Imaging a Beam with Synchrotron Radiation

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Beam Diagnostics Using Synchrotron Radiation:
Theory and Practice
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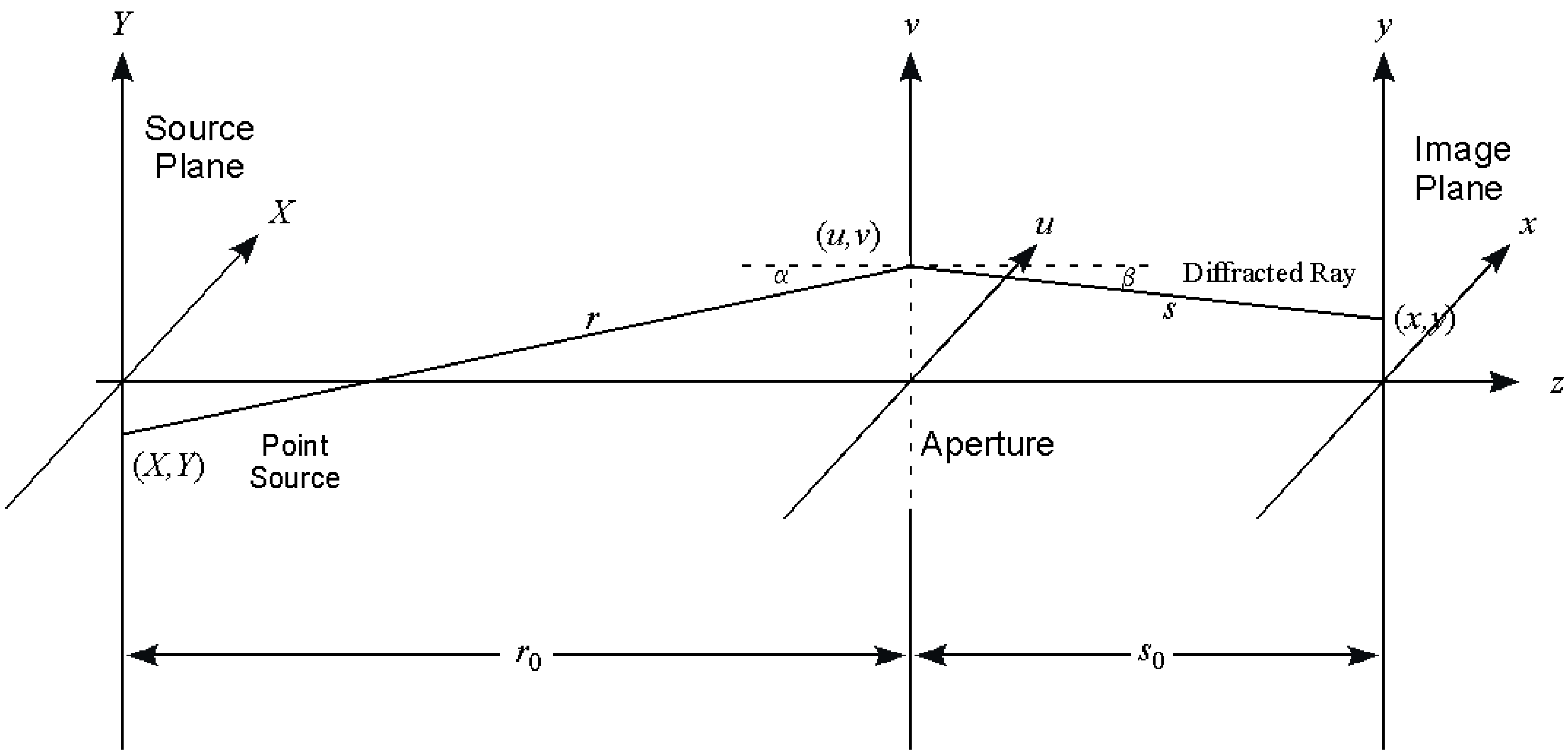
Diffraction

- Diffraction limits the resolution at long wavelengths.
  - An important consideration when the beam is small (usually in *y*).
    - Image a point near a defocusing quad, where the beam is largest vertically.
  - A cold finger or slot also adds diffraction.
- Small beams drive the design toward shorter wavelengths.
  - Blue rather than red, but often ultraviolet or x rays.
  - More about using these wavelengths later.

Diffraction by an Aperture



Diffraction by an Aperture

- All points in an aperture are considered point sources, reradiating light incident from a point source at (X,Y)
  - Wavelength is $\lambda = 2\pi/k$.
- The field at (x,y) is given by a Fresnel-Kirchhoff integral over the (small) aperture:

$$E(x,y) = -\frac{Ai}{2\lambda}\iint_{\text{aperture}} \frac{e^{ik(r+s)}}{rs}(\cos\alpha + \cos\beta)\,dS$$

$$\approx -\frac{Ai}{2\lambda r_0 s_0}(\cos\alpha + \cos\beta)\iint_{\text{aperture}} e^{ik(r+s)}\,dS$$

  - Everything is essentially constant except the phase from each point in the aperture.

Expanding the Phase

$$r = \sqrt{(X-u)^2 + (Y-v)^2 + r_0^2} \approx r_0 + \frac{(X-u)^2 + (Y-v)^2}{2r_0}$$

$$s = \sqrt{(x-u)^2 + (y-v)^2 + s_0^2} \approx s_0 + \frac{(x-u)^2 + (y-v)^2}{2s_0}$$

$$e^{ik(r+s)} \approx \exp\left[ik\left(r_0 + s_0 + \frac{X^2+Y^2}{2r_0} + \frac{x^2+y^2}{2s_0}\right)\right]\exp\left[ik\left(\frac{u^2+v^2}{2r_0} + \frac{u^2+v^2}{2s_0}\right)\right]\exp\left[-ik\left(\frac{Xu+Yv}{r_0} + \frac{xu+yv}{s_0}\right)\right]$$

- First factor: Independent of the aperture coordinates *u*, *v*.
  - Contributes only an overall phase to the *uv* integral over the aperture.
- Second: Quadratic in *u* and *v*. Neglegible since the aperture is small.
- Third: Products of *u*, *v* with cosines (X/r_0, etc.) of the ray angles from the source or measurement points to the horizontal and vertical axes.
- The only factor that matters in the integral over the aperture is:

$$e^{ik(r+s)} \approx \exp\left[-ik\left(pu+qv\right)\right]$$

where $p = X/r_0 + x/s_0$ and $q = Y/r_0 + y/s_0$

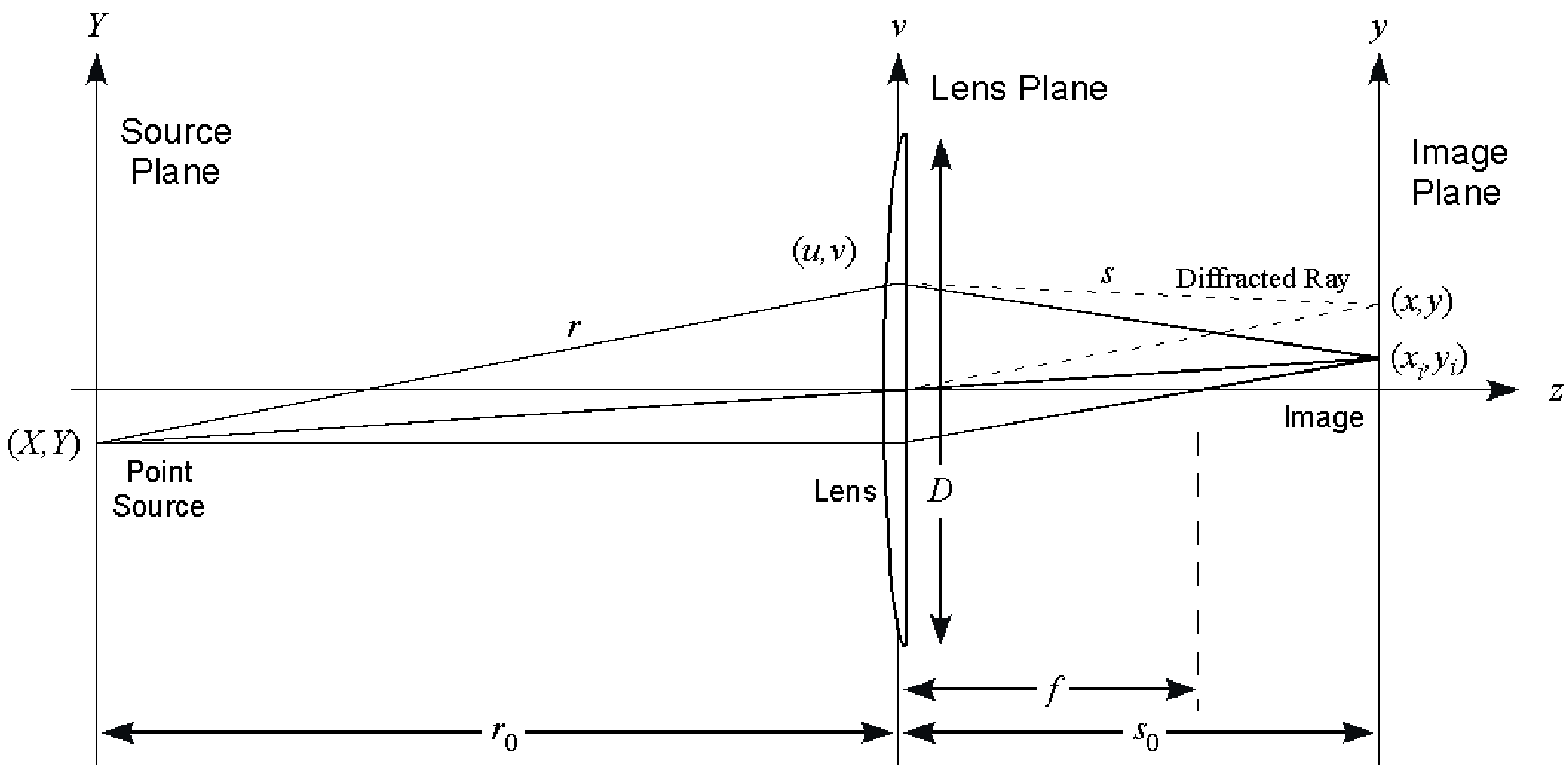
Spatial Fourier Transform

- The diffraction pattern on the *xy* plane becomes a **Fourier transform** in the spatial coordinates *uv* of the aperture:

$$E(x,y) = -\frac{Ai}{2\lambda r_0 s_0}(\cos\alpha + \cos\beta)\iint_{\text{aperture}} e^{-ik(pu+qv)}\,du\,dv$$

- One example of this principle is a *spatial filter:*
  - Laser light is sometimes focused through a small hole to remove noisy, non-Gaussian parts of the beam’s transverse profile.
  - Since the noise is found at high *spatial frequencies*, which appear at larger values of *u* and *v*, it can be clipped by a properly sized hole, which acts as a spatial filter.

Diffraction by a Lens



Diffraction by a Lens: Path Length

- The length S of each optical path from source (X,Y) to image (x_i,y_i) is *equal*.

$$S = \int_{(X,Y)}^{(x_i,y_i)} n(s)\,ds$$

  - The integral along each path element ds is scaled by the index of refraction n.
  - This is a fundamental property of geometric imaging.
- The phase difference in the uv integral arises from the different paths from (X,Y) to (x,y), compared to the equal paths from (X,Y) to (x_i,y_i).
  - It is helpful to subtract this reference path, so that the phase difference becomes the difference between (u,v) to (x,y) and (u,v) to (x_i,y_i).

$$\sqrt{(x-u)^2+(y-v)^2+s_0^2}-\sqrt{(x_i-u)^2+(y_i-v)^2+s_0^2}$$
$$\approx -\frac{(x-x_i)u+(y-y_i)v}{s_0} = -\frac{\rho w}{s_0}\cos(\phi-\psi)$$

  - Here we used polar coordinates: $(u,v) \to (w,\psi)$ and $(x - x_i\,, y - y_i) \to (\rho,\phi)$

Diffraction by a Lens: Result

- The diffraction integral (neglecting constants) becomes:

$$E(x,y) = \int_0^{2\pi}\int_0^{D/2} \exp\left[-ik\frac{\rho w}{s_0}\cos(\phi-\psi)\right] w\,dw\,d\psi$$

$$= 2\pi\int_0^{D/2} J_0\left(\frac{k\rho w}{s_0}\right) w\,dw = \left(\frac{\pi D^2}{4}\right)\frac{2J_1\left(\frac{k\rho D}{2s_0}\right)}{\frac{k\rho D}{2s_0}}$$

where we have used two Bessel-function identities.

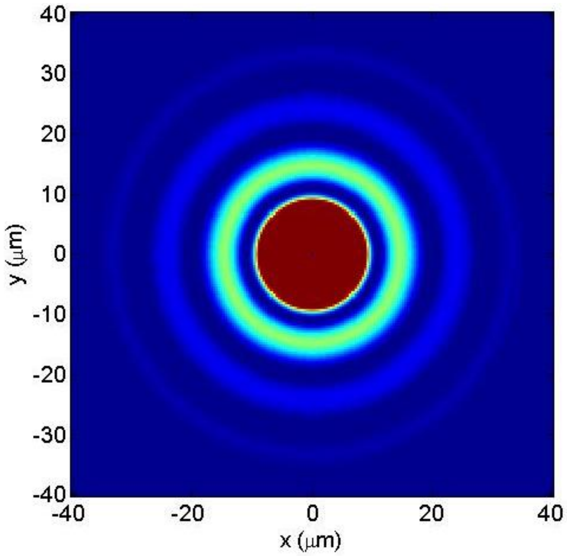
- This is called the Airy diffraction pattern.

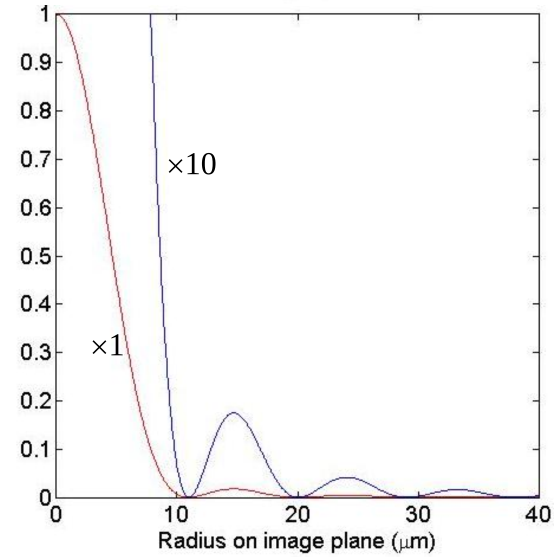
Diffraction by a Lens: Airy Pattern

- Concentric circles, with the first minimum at radius r_A:

$$r_A = 1.22\frac{s_0}{D}\lambda = 0.61\frac{\lambda}{\theta} \approx 1.22\frac{f}{D}\lambda = 1.22F\lambda$$

- θ is the half angle of the light cone exiting the lens.
- F is called the "F-number" of the lens.
- We plot this pattern for λ = 450 nm, D = 50 mm, and s_0 = 1 m
  - Top right: The central circle is saturated by a factor of 30 to highlight the faint rings.
  - Bottom right: The blue curve is multiplied by 10 to highlight the rings.
- r_A is the resolution of the imaging system.
  - Compare it to the size of the geometric image to see if diffraction is a problem.





Diffraction of Dipole Radiation

- For the half angle θ, substitute the Gaussian approximation for dipole radiation given earlier:

$$r_d \approx 0.61\frac{\lambda}{\theta} = \frac{0.61\lambda}{0.60\gamma^{0.062}\left(\lambda/\rho\right)^{0.354}} \approx \rho^{\frac{1}{3}}\lambda^{\frac{2}{3}}$$

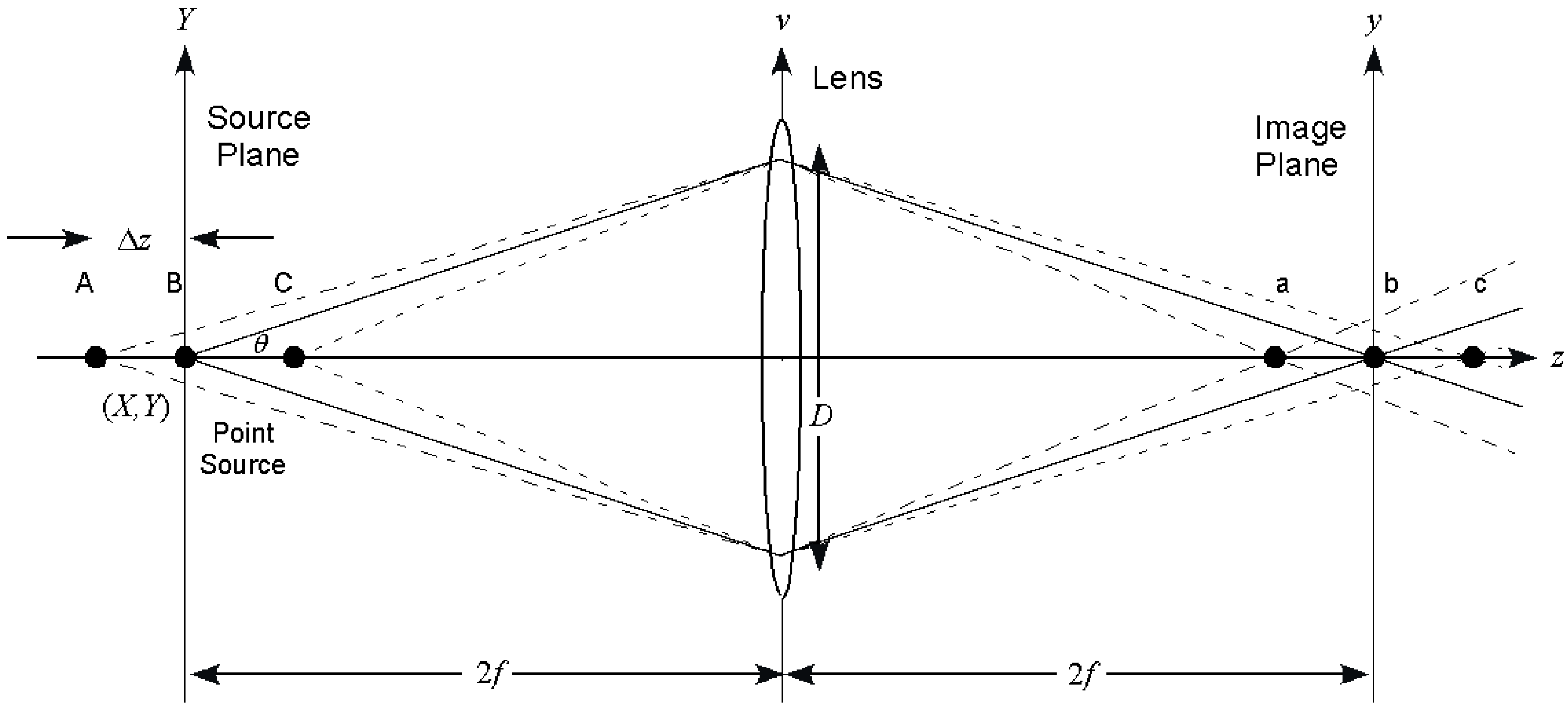
  - Short wavelengths: In the visible, choose blue at 400 nm (or use UV or x rays).
  - Large opening angles: In the LHC at high energy, edge radiation is too narrow.
  - A difficult case: The HER of PEP-II has ρ = 165 m. At 400 nm, r_d = 0.25 mm.
- More thoroughly, use the SR power spectral density from a point source in a Fraunhofer diffraction integral over the area of the lens illuminated through the beamline aperture, to find the field at (x', y') on the image:

$$E(x',y') = A\int_{-x_a}^{x_a} dx \int_{-y_a}^{y_a} dy \frac{\gamma P_s}{\omega_c} F_s\left(\omega,\psi\right) e^{-ik(ux+vy)}$$

  - The first minimum of the intensity then gives the resolution.
- Optics software like Zemax does (monochromatic) diffraction calculations.

Depth of Field

- A dipole emits light along a gradual arc, not from a single plane.
  - What is the source distance?
  - Can it all be in focus?
  - How do you avoid blurring the measurement?



Depth of Field: A Quick Derivation

- Diameters of A and C images as they cross the *xy* plane, based on typical rays at angles $\pm\theta/2$:

$$d = 2\left|\frac{D/4}{2f \mp \Delta z}(\pm\Delta z)\right| \approx \frac{D\Delta z}{4f} = \theta\Delta z$$

- The vertical angle θ lighting the lens is roughly $2\sigma_\lambda$.
- If we capture a similar portion of a horizontal arc:

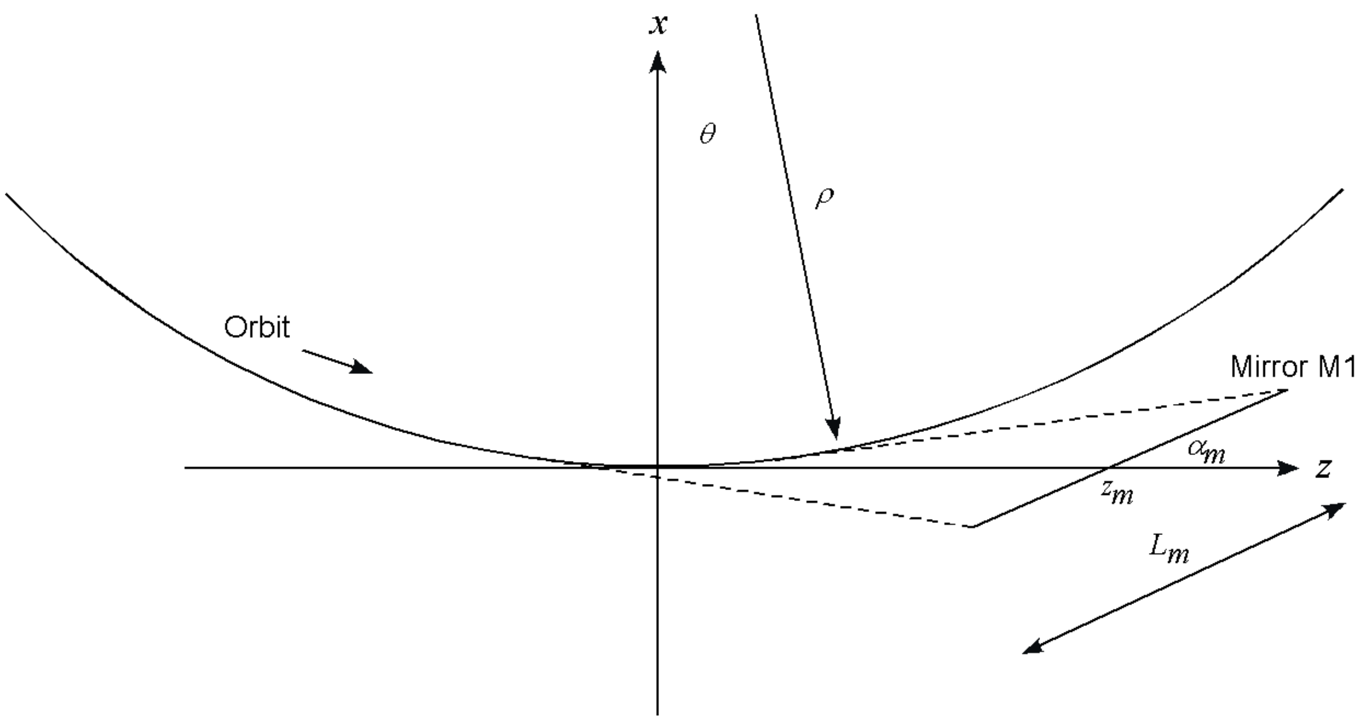
$$\Delta z = \rho\sigma_\lambda$$

$$d = \theta\Delta z = 2\rho\sigma_\lambda^2 \approx 0.7\rho^{\frac{1}{3}}\lambda^{\frac{2}{3}}$$

  - This expression is similar to the diffraction resolution.
  - As before, short wavelengths are preferable.
  - But this time, small opening angles are better.
    - If the source is dipole radiation, the angle and the wavelength are not independent.
- But how much of the orbit do we actually capture?

From Horizontal Space to Phase Space

- Consider the beam's orbit both in the horizontal plane (xz) and in horizontal *phase space* (xx').
  - x' is the beam's angle to the direction of motion z.
- Which rays, at which angles, are reflected by M1?



Horizontal Phase Space

- A point on the orbit near the *xz* origin is given by:

$$(x,z) = (\rho - \rho\cos\theta, \rho\sin\theta) \approx (\tfrac{1}{2}\rho\theta^2, \rho\theta) = (\tfrac{1}{2}\rho x'^2, \rho x')$$

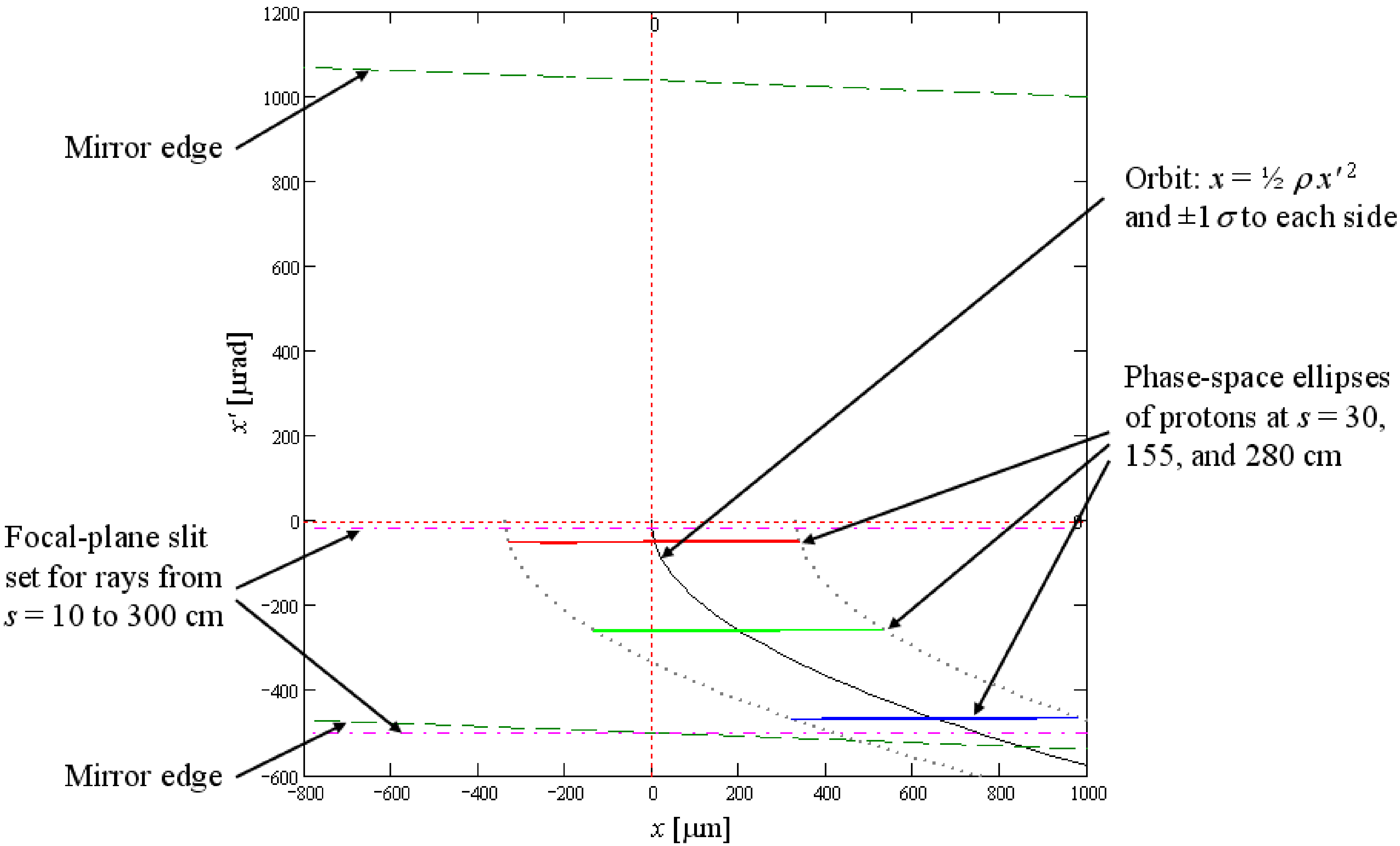
  - For a point on the orbit, the angle x' to the z axis is equal to θ.
- The rays striking the $+x$ and $-x$ ends of M1 are given by:

$$x + x'\left(z_m \pm \frac{L_m}{2}\cos\alpha_m - z\right) = \pm\frac{L_m}{2}\sin\alpha_m$$

$$x + x' z_m \approx \pm\frac{L_m}{2}\sin\alpha_m$$

- We plot these curves in phase space, along with the beam's 1-sigma phase-space ellipse at three points along its orbit.

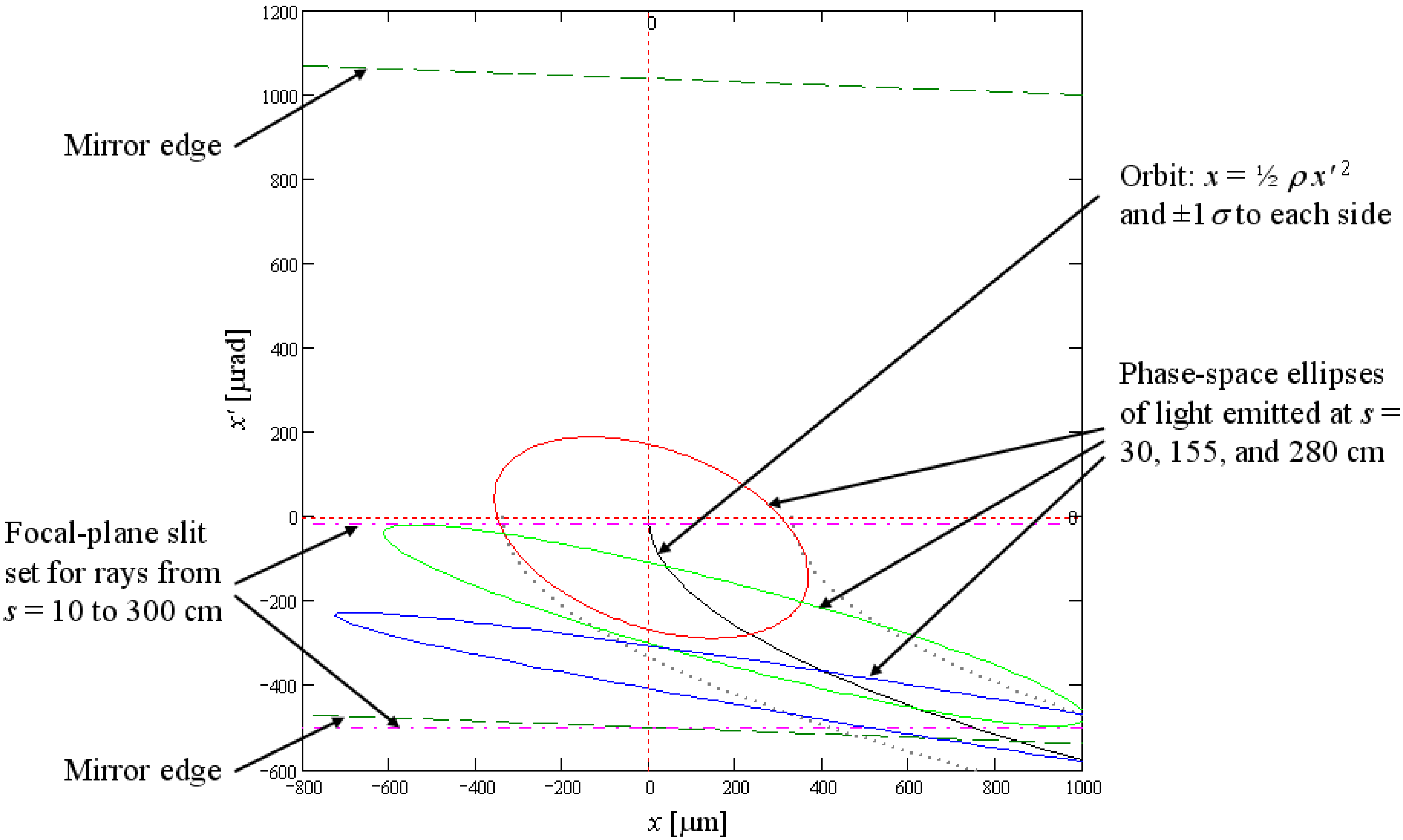
LHC: *x* Phase Space



LHC: *x* Phase Space

- The two mirror edges appear as slanted lines.
- Because the radius of curvature is so long, the mirror receives light from the first 3 m of the dipole.
- The path can be shortened by adding a slit *one focal length* from the first focusing optic (mirror or lens).
  - The position of a ray on this plane corresponds only to its angle x' at the source.
  - We can select light from an adjustable horizontal band across the plot.
- The x positions of the proton ellipses shift along this 3-m path.
  - Project light from each ellipse onto the x axis
  - The combined light is smeared out along x and so blurs the resolution.
- But each proton emits light with an opening angle.
  - We need the photon ellipse, not the proton ellipse.
  - A convolution of the proton ellipse with the opening angle.

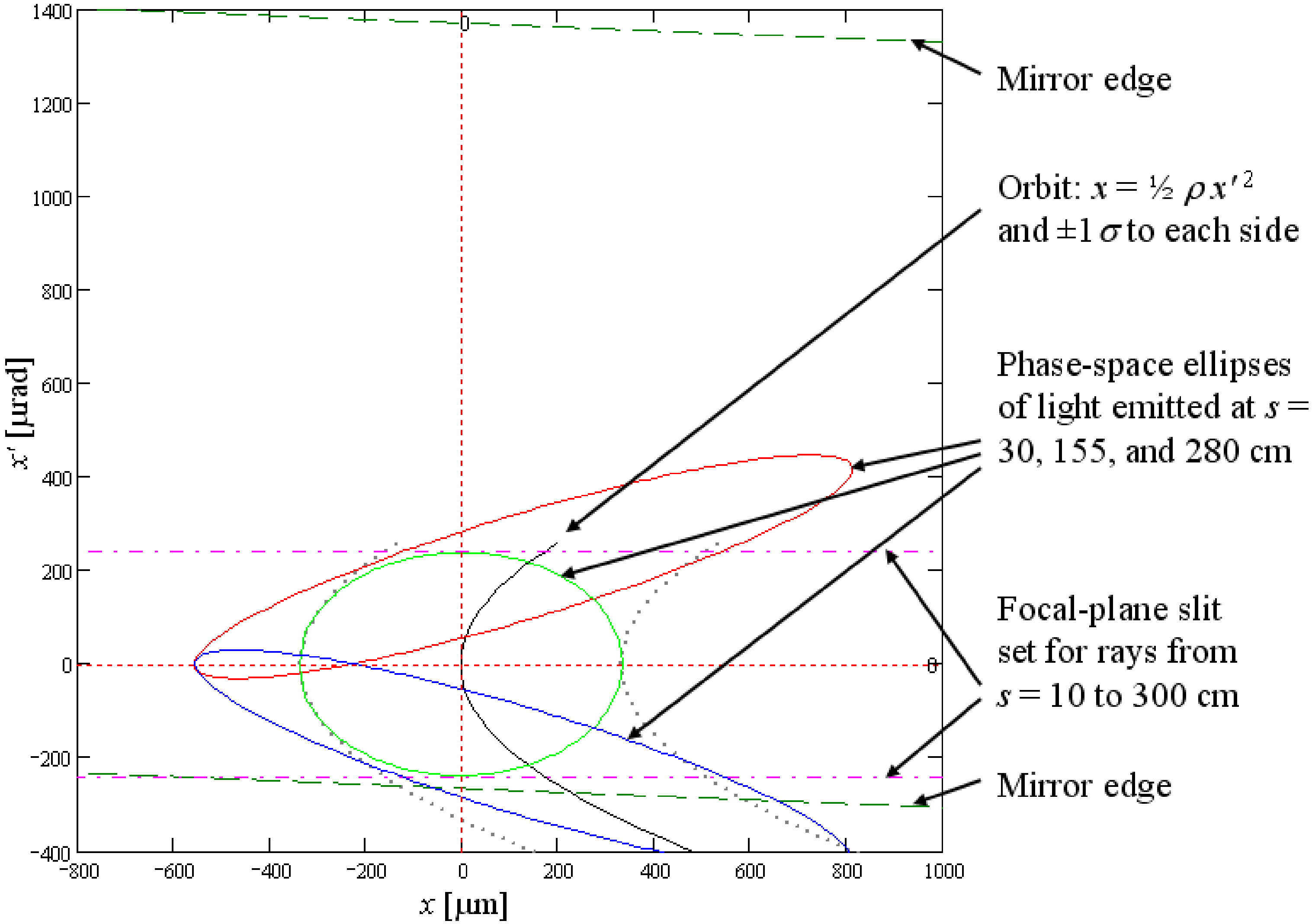
LHC: *x* Phase Space with Light Ellipses



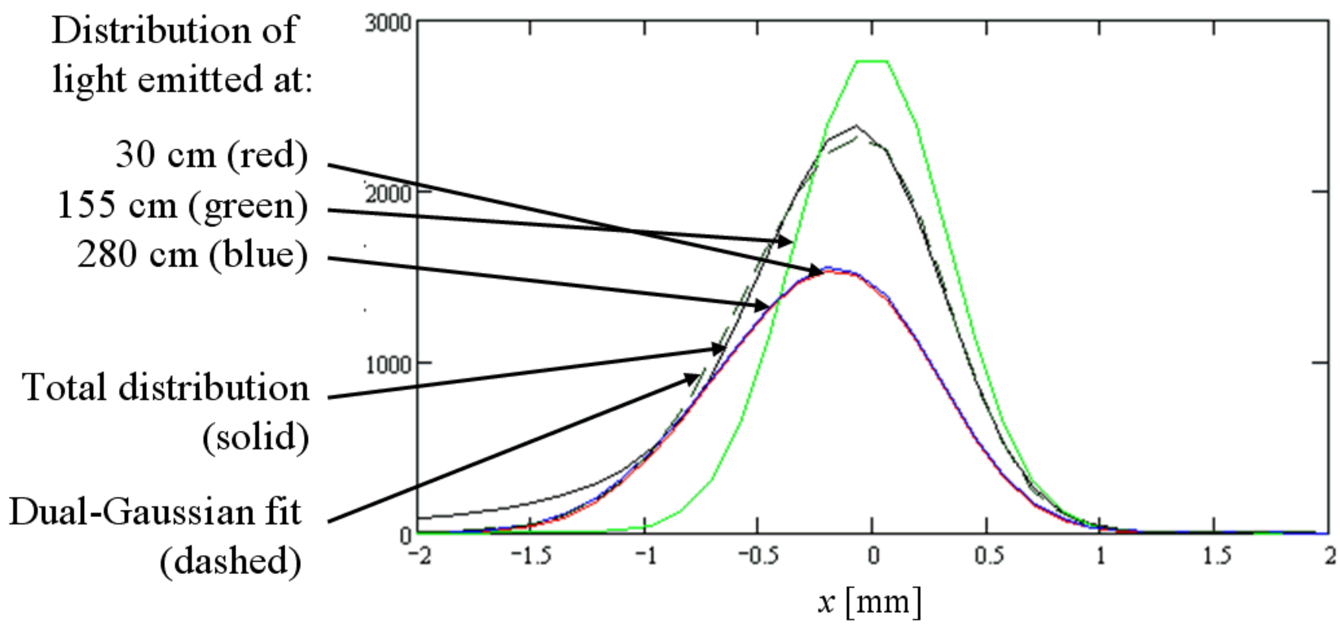
LHC: *x* Phase Space with Light Ellipses

- The ellipses are much bigger, going well outside the slit.
- They get increasing tilted and elongated with distance from the entrance to the dipole.
  - But this plot assumes that the optics are focused at the dipole entrance.
  - Move the focus to the midpoint of the 3-m path.

LHC: x Phase Space, Focus at Midpoint

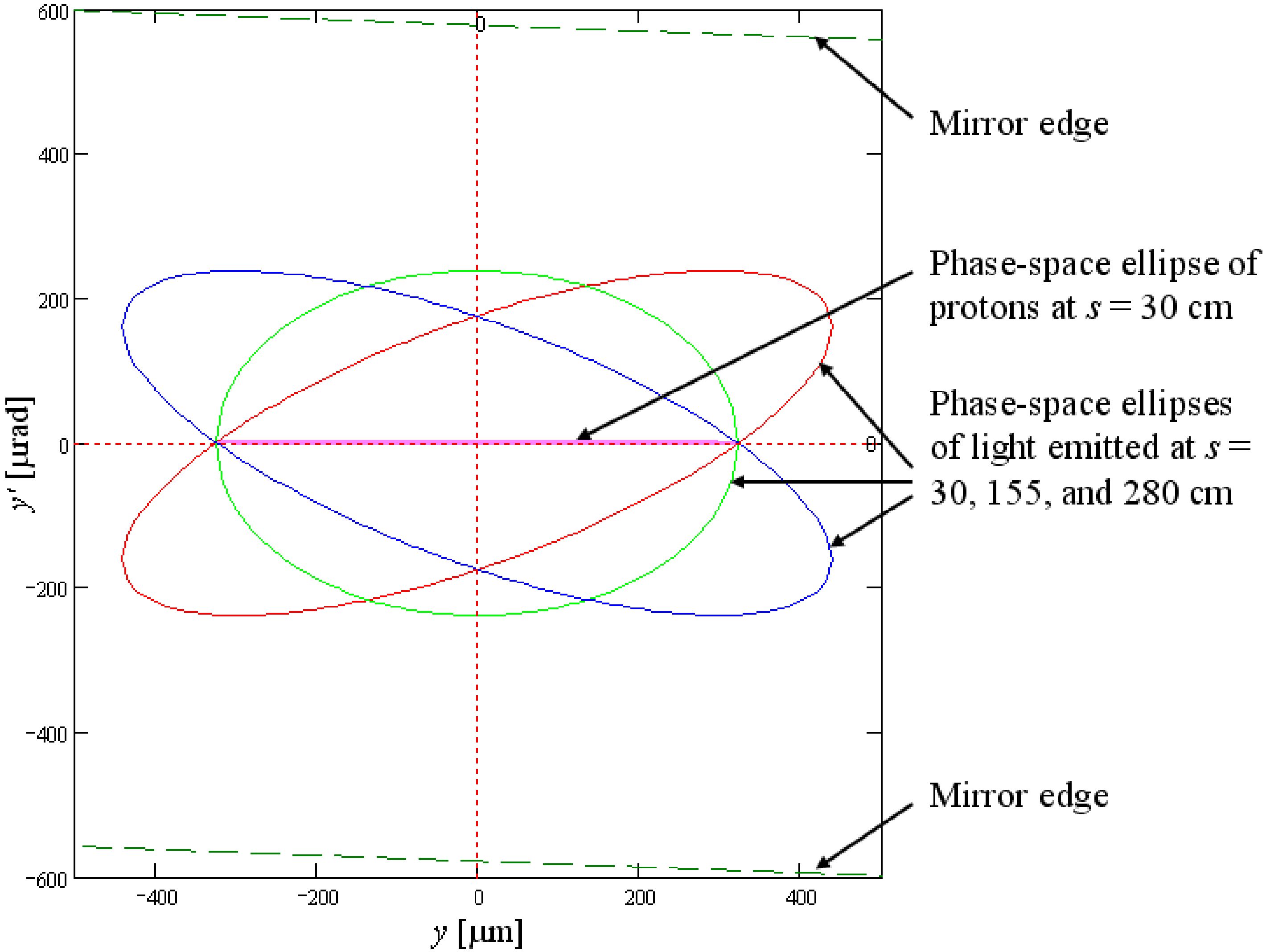


LHC: *x* Phase Space, Projection onto *x*

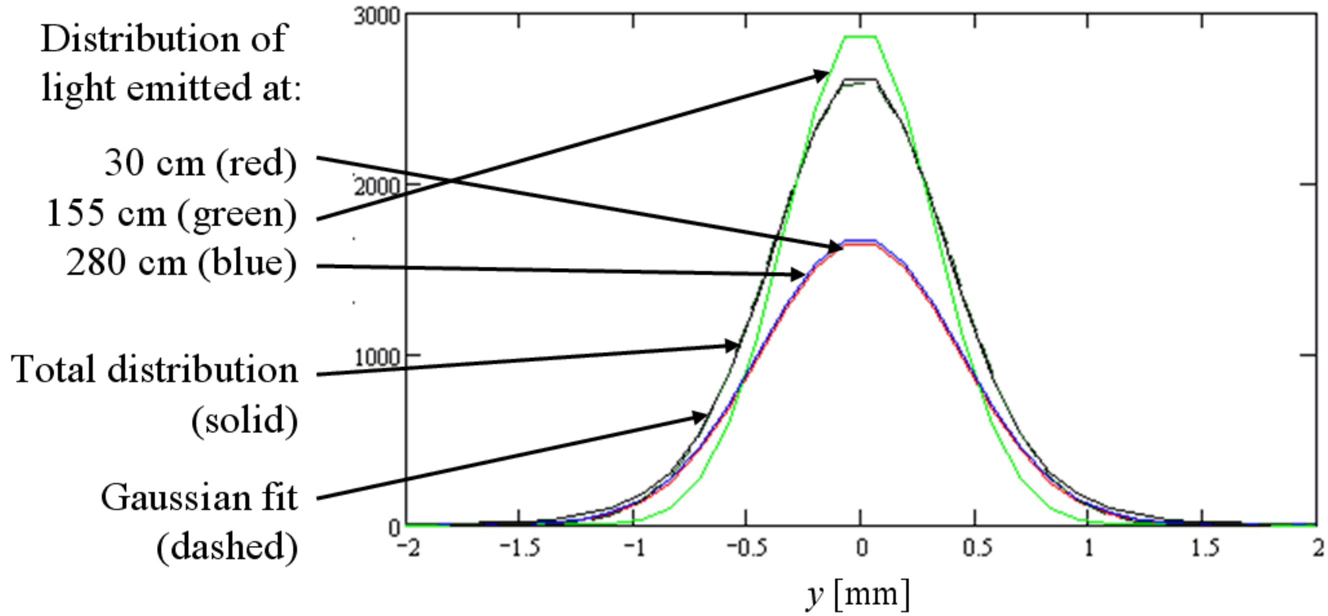


- Slit excludes the right (+*x*) side of the elongated ellipses, but includes the left side.
  - Tail on the left side due to depth of field.
  - Right side of total distribution is a good measure of the true beam size.
- This answer pertains only to the LHC. Each machine requires a careful study of depth of field.

LHC: *y* Phase Space, Focus at Midpoint



LHC: *y* Phase Space, Projection onto *y*

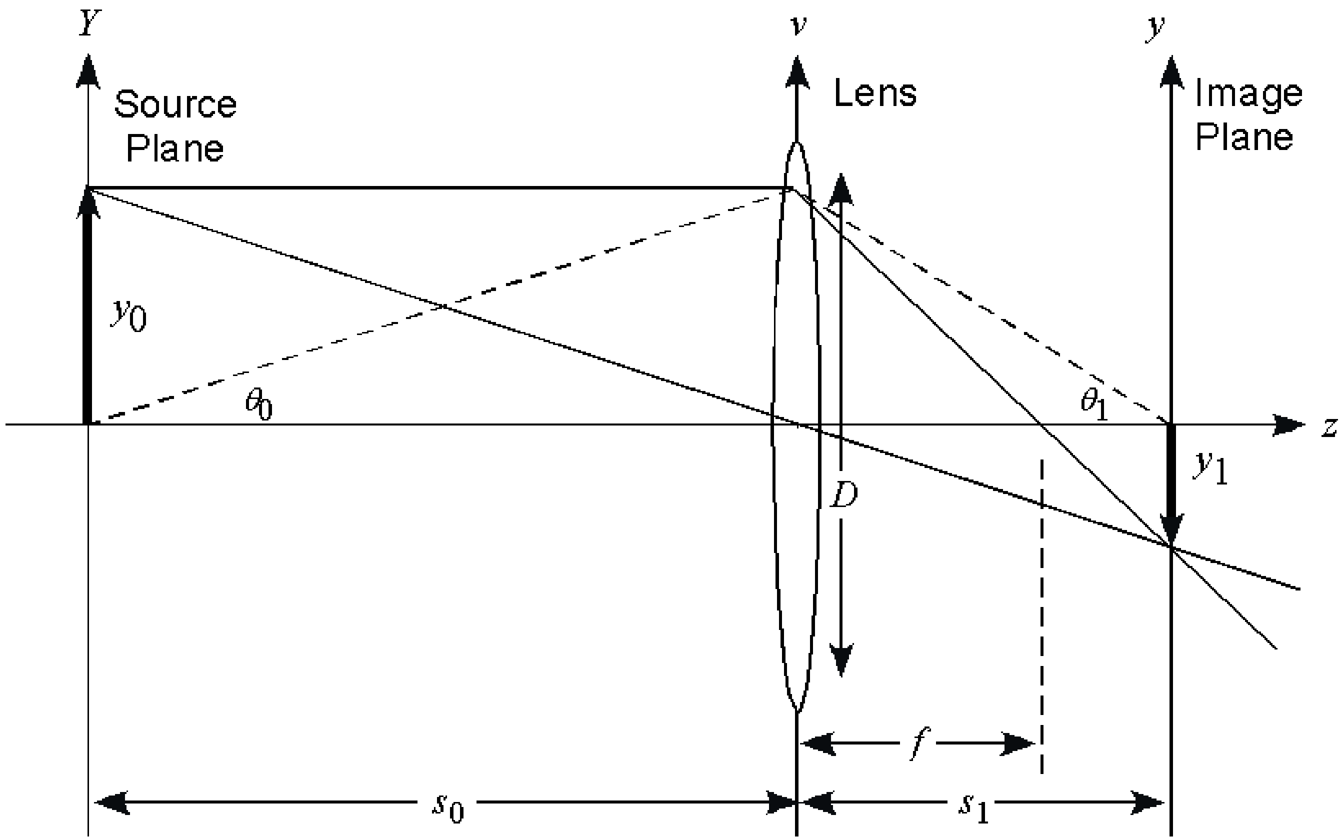


- The elongated ellipses create a tail on both sides.
  - The vertical measurement is more affected by depth of field than the horizontal.
    - Broadens the result by as much as 20% at 7 Tev.
  - Adding a vertical slit does little to reduce the effect.

Photon Emittance (Brightness)

- Accelerator people know that Liouville's theorem conserves the emittance of a beam in a transport line.
  - The phase-space ellipse changes shape, but not area.
    - At each waist, the size-angle product $\sigma_x \sigma_{x'}$ is constant.
  - (But for electrons in a ring, dissipation by synchrotron radiation allows damping that "cheats" Liouville.)
- Light in an optical transport line has an emittance too.
  - At each image, the product of size and opening angle (light-cone angle) is constant.
    - Magnification makes the image bigger, but the angle smaller.
  - The area of the light's phase-space ellipse—the brightness of the source—is conserved.

Conservation of Brightness



Lens Equations

$$\frac{1}{f} = \frac{1}{s_0} + \frac{1}{s_1}$$
$$m = s_1 / s_0$$
$$y_1 = y_0 m$$
$$\theta_1 = \theta_0 / m$$

Minimum Photon Emittance

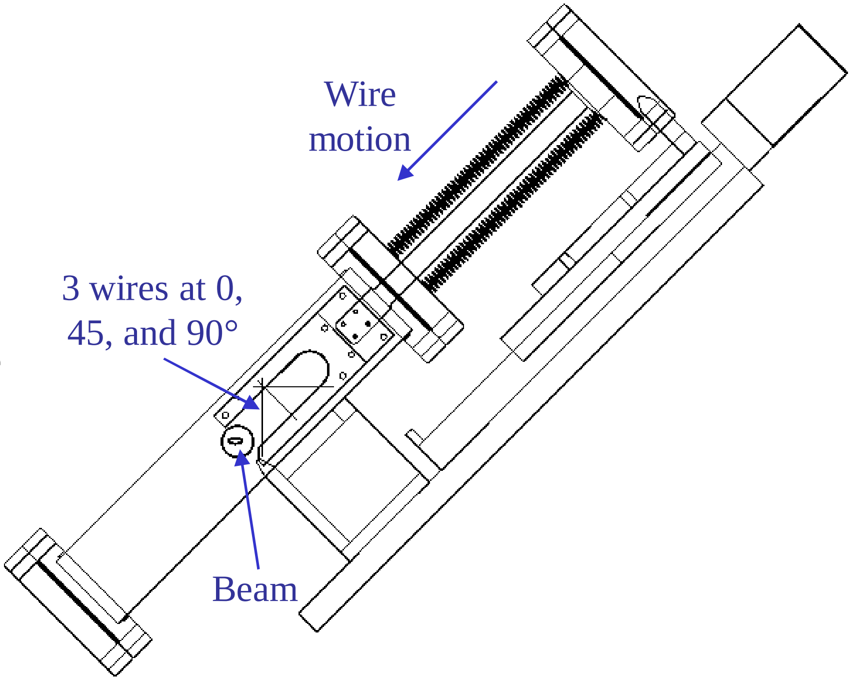
- The minimum emittance for a light beam is that of the lowest-order Gaussian mode (TEM_{00}) of a laser.
  - ω is the beam radius.
    - In the usual definition (where ω is not the one-sigma value):
      - The electric field follows $E(r) = E_0 \exp(-r^2/\omega^2)$
      - The intensity (power) is the square: $I(r) = I_0 \exp(-2r^2/\omega^2)$
  - ω_0 is the radius at the waist (the focus).
    - This size is nonzero due to diffraction.
  - $z_R = \pi\omega_0^2/\lambda$ is called the Rayleigh length.
    - Characteristic distance for beam expansion due to diffraction.
  - The expansion is given by $\omega^2(z) = \omega_0^2(1+z^2/z_R^2)$
  - The angle (for $z >> z_R$) is $\theta = \omega/z = \omega_0/z_R = \lambda/\pi\omega_0$
  - The product of waist size and angle is then $\omega_0\theta = \lambda/\pi$
  - One-sigma values for the size and angle of I give an emittance of $\lambda/4\pi$

Measuring Small Beams

- Third-generation light sources (SLS, SOLEIL, Diamond, SSRF, ALBA, PETRA-III, NSLS2…), future HEP accelerators (ILC damping rings, Super-*B*, ERLs) and prototypes (ATF at KEK) have *very* low emittances:
  - Typical emittances: $\varepsilon_x \approx 1$ nm $\varepsilon_y \approx 10$ pm
  - Typical beam sizes: $\sigma_x < 100$ µm $\sigma_y < 10$ µm
- An SLM images a beam from many meters away. (It's really a telescope.)
  - F-number must be large: $F = f\,/\,D \sim (10\text{ m})/(50\text{ mm}) = 200$
  - Resolution with blue light: $r_A = 1.22\,F\lambda \sim 100$ µm
- Techniques for measuring small beams:
  - In this lecture:
    - Imaging with ultraviolet synchrotron radiation
    - Imaging with x rays
    - Other methods that do not use synchrotron radiation (briefly)
  - Wednesday:
    - Synchrotron-light interferometry (~10 µm resolution)
    - Vertical beam size using the null in vertically polarized light
      - Not in Wednesday's lecture, but similar in concept to an interferometer

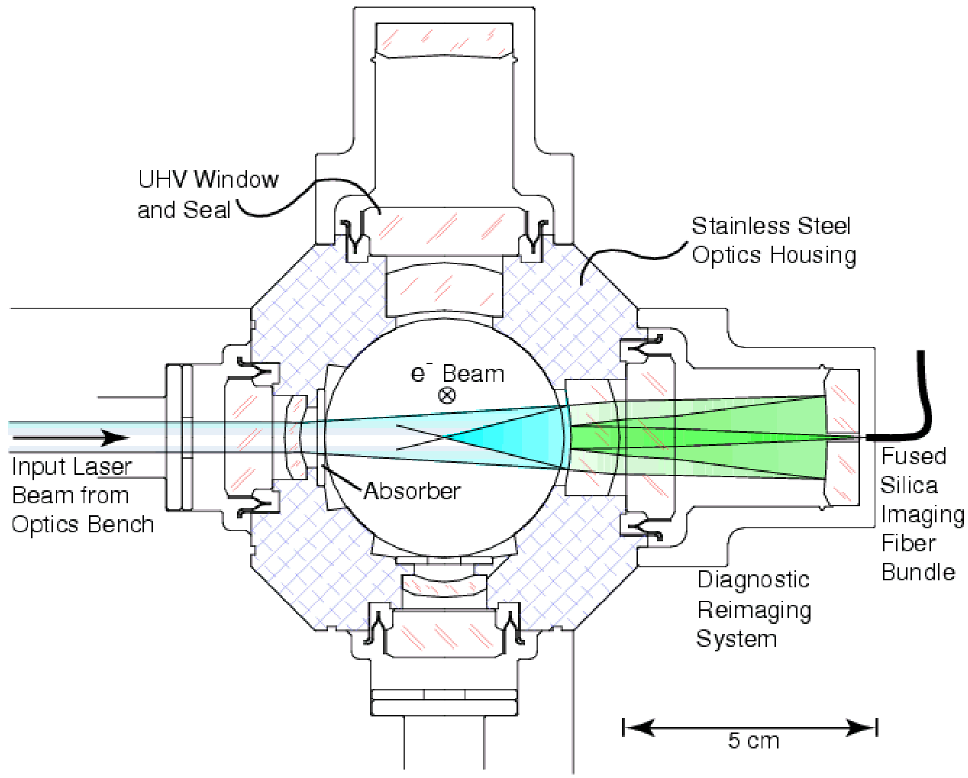
Without Synchrotron Light: Wire Scanner

- Methods that don't use synchrotron light are also useful. They're outside the scope of this class, but…
- Wire scanner:
  - While a thin stretched wire is scanned across the (wider) beam, measure scattered radiation or lost electrons vs. wire position.
  - Gives a projection of the beam in the scan direction.
  - Three wires at 0, 45, 90 degrees give major and minor axes and tilt of beam ellipse.
  - Wire size (≥ 4 μm), limited by wire erosion, sets resolution.
  - Multiple measurements: beam jitter
  - Can be destructive to stored beams



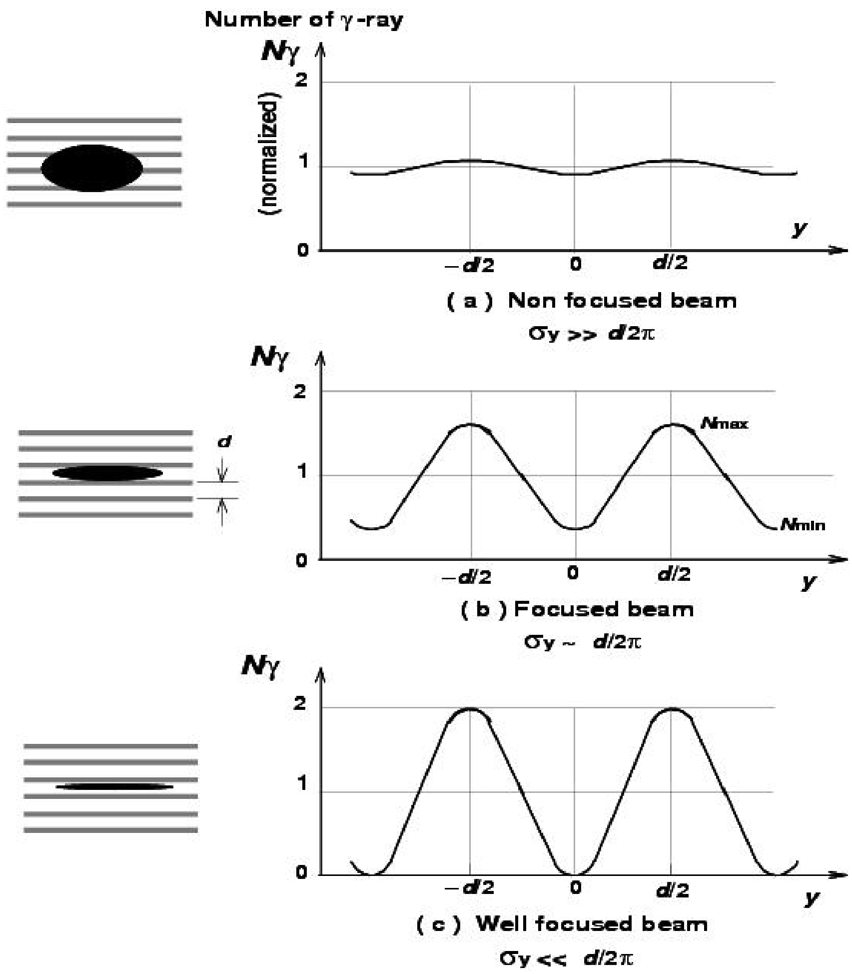
Without Synchrotron Light: Laser Wire

- Laser crosses electrons at a waist smaller than the *e*-beam.
- Focus with a small F-number to get resolution $\approx\lambda \geq 300$ nm.
- Like wire scanner, look for scattered radiaton.
- Compared to wire scanner, better resolution and non-destructive. Still needs many measurements.



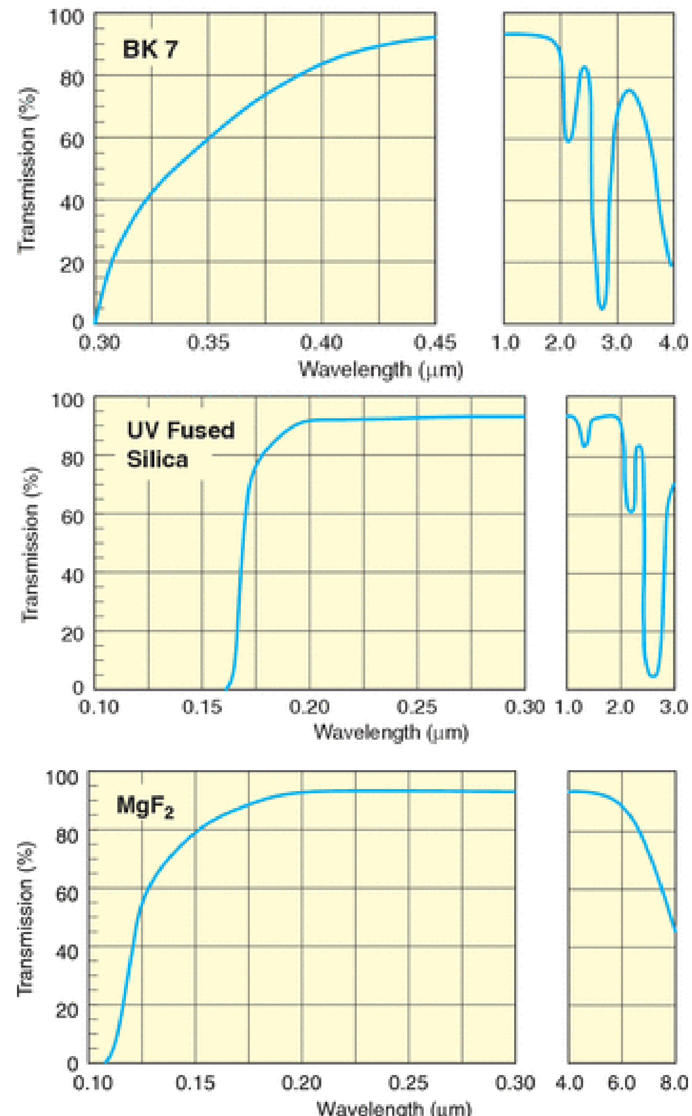
Laser Interferometer

- Split a laser beam. Intersect both parts at an angle as they cross the electron beam.
- Interference fringes with maxima and minima across the electrons.
- Move the beam relative to the fringe pattern.
- When the beam is small compared to the fringe spacing, the scatter is heavily modulated by the shift in the fringes.
- Can measure down to tens of nm



Imaging with UV Synchrotron Light

- Can't go far into the UV without problems.
  - Window and lens materials become opaque:
    - Glasses (like BK7 at right) are useful above ~330 nm.
    - Fused silica works above ~170 nm.
    - Special materials like MgF_2 work above ~120 nm.
  - Absorption in air below ~100 nm
    - Must use reflective optics in vacuum.



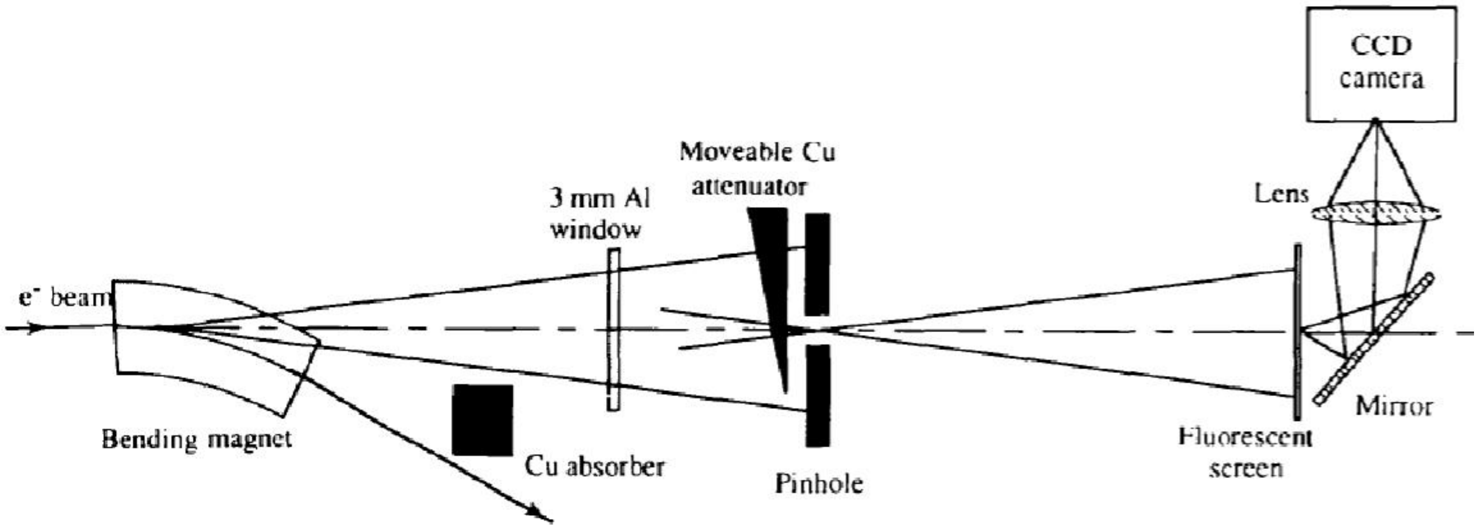
Imaging with Synchrotron X Rays

- The good news: Most of the beam's emission is in the x-ray region.
- The bad news: How do you form an image?
- We'll discuss some techniques:
  - Pinhole cameras
  - Zone plates
  - Grazing-incidence optics
  - X-ray lenses
- Labs later today on imaging with pinholes and zone plates (along with ordinary lenses)

Imaging X Rays with a Pinhole Camera

- Resolution σ on image plane with a pinhole of radius r: $\sigma = \sqrt{\sigma_g^2 + \sigma_d^2}$
  - Distances: a from source to pinhole, b from pinhole to image:
  - Geometric optics: Pinhole should be << beam size $\sigma_g = \frac{r}{\sqrt{3}} \frac{a+b}{a}$
  - Diffraction blurs image if the pinhole is too small: $\sigma_d = \frac{5}{8\pi} \frac{\lambda b}{r}$
  - Pinhole size for best resolution: $r^2 = \frac{5\sqrt{3}}{8\pi} \lambda \frac{ab}{a+b}$
    - Geometric mean of λ (~0.2 nm) and a or b (~10 m): $r \approx 20$ µm
- Optimum resolution on the *source* plane: $\sigma_{\text{opt}}^2 = \frac{5\lambda}{4\pi\sqrt{3}} \left(1 + \frac{a}{b}\right) a$
  - Want small λ, small a, and large magnification b/a
- On image plane, a scintillator converts x rays to visible light.
- Make "pinhole" with a sheet of heavy metal thick enough to stop x rays.
- X-rays surrounding the hole must be blocked upstream, so that pinhole get too hot and deform.
- Most of the x rays are not used for the image

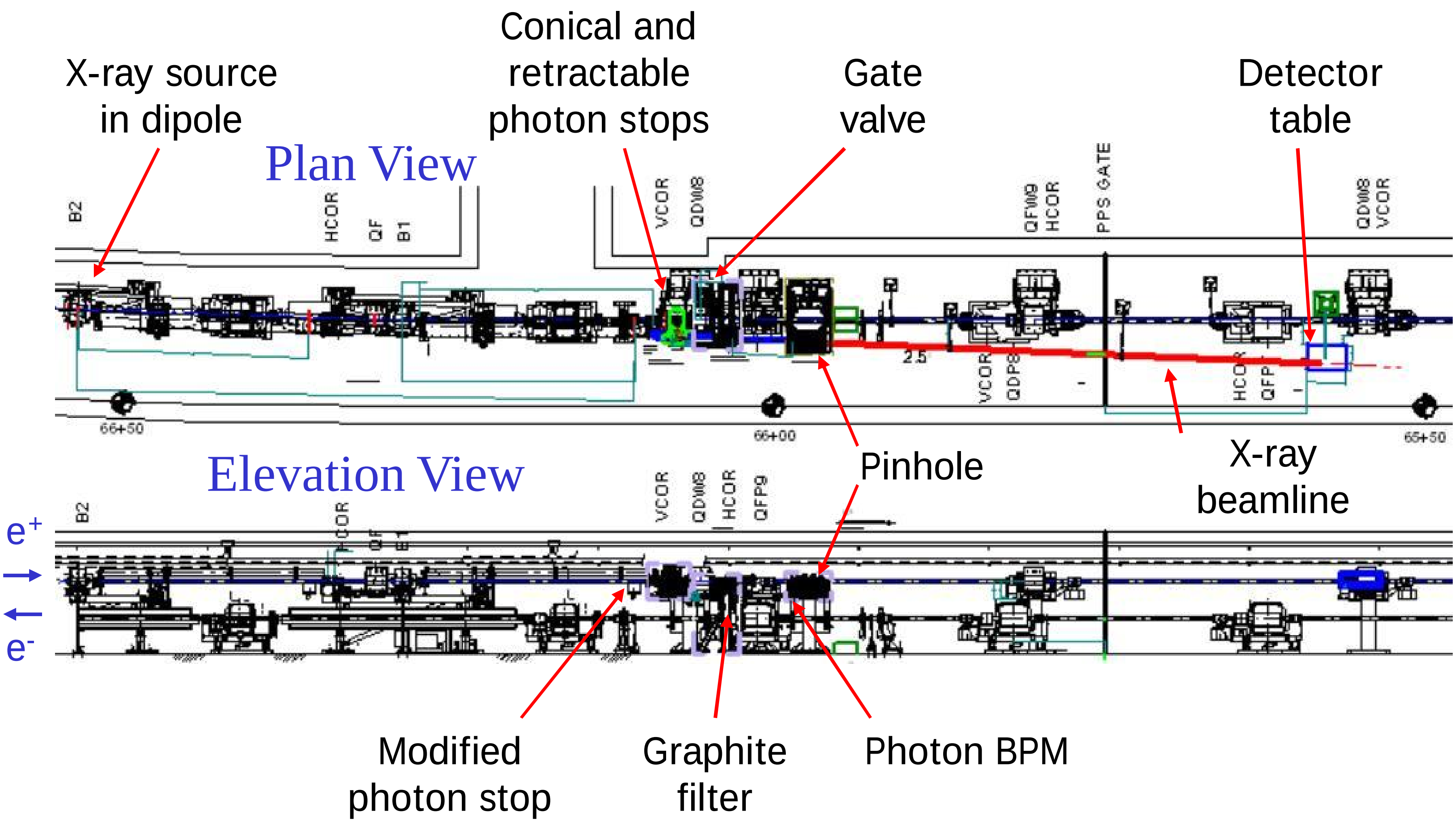
Pinhole Camera at ESRF



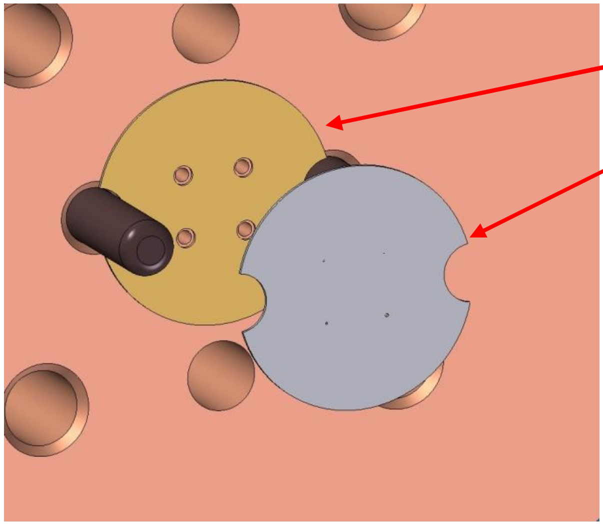
$$\Sigma = \left(\sigma_x^2 + \sigma_{screen}^2 + \sigma_{camera}^2 + \sigma_{diffraction}^2 + \sigma_{pinhole}^2\right)^{1/2}$$

Their 10-μm pinhole gives a resolution of 13 μm.

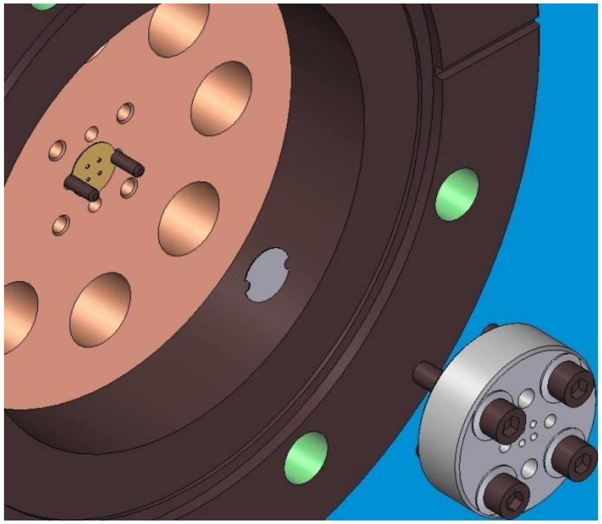
X-Ray Pinhole Camera in the PEP-2 LER

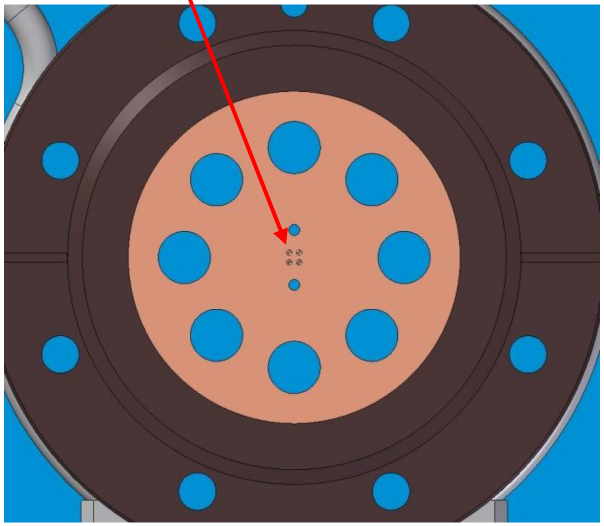


Design of the Pinhole Assembly

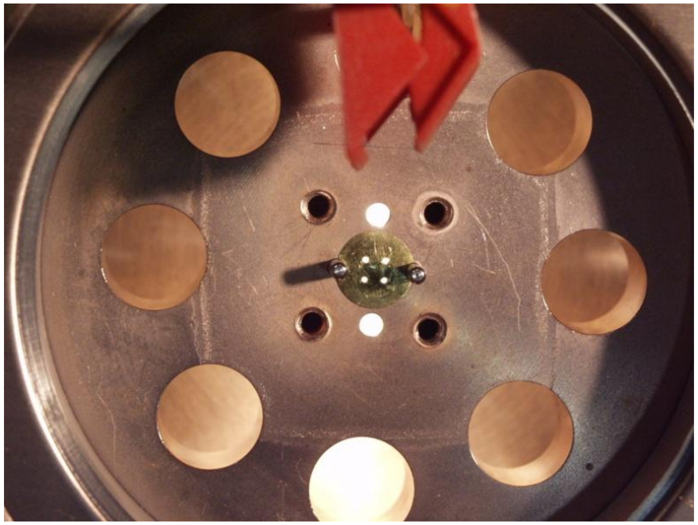


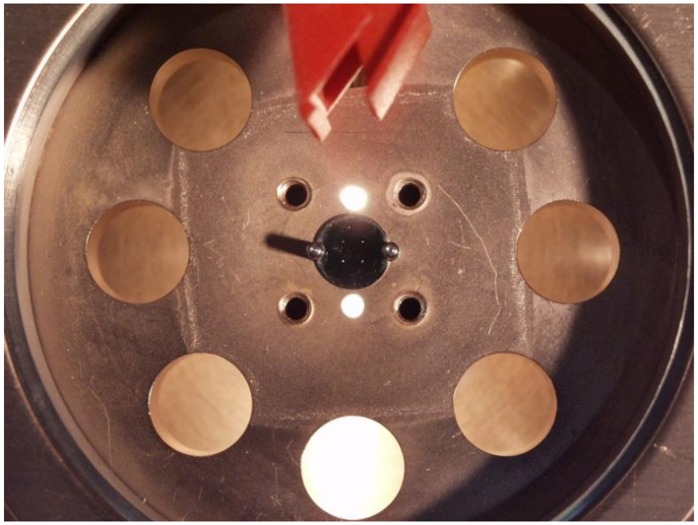
- Gold disk for heat transfer
- Pt:Ir (90:10) disk with 4 pinholes.
  - Diameters of 30, 50, 70, and 100 μm.
- Front: Glidcop with 4 larger holes

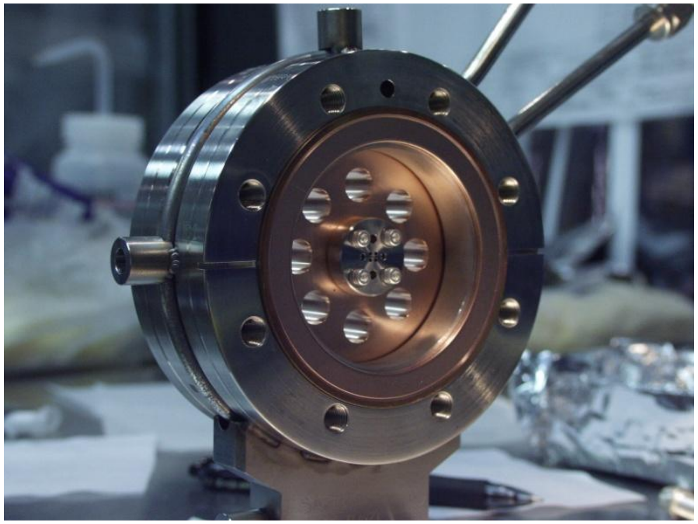


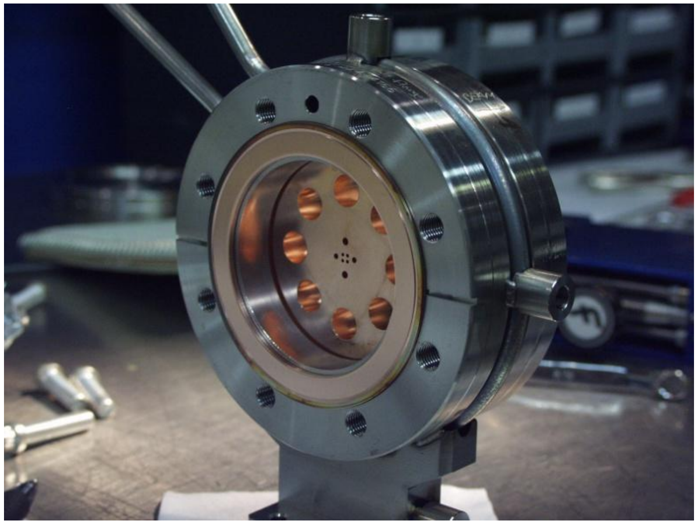


Photos of the Pinhole Assembly

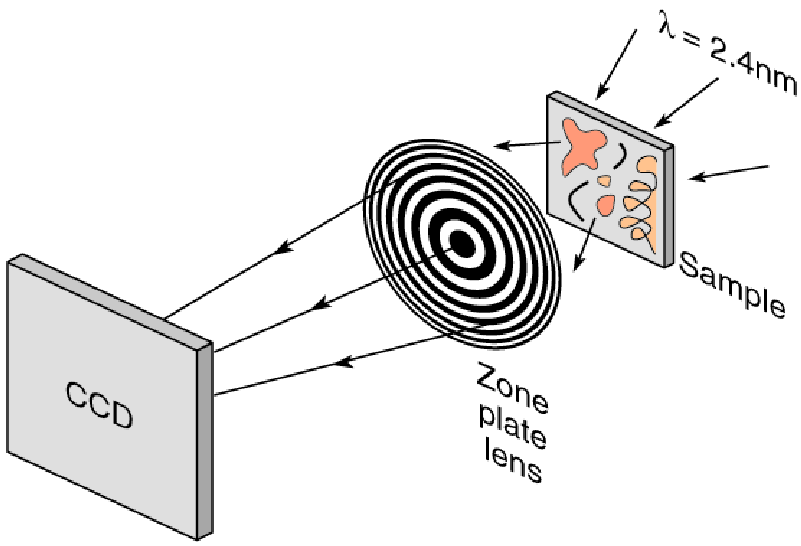


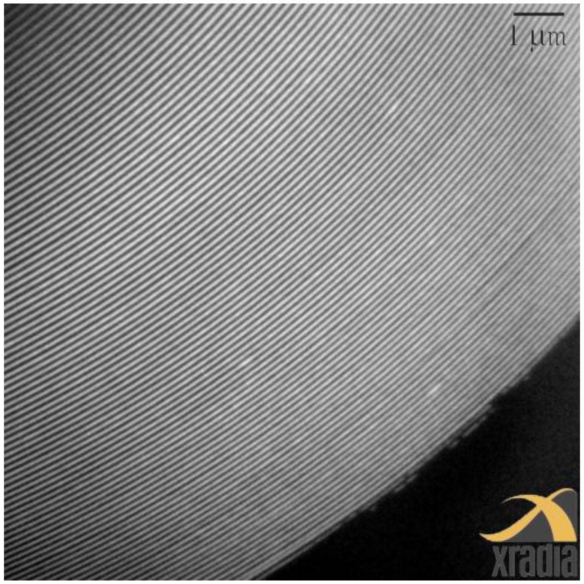






Imaging X Rays with a Zone Plate

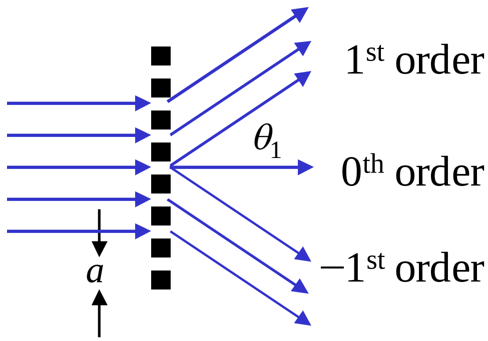


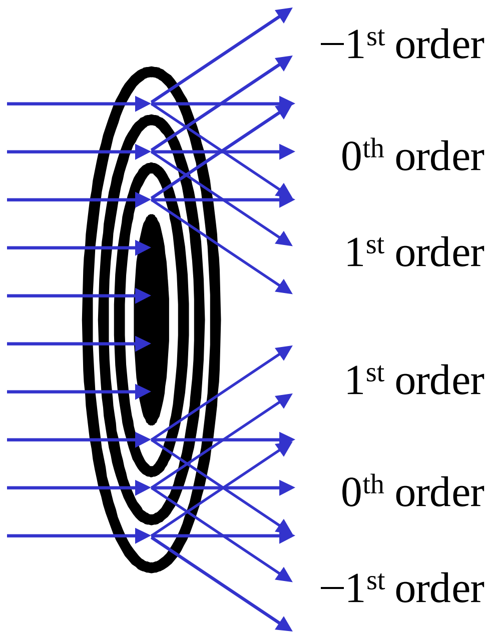


- A diffractive lens, made by microlithography
- Rings of a high-*Z* metal (gold) deposited on a thin low-*Z* membrane (SiN)
  - Ring widths as narrow as 50 nm are possible
- Power must be kept low, and bandwidth must be narrow (≈1%)
  - Precede with a pair of multilayer x-ray mirrors, which reflect a narrow band and absorb the out-of-band power.

How Does a Zone Plate Work?

- Consider a transmissive diffraction grating.
  - Parallel opaque lines on a clear plate, with period *a*
  - Parallel rays of wavelength λ passing through adjacent lines and exiting at an angle θ have a difference in optical path of $a \sin \theta$.
  - They are in phase if this difference is $n\lambda$, giving the n^{th}-order diffraction maximum: $\sin\theta_n = n\lambda/a$
- Now wrap these grating lines into a circle.
  - 1^{st} order bends toward center: focusing
  - -1^{st} order bends away from center: defocusing
  - 0^{th} order continues straight ahead
  - Make central circle opaque to block 0^{th}-order light around the focus (a "central stop").
- But the 1^{st}-order rays are parallel and so don't focus
  - Vary the zone spacing as a function of ring radius *r* so that all the exiting rays meet at a focal point a distance *f* from the zone plate.





How the Zone Widths Vary

- To focus at f, the ray at radius r_n must exit at an angle θ_n with: $r_n = f\tan\theta_n$
- First-order diffraction gives $\lambda = a_n \sin\theta_n$
- The grating period a now varies too: $a_n = r_{n+1} - r_{n-1}$
- There are many, closely spaced zones, and so we treat n as a continuous variable: $a(n) = \Delta \mathrm{n}\, dr(n)/dn = 2dr(n)/dn$
- We use the expression for $\tan\theta(n)$ to substitute for $\sin\theta(n)$:

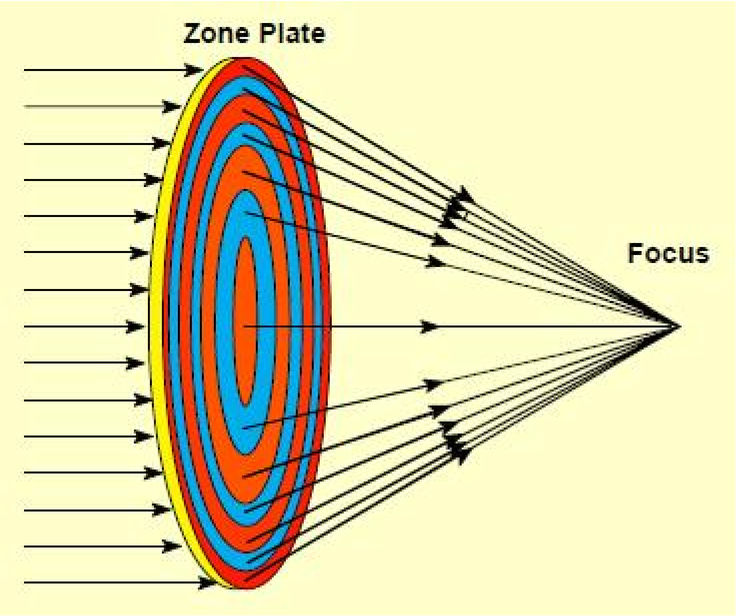
$$\sin^2\theta = \frac{1}{\cot^2\theta + 1} = \frac{1}{1 + f^2/r^2} = \frac{\lambda^2}{a^2} = \lambda^2 \Big/ \left(2\frac{dr}{dn}\right)^2$$

$$\frac{d}{dn}\left(\frac{r^2}{f^2}\right) = \frac{\lambda}{f}\sqrt{1 + \frac{r^2}{f^2}} \qquad \int_0^n \frac{\lambda}{f}\, dn' = \frac{\lambda n}{f} = \int_0^{r^2/f^2} \frac{dx}{\sqrt{1+x}} = 2\sqrt{1 + \frac{r^2}{f^2}} - 2$$

$$\frac{r^2}{f^2} = \left(\frac{\lambda n}{2f} + 1\right)^2 - 1 \qquad r^2 = n\lambda f + \frac{n^2\lambda^2}{4}$$

Zone-Plate Formulas

- λ = wavelength (monochromatic light)
- $\Delta\lambda$ = bandwidth
- f = focal length of lens at λ
- N = number of zones
  - Counting both clear and opaque zones
- r_n = radius of n^{th} zone boundary
- $\Delta r = r_N - r_{N-1}$ = thickness of outer zone
- $D = 2r_N$ = outer diameter
- F = F-number
- r_A = (Airy) resolution



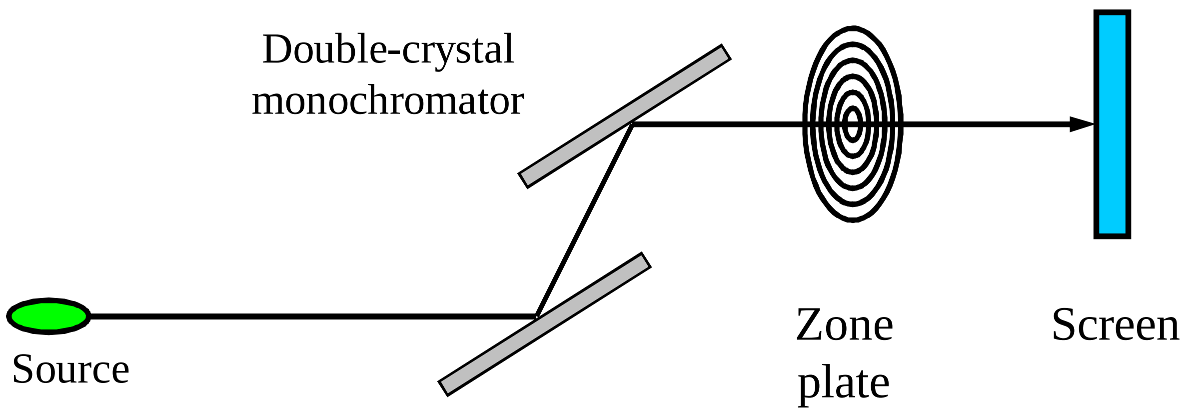
$$r_n = \sqrt{nf\lambda + n^2\lambda^2/4} \approx \sqrt{nf\lambda}$$

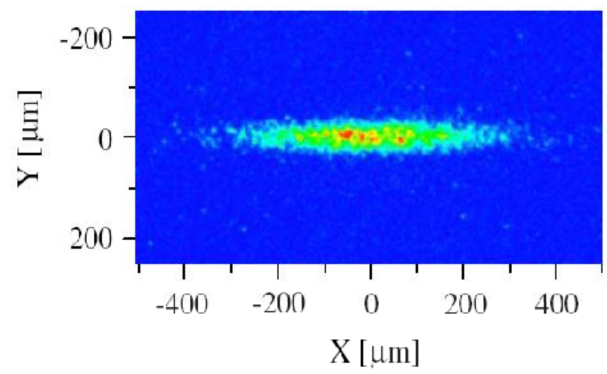
$$f = 4N(\Delta r)^2/\lambda \qquad D = 2r_N = 4N\Delta r$$

$$F = f/D = \Delta r/\lambda \qquad r_A = 1.22F\lambda = 1.22\Delta r$$

$\Delta\lambda < \lambda/N$ to avoid chromatic blurring

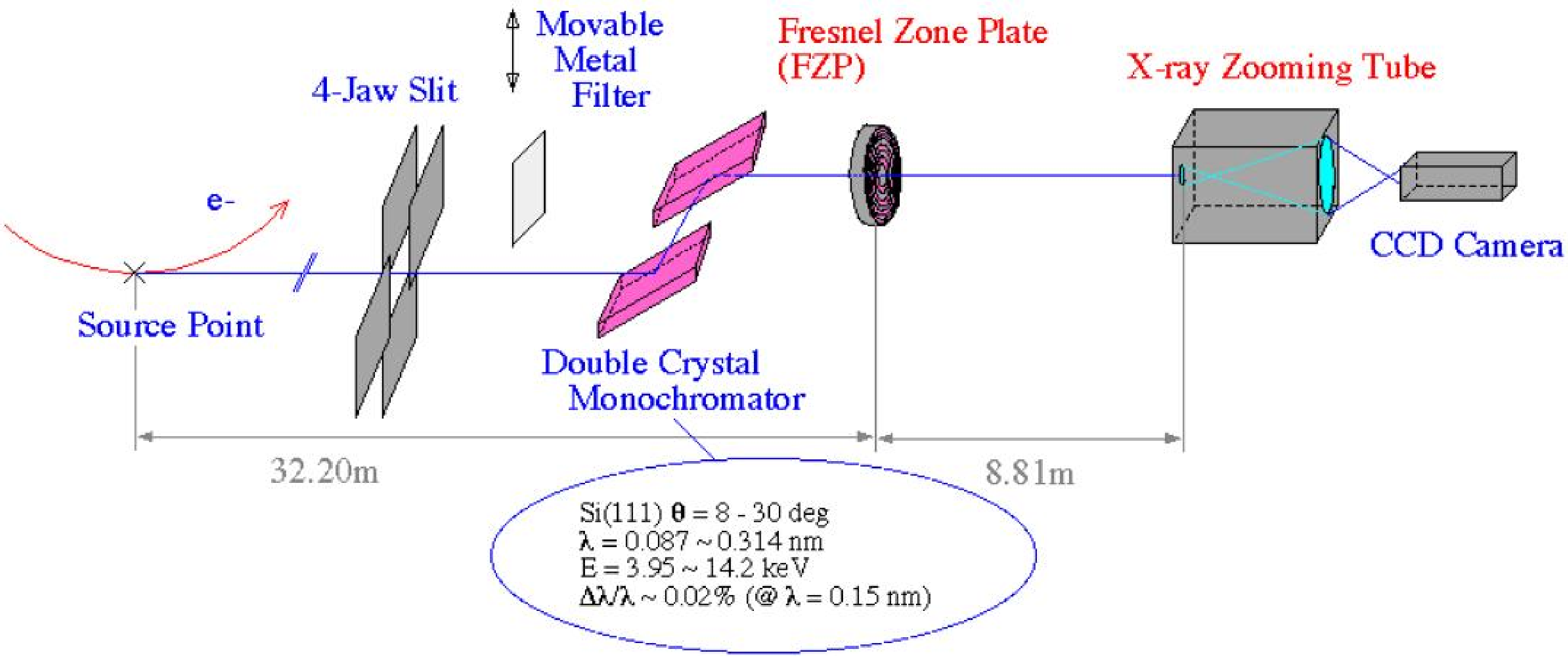
Imaging with a Fresnel Zone Plate





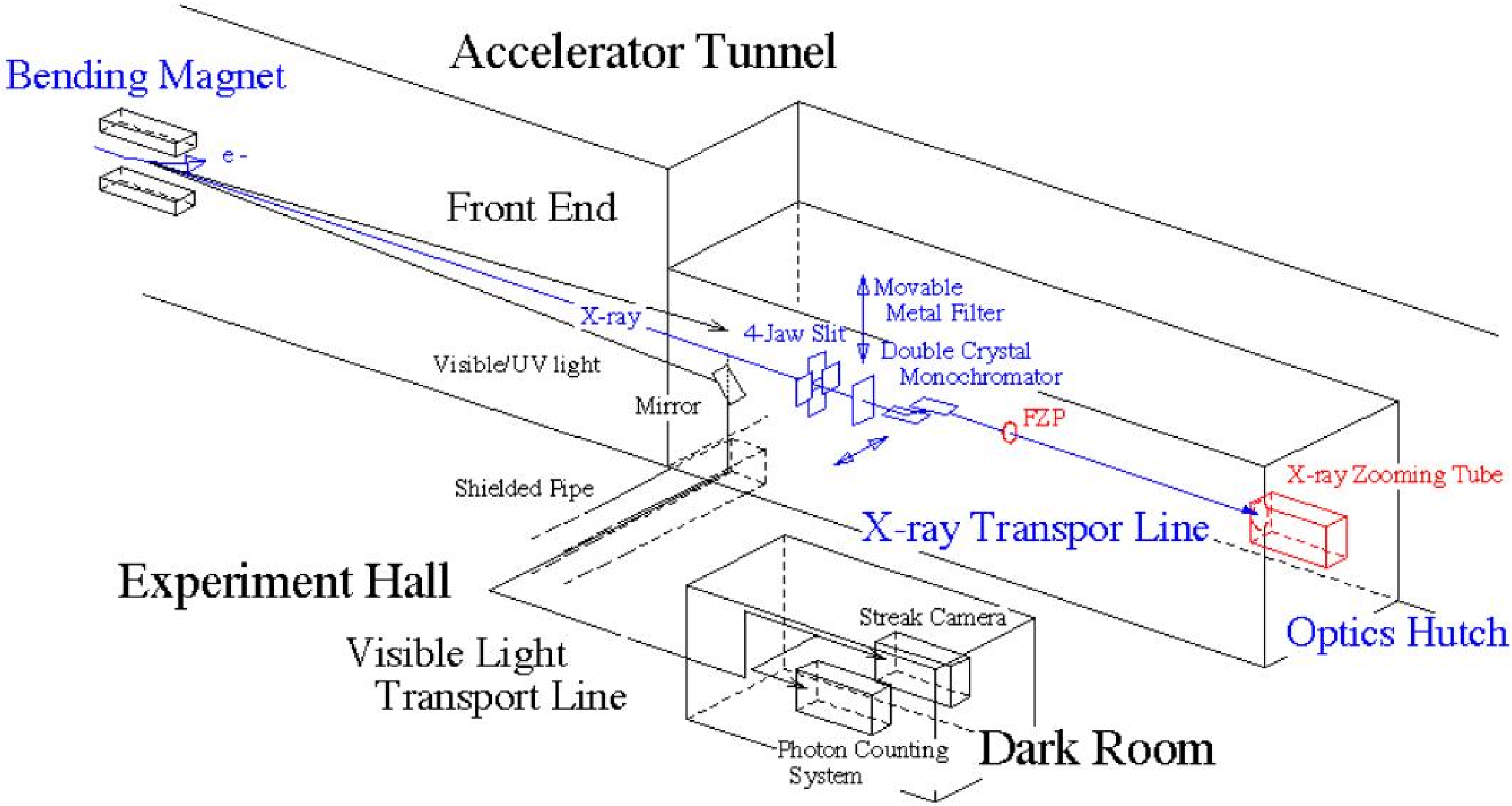
- A zone plate is designed to focus at a single wavelength.
  - This is called “strong chromatic aberration”.
  - Insert a monochromator, to limit bandwidth and to absorb power at other wavelengths.
  - With two crystals, the entering and exiting rays are parallel.

Zone Plate at SPring-8

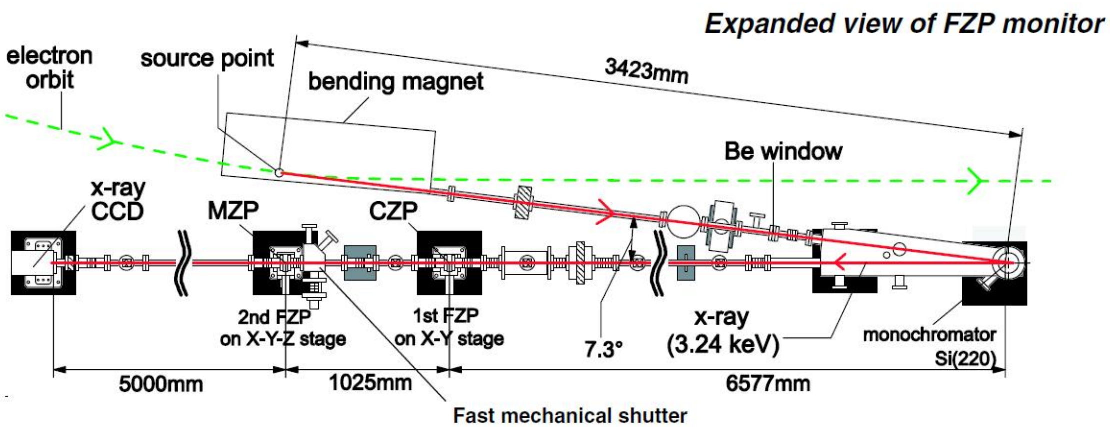


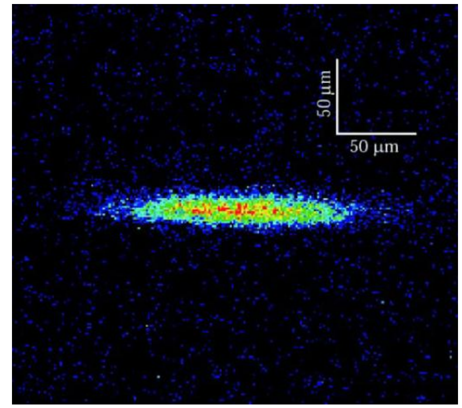
- Monochromator transmits 8.2-keV photons (λ = 0.151 nm)
- Total magnification = 13.7 (0.2737 by FZP, 50 by XZT)
- 4-μm resolution with the help of the x-ray zooming tube
- Observed a transient in the beam size during top-off operation

SPring-8 Diagnostic Beamline

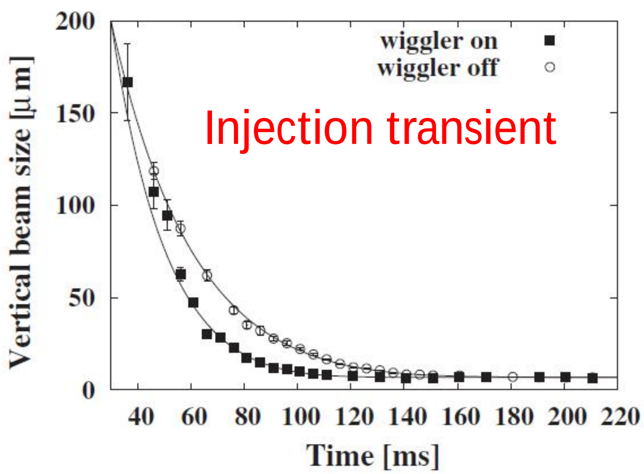


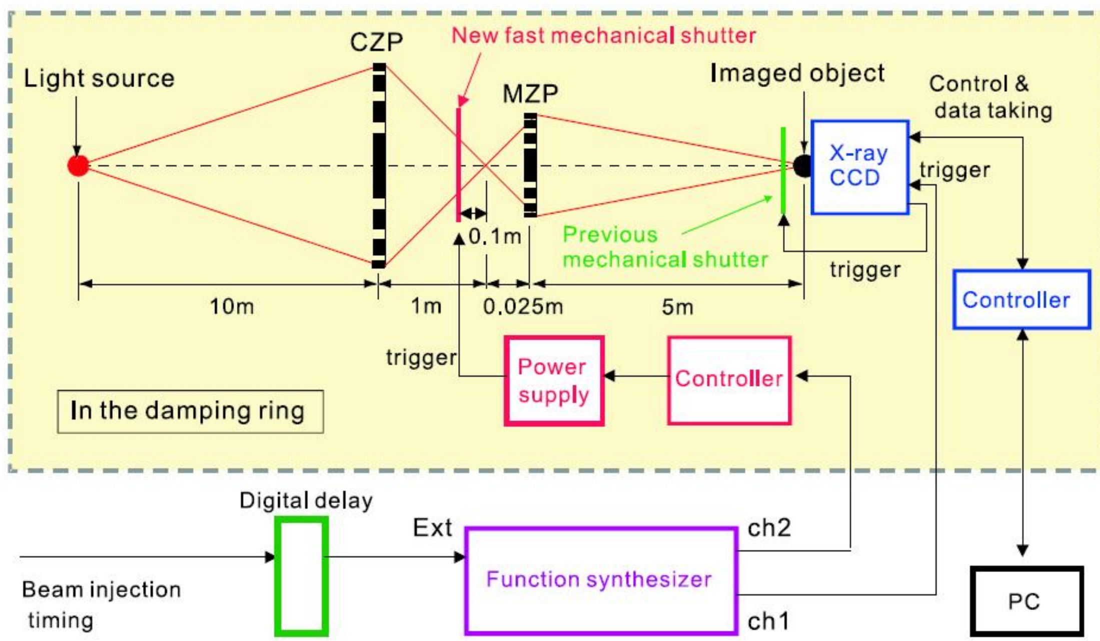
Zone-Plate Imaging at the ATF at KEK





< 1ms exposure time





Specifications of the ATF Zone Plates

TABLE II. Specifications of the two FZPs.

Fresnel zone plate	CZP	MZP
Total number of zone	6444	146
Radius	1.5 mm	37.3 μm
Outermost zone width Δr_N	116 nm	128 nm
Focal length at 3.24 keV	0.91 m	24.9 mm
Magnification	$M_{CZP} = 1/10$	$M_{MZP} = 200$

- Total magnification = 20
- Detecting 3.24-keV photons (λ = 0.383 nm)
  - Where's the monochromator? A pinhole at the intermediate waist can be used to reject defocused light at other wavelengths.

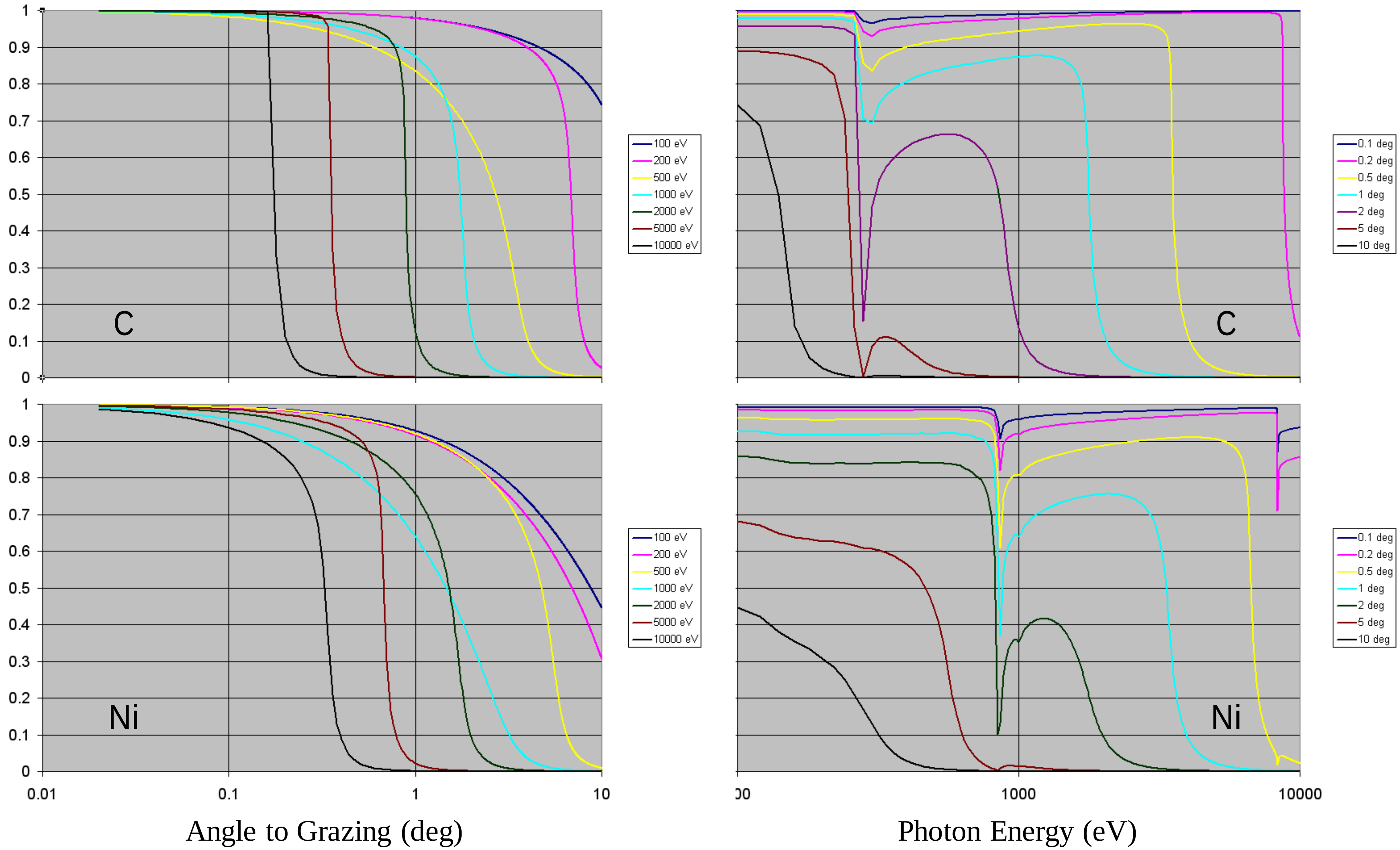
Grazing-Incidence X-Ray Mirrors

- A plasma with electron density n_e has characteristic oscillations of charge and electric field at the "plasma frequency" $\omega_p = \sqrt{n_e e^2 / (\varepsilon_0 m_e)}$
- The index of refraction of the plasma is $n = 1 - \omega_p^2 / \omega^2 < 1$
- The free electrons in a metal act like a plasma.
  - Visible frequencies are cut off: $\omega << \omega_p$ and so $n < 0$
  - X rays are transmitted: $\omega >> \omega_p$ and so n is slightly below 1
- Snell's law: $n_1 \sin\theta_1 = n_2 \sin\theta_2$, or $n_1 \cos\alpha_1 = n_2 \cos\alpha_2$
  - Here $\theta_1 = \pi/2 - \alpha_1$ and $\theta_2 = \pi/2 - \alpha_2$ are the ray angles to the normal.
- Total internal reflection occurs in medium 1 when $n_1 \sin\theta_1 > \mathrm{n}_2$
  - For x-rays in vacuum striking metal at an angle α to grazing:

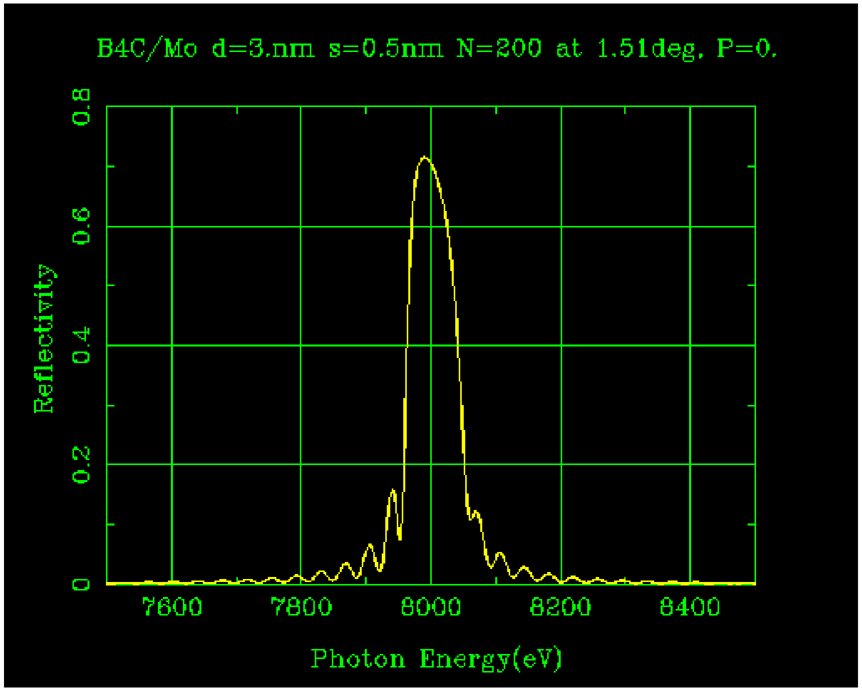
$$\cos\alpha > 1 - \omega_p^2 / \omega^2, \text{ or } \alpha < \sqrt{2}\,\omega_p / \omega \ll 1$$

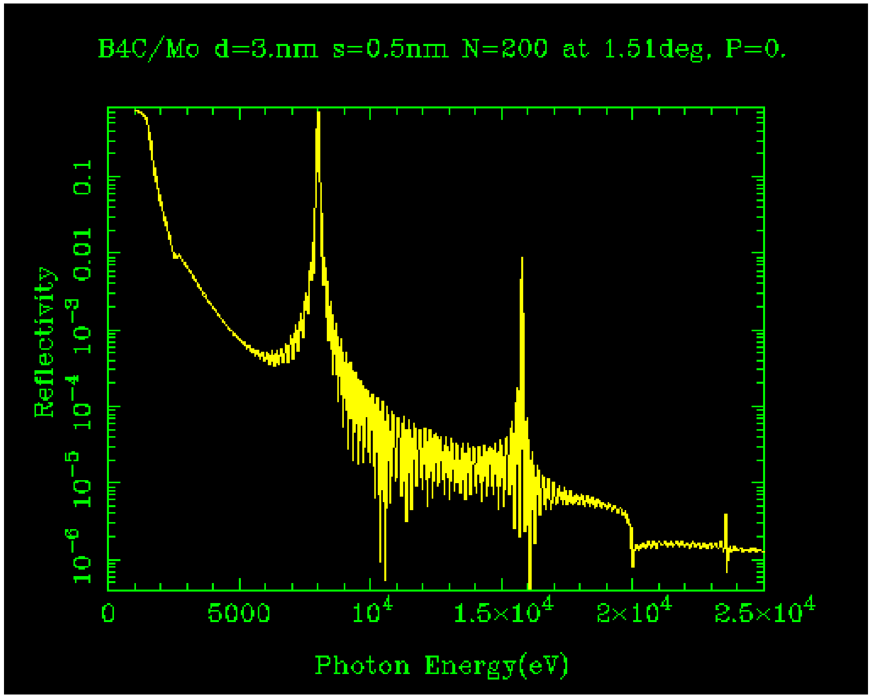
  - Mirrors at small angles to grazing can reflect x rays
    - Flat grazing-incidence mirrors
    - Multilayer mirrors that use interference to get a narrow bandwidth
    - Telescopes and imaging systems using off-axis conic surfaces

X-Ray Reflectivity of Materials



Multilayer X-Ray Mirrors



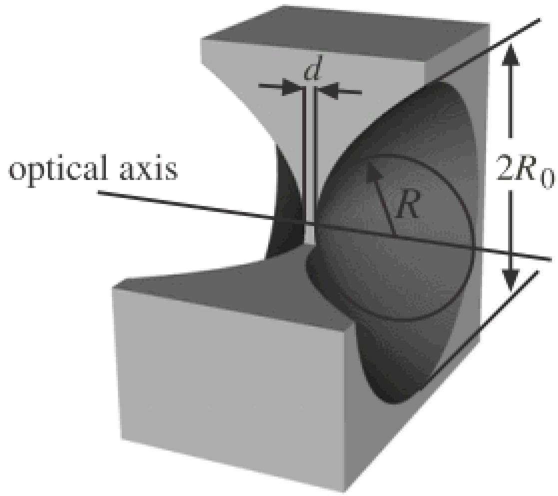


- Multilayer mirror designed for a narrow passband at 8 keV
  - Unpolarized light incident at 1.51 degrees
  - 200 periods with alternating layers of low- and high-Z materials: B_4C and Mo
  - 3-nm spacing: 2.1 nm of B_4C and 0.9 nm of Mo, with an interdiffusion thickness of 0.5 nm

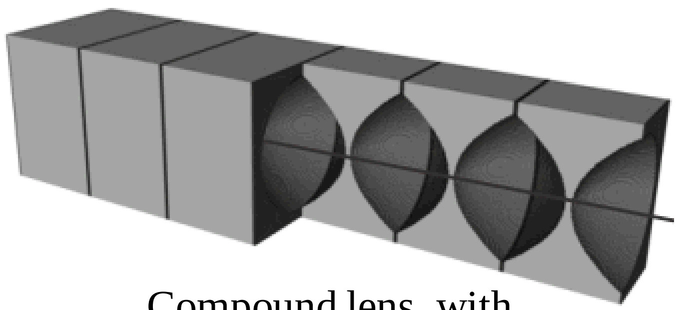
Refractive X-Ray Lenses

- The refractive index for x rays is slightly below 1 for any material.
- When a ray in vacuum strikes a material at a non-grazing angle, the transmitted ray enters with a deflection following Snell's law.
- A spherical surface can make an x-ray lens!
  - Collimated x rays: Use a parabolic surface
- $n < 1$: A focusing lens must be **concave.**
- $1 - n << 1$: Small deflection, long focal length
  - Stack many lenses for a shorter overall focal length.
  - To avoid absorption: Low-Z material (Be)
- $1 - n = \omega_p^2/\omega^2$: Strong chromatic aberration
  - Only monochromatic x-rays can focus.

Single focusing lens



Paraboloidal surface, typically with R = 0.2 mm and R_0 = 0.5 mm



Compound lens, with up to 300 elements

Beamline Design: Machine Constraints

- Distance to the first mirror (M1)
  - Ports and M1 itself introduce wakefields and impedance.
    - Is M1 flush with the vacuum-chamber wall?
  - The heat load on M1 is reduced by distance.
    - Is the mirror far down a synchrotron-light beamline?
- Distance to the imaging optics
  - In a hutch: Adds distance to get outside the shielding
  - In the tunnel: Inaccessible, but often necessary for large colliders.
- Size and location of the optical table
  - What measurements are needed?
  - Which instruments are available (affordable)?

Beamline Design: Optical Constraints

- Choose a source point with a large *y* size, to lessen effect of diffraction (for visible light).
- Magnification: Transform expected beam size to a reasonable size on the camera.
  - $6\sigma <$ camera size $< 12\sigma$: Uses many pixels; keeps the image and the tails on the camera; allows for orbit changes.
  - Needs at least two imaging stages: Since the optics are generally far from the source, the first focusing element strongly demagnifies.
- Optics: Use standard components whenever possible.
  - For example, adjust the design to use off-the-shelf focal lengths from the catalog of a high-quality vendor.
  - Use a color filter to avoid dispersion in lenses (or use reflective optics).
  - Correct lens focal lengths (specified at one wavelength) for your color.

Basic Design Spreadsheet

- You can iterate a lot of the basic design in a simple spreadsheet.
  - Enter the fixed distances.
  - Specify the desired magnifications.
  - Solve the lens equations, one stage at a time, to find lenses giving the ideal magnifications.
  - Change the lenses to catalog focal lengths.
  - Correct their focal lengths (using the formula for each material as found in many catalogs).
  - Iterate the magnifications and distances.
- Then optimize your design with optics software
  - I used Zemax for the CERN design.