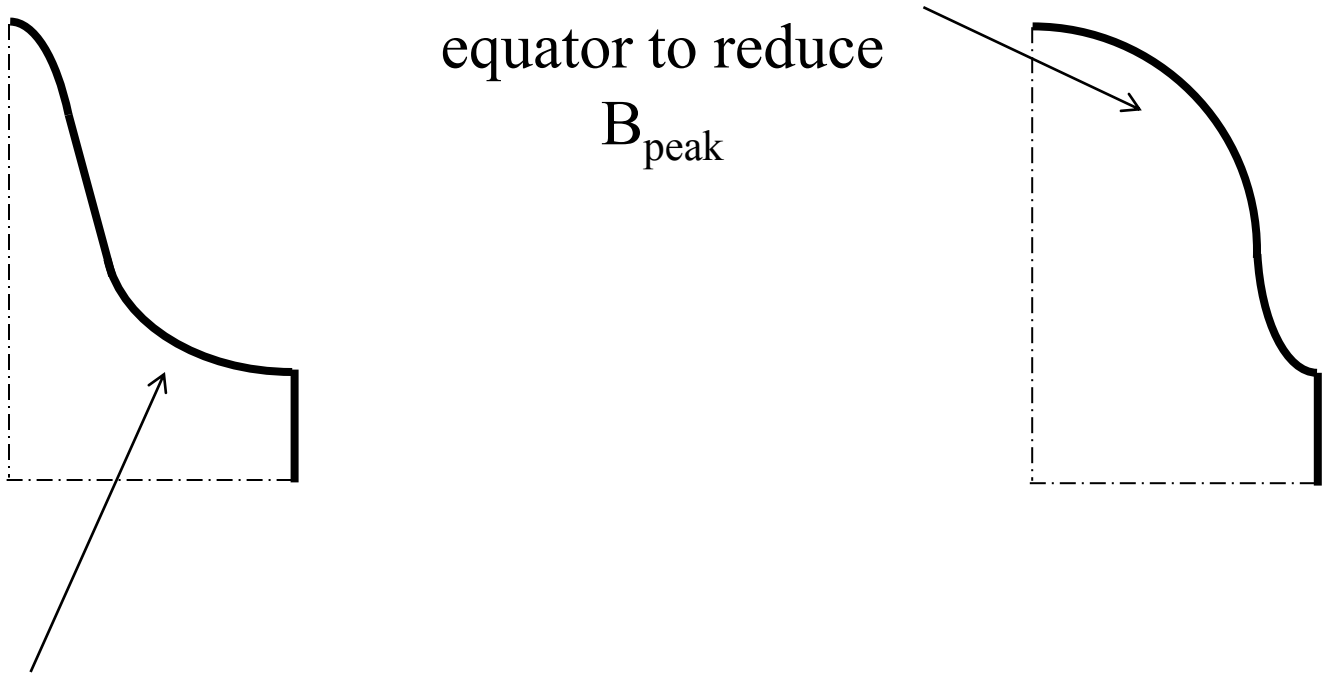
but unfortunately a smaller aperture makes:

- *HOMs impedances* ($k_{\perp}$, $k_{\parallel}$) *higher*
- *cell-to-cell coupling* (k_{cc}) *weaker*

} **(-)**

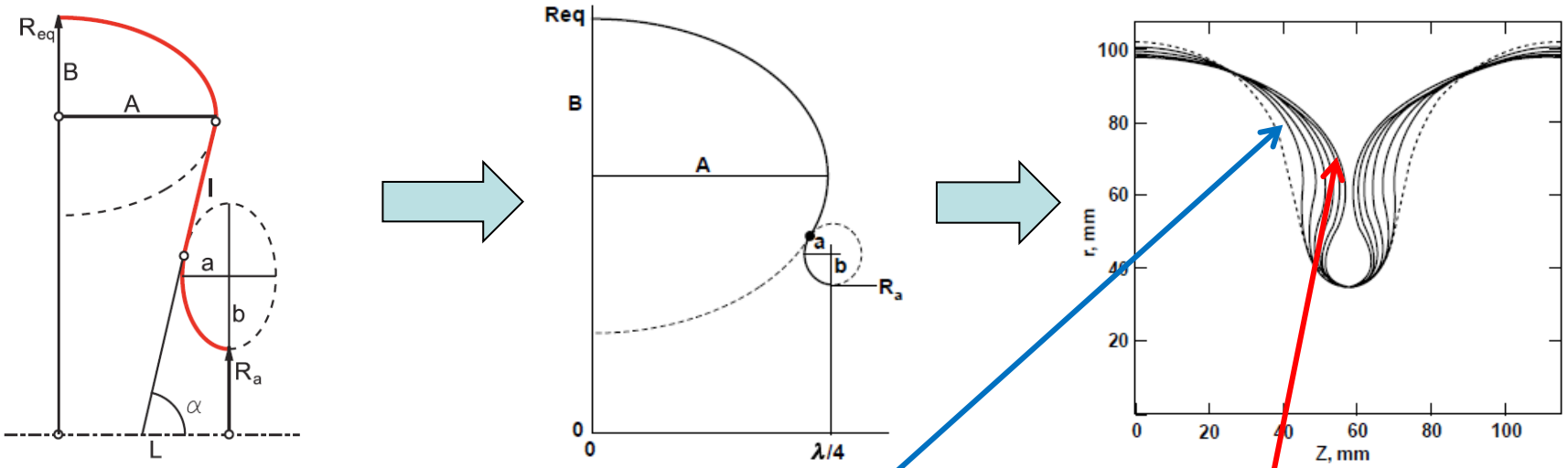
"Rule of thumb" for Optimizing Peak Surface Fields

Add "*magnetic volume*" at the equator to reduce B_{peak}

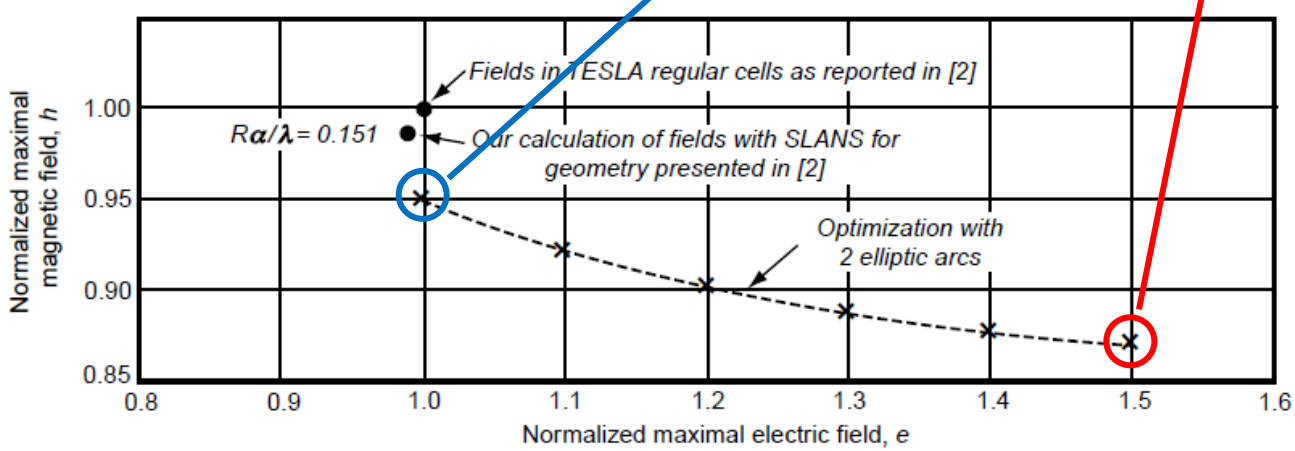


Add "*electric volume*" at the iris to reduce E_{peak}

Pushing the Design: Reentrant cavity

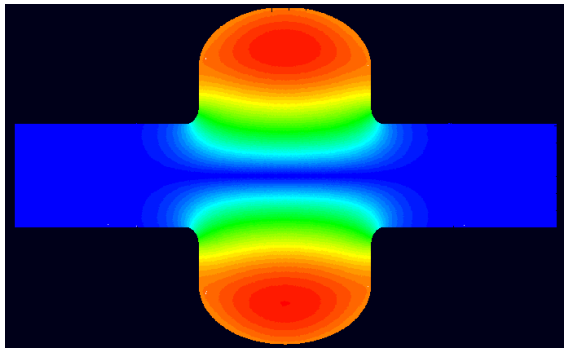


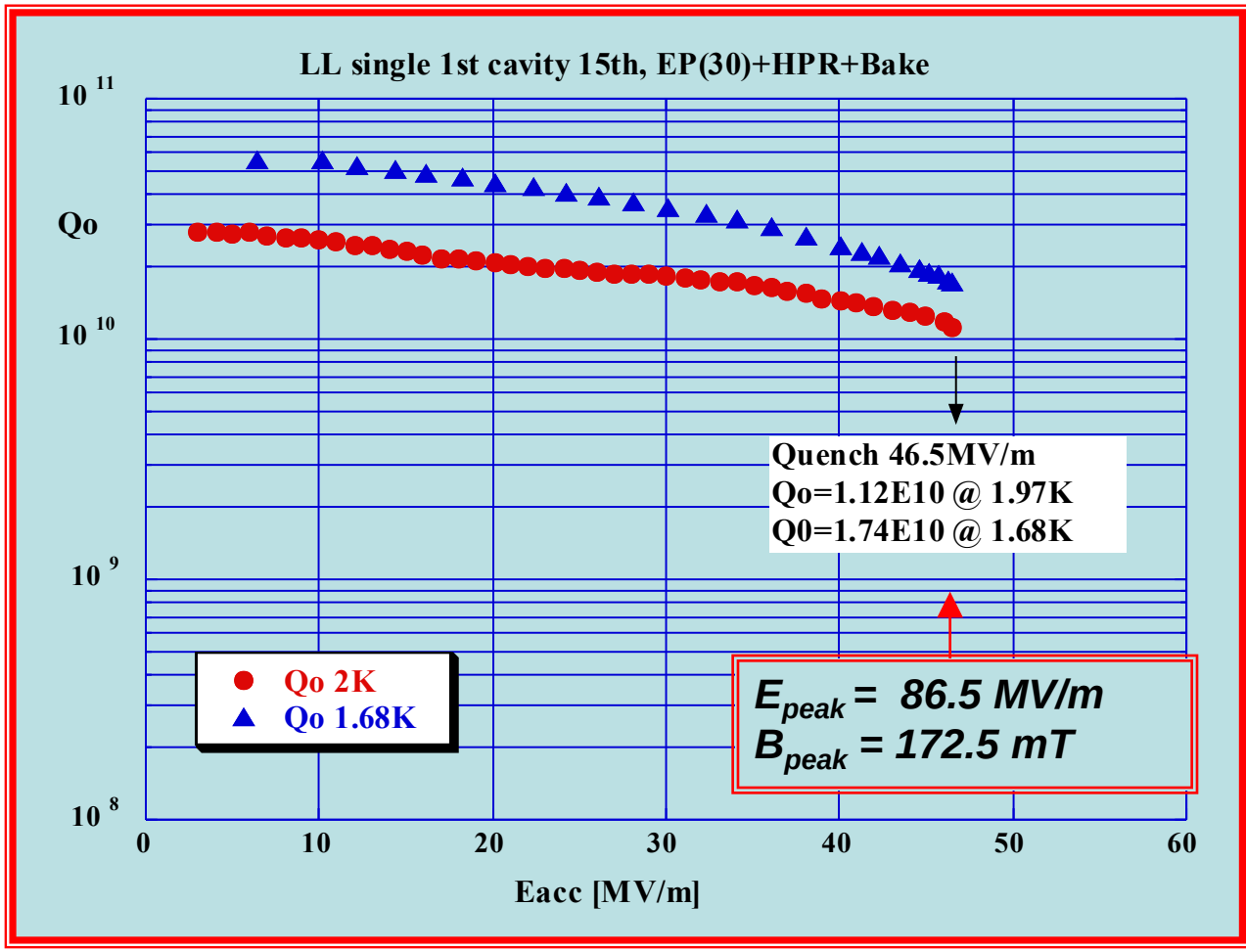
- 3 independent parameters: A, B, a
- potential issue with cavity forming and cleaning



RF Tests of New Cavity Shapes: LL

		LL
f_{π}	[MHz]	1286.6
E_{peak}/E_{acc}	-	1.86
B_{peak}/E_{acc}	[mT/(MV/m)]	3.71
R/Q	[Ω]	130.0
G	[Ω]	279
$Ø_{iris}$	[mm]	61

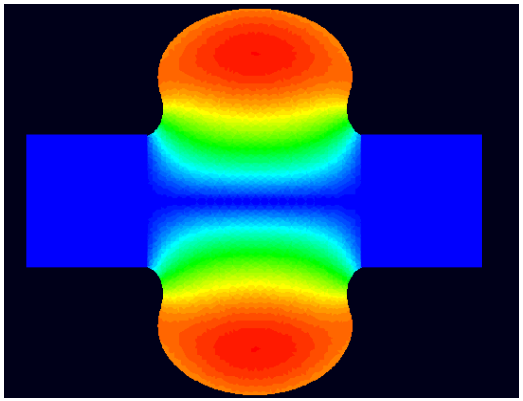


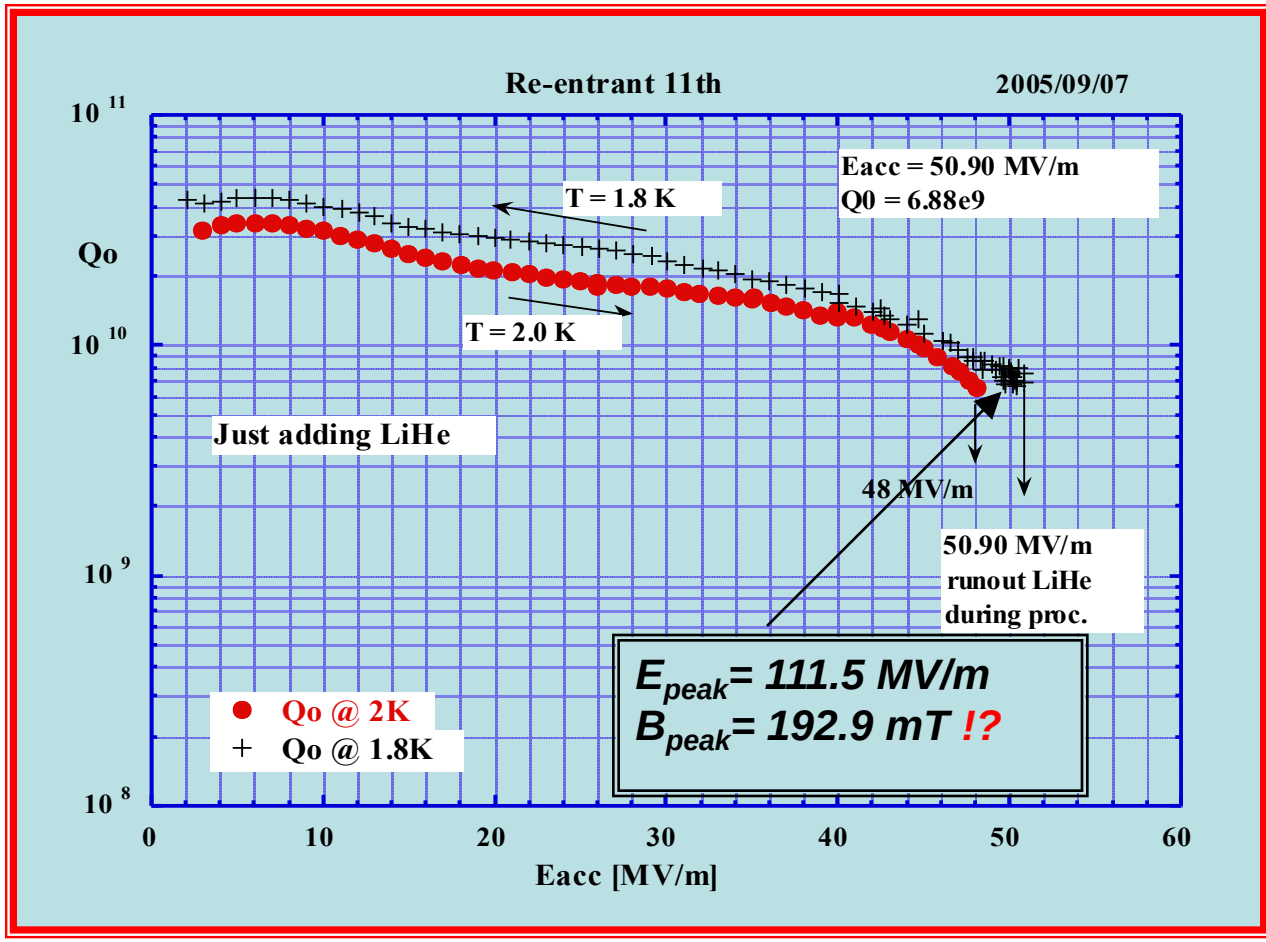


9-cell LL cavity was tested at JLab up to E_{acc}=36MV/m

RF Tests of New Cavity Shapes: RE

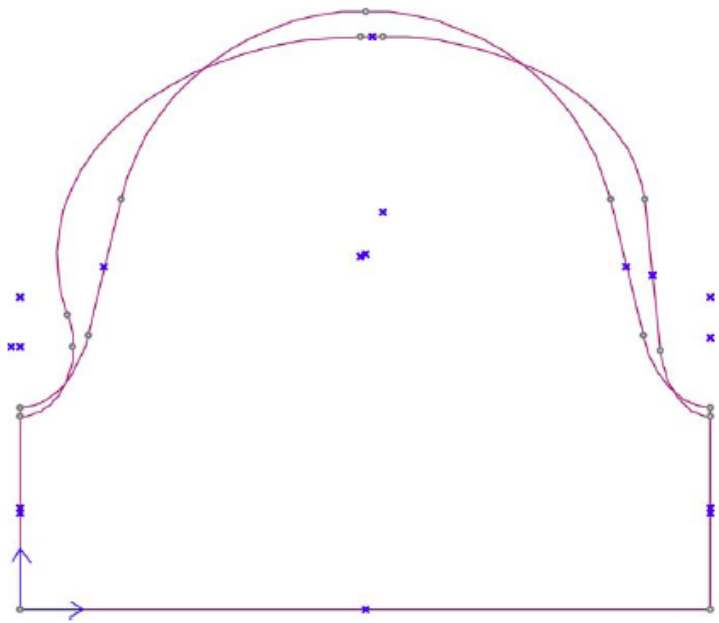
		RE
f_{π}	[MHz]	1278.6
E_{peak}/E_{acc}	-	2.19
B_{peak}/E_{acc}	[mT/(MV/m)]	3.79
R/Q	[Ω]	126.0
G	[Ω]	278
$Ø_{iris}$	[mm]	68

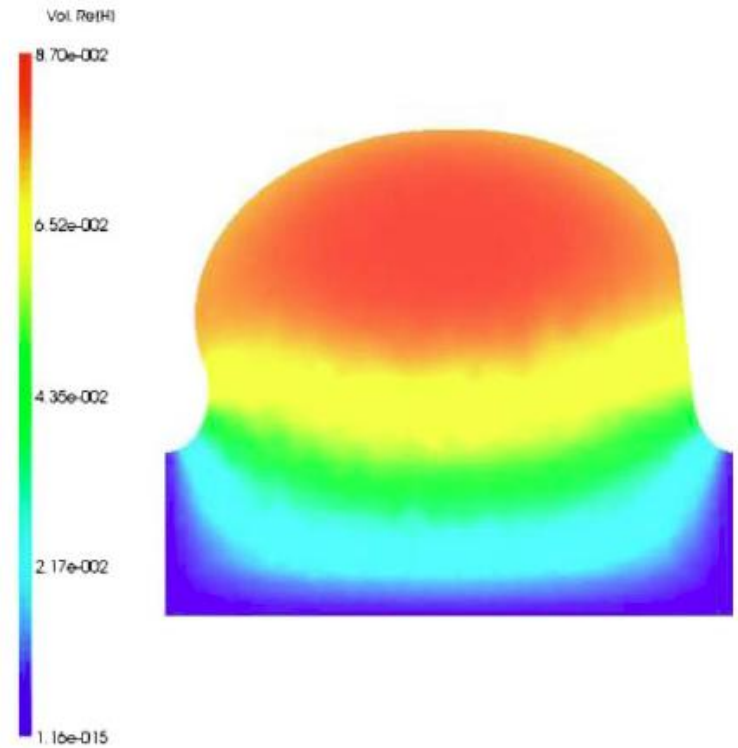




9-cell RE cavity was tested at Cornell up to E_{acc}=28MV/m

Want more?...Half-Reentrant Cavity



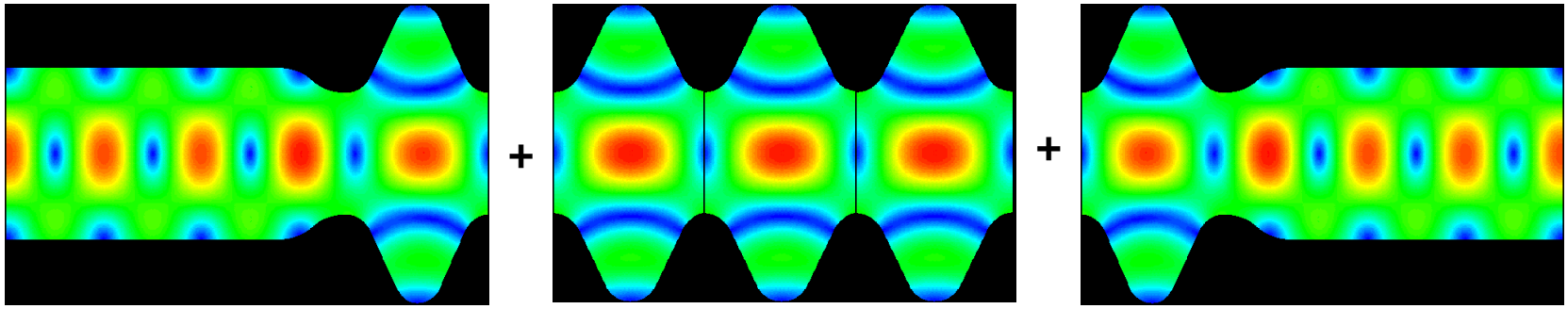


Parameters of the two proposed half-reentrant inner cells compared to the proposed Low Loss ILC geometry

	High-k_{cc} HR	Low-k_{cc} HR	Low Loss ILC
Frequency (MHz)	1300	1300	1300
Wall angle (°)	6	6	0.165
E_{peak}/E_{acc} (−)	2.40	2.38	2.36
B_{peak}/E_{acc} $\left(\frac{mT}{MV/m}\right)$	3.78	3.60	3.61
R/Q (Ω)	123	135	134
G (Ω)	283	283	284
$(R/Q) \cdot G$ (Ω^2)	34,673	38,021	37,970
k_{cc} (%)	2.09	1.51	1.52
r_i (cm)	3.34	2.97	3.00

End-Cell Design

The geometry of end-cells differs from the geometry of inner cells due to the attached beam tubes



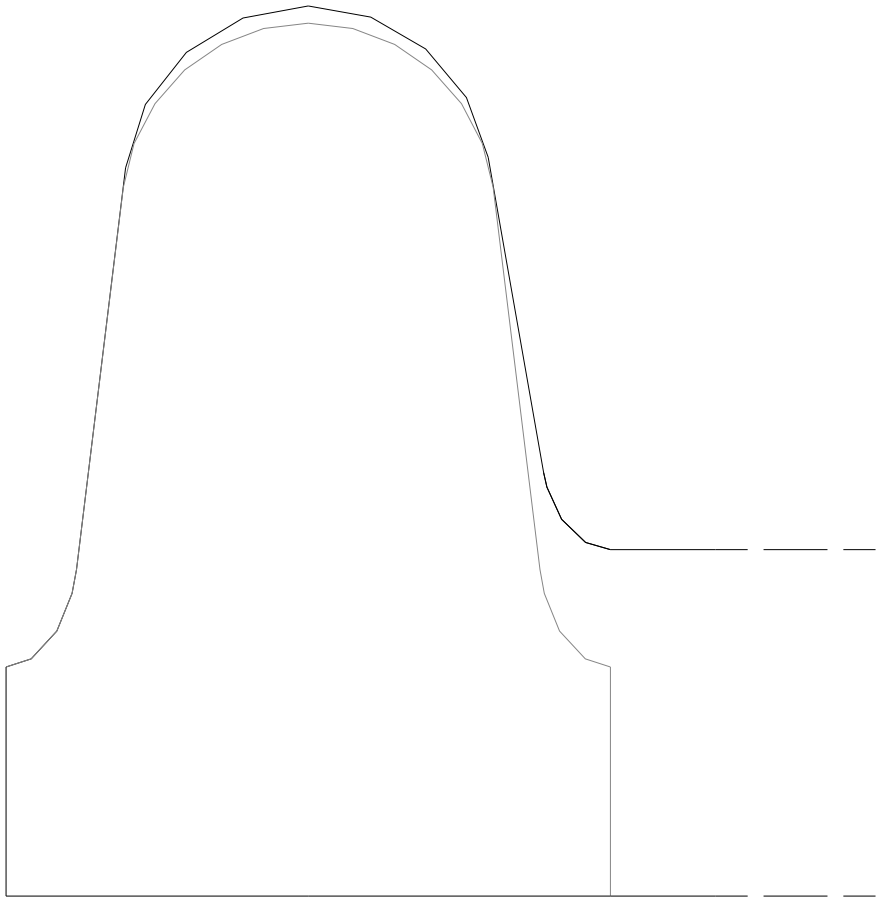
Their function is multi-folded and their geometry must fulfill three requirements:

- field flatness and frequency of the accelerating mode
- field strength of the accelerating mode at FPC location enabling operation with matched Q_{ext}
- fields strength of dangerous HOMs ensuring their required damping by means of HOM couplers or/and beam line absorbers.

All three make design of the end-cells more difficult than inner cells.

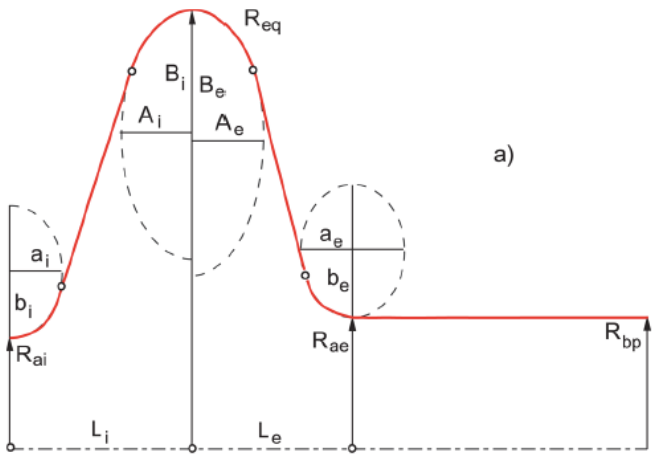
Example: SNS MB cavity

Optimization done with BuildCavity

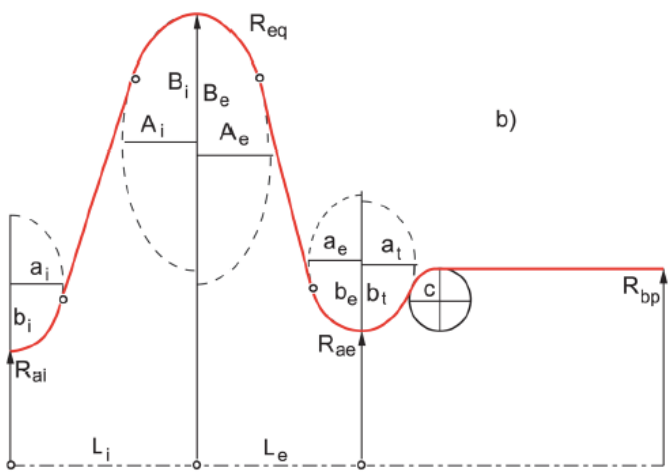


- R_{iris} set to 65 mm to have enough field at the power coupler antenna
- d set 1 mm lower than the in-cell
- optimization of r = b/a at iris
- α set to 10 deg to have the necessary stiffening
- Slater compensation (increase of the magnetic volume) of the cut-off tube ($\downarrow$f), d reduction ($\downarrow$f), α and R_{iris} increase ($\uparrow$f) by increasing the equator radius ⇨ 4 dies
- the frequency of end cell + tube is about 40 kHz lower than the in-cell's due to the asimmetry

More Examples of End Cell Optimizations

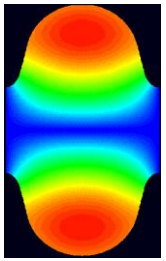


- Same R_{eq} as inner cell, use L_e as parameter to adjust the frequency
- Adjust parameters A_e, B_e, a_e, b_e and α to minimize either E_{peak}/E_{acc} or losses



- By adding more parameters (a_t, b_t, c, R_{bp}) it is possible to optimize the propagation of unwanted HOM, without increasing E_{peak}/E_{acc} or losses for the fundamental mode

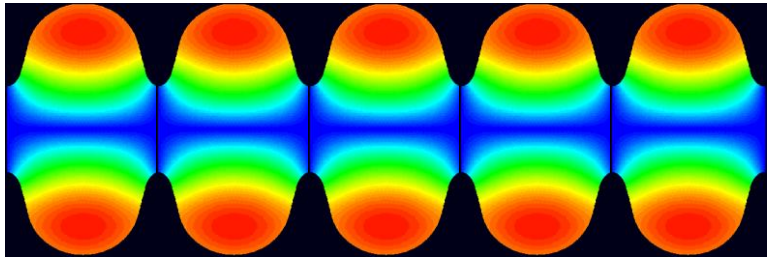
Multicell Cavities



Single-cell is attractive from the RF-point of view:

- Easier to manage HOM damping
- No field flatness problem.
- Input coupler transfers less power
- Easy for cleaning and preparation

- ***But it is expensive to base even a small linear accelerator on the single cell. We do it only for very high beam current machines.***



A multi-cell structure is less expensive and offers higher real-estate gradient but:

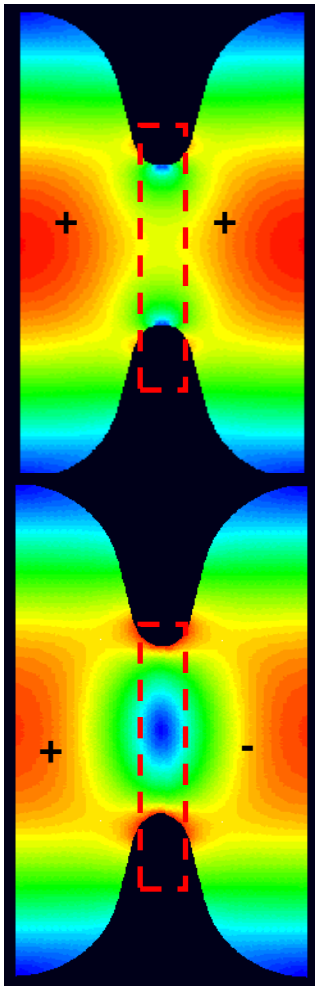
Field flatness (stored energy) in cells becomes sensitive to frequency errors of individual cells

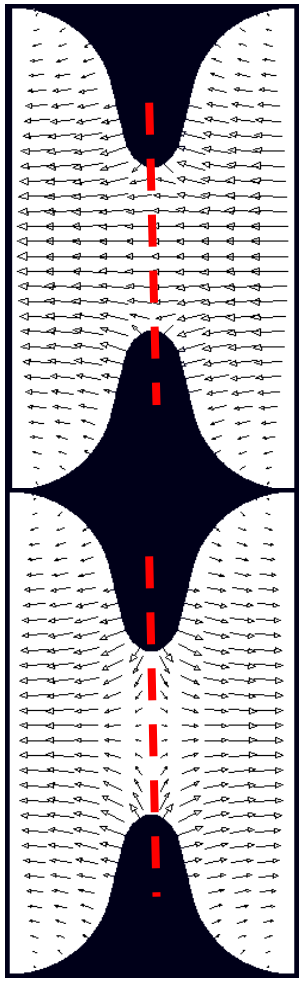
- ***Other problems arise: HOM trapping…***

Coupling between cells

ω_o

ω_π





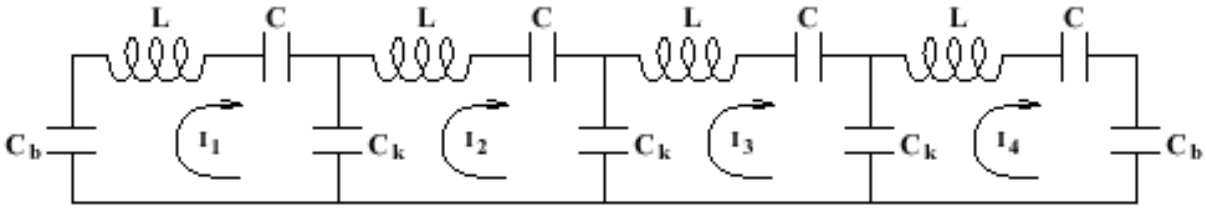
Symmetry plane for the H field

Symmetry plane for the E field which is an additional solution

The normalized difference between these frequencies is a measure of the energy flow via the coupling region

$$k_{cc} = \frac{\omega_\pi - \omega_0}{\frac{\omega_\pi + \omega_0}{2}}$$

Equivalent Circuit



$$k = \frac{C}{C_k} \qquad C_b = C_k / 2$$

Mode frequencies:

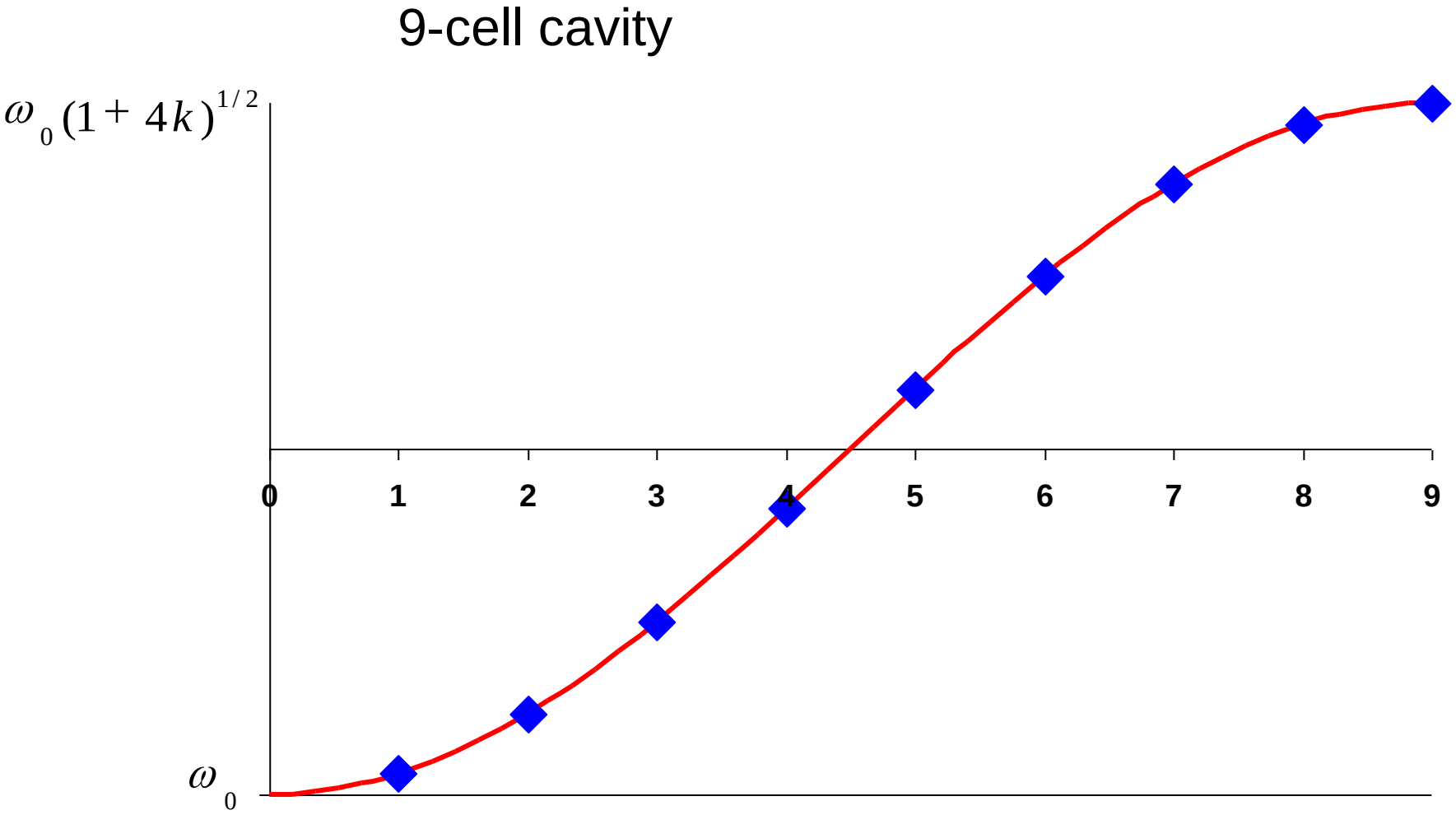
$$\frac{\omega_m^2}{\omega_0^2} = 1 + 2k\left(1 - \cos\frac{\pi m}{N}\right)$$

$$\frac{\omega_N - \omega_{N-1}}{\omega_0} \simeq k\left(1 - \cos\frac{\pi}{N}\right) \simeq \frac{k}{2}\left(\frac{\pi}{N}\right)^2$$

Voltages in cells:

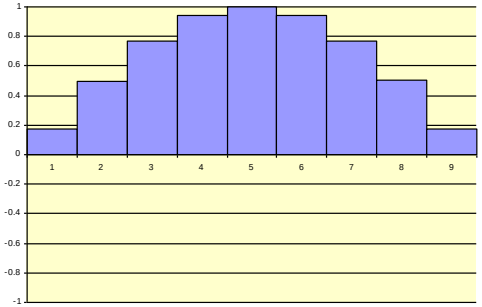
$$V_j^m = \sin\left(\pi m \frac{2j-1}{2N}\right)$$

Pass-Band Modes Frequencies

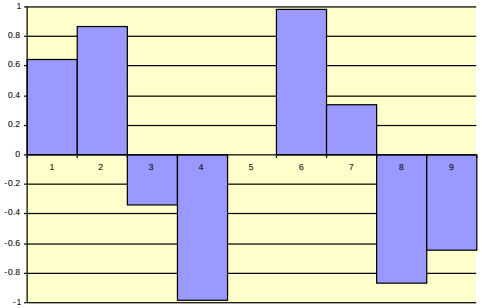


Cell Excitations in Pass-Band Modes

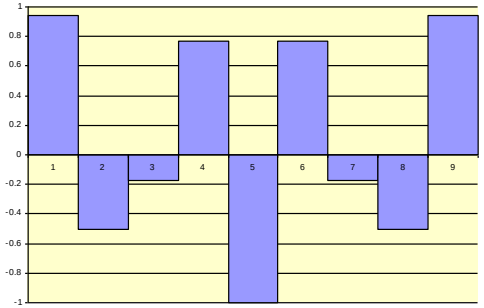
9 Cell, Mode 1



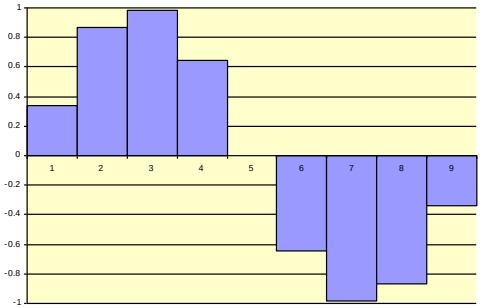
9 Cell, Mode 4



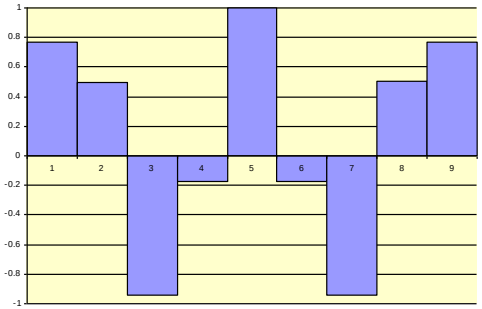
9 Cell, Mode 7



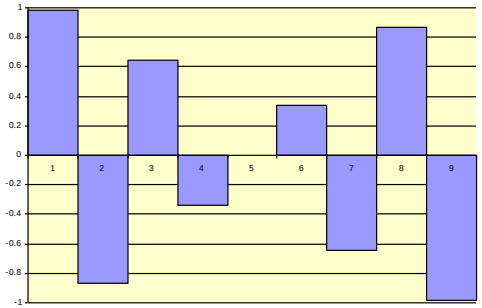
9 Cell, Mode 2



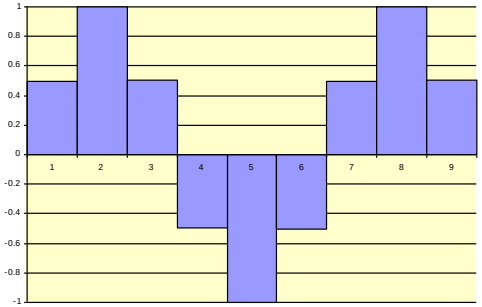
9 Cell, Mode 5



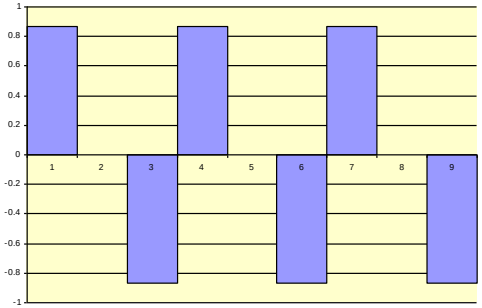
9 Cell, Mode 8



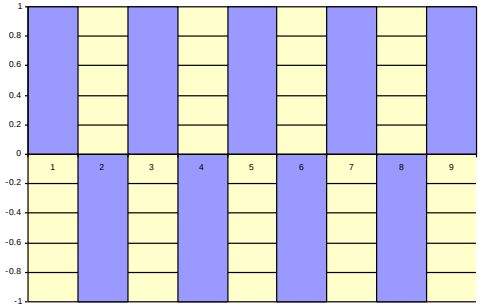
9 Cell, Mode 3



9 Cell, Mode 6



9 Cell, Mode 9

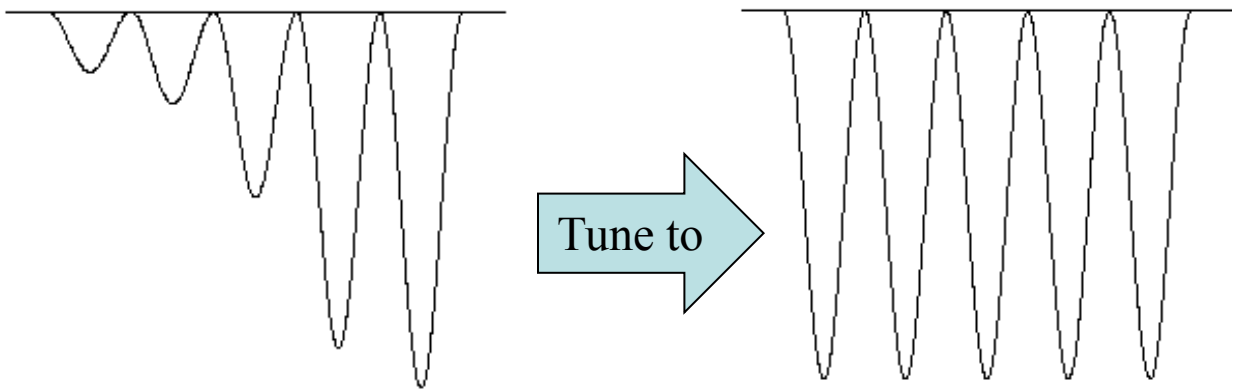


Field Flatness

Geometrical differences between cells causes a mixing of the eigenmodes

Sensitivity to mechanical deformation depends on mode spacing

$$\frac{\omega_N - \omega_{N-1}}{\omega_0} \simeq k\left(1 - \cos\frac{\pi}{N}\right) \simeq \frac{k}{2}\left(\frac{\pi}{N}\right)^2$$



Field flatness factor for elliptical cavities with arbitrary ß=v/c $a_{ff} = \frac{N^2}{k_{cc} \cdot ß}$

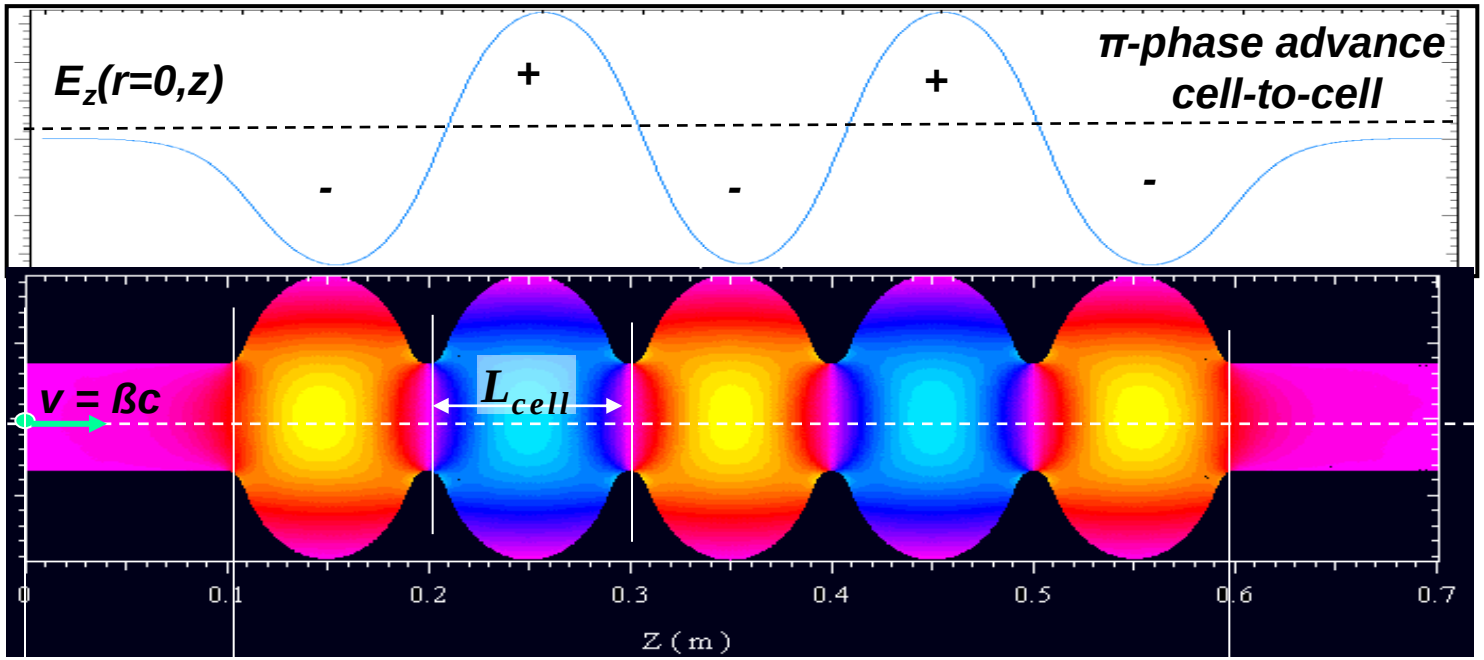
Pros and Cons of Multicells

- Cost of accelerators are lower (less auxiliaries: LHe vessels, tuners, fundamental power couplers, control electronics)
- Higher real-estate gradient (better fill factor)

- Field flatness vs. N
- HOM trapping vs. N
- Power capability of fundamental power couplers vs. N
- Chemical treatment and final preparation become more complicated
- The worst performing cell limits whole multi-cell structure

Beam Acceleration

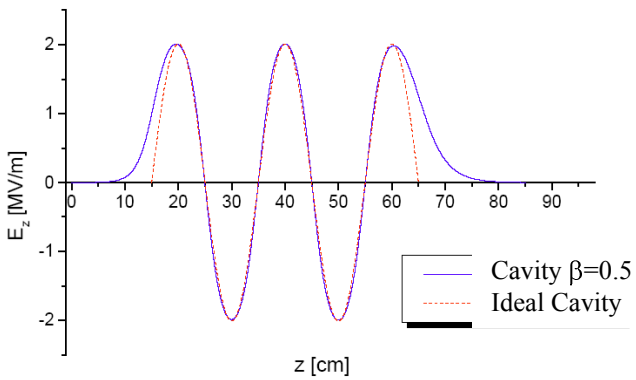
Accelerating mode in a multi-cell structure



s L_{active}

Synchronic acceleration and max of $(R/Q)_{acc} \leftrightarrow L_{active} = NL_{cell} = Nc\beta/(2f)$ and the injection takes place at an optimum phase φ_{opt} which ensures that particles will arrive at the mid-plane of the first cell when E_{acc} reaches its maximum (+q passing to the right) or minimum (-q passing to the right).

β<1 Cavities: Transit Time Factor



LINAC design is made assuming "ideal" cavity with sinusoidal field on-axis

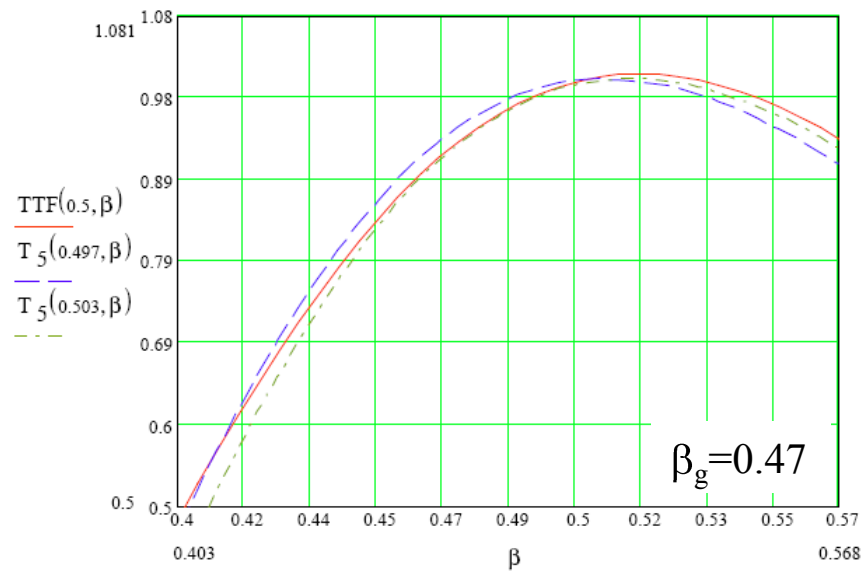
$$E_{z,ideal}(z) = E_0 \sin\left(\frac{2\pi}{\lambda\beta_g} z\right) \qquad \beta_g = \frac{2L}{\lambda}$$

The "real" β, β_c, of a multi-cell cavity is defined as the β_g of an ideal cavity that has an the energy gain curve for the synchronous particle closest to the one of the real cavity

$$\delta W(\beta,\beta_c) = q\int_0^{L_{tot}} E_z(z)\cos\left[\frac{\omega}{\beta c}(z - z_c)\right]dz$$

$$T^{(N)}(\beta,\beta_c) = \frac{\delta W(\beta,\beta_c)}{\delta W(\beta = \beta_c,\beta_c)}$$

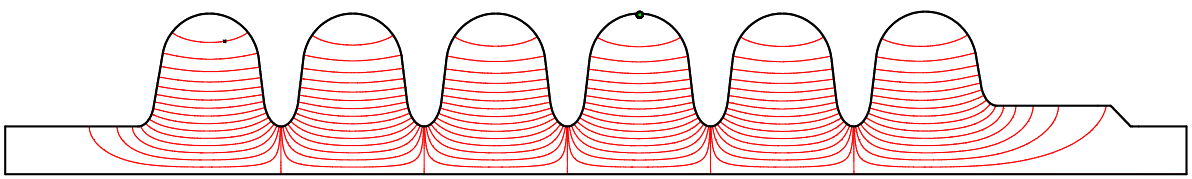
$$\delta W = q\Delta V = qE_{acc}L_{att}T^{(N)}(\beta,\beta_c)\cos\phi_s$$



β_g = 0.81 Cavity for SNS

Effective β that matches the TTF curve = 0.832

E_p/E_{acc}	2.19 (2.14 inner cell)
B_p/E_{acc} [mT/(MV/m)]	4.72 (4.58 inner cell)
R/Q [Ω]	484.8
G [Ω]	233
k [%]	1.52
Q_{BCS} @ 2 K [10^9]	36.2
Frequency [MHz]	805.004
Field Flatness [%]	1.1



KL70 = -0.7 [Hz/(MV/m)^2] KL80 = -0.8 [Hz/(MV/m)^2]

Nb thickness = 3.8 mm-

Geometrical Parameters

	Inner cell	End Cell Left	End Group (coupler) Left	End Group (coupler) Right
L [cm]	**7.55**	**7.55**	**7.55**	
R_{iris} [cm]	**4.88**	**4.88**	**4.88**	**7.0**
D [cm]	**16.415**	**16.415**	**16.611**	
d [cm]	**1.5**	**1.3**	**1.5**	**1.3**
r	**1.8**	**1.6**	**1.8**	**1.6**
R	**1.0**	**1.0**	**1.0**	
α [deg]	**7.0**	**10.072**	**7.0**	**10.0**

Multipacting Simulations

Once the cavity shape has been designed, multipacting simulations have to be done:

- get the fields on the contour
- electrons are launched from given initial sites at given phases of the RF field
- for a fixed field level the electron trajectories are calculated by integrating the equations of motion, until the electrons hit the wall
- record the location, phase, and impact energy
- the number of secondary electrons is determined, given the SEY function
- the trajectory calculation is continued if the field phase is such as secondary electrons leave the wall
- after a given number of impacts N the No. of free electrons and their avg. impact energy and the No. of secondary electrons is calculated

Counter function

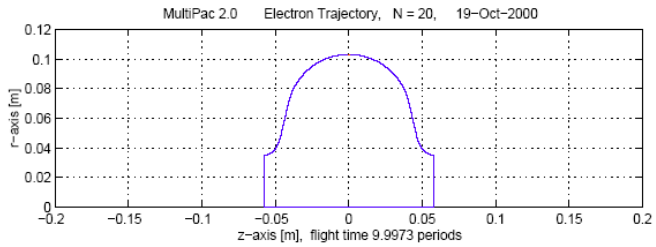
Enhanced counter function

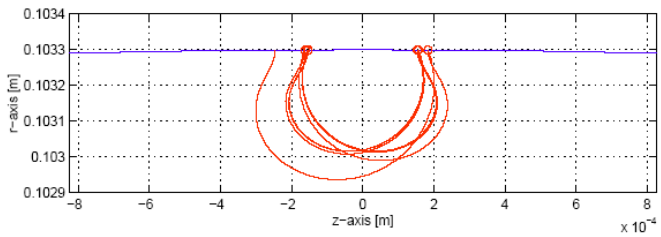
Counter function: field levels at which resonant conditions are satisfied

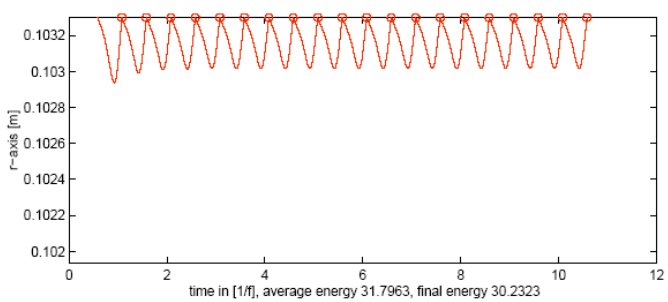
At field levels where Enhanced counter function > No. initial electrons: Multipacting

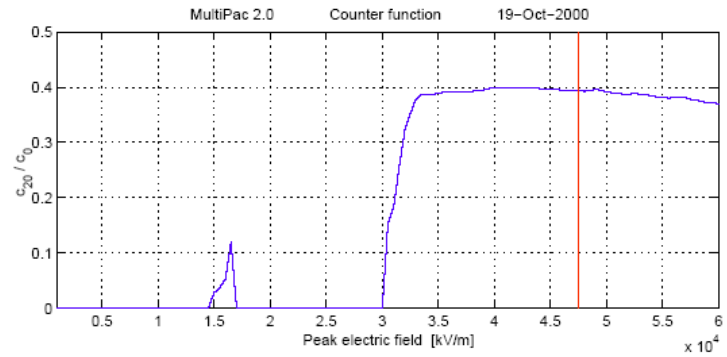
MultiPac

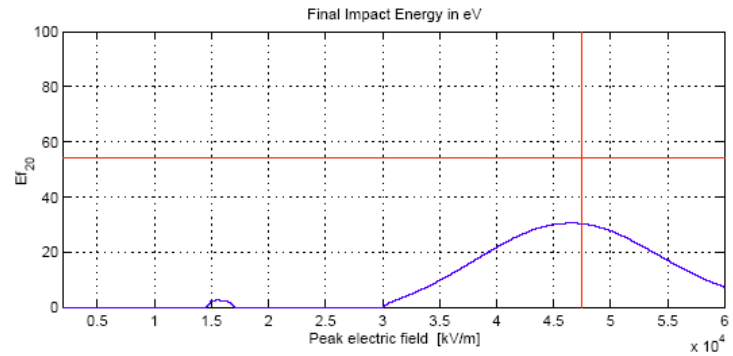
- 2D code, has it's own FEM field solver
- Runs on Linux
- MATLAB user interface

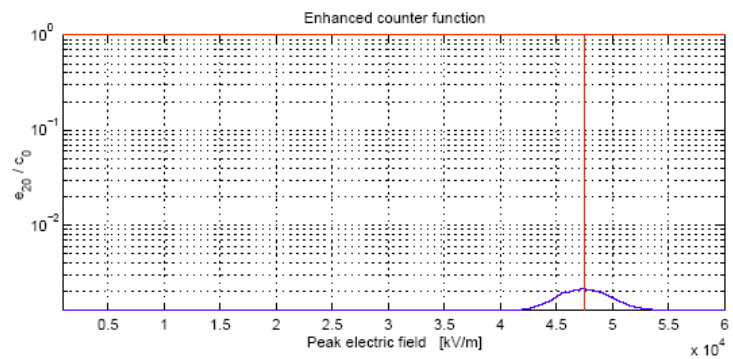






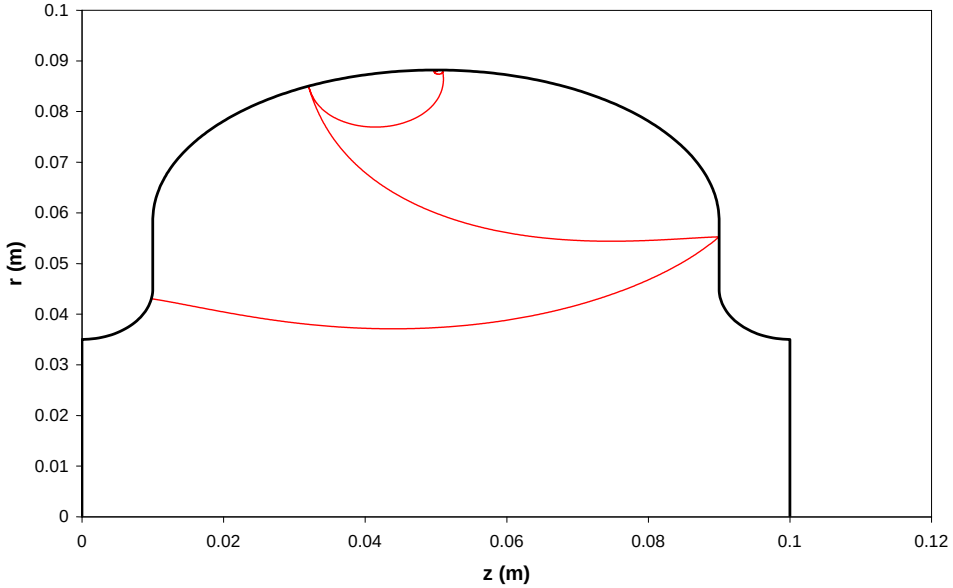


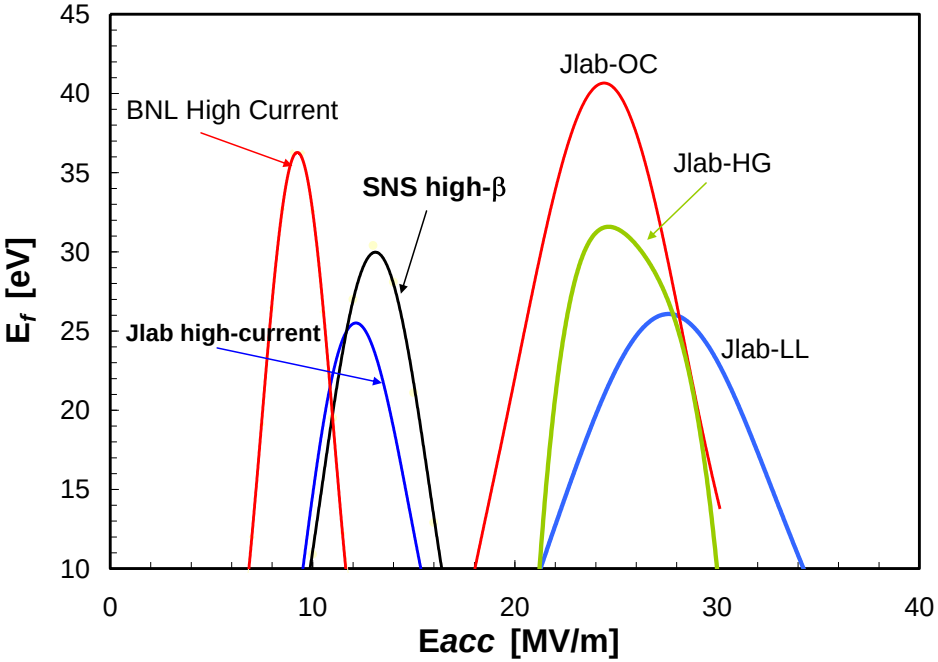




FishPact

- 2D code, uses SUPERFISH to compute surface fields
- Runs on PC

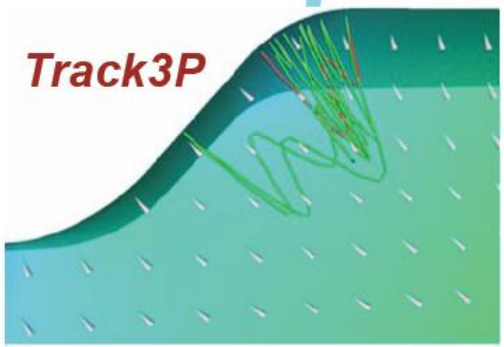


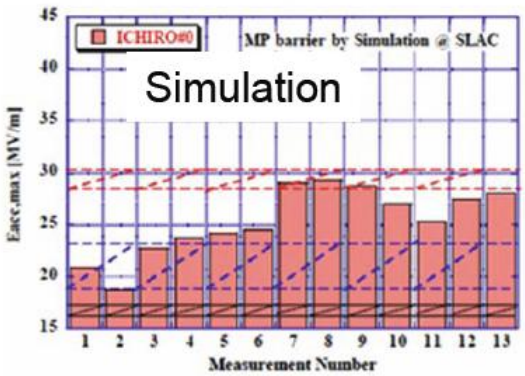


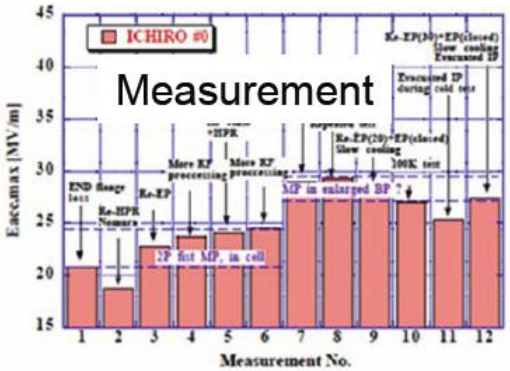
Track3P

- 3D code, uses Omega3P for field solver
- Runs on Supercomputer, user interface not fully developed

Example: Multipacting found in beam pipe step of LL cavity

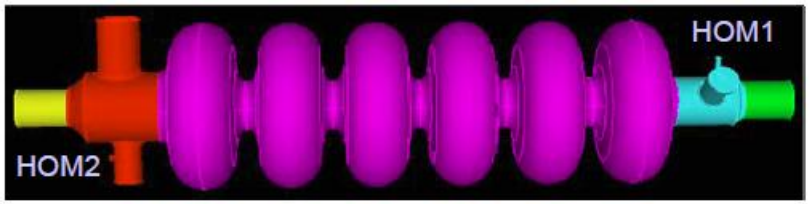




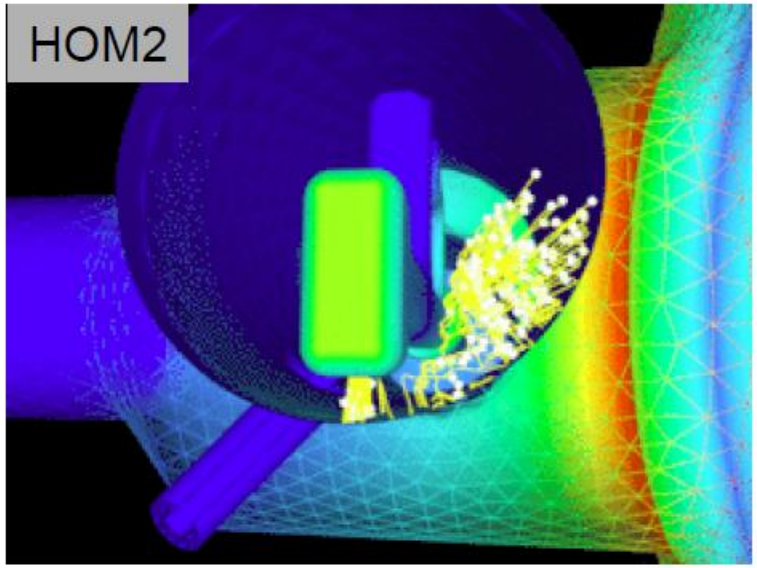


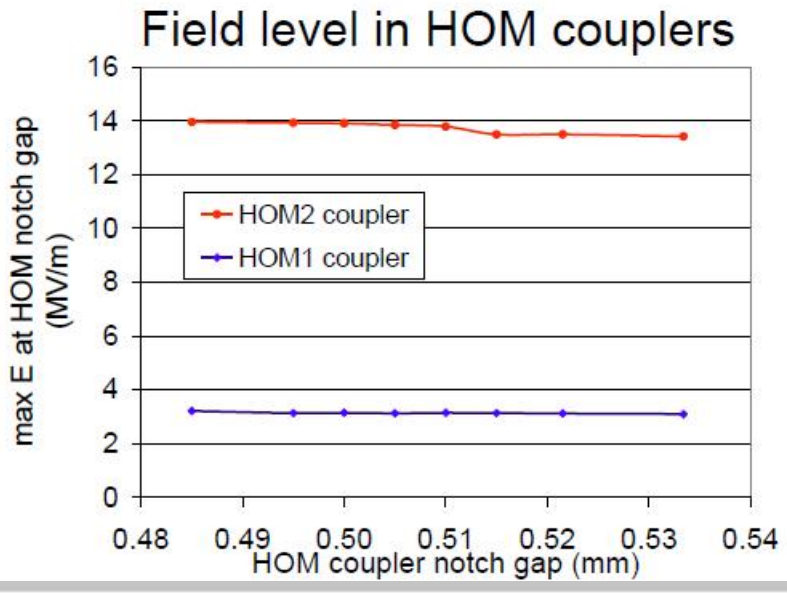
(Left) MP barriers in 9-cell ICHIRO cavity calculated with Track3P, (Right) MP barriers measured on ICHIRO prototype (K.

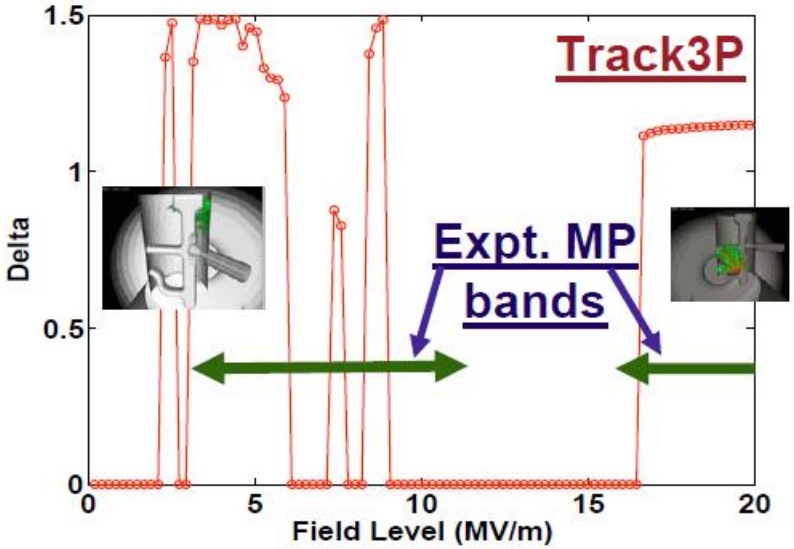
Example: Multipacting in SNS HOM Coupler



- SNS SCRF cavity experienced RF heating at HOM coupler
- 3D MP simulations showed MP barriers closed to measurements
- Similar analysis are carried out for ILC ICHIRO and crab cavity





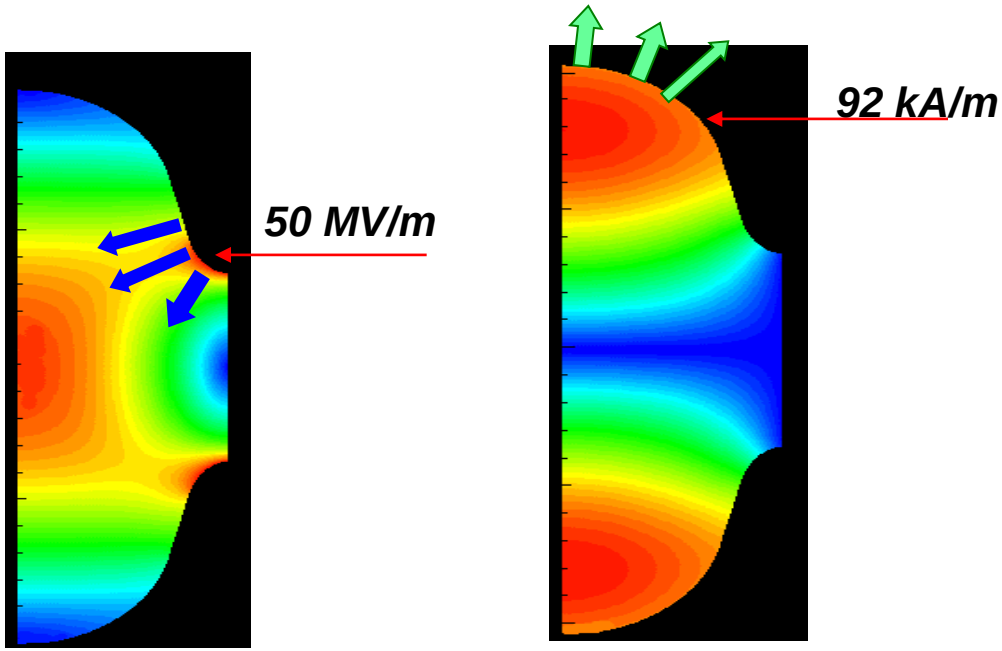


Mechanical Analysis

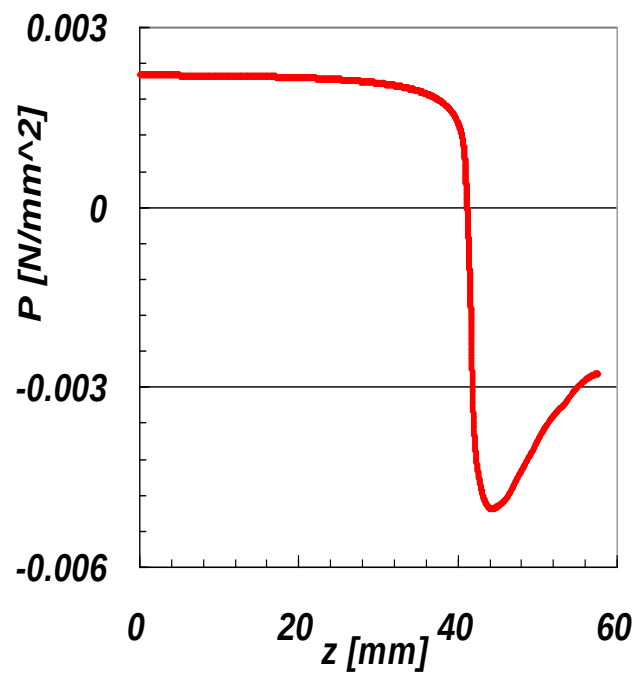
The mechanical design of a cavity follows its RF design:

- Lorentz Force Detuning
- Mechanical Resonances
- Structural stability under different load conditions

Lorentz Force Detuning



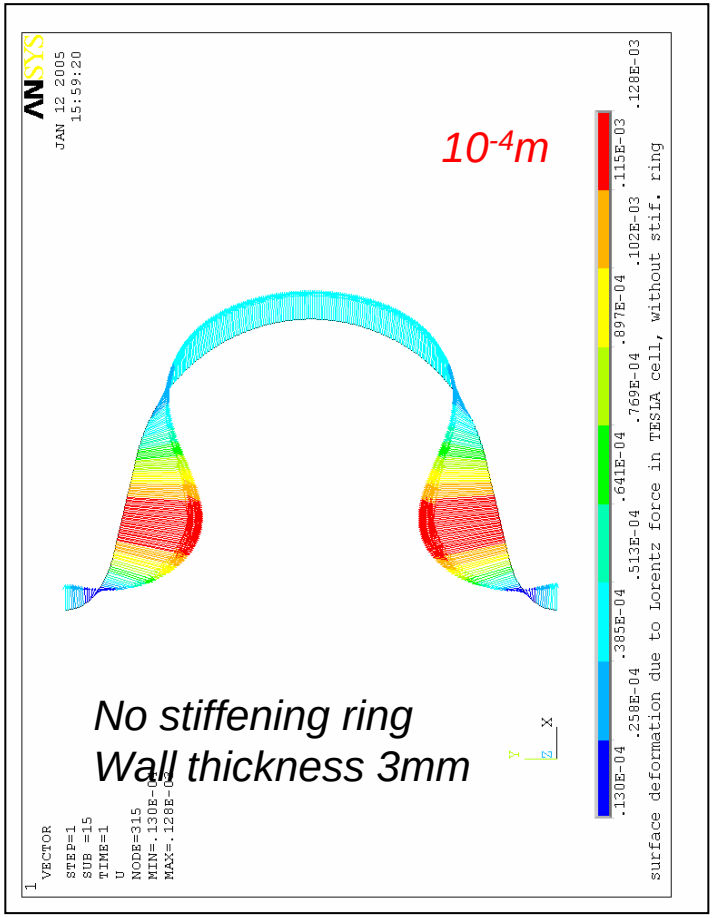
$$P = \frac{\mu_0 H_s^2 - \varepsilon_0 E_s^2}{4}$$

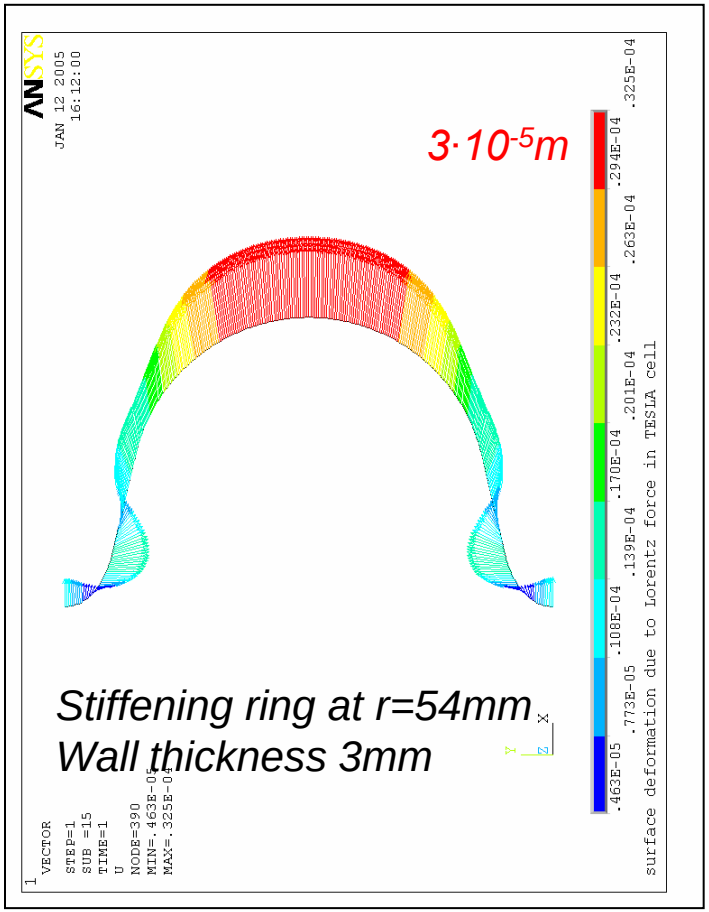


E and H at E_{acc} = 25 MV/m in TESLA inner-cup

Stiffening

Surface deformation without and with stiffening ring



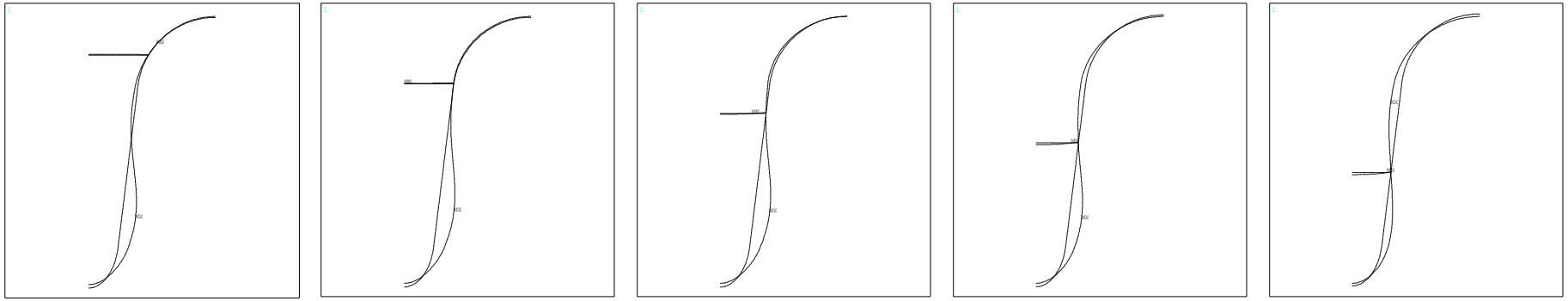


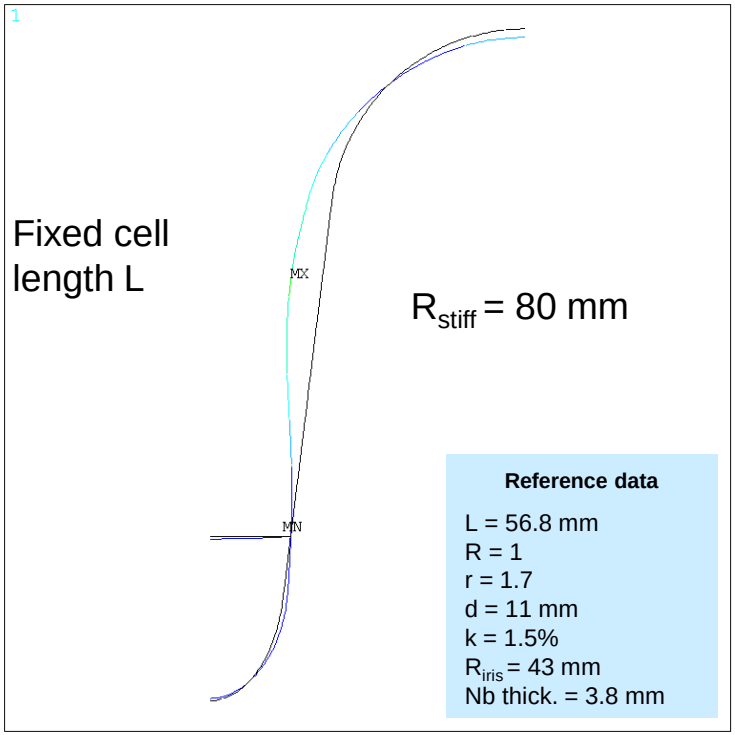
Essential for the operation of a pulsed accelerator

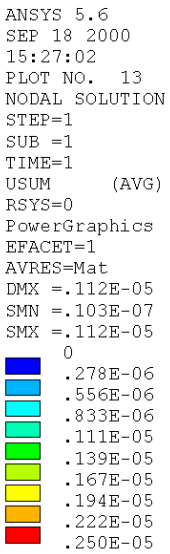
$\Delta f = k_L (E_{acc})^2$

$k_L = -1\ Hz/(MV/m)^2$

Optimal stiffening ring position

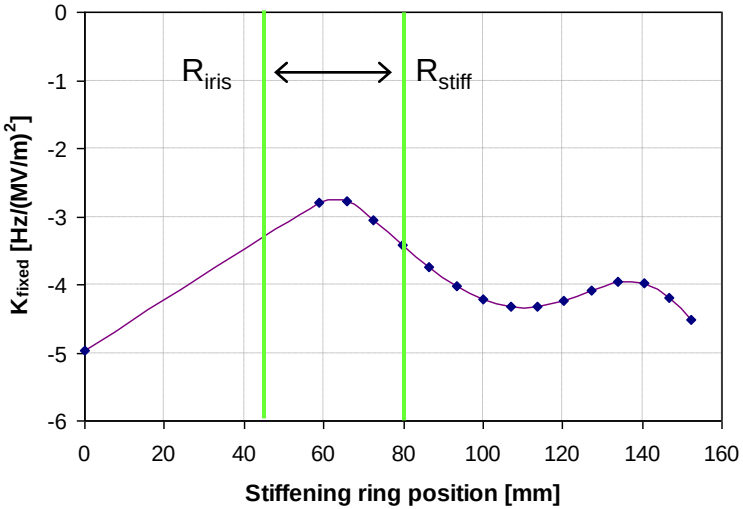






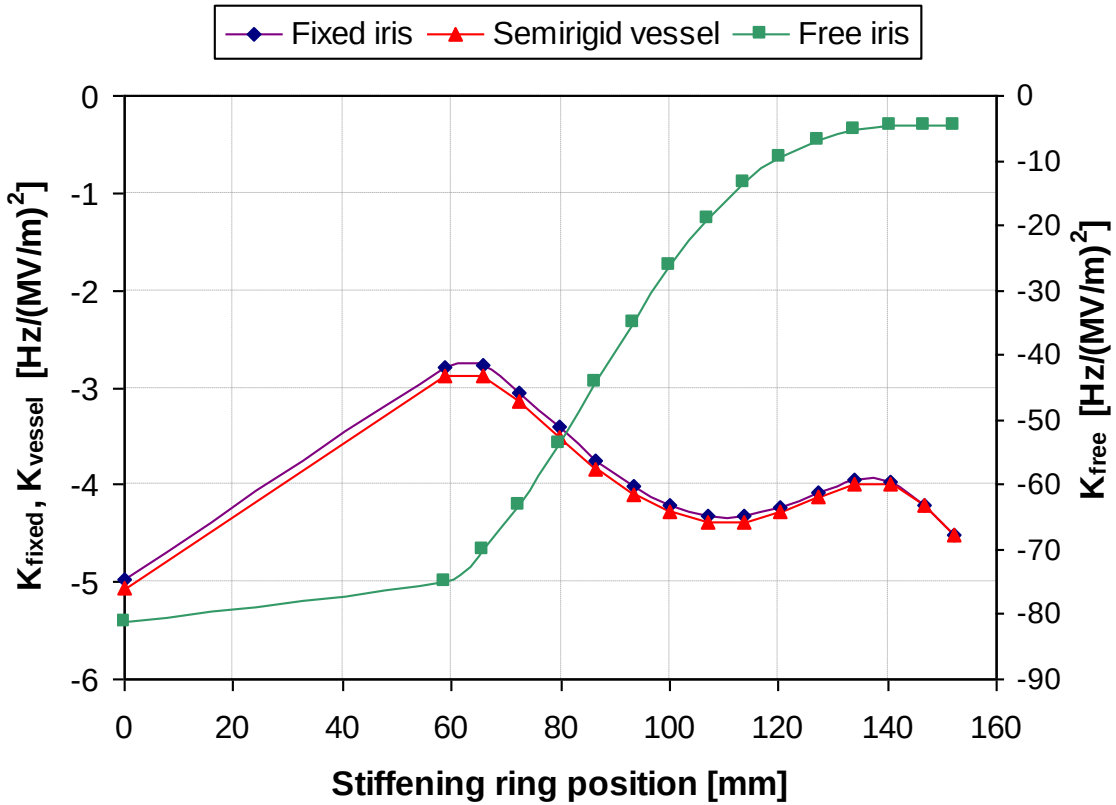
Displacements [mm]

The Lorentz forces coefficients for 15 different stiffening ring positions are evaluated automatically with ANSYS, preparing the geometry and reading the fields from the SFO output from SUPERFISH



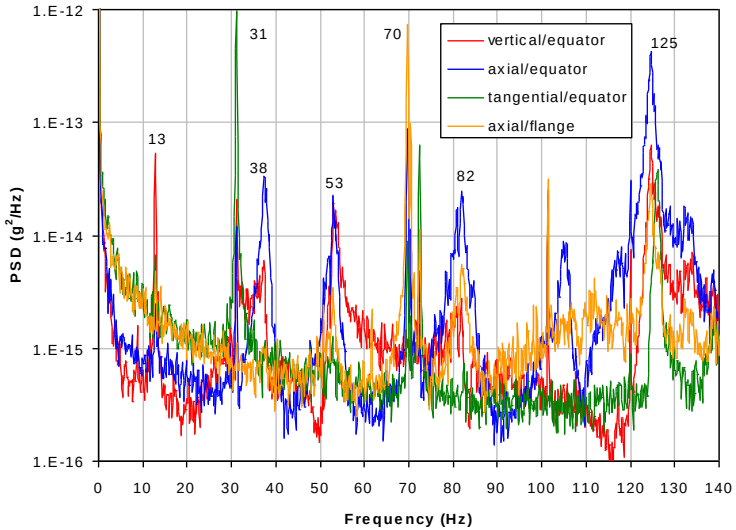
K_L for different boundary conditions

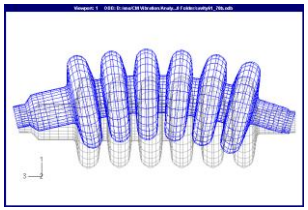
- The estimate for K_L strongly depends on the cell boundaries. We compute it for 3 different cases:
  - Fixed cell length
  - Free cell length
  - Helium Vessel/Tuning System (= 3 tubes with diameter 30 mm and thickness 2 mm)



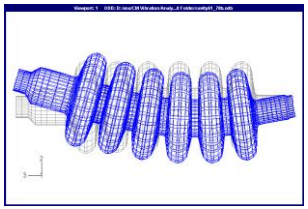
Mode Analysis

Calculate mechanical resonances of a multi-cell cavity as they modulate frequency of the accelerating mode. Sources of their excitation: vacuum pumps, ground vibrations…

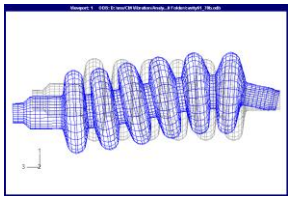




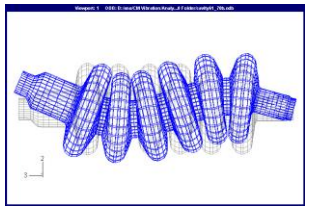
Mode 1 – 14 Hz



Mode 2 – 26 Hz



Mode 3 – 40 Hz



Mode 5 – 72 Hz

	Natural Frequency (Hz)	
Mode	**Test Data**	**FE Analysis**
1	13	14
2	31	26
3	38	40
4	53	48
5	70	72
6	82	83
7	125	124

Typical Cavity Mechanical Design Requirements

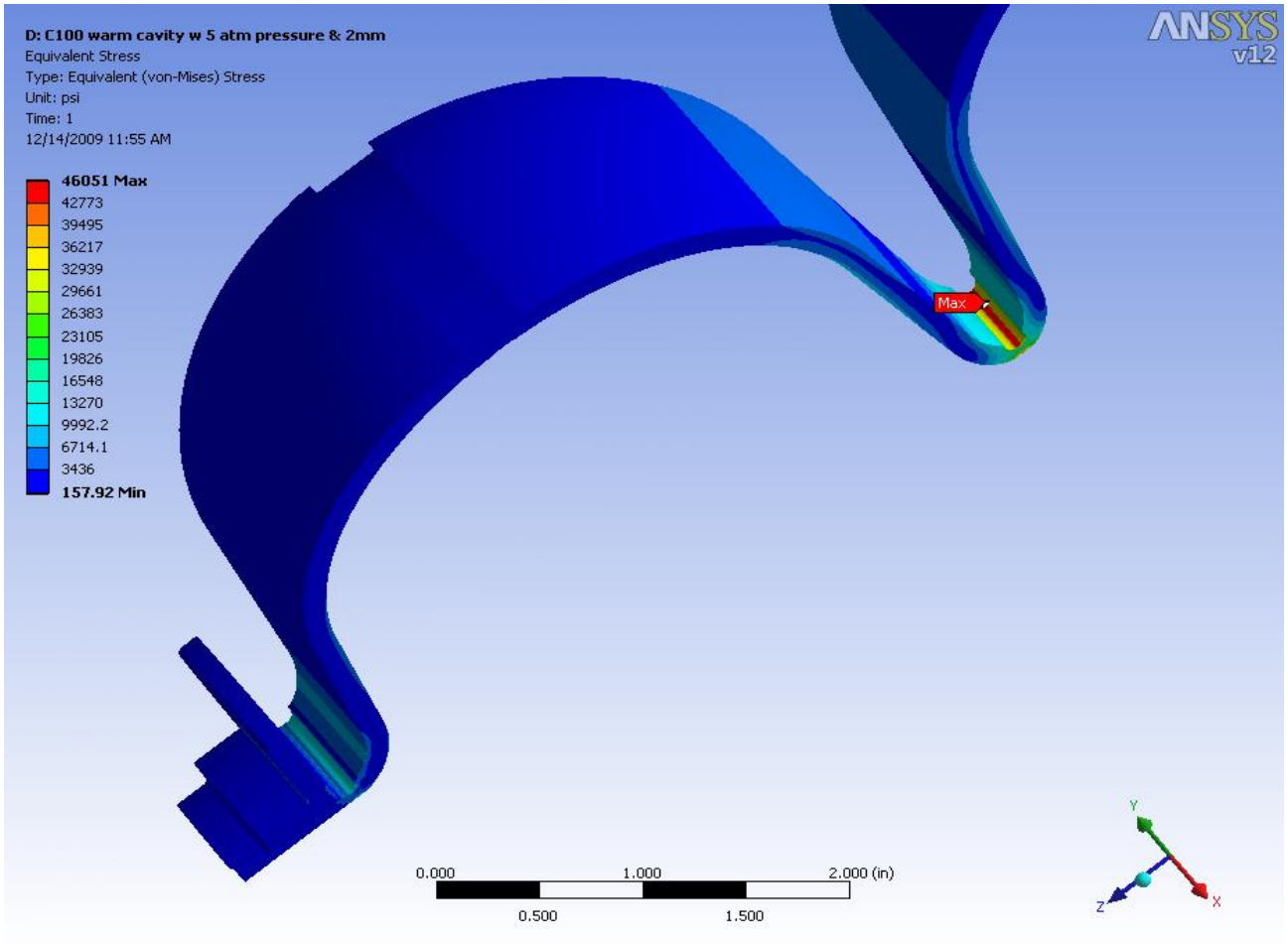
- Minimize/prevent microphonics
- Withstand loss of vacuum accident up to 5 atm
- Withstand cool down at 1.65 atm
- Adhere to intent of ASME B&P Code
  - Allowable Stress (Sm) = 2/3 Yield Stress
  - Primary Membrane Stress (Pm) <= (Sm)
  - Pm + Bending <= 1.5*Sm
  - Pm + Bending + Secondary Stress <= 3*Sm
    - Allowable Stresses

      »Warm Niobium = 4,667 psi

      »Cold Niobium = 53,333 psi

Mechanical analysis tools

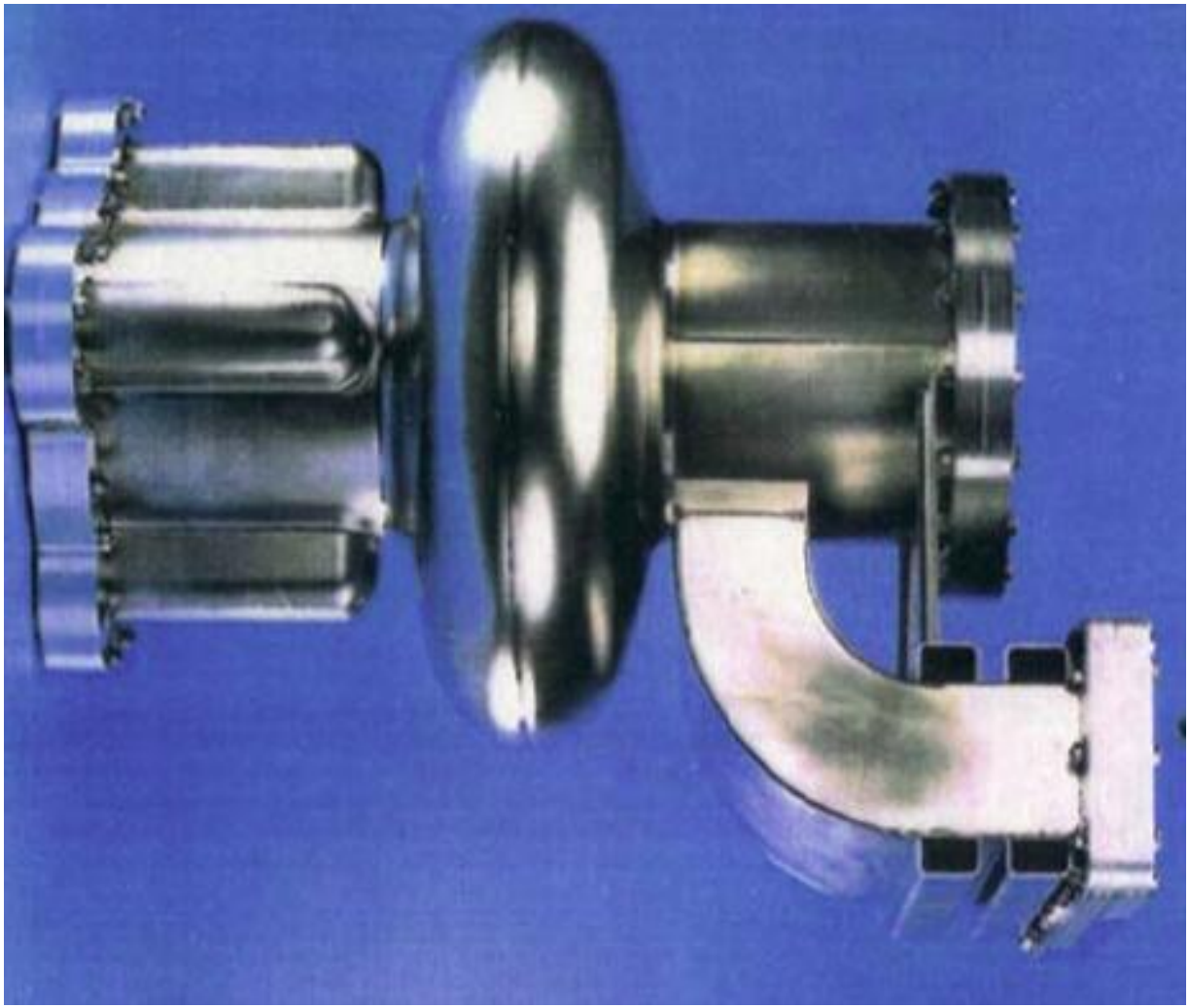
- ANSYS: FEM multiphysics solver



Peak von Mises stress in cold cavity with 5 atm pressure and 2 mm tuning displacement, calculated on CEBAF LL Upgrade cavity

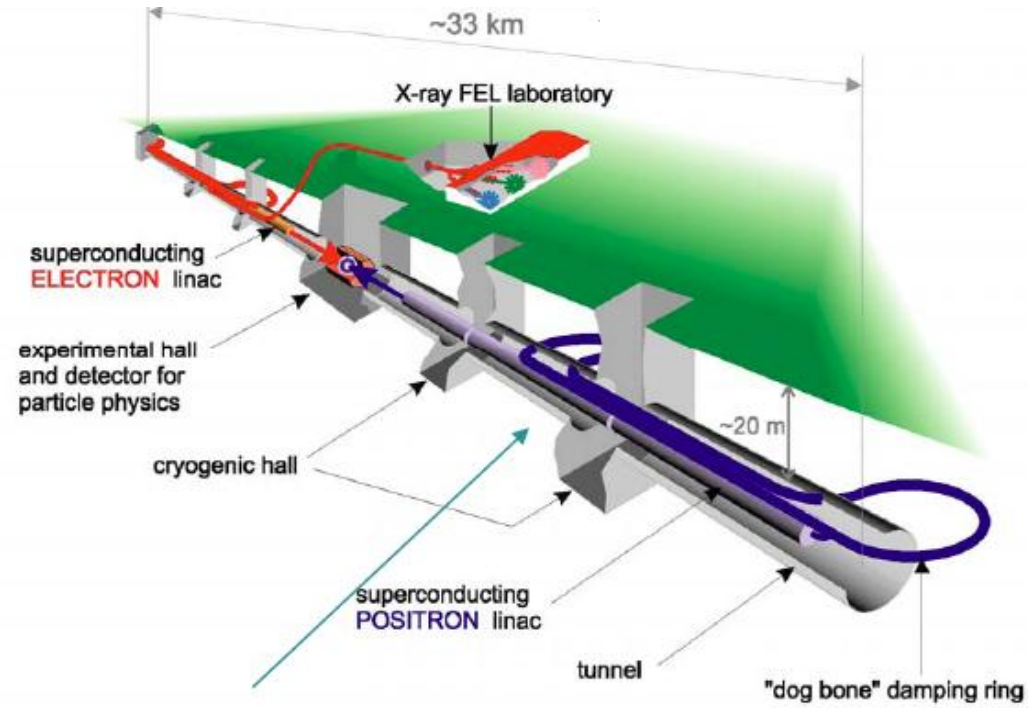
Cavities, large and small…

500 MHz, Single-cell

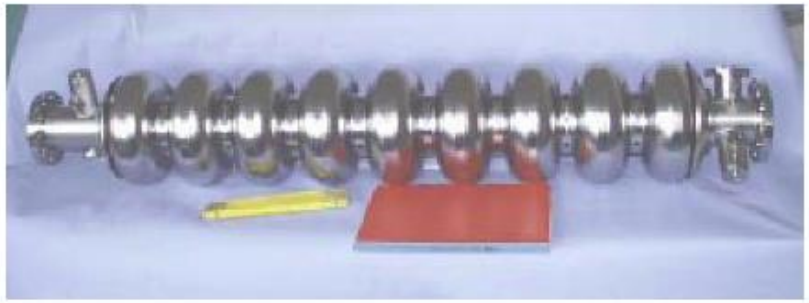


Pulsed LINACs (ILC, XFEL)

- High gradient (≥ 25 MV/m)
- Moderate HOM damping ($Q_{ext} = 10^4 – 10^5$)
- High peak (> 250 kW), low average RF power (~ 5 kW)

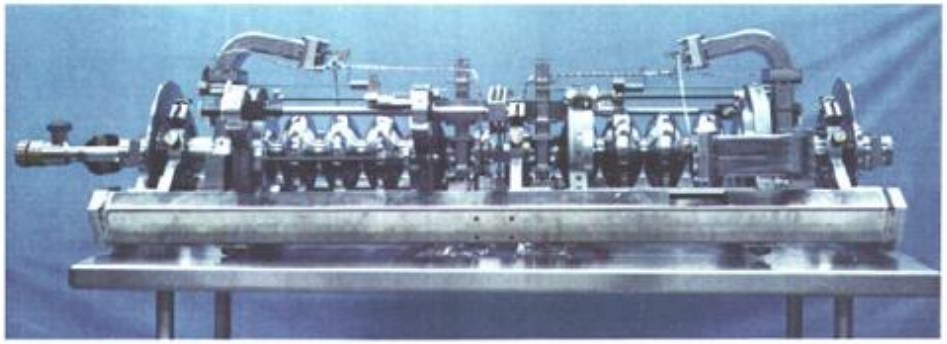


ILC: 21,000 cavities!

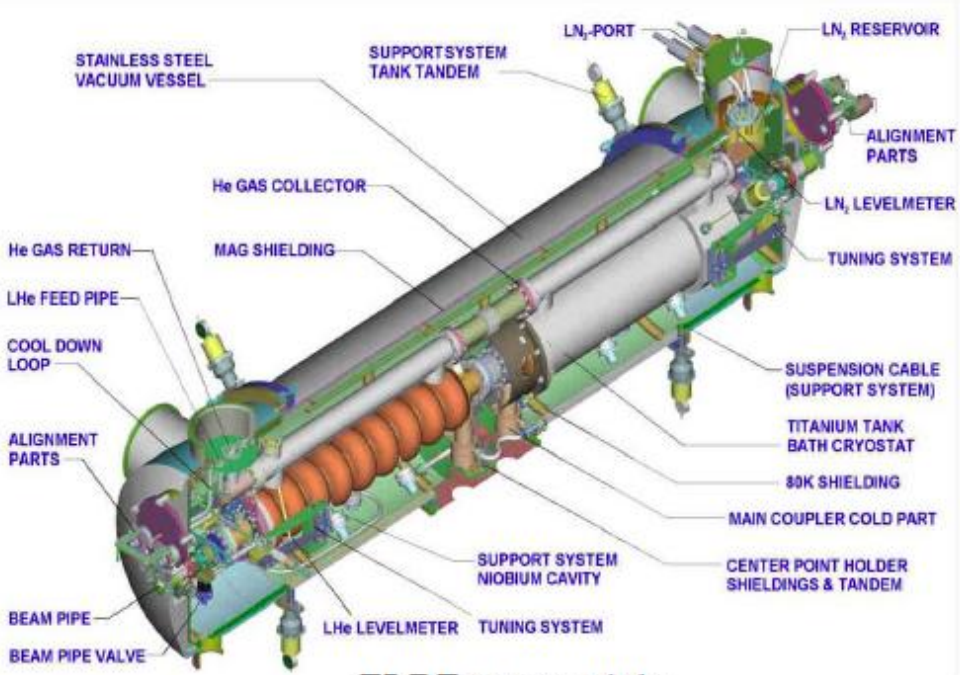


CW Low-Current LINACs (CEBAF, ELBE)

- Moderate to low (8 – 20 MV/m)
- Relaxed HOM damping requirements
- Low average RF power (5 – 13 kW)



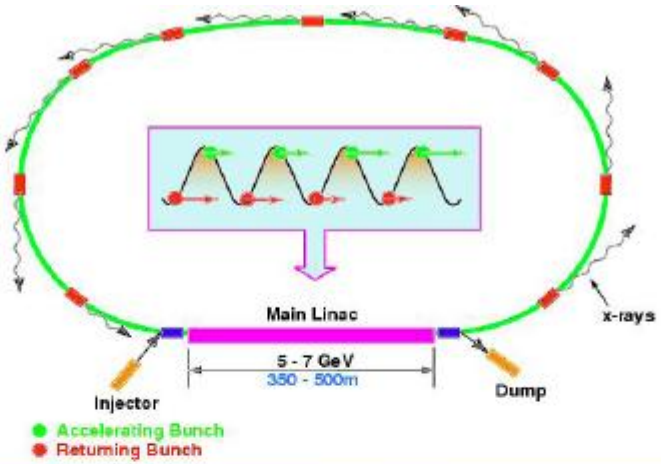
CEBAF cavities

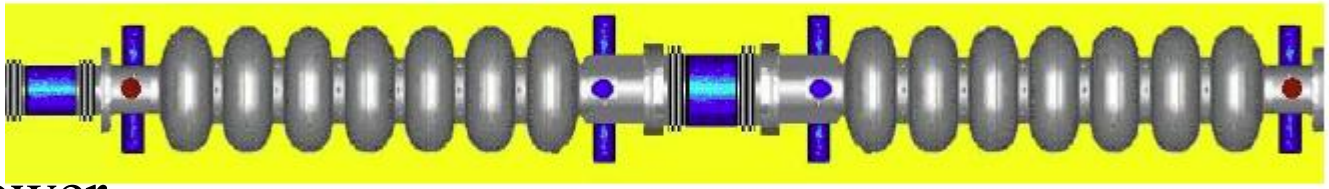


ELBE cryomodule

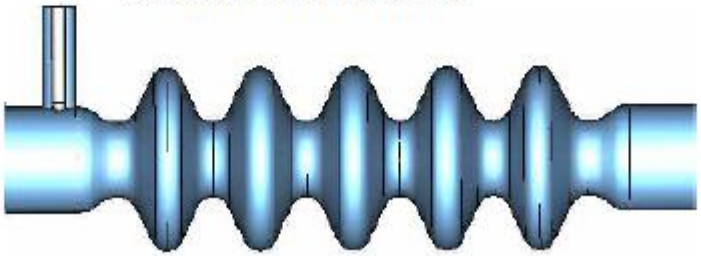
CW High-Current ERLs

- Moderate gradient (≥ 15 - 20 MV/m)
- Strong HOM damping ($Q_{ext} = 10^2 – 10^4$)
- Low average RF power (few kW)



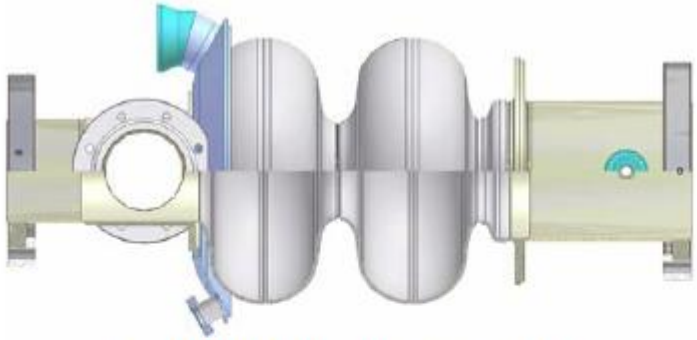


Cornell ERL cavities



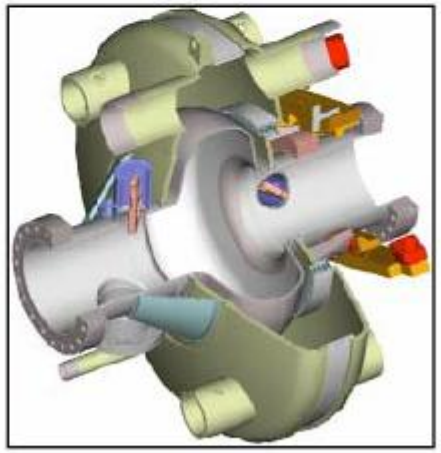
BNL ERL cavity

CW High-Current Injectors for ERLs



Cornell ERL injector cavity

- Moderate to low gradient (5 - 15 MV/m)
- Strong HOM damping ($Q_{ext} = 10^2 – 10^4$)
- High average RF power (50 - 500 kW)



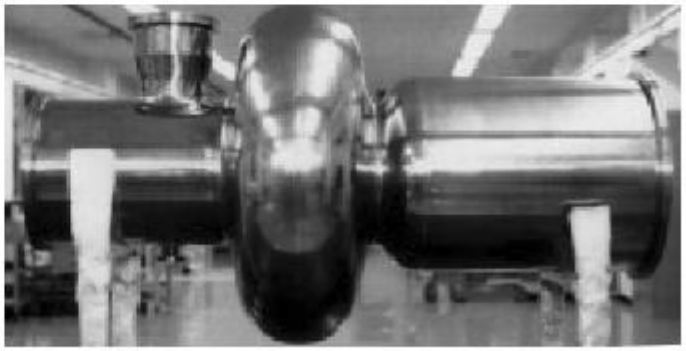
JLab FEL 100 mA injector cavity

CW High-Current Storage Rings

- Relatively low gradient (5 - 9 MV/m)
- Strong HOM damping ($Q_{ext} \sim 10^2$)
- High average RF power (up to 390 kW)

CESR cavities

LHC cavity



KEK cavity

Summary

Cavity DESIGN: be creative

and have fun with it!