Basics of photon detection and statistics

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Beam Diagnostics Using Synchrotron Radiation:
Theory and Practice
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Outline

- Pulsed LED to simulate the synchrotron radiation light
- Photodiode used as sensor to detect the light
- Statistics basics

Synchrotron Radiation Pulse

- Several ps to 100ps pulse lenth (RMS) from most of third generation light sources
- Broadband from IR to x-ray
- Huge number of photons per pulse (depends on current, magnet field strength, aperture, filters etc)
- Spatial distribution (σ, π – mode from dipole)

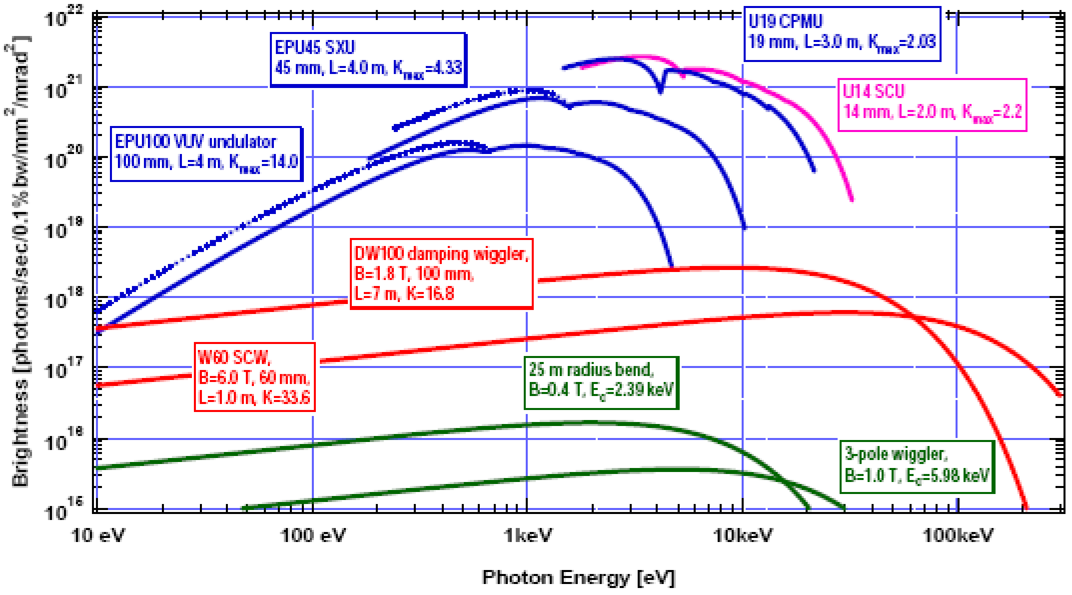
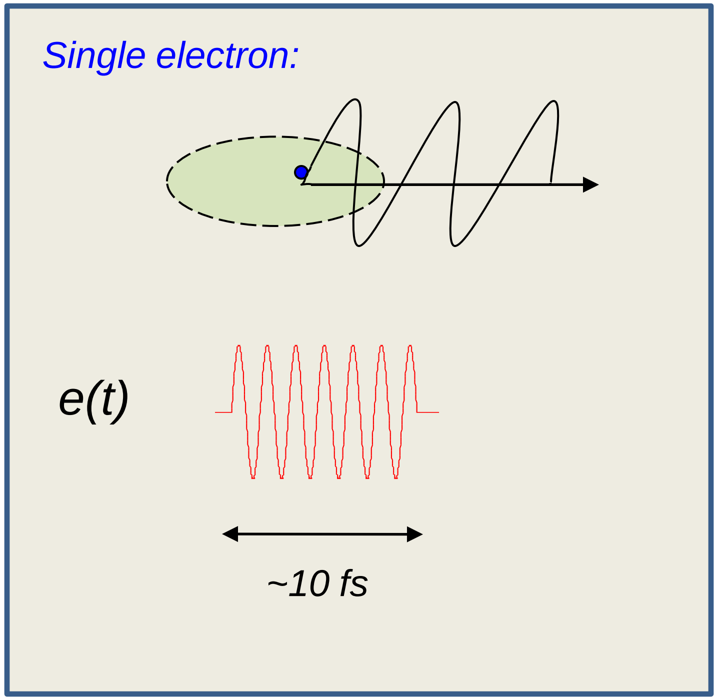


Figure 1. Brightness vs. photon energy for various devices at NSLS-II.

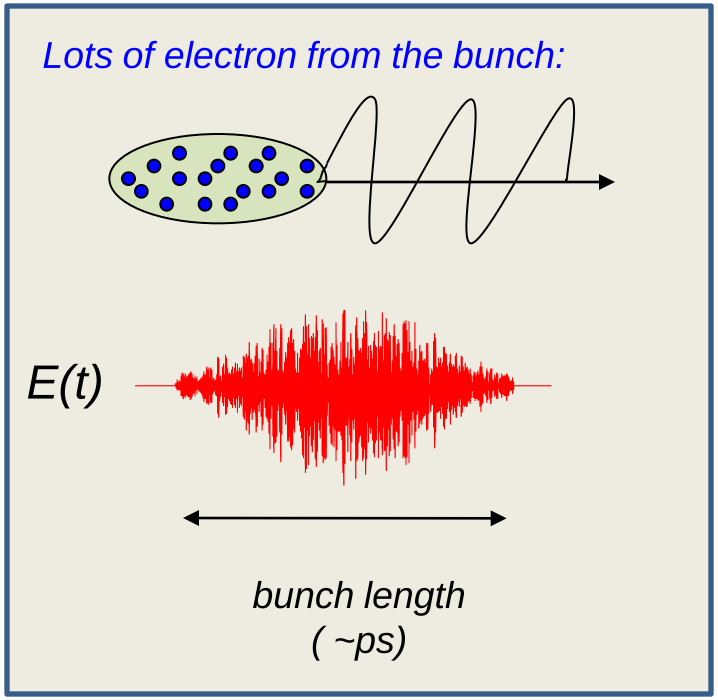
*These broadband spectrum are **average effect** of many electrons' radiation, or one electron radiation for many photons in long time scale.*

Radiation Theory (time-domain)

- One electron (*j*-th) in the bunch radiates an electromagnetic pulse ***e(t$_j$)***
- Total radiated field is $E(t)=\sum_{j=1}^{N} e(t-t_j)$



Single electron:



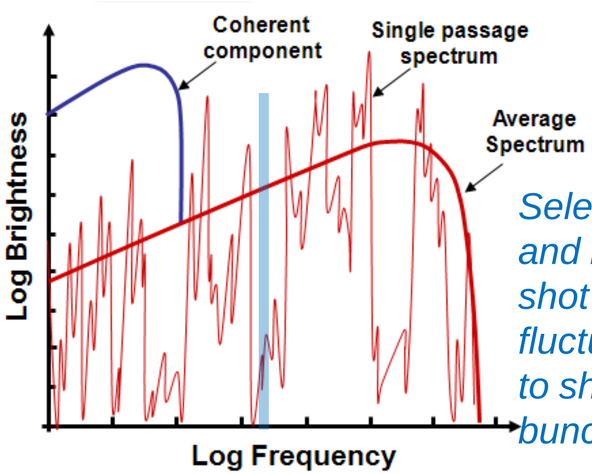
Lots of electron from the bunch:

Radiation Theory (frequency-domain)

$$\hat{E}(\omega)=\int_{-\infty}^{\infty}E(t)\,e^{i\omega t}dt=\hat{e}(\omega)\sum_{k=1}^{N}e^{i\omega t_k}$$

Radiation power per passage:

$$\boxed{P(\omega)\propto\left|\hat{E}(\omega)\right|^2=\left|\hat{e}(\omega)\right|^2\sum_{k=1}^{N}\sum_{l=1}^{N}e^{i\omega(t_k-t_l)}}$$



Select a narrow band and measure shot to shot radiations, the fluctuation of the shot to shot is related to bunch length.

Average power:

$$\langle P(\omega)\rangle\propto\left|\hat{e}(\omega)\right|^2\sum_{k,l=1}^{N}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}dt_k\,dt_l\,f(t_k)f(t_l)e^{i\omega(t_k-t_l)}=\left|\hat{e}(\omega)\right|^2\left[N+N(N-1)\left|\hat{f}(\omega)\right|^2\right]$$

Incoherent term

Coherent

$f(t)$ *is bunch distribution*

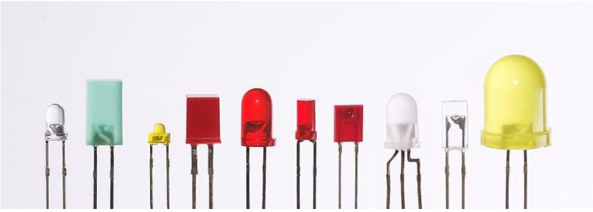
$$f(t)=\frac{1}{\sqrt{2\pi}\sigma_z}\exp\left(-\frac{z^2}{2\sigma_z^2}\right)$$

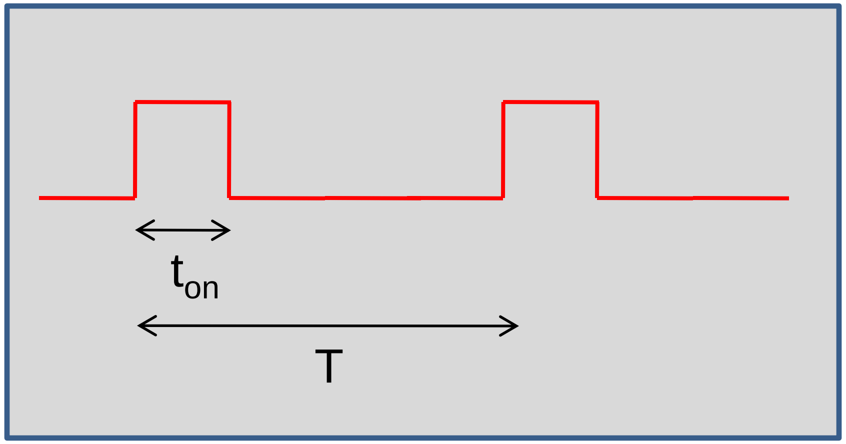
$$\hat{f}(\omega)=\int_{-\infty}^{\infty}f(t)e^{i\omega t}dt=\exp\left(-2\pi^2\left(\frac{\sigma_z}{\lambda}\right)^2\right)$$

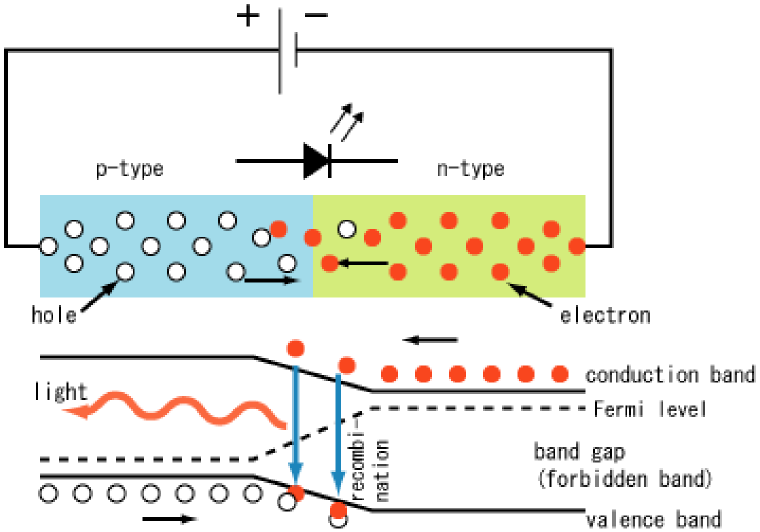
Simulated pulse from LED

LED – Light Emitting Diode

- Semiconductor light source
- Visible, UV, IR wavelength
- High brightness



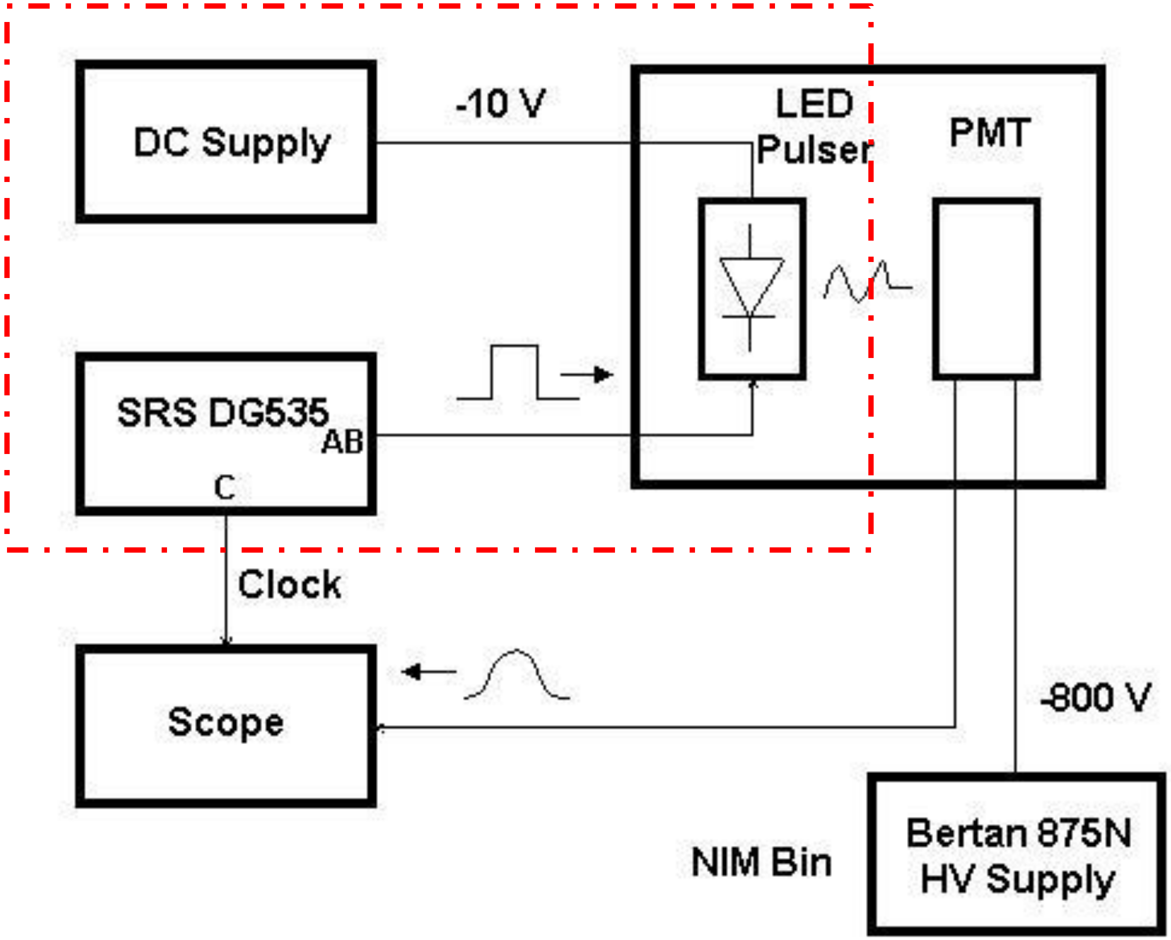




Pulse driver

50ps FWHM minimum laser pulse
Repetition rate 31.25kHz to 80MHz
Pulse energy adjustable
Laser heads from 375nm to 1550nm
LED heads from 255nm to 600nm
External trigger / sync output

Pulse LED with DG535



Typical rise time 2-3ns, hard to get pulse less than 1ns

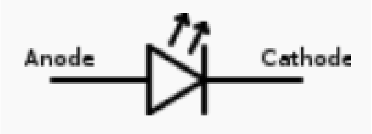
With fast rise time option, could get 100ps (20% - 80% amplitude)

Use ~10ns minimum due to LED limitations

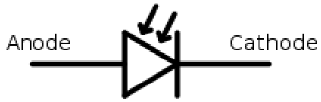
Photodiode

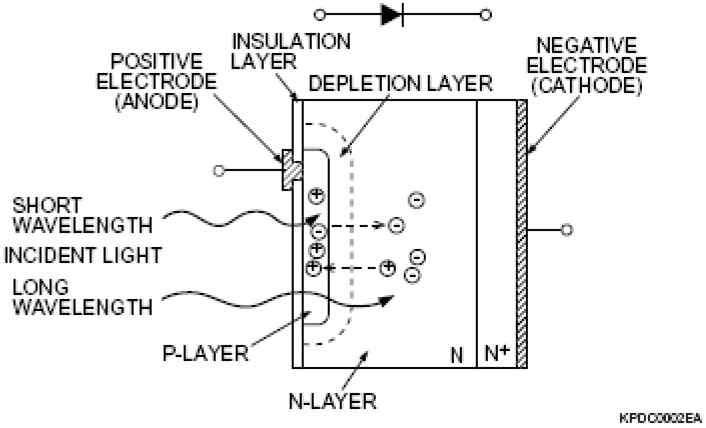
- Convert light to current/voltage signal
- Similar to regular semiconductor diode
- Photon hit the diode => create electron/hole pair
- Positive charge collected at P-layer, negative charge in the N-layer

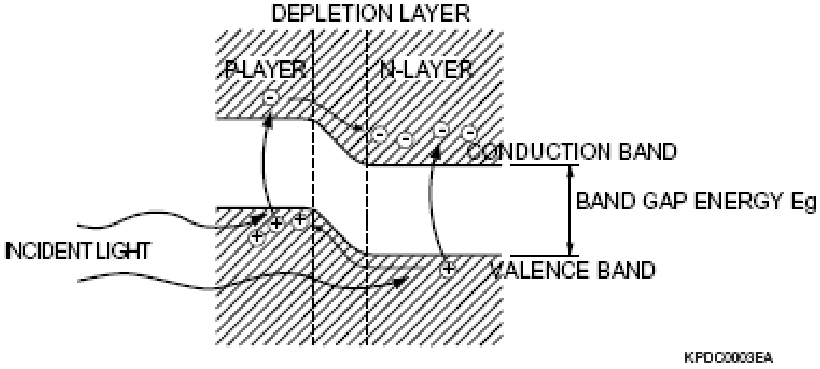
LED symbol



Photodiode symbol







APD principle

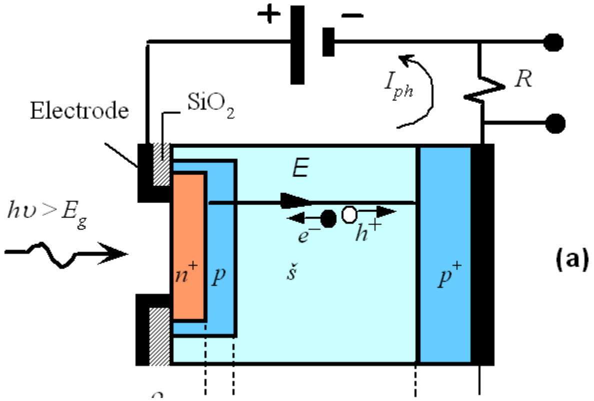
Diode operates in reverse bias mode

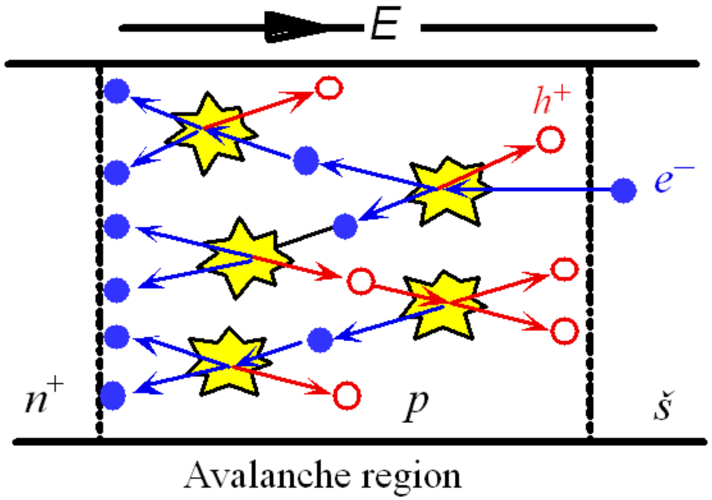
Electrons drift to N-layer with large kinetic energy, holes drift to P-layer
Impact ionization will generate mode electron-hole pair

Avalanche of impact ionization

Internal gain (typical gain 100 ~1000)

A single electron entering the p-layer can generate a large number of electron-hole pairs which contribute to an observed photocurrent



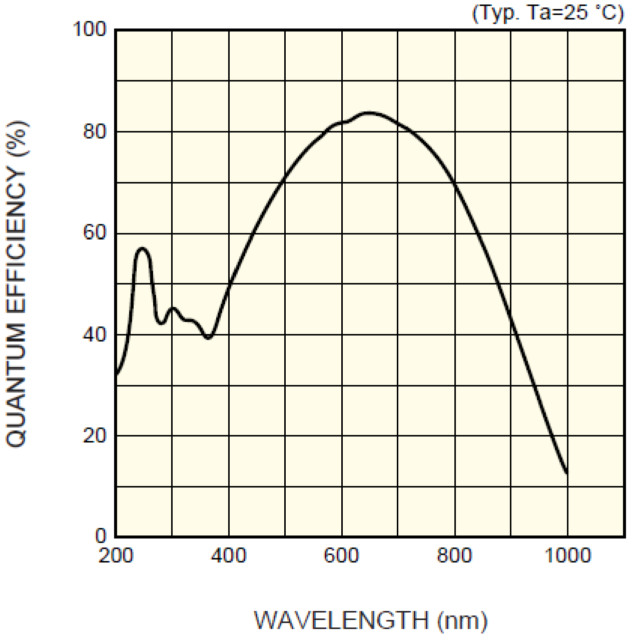


SPAD (Single Photon Avalanche Diode)

- SPAD is APD working in Geiger mode
- Reverse bias voltage well above the breakdown voltage. (APDs operate at a bias lesser than the breakdown voltage)
- Able to detect low intensity signals (down to the single photon)
- APD a single photon produces only tens or few hundreds of electrons, but in a SPAD a single photon triggers a current in the mA region (billions of billions of electrons per second) that can be easily "counted".
- APD is a linear amplifier for the input optical signal with limited gain (~hundreds), the SPAD is a trigger device so the gain concept is meaningless.
- SPAD used for bunch purity monitoring

APD Examples

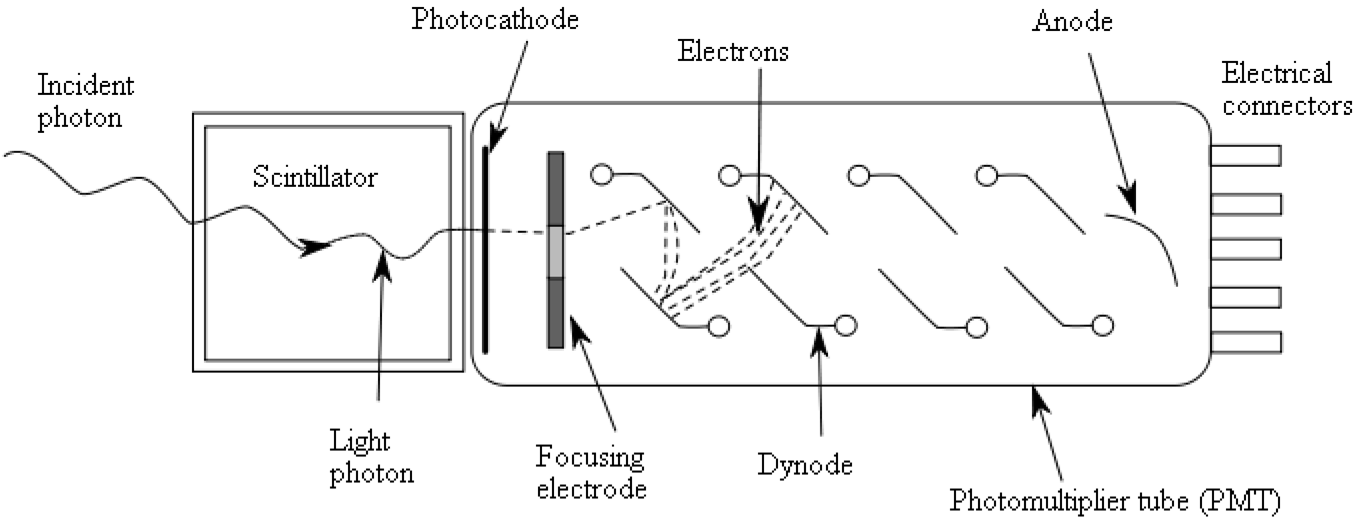
Hamamatsu Photonics S5343



PerkinElmer C30902

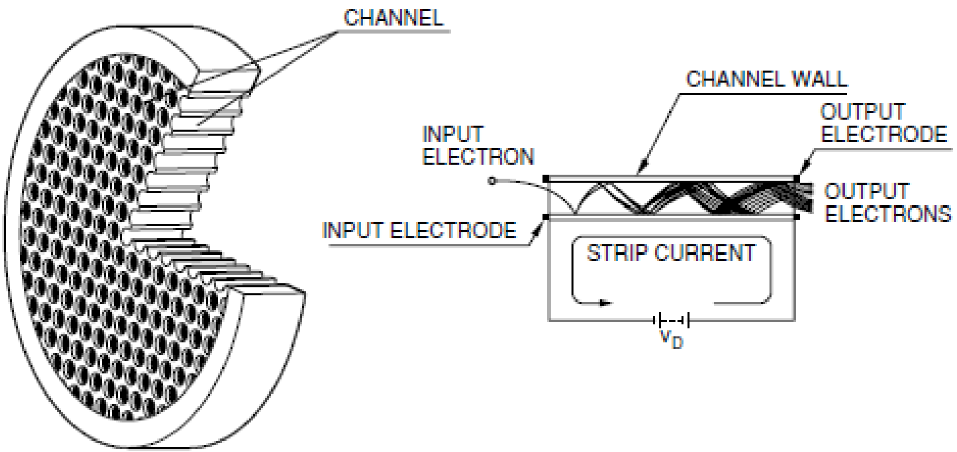
200nm – 1000nm, peak at 620nm

Photomultiplier tube

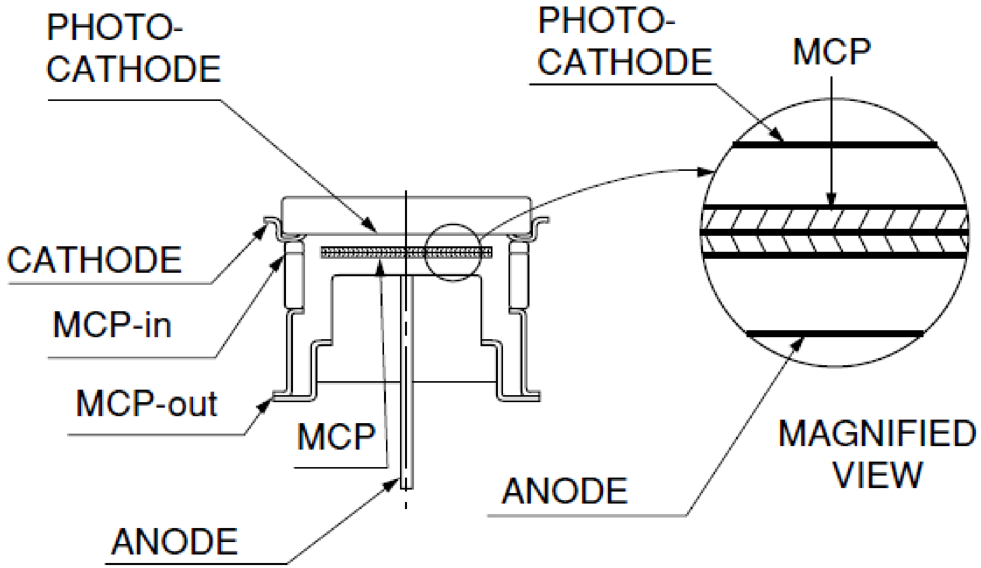


Photodiode	Photomultiplier
Good linearity Wide response spectral Low noise Low cost	High gain (~10^8) Fast response time High sensitivity (detecting low level light, 30 photons per second)

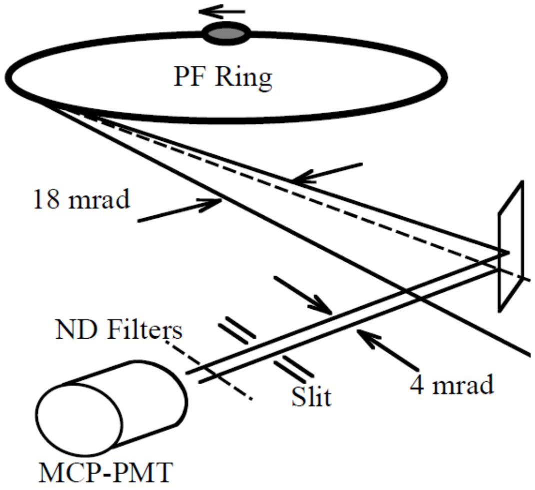
MCP-PMT



High gain
High spatial resolution
Fast time response

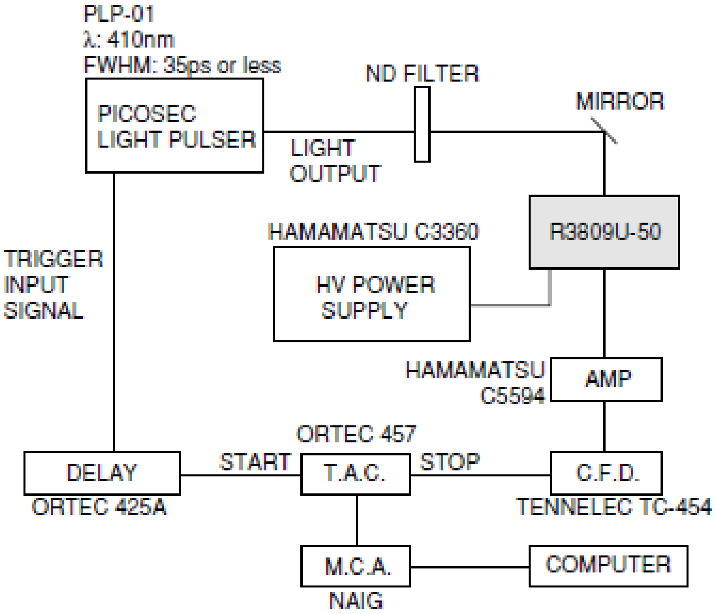


MCP-PMT used for bunch purity and filling pattern measurement



Hamamatsu R2809U-06 (or R3809U-52), used at PF, Spring-8, Diamond, ESRF etc. for the bunch purity and filling pattern monitor

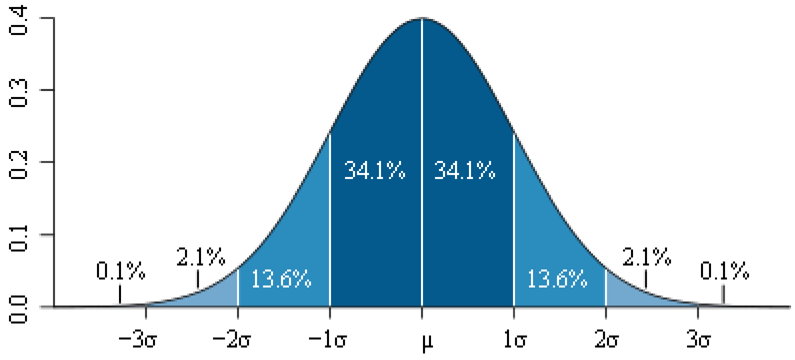
Transit Time Spread = 25ps, 160nm to 650nm peak at 400nm



Time Correlated Single Photon Counting (TCSPC)

Statistic of sampled data

Gaussian distribution



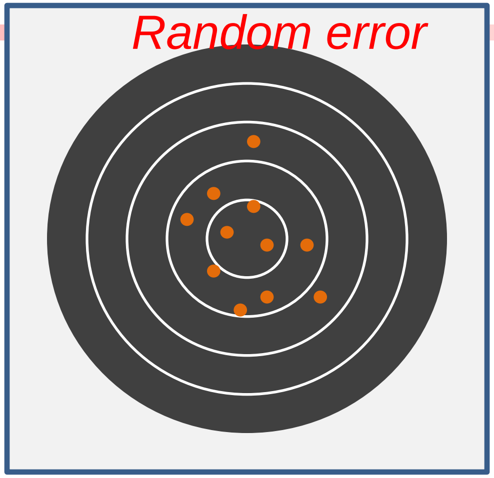
N samples,

Mean: $$\overline{x} = \frac{x_1 + x_2 + \cdots + x_N}{N} = \frac{1}{N}\sum_{i=1}^{N} x_i$$

Std: $$\sigma = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_i - \overline{x})^2}$$

$$\sigma = \sqrt{\frac{1}{N}\left(\left(\sum_{i=1}^{N} x_i^2\right) - N\overline{x}^2\right)} = \sqrt{\frac{1}{N}\left(\sum_{i=1}^{N} x_i^2\right) - \overline{x}^2}$$

Std of the mean: $$\sigma_{\mathrm{mean}} = \frac{\sigma}{\sqrt{N}}$$ *N* – number of samples


Random error

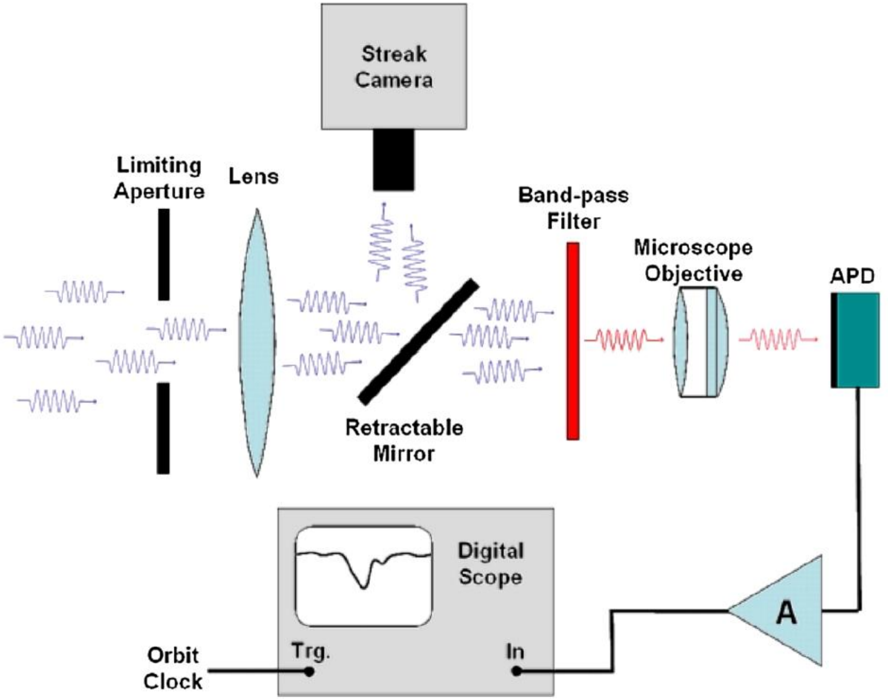

Systematic error

$$\sigma_{mean} = \frac{\sigma}{\sqrt{N}}$$

$$\delta x_{rand} = \sigma_{mean}$$

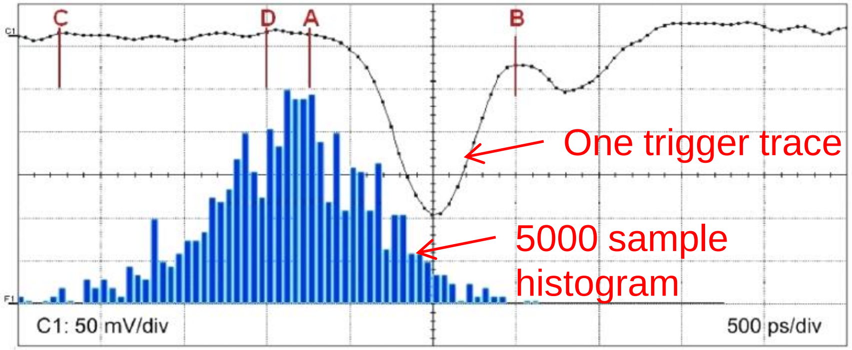
$$\delta x_{total} = \sqrt{(\delta x_{rand})^2 + (\delta x_{sys})^2}$$

Statistics in the fluctuation bunch length measurement



ALS Fluctuation Bunch Length Measurement
(1nm FWHM band pass filter at 632.8nm)

APD output signal



One trigger trace

5000 sample histogram

$$\sigma_{\delta^2}/\delta^2 = \sqrt{2/(N_S - 1)}.$$

Ns – number of sample, > 5000 to get <2% measurement error of δ^2

$$\delta^2 \simeq \frac{1}{\sqrt{1 + 4\sigma_\omega^2 \sigma_t^2}}$$

$$\delta^2 = \frac{\sigma_W^2}{\langle W \rangle^2}$$

W – radiation energy of per passage, represented by APD pulse amplitude or area