Beam-Beam Effects in Particle Colliders

Mauro Pivi

SLAC

US Particle Accelerator School

January 17-21, 2011 in Hampton, Virginia

Center of mass energy in a collision

Particle physicist usually assume $\hbar$=c=1, in this context (next 5 pages only).

To compute the collision energy between particles, let's define a new entity: the particle energy E and the classical 3-momentum ***p*** form a (Jargon point!) "4-momentum tensor":

$$\hat{p} = (E, \boldsymbol{p})$$

Center of mass energy in a collision

The scalar product between two particles momenta is defined as:

$$\hat{p}_1 \cdot \hat{p}_2 = E_1 E_2 - \boldsymbol{p_1} \cdot \boldsymbol{p_2}$$

And it is an "invariant" (reference frame independent).

Thus note that, since

$$E = \sqrt{|\boldsymbol{p}|^2 c^2 + m^2 c^4} = \sqrt{|\boldsymbol{p}|^2 + m^2}$$

where ($\hbar$=c=1!), then the product of the four-vector by itself is:

$$\hat{p} \cdot \hat{p} = E^2 - |\boldsymbol{p}|^2 = m^2$$

Center of mass energy in a collision

In a particle collision, the quantity:

$$s = (\hat{p}_1 + \hat{p}_2)^2$$

is a Lorentz-invariant. The center-of-mass energy available for physics experiments is:

$$\boxed{E_{cm} = \sqrt{s}}$$

Center of mass energy in a collision

Also:

$$s = (\hat{p}_1 + \hat{p}_2)^2 = (\hat{p}_1 + \hat{p}_2) \cdot (\hat{p}_1 + \hat{p}_2) =$$

$$= \hat{p}_1^2 + \hat{p}_2^2 + 2\hat{p}_1 \cdot \hat{p}_2 =$$

$$= E_1^2 - |\boldsymbol{p}_1|^2 + E_2^2 - |\boldsymbol{p}_2|^2 + 2E_1E_2 - 2\boldsymbol{p}_1\boldsymbol{p}_2 =$$

$$= (E_1 + E_2)^2 - (\boldsymbol{p}_1 + \boldsymbol{p}_2)^2 =$$

$$= m_1^2 + m_2^2 + 2E_1E_2(1 - \beta_1\beta_2 \cos\theta)$$

Where the relativistic $\beta = \boldsymbol{p}/E$ *and* θ is the collision angle (and $\theta = 0$ represents an head-on particle collision). Remember: $c = 1$.

Center of mass energy in a collision

Check point

In a linear collider with $\boldsymbol{p}_1 = -\boldsymbol{p}_2$ the c.m. energy is …

$$E_{cm} = \sqrt{(E_1 + E_2)^2}$$

Note: due to “beamstrahlung” and “crossing angle” (see later), generally the center of mass energy is lower ….

$$E_{cm} < 2\,E$$

Thus, the collision is not a “clean monochromatic” process but there are spurious effects that lower the collision energy and introduce an energy spread. This results in a “luminosity spectrum dilution”

Luminosity

Why is the **Luminosity** the most important parameter* in the design of a particle collider? (*with the Energy)

During particles collision, for sufficiently high energy, several fundamental physics processes occur.

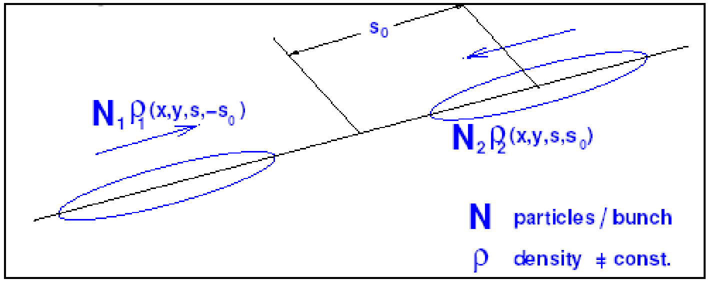
The event rate of a specific process is given as

$$\frac{dN_{ev}}{dt}(\sec^{-1}) = L(cm^{-2}\sec^{-1})\ \sigma_{ev}(cm^{2})\ Br\ \varepsilon$$

"Br" is the branching ratio, ε detection efficiency. The cross section σ is a fixed number dependent on the specific physics process only, while the Luminosity "L" is controlled by the beam parameters of the collider.

Luminosity

Generally, the luminosity is defined as:



$$L = \frac{N^2}{A} n_b f$$

where *N* is the number of particle per bunch, *A* is the transverse area collision area, n_b is the number of bunches in a train and *f* is the collider frequency.

For transverse Gaussian beam distributions

$$L = \frac{N_1 N_2}{4\pi\sigma_x^* \sigma_y^*} n_b f \ H_D$$

where we have introduced H_D, an enhancement factor due to the "pinching" of particles when they cross the field of the opposite bunch [see later].

LC parameters

Parameter	ILC 500 GeV	ILC 1TeV	CLIC 3TeV
E beam [GeV]	250	500	1500
$\gamma\varepsilon_x$ / $\gamma\varepsilon_y$ [10^{-6} m]	10 / 0.04	10 / 0.04	0.66 / 0.02
N [10^{10}]	2	2	0.372
σ_x / σ_y [nm]	655 / 5.7	554 / 3.5	45 / 1
σ_z [um]	300	300	44
N_b bunches	2820	2820	312
bs [ns]	307.7	307.7	0.5
f [Hz]	5	4	50
θ_c [mrad]	14	14	20
L [10^{34} $cm^{-2}s^{-1}$]	2.0	2.0	5.9

Comparison of beam-beam effects in Ring Colliders and Linear Colliders

Beam-beam effects are dramatically different in linear colliders and storage rings

In Ring Colliders

Beams are re-used

Single collisions are “gentle” and multiple turns are important ➔ resonances

⇩

Luminosity is made with large *f*, and large collision area

$$L = \frac{N^2}{4\pi\sigma_x\sigma_y} f$$

Wealth of experimental experience

Comparison of beam-beam effects in Ring Colliders and Linear Colliders

Beam-beam effects are dramatically different in linear colliders and storage rings

In linear colliders

Used only once

Single collisions are "violent"

Drastic bunch deformation during collision

$\Downarrow$

Luminosity is made with small *f*, and small collision area

$$L = \frac{N^2}{4\pi\sigma_x\sigma_y} f$$

Limited experimental experience

Check: what is this f in a LC depend on? Why it is smaller and different than storage rings?

Beam-beam in Storage Rings

Let’s build the one-turn linear transfer map combination of the linear optics and the effect of beam-beam at IP.

For a (vertical) tune Q_y, and β_y at IP the one-turn transfer map for the vertical coordinates is given by:

$$M_1 = \begin{pmatrix} \cos 2\pi Q_y & \beta_y \sin 2\pi Q_y \\ -\dfrac{1}{\beta_y} \sin 2\pi Q_y & \cos 2\pi Q_y \end{pmatrix}$$

The (vertical) beam-beam kick of strength δ can be represented by a quadrupole-like kick:

$$B = \begin{pmatrix} 1 & 0 \\ -\delta & 1 \end{pmatrix} \qquad \delta = \frac{2Nr_0}{\gamma\sigma_y(\sigma_x + \sigma_y)}$$

Beam-beam in Storage Rings

Thus, the (vertical) one-turn matrix with the beam-beam kick is:

$$M = M_1 \cdot B = \begin{pmatrix} \cos(2\pi Q_y) - \delta\beta_y \sin(2\pi Q_y) & \beta_y \sin(2\pi Q_y) \\ -\dfrac{1}{\beta_y}\sin(2\pi Q_y) - \delta\cos(2\pi Q_y) & \cos(2\pi Q_y) \end{pmatrix} =$$

or also ...

$$M = \begin{pmatrix} \cos(2\pi(Q_y + \xi_y)) & \beta_y \sin(2\pi(Q_y + \xi_y)) \\ -\dfrac{1}{\beta_y}\sin(2\pi(Q_y + \xi_y)) & \cos(2\pi(Q_y + \xi_y)) \end{pmatrix}$$

Where we have assumed that the new tune is a small perturbation ($Q_y + \xi_y$). Now, we take the Trace of the turn map to extract the new perturbed tune:

$$Tr(M) = 2\cos 2\pi(Q_y + \xi_y) = 2\cos 2\pi Q_y - \beta_y \delta \sin 2\pi Q_y$$

Beam-beam in Storage Rings

Where the so called incoherent "beam-beam" tune shift $\xi_y \ll 1$ is

$$\xi_y = \beta_y \delta / 4\pi = \frac{N r_0 \beta_y}{2\pi\gamma\sigma_y(\sigma_x + \sigma_y)}$$

Assuming $\sigma_y \ll \sigma_x$ a Ring collider luminosity is proportional to the beam-beam tune shift

$$\boxed{L \approx \frac{N f k \xi_y \gamma}{2 r_0 \beta_y}}$$

Where typically:

ξ_{max} ~.04 for e^+e^-
~.01 for pp

Beam-beam effects in Linear Colliders: the Good, the Bad and the Ugly

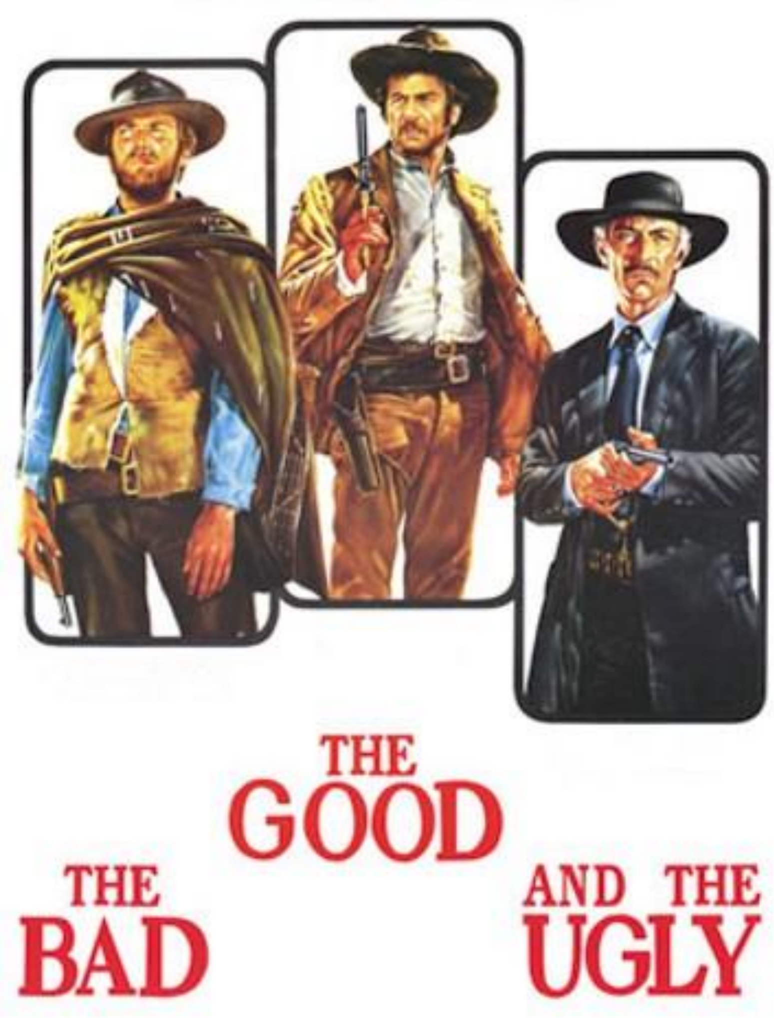
In Linear Colliders, to get the maximum luminosity out of the collision one needs small nanometer beam sizes.

$$L \propto \frac{N^2}{\sigma_x^* \sigma_y^*}$$

When the beams are small, the particle densities are very high.

Also, the strong beam *demagnification* at the IP to obtain nanometer beam sizes requires a very strong focusing (high magnetic fields) by the final doublet quadrupoles.

Beam-beam effects in Linear Colliders: the Good, the Bad and the Ugly



Beam-beam effects in Linear Colliders: the Good, the Bad and the Ugly

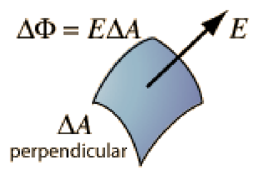
High beam densities lead to the following good or bad beam-beam effects:

- GOOD: Strong *pinching* effect of the bunches enhance luminosity
- BAD: Instability effects sets tight collision tolerances at IP
- BAD: high *beamstrahlung* radiation with luminosity spectrum dilution
- UGLY: pairs production, e^+ e^- generated by the radiation propagating in the strong field of the bunches are source of Detector Background

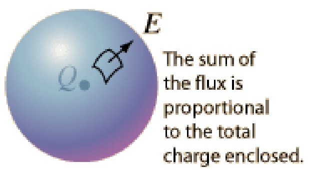
Beam-beam effects in linear Colliders: the Good, the Bad and the Ugly

Reminder: Gauss' Law

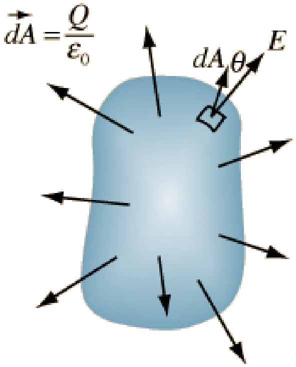
The total of the electric flux out of a closed surface is equal to the charge enclosed divided by the permittivity.



$$\Phi_{electric} = \frac{Q}{\varepsilon_0}$$



$$\oint \vec{E} \cdot \vec{dA} = \frac{Q}{\varepsilon_0}$$



The electric flux through an area is defined as the electric field multiplied by the area of the surface projected in a plane perpendicular to the field.

For geometries of sufficient symmetry, it simplifies the calculation of the electric field.

Example: Electric Field of a infinite cylinder of charge

Let’s compute the electric field of an infinite cylinder of uniform volume charge density and radius R. Use Gauss’ law. This example will be useful later to compute the electric field for a beam of particles.

First: Consider a surface in the form of a cylinder at radius r > R, the electric field is directed outward. The electric flux is then just the electric field times the area of the cylinder. Then: do the same for r < R.

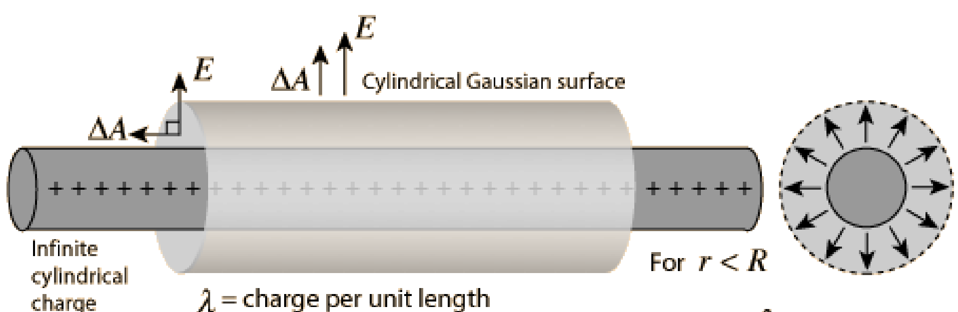
$$\Phi = E2\pi rL = \frac{\lambda L}{\varepsilon_0}$$

For $r \geq R$

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

This expression is a good approximation for the field close to a long cylindrical charge.

…plot E for r∈[-10R,10R] (assume λ=1e-9 C/m)



$$E = \frac{\lambda r}{2\pi\varepsilon_0 R^2}$$

Exercise (1): prove this

Electric and Magnetic fields in a beam

First we compute the Electric field E.

To compute the Electric field of a relativistic beam, we consider three cases for the transverse beam charge distribution:

– Round Gaussian beam

– Flat Gaussian beam (exact)

– Flat Gaussian beam (approximation)

Electric and Magnetic Field for a Round Gaussian beam

The beam-beam interaction is a purely Coulomb interaction of particles in the electric and magnetic field of the opposite bunch.

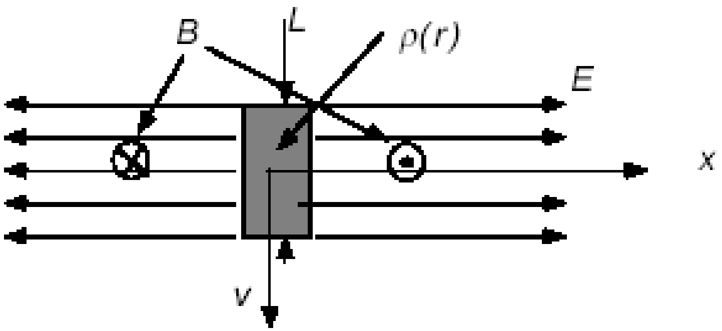
Let's have a closer look at the electric and magnetic field of a relativistic bunch and, to simplify the description, with a Round σx=σy beam size and a Gaussian charge distribution in the transverse direction:

$$\rho(r) = \frac{Ne}{2\pi\sigma^2} \exp\left(-\frac{r^2}{2\sigma^2}\right)$$

Let's apply Gauss' law to compute the field at a distance *r* from the bunch axis, assume a bunch of length L

Gauss' law →

$$\boxed{\int_S \vec{E} \cdot \vec{n}\, da = \frac{Q}{\varepsilon_0}}$$



Electric Field for a Round Gaussian beam

The electric charge contained in the bunch at a radial distance *r* is obtained by integrating the charge distribution:

$$Q = \int\int \rho(r)\, r\, dr d\phi = \frac{Ne}{2\pi\sigma^2}\int_0^{2\pi}\int_0^r r \exp\left(-\frac{r^2}{2\sigma^2}\right) dr d\phi$$

We use Gauss' Law to obtain the Electric field *E* at the radial distance *r* which contains the electric charge Q

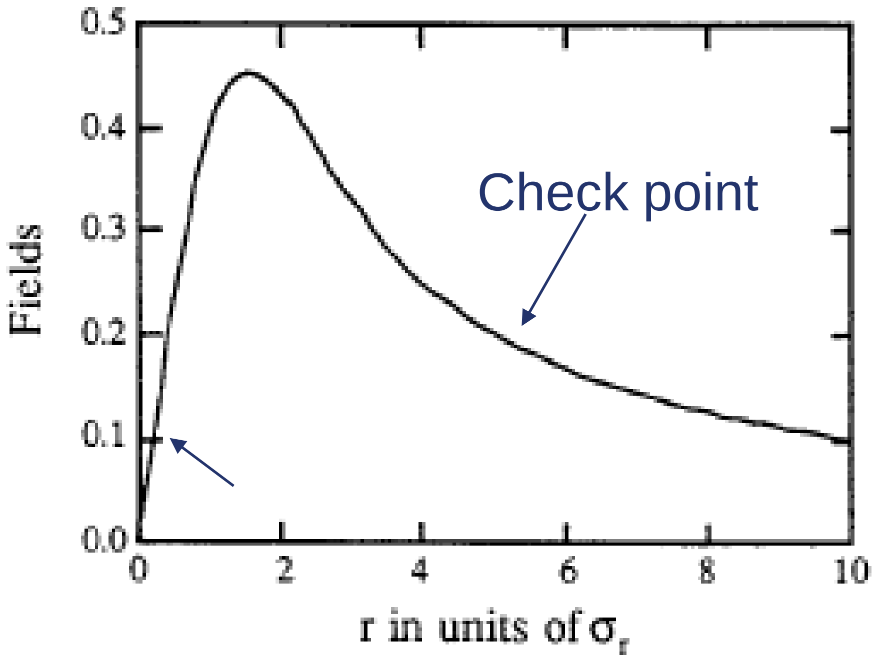
$$\int_S \vec{E}\cdot\vec{n}\, da = E 2\pi r L = \frac{Q}{\varepsilon_0} = \frac{Ne}{2\pi\varepsilon_0\sigma^2}\int_0^{2\pi}\int_0^r r \exp\left(-\frac{r^2}{2\sigma^2}\right) dr\, d\phi$$

Electric Field for a Round Gaussian beam

Finally from the previous equation, the electric field is:

$$\vec{E} = \frac{Ne}{2\pi\varepsilon_0 r L}\left(1 - \exp\left(-\frac{r^2}{2\sigma^2}\right)\right)\hat{r}$$

This is the electric field in the radial direction and at a distance *r*

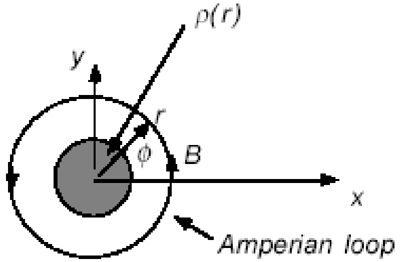


Electric field of a bunch with Gaussian charge distribution (radial distance in units of transverse beam size)

Electric and Magnetic Field for a round Gaussian beam

Similarly, we compute the magnetic field *B* by applying the Ampere's law

Ampere's law → $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$



$$\oint \vec{B} \cdot d\vec{l} = B 2\pi r = \mu_0 I = \mu_0 \rho v = \mu_0 \frac{Nev}{2\pi\sigma^2 L} \int_0^{2\pi} \int_0^{r} r \exp\left(-\frac{r^2}{2\sigma^2} \right) dr\, d\phi$$

check

$$\vec{B} = \mu_0 \frac{Nev}{2\pi r L} \left(1 - \exp\left(-\frac{r^2}{2\sigma^2} \right) \right) \hat{\phi}$$

$$B_\phi = \beta E_r / c$$

Force and angular kick round Gaussian beam

The Lorentz force experienced by a particle of charge –*e* (electron) when travelling in the $-\hat{s}$ direction through the opposite (positron) bunch is

$$\vec{F} = -e(\vec{E} + \vec{v} \times \vec{B}) = -\frac{Ne^2}{2\pi\varepsilon_0 rL}\left(1 - \exp\left(-\frac{r^2}{2\sigma^2}\right)\right)\left(\hat{r} - \hat{s} \times \hat{\phi}\frac{v^2}{c^2}\right) =$$

$$= -\frac{Ne^2}{2\pi\varepsilon_0 rL}\left(1 - \exp\left(-\frac{r^2}{2\sigma^2}\right)\right)\left(1 + \frac{v^2}{c^2}\right)\hat{r} \approx -\frac{Ne^2}{\pi\varepsilon_0 rL}\left(1 - \exp\left(-\frac{r^2}{2\sigma^2}\right)\right)\hat{r}$$

where we have used the relations $c = 1\big/\sqrt{\varepsilon_0 \mu_0}$ **and** $\beta = v/c \approx 1$

This force is for <u>round</u> Gaussian distribution only.

[Exercise (2). Can you show this? For particles in the same bunch: the electric and magnetic fields “tend to” cancel each others (!) and the force scale as γ^{-2} … so that the total force is negligible.]

Angular kick during collision (Round Gaussian beam)

Force is computed for a longitudinal bunch of length L. The transverse angular kick is obtained by integrating over the bunch length and considering that the relative speed of the two beams is *2c*

$$p_r = \int_{-\infty}^{+\infty} F(2ct)\,dt = \frac{1}{2c}\int_{-\infty}^{+\infty} F(z)\,dz \quad \text{or}$$

$$p_r = \frac{1}{2c}F(z)\Delta s = \frac{1}{2c}F(z)L = -\frac{Ne^2}{2\pi\varepsilon_0 cr}\left(1-\exp\left(-\frac{r^2}{2\sigma^2}\right)\right)$$

the angular kick →

$$\Delta r' = \frac{p_r}{p_0} = -\frac{2Nr_e}{\gamma}\frac{1}{r}\left(1-\exp\left(-\frac{r^2}{2\sigma^2}\right)\right)$$

in the vertical direction

$$\Delta y' = -\frac{2Nr_e}{\gamma}\frac{y}{x^2+y^2}\left(1-\exp\left(-\frac{x^2+y^2}{2\sigma^2}\right)\right)$$

$r_e = e^2/4\pi\varepsilon_0 mc^2$

$\Delta x'$ obtained exchanging x → y.

Electric and Magnetic fields in a beam

First we compute the Electric field E.

To compute the Electric field of a relativistic beam, we consider three cases for the transverse beam charge distribution:

- Round Gaussian beam
- Flat Gaussian beam (exact)
- Flat Gaussian beam (approximation)

Electric field for a relativistic flat beam (exact)

The electric field can be computed exact with Gauss law in a closed form, for example Basetti-Erskine formula with σx > σy :

$$E_x = \frac{Ne}{2\varepsilon_0 \varsigma} \mathrm{Im}\left(W\left(\frac{x+iy}{\varsigma}\right) - e^{-\left(\frac{x^2}{2\sigma_x^2}+\frac{y^2}{2\sigma_y^2}\right)} W\left(\frac{x\frac{\sigma_y}{\sigma_x}+iy\frac{\sigma_x}{\sigma_y}}{\varsigma}\right)\right) \frac{e^{-\left(\frac{(z-ct)^2}{2\sigma_z^2}\right)}}{\sqrt{2\pi}\sigma_z}$$

$$E_y = \frac{Ne}{2\varepsilon_0 \varsigma} \mathrm{Re}\left(W\left(\frac{x+iy}{\varsigma}\right) - e^{-\left(\frac{x^2}{2\sigma_x^2}+\frac{y^2}{2\sigma_y^2}\right)} W\left(\frac{x\frac{\sigma_y}{\sigma_x}+iy\frac{\sigma_x}{\sigma_y}}{\varsigma}\right)\right) \frac{e^{-\left(\frac{(z-ct)^2}{2\sigma_z^2}\right)}}{\sqrt{2\pi}\sigma_z}$$

where $\varsigma = \sqrt{2\left(\sigma_x^2 - \sigma_y^2\right)}$ and W the complex error function

$$W(z) = e^{-z^2}(1 + i\ Erfi(z));\ Erfi(z) = Erf(iz)/i;\ Erf(z) = \frac{2}{\sqrt{\pi}}\int_0^z e^{-t^2}dt$$

Electric field for a relativistic flat beam (use approximation)

- Flat beam $\sigma_x >> \sigma_y$
- Assume
  - infinitely wide beam with constant density per unit length in *x*:

$$\rho(x) \approx \frac{1}{\sqrt{2\pi}\sigma_x}$$

  - Gaussian charge distribution in *y*:

$$\rho(y) \approx \frac{1}{\sqrt{2\pi}\sigma_y} \exp\left[-\frac{1}{2}\left(\frac{y}{\sigma_y}\right)^2\right]$$

Electric Field from a Relativistic Flat Beam

Use Gauss' theorem: $\oiint_s \mathbf{E}\cdot\mathbf{ds} = \dfrac{q}{\varepsilon_0}$

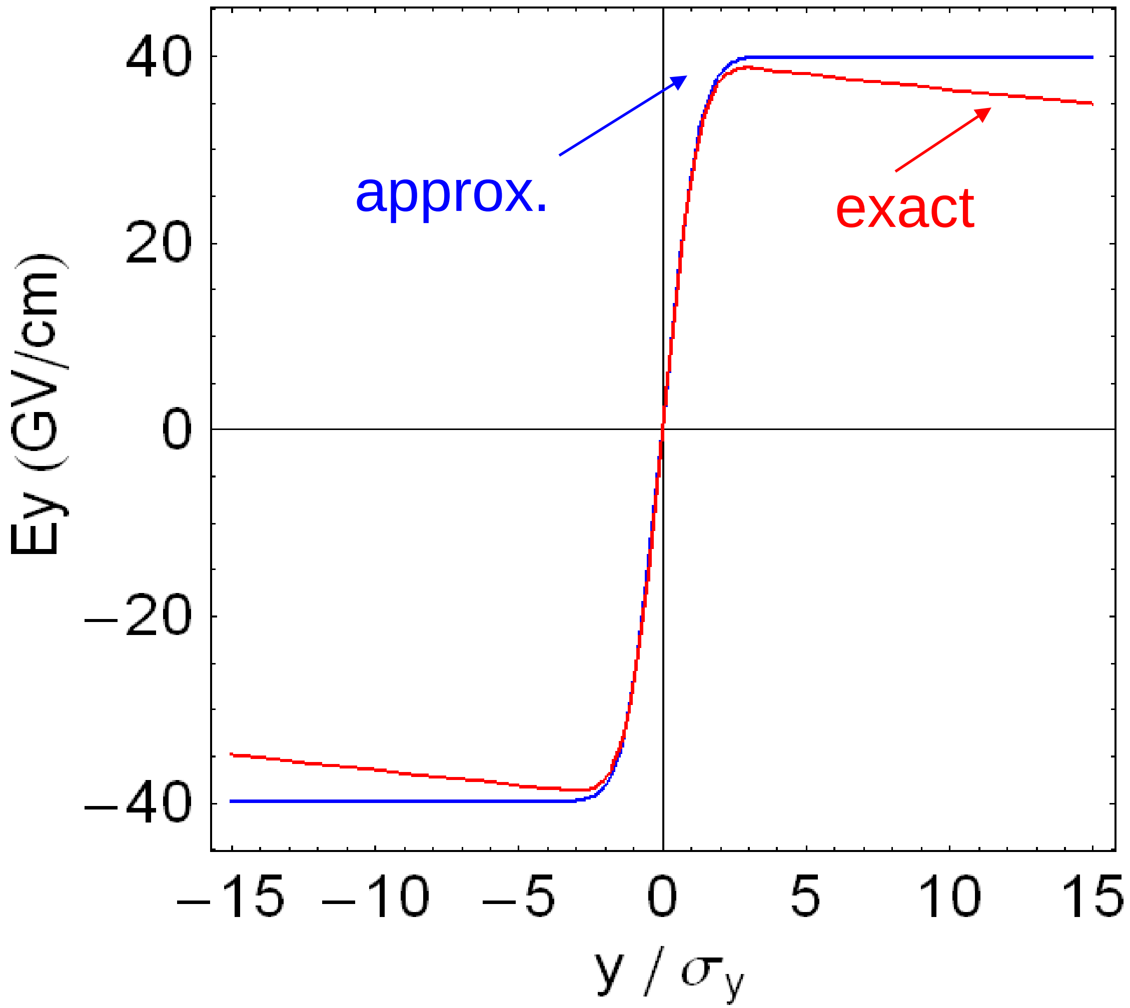
$$E_y(y,z)\Delta x\Delta z \approx \frac{qN\rho(x)\rho(z)\Delta x\Delta z}{\varepsilon_0}\int_{y'=0}^{y}\rho(y')dy'$$

$$E_y(y,z) = \frac{qN}{2\sqrt{2\pi}\,\varepsilon_0\sigma_x}\mathrm{Erf}\left(\frac{y}{\sqrt{2}\sigma_y}\right)\rho(z)$$

Assuming Gaussian distribution for *z*, the peak field is given by

$$\hat{E}_y = \frac{qN}{4\pi\varepsilon_0\sigma_x\sigma_z}$$

Electric field for a relativistic flat beam



a set of parameters for CLIC

N = 2.56 10^9

σx = 60 nm

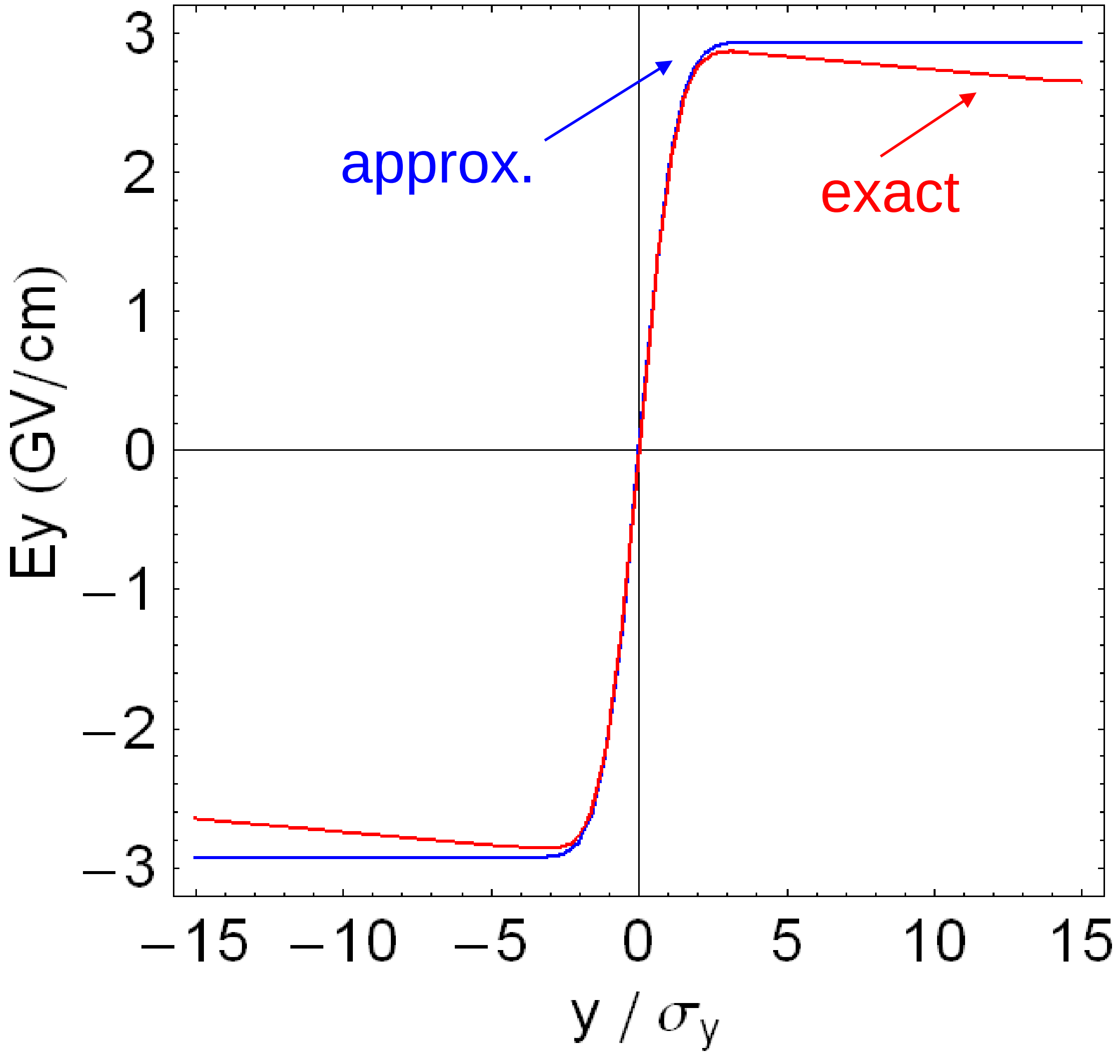
σy = 0.7 nm

σz = 30.8 um

peak field ~40 GV/cm!

2xEy plotted at peak of longitudinal density z=0

Electric field for a relativistic flat beam



ILC

N = 2. 10^{10}

σx = 655 nm

σy = 5.7 nm

σz = 300 um

peak fields
ILC ≤ CLIC / 10

Check point

2xEy plotted at peak of longitudinal density z=0

Linear approximation and Disruption

Fields near axis are linear. Then,

again, consider the angular kick for a <u>round</u> beam

$$\Delta r' = \frac{p_r}{p_0} = -\frac{2Nr_e}{\gamma}\frac{1}{r}\left(1 - \exp\left(-\frac{r^2}{2\sigma^2}\right)\right)$$

taking the linear field approximation, for a particle near the axis of the opposite beam

$$\Delta r' = -\frac{Nr_e}{\gamma\sigma^2} r$$

In the more general case of a Gaussian transverse beam distribution near the axis the beam kick results in the final particle angle:

$$\Delta x' = \frac{dx}{ds} = -\frac{2Nr_e}{\gamma\sigma_x\left(\sigma_x + \sigma_y\right)} \cdot x \qquad \Delta y' = \frac{dy}{ds} = -\frac{2Nr_e}{\gamma\sigma_y\left(\sigma_x + \sigma_y\right)} \cdot y$$

Linear approximation and Disruption

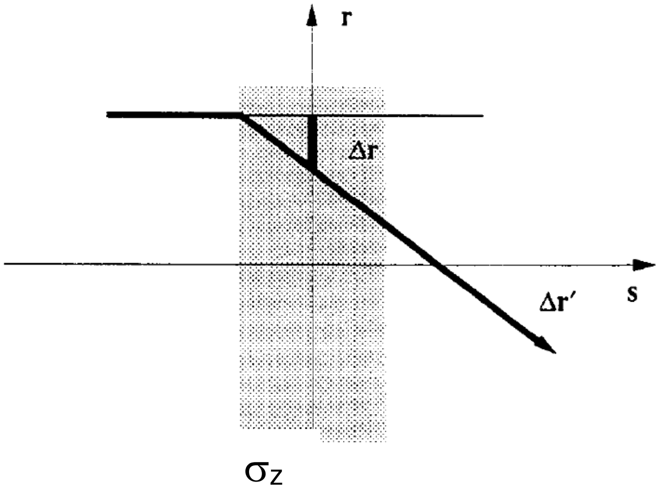
The bunch behaves like a focusing lens for a particle traveling in the opposite direction and is equivalent to a lens with focal lengths

$$\frac{1}{f_{x,y}} = \frac{2Nr_e}{\gamma\sigma_{x,y}\left(\sigma_x + \sigma_y\right)}$$

for CLIC, ILC:

f_y ~ 10um << σ_z !

if "lens" is strong as in linear colliders, both particle slope *Δr'* and position *r* change during bunch passage



$$\Delta r = \Delta r' \cdot \sigma_z$$

Since, from previous slide

$$\Delta r' = -\frac{1}{f} r$$

then

$$\Delta r = -\frac{\sigma_z}{f} r$$

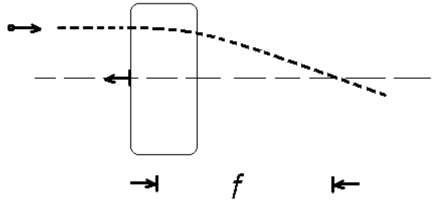
Initial radial position

Disruption limits

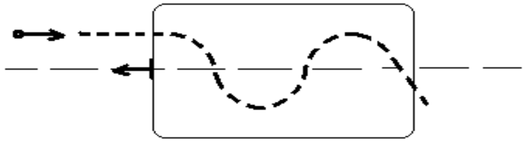
In the context of the linear colliders, the relative change in the transverse position is defined as the *disruption parameter*

$$D = \frac{\Delta r}{r} = -\frac{\sigma_z}{f} \qquad \Rightarrow \qquad \boxed{D_{x,y} \equiv \frac{2Nr_e}{\gamma} \frac{\sigma_z}{\sigma_{x,y}\left(\sigma_x + \sigma_y\right)}}$$

- A small disruption $D < 1$ means that the incoming beam is acting as a thin lens



- When fields are strong, disruption $D \gg 1$ and particles oscillate while passing through other beam potential. Notion breaks down $D \approx -\sigma_z/f$



n number of plasma oscillations

- Calculation of disruption effects is done with computer simulations
- If D is too big, an instability may take place that significantly reduces luminosity in the presence of small beam beam offsets

Disruption angle

The disruption angle is characterized by nominal deflecting angle

$$\theta_0 \equiv \frac{2Nr_e}{\gamma(\sigma_x + \sigma_y)} = \frac{D_x\sigma_x}{\sigma_z} = \frac{D_y\sigma_y}{\sigma_z}$$

The maximum and rms disruption angles obtained from computer simulations scaling laws for flat beams and in the limit $A_y \equiv \sigma_z / \beta_y \to 0$

$$\theta_{y,rms} \sim \frac{0.55\ \theta_0}{[1 + (0.5\ D_y)^5]^{1/6}}$$

$$\theta_{y,\max} \sim 2.5\ \theta_{y,rms}$$

Beam-beam deflections

One of the most important success of the Stanford Linear Collider SLC is the use of the beam-beam deflections as a tool for steering the micron beam-size beams into collision, for maintaining collision and for monitoring and tuning the transverse beam size at the IP and beams overlap.

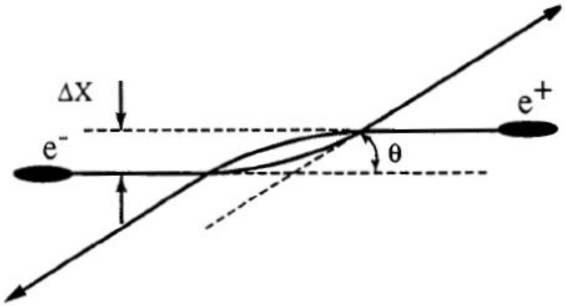


Fig. 5 Geometry of the beam-beam deflections

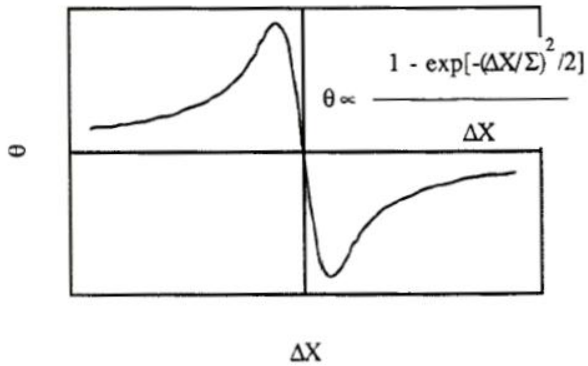
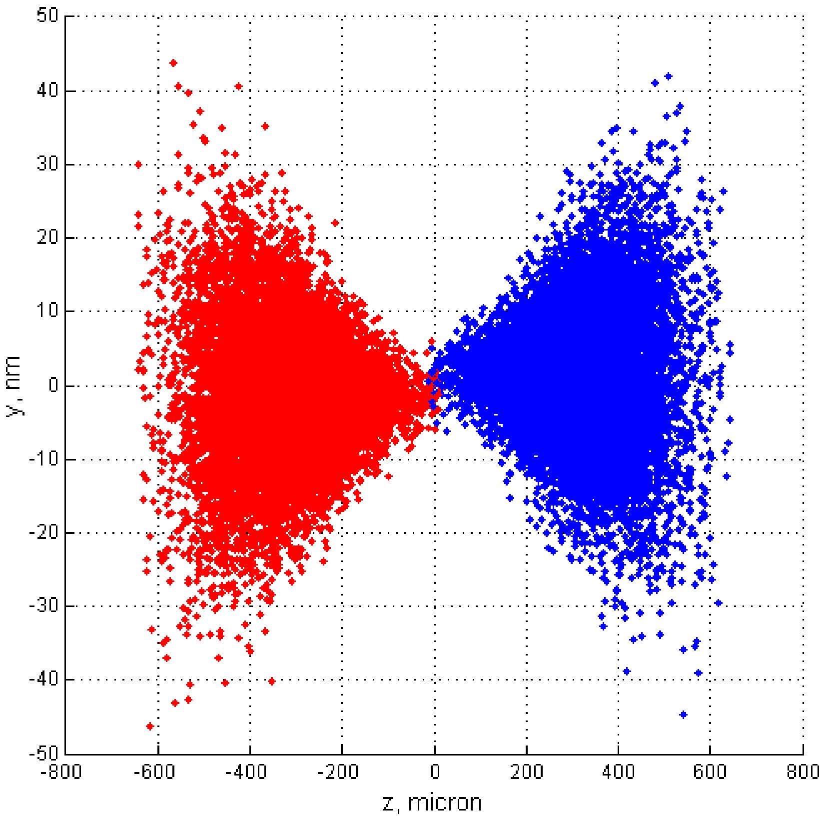


Fig. 6 Beam-beam deflection angle as a function of the transverse beams offset

Beam-beam simulations (with guinea-pig)

Note on units in **acc.dat**:

$ACCELERATOR:: YOURLC1
{ energy = 500 ; GeV
particles = 0.75 ; e10
sigma_x = 250 ; nm
sigma_y = 2.0 ; nm
sigma_z = 100 ; micron
beta_x = 5.0 ; mm
beta_y = 0.2 ; mm
offset_x = 0 ; nm (total offset will be 2*offset_x)
offset_y = 0 ; nm (-//-)
}



Analysing the results
Look for these useful numbers in the output file **gp.out**:

lumi_fine -- luminosity [1/m^2]
E_cm and E_cm_var -- CM energy and energy spread due to beamstrahlung [GeV]
bpm_vx, bpm_vy -- average angular beam deflection after collision [microrad]
upsmax -- max value of Upsilon parameter

to extract **luminosity spectrum** from gp.out run:
gpv.exe gp.out lumi_ee

Example of output is in gp.out and luminosity spectrum is shown in lumi_ee.dat

Simulation Procedure

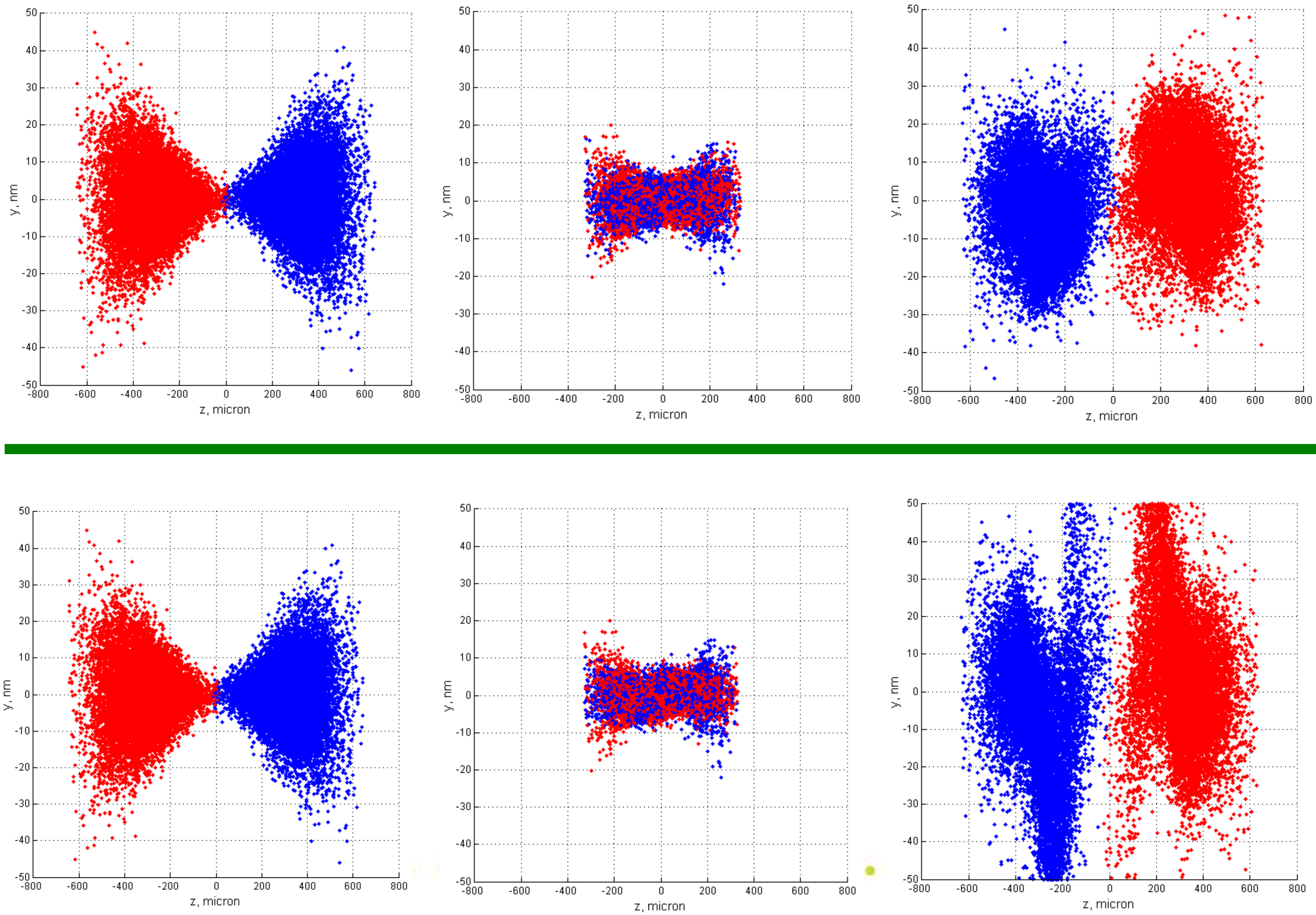
Two widely spread codes to simulate the beam-beam interaction are CAIN (K. Yokoya et al.) and GUINEA-PIG (D. Schulte et al.)

- The beam is represented by macro particles
- It is cut longitudinally into slices
- Each slice interacts with one slice of the other beam at a given time
- The slices are cut into cells
- The simulation is performed in a number of time steps in each of them
  - The macro-particle charges are distributed over the cells
  - The forces at the cell locations are calculated
  - The forces are applied to the macro particles
  - The particles are advanced

BEAM-BEAM SIMULATIONS

Beam-beam effects
H_D and instability

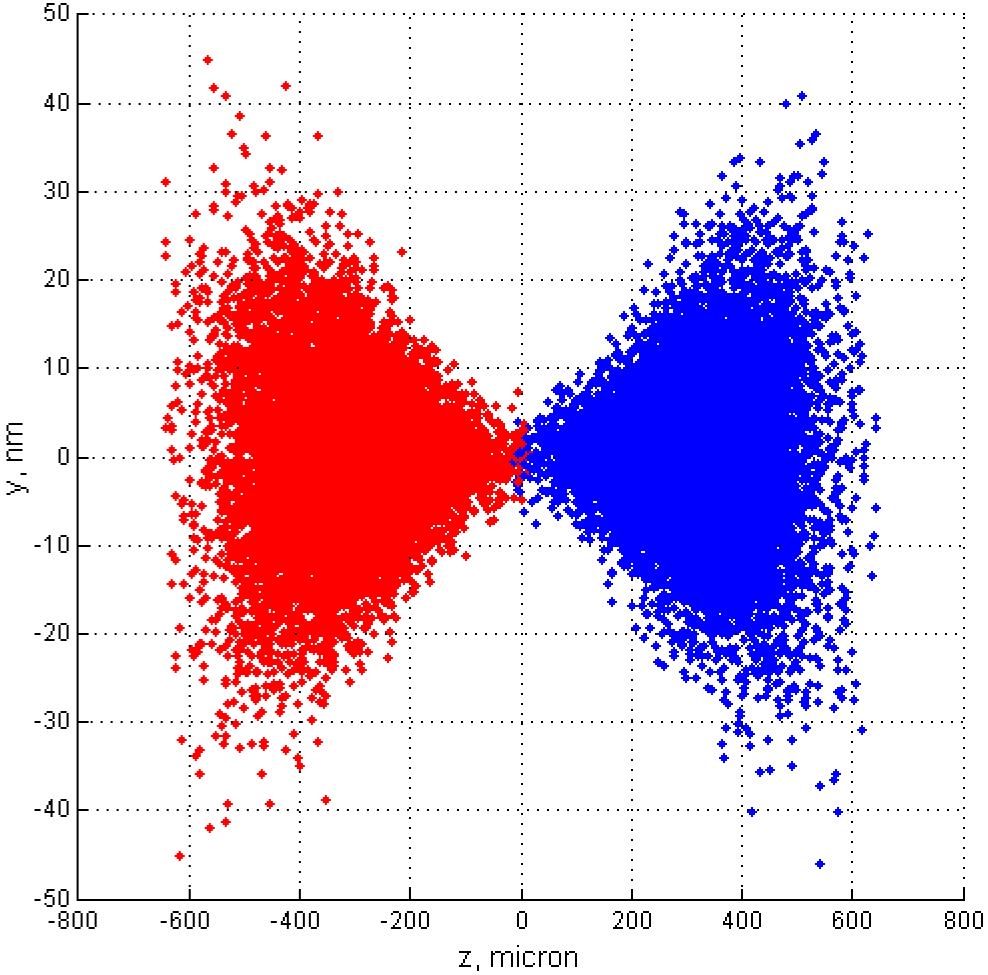
$$D_y = \frac{2r_e}{\gamma}\frac{N\sigma_z}{\sigma_x\sigma_y}$$



LC parameters
D_y~12

Nx2
D_y~24

Beam-beam effects
H_D and instability



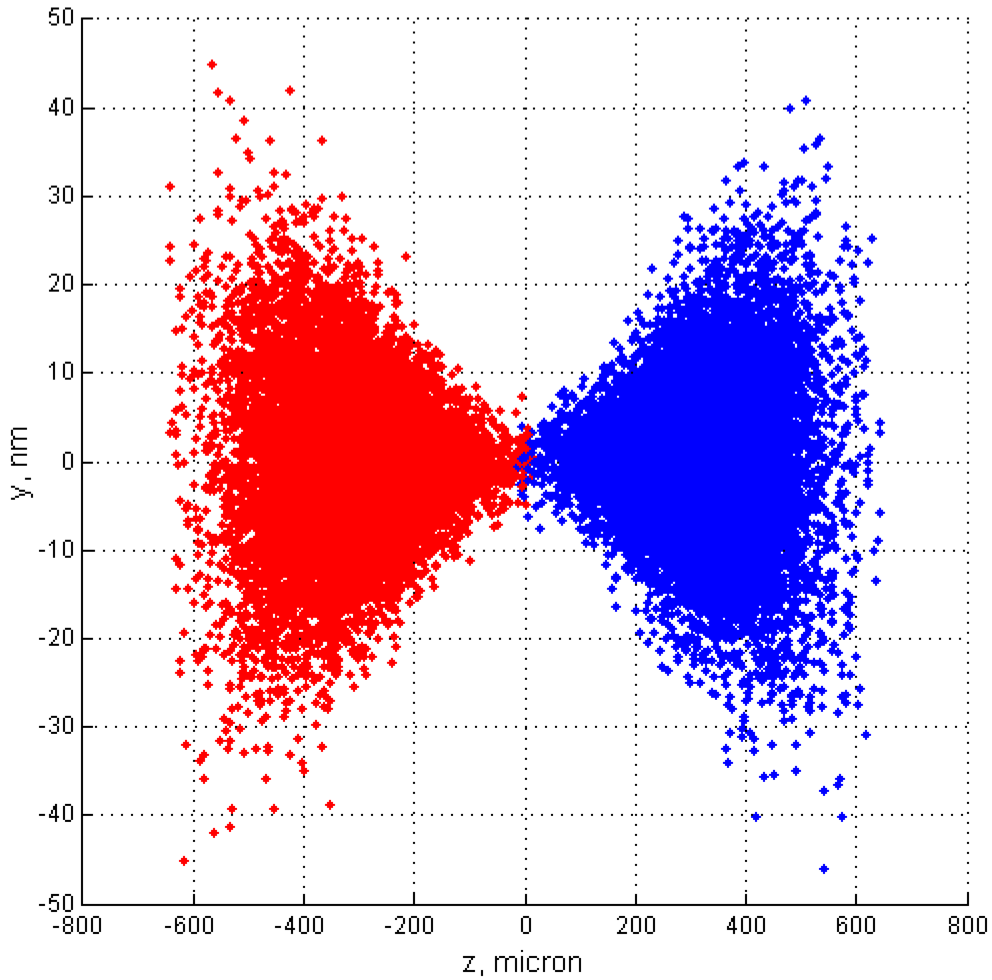
LC parameters
D_y~12

Luminosity enhancement
H_D ~ 1.4

Not much of an instability

Beam-beam effects

H_D and instability

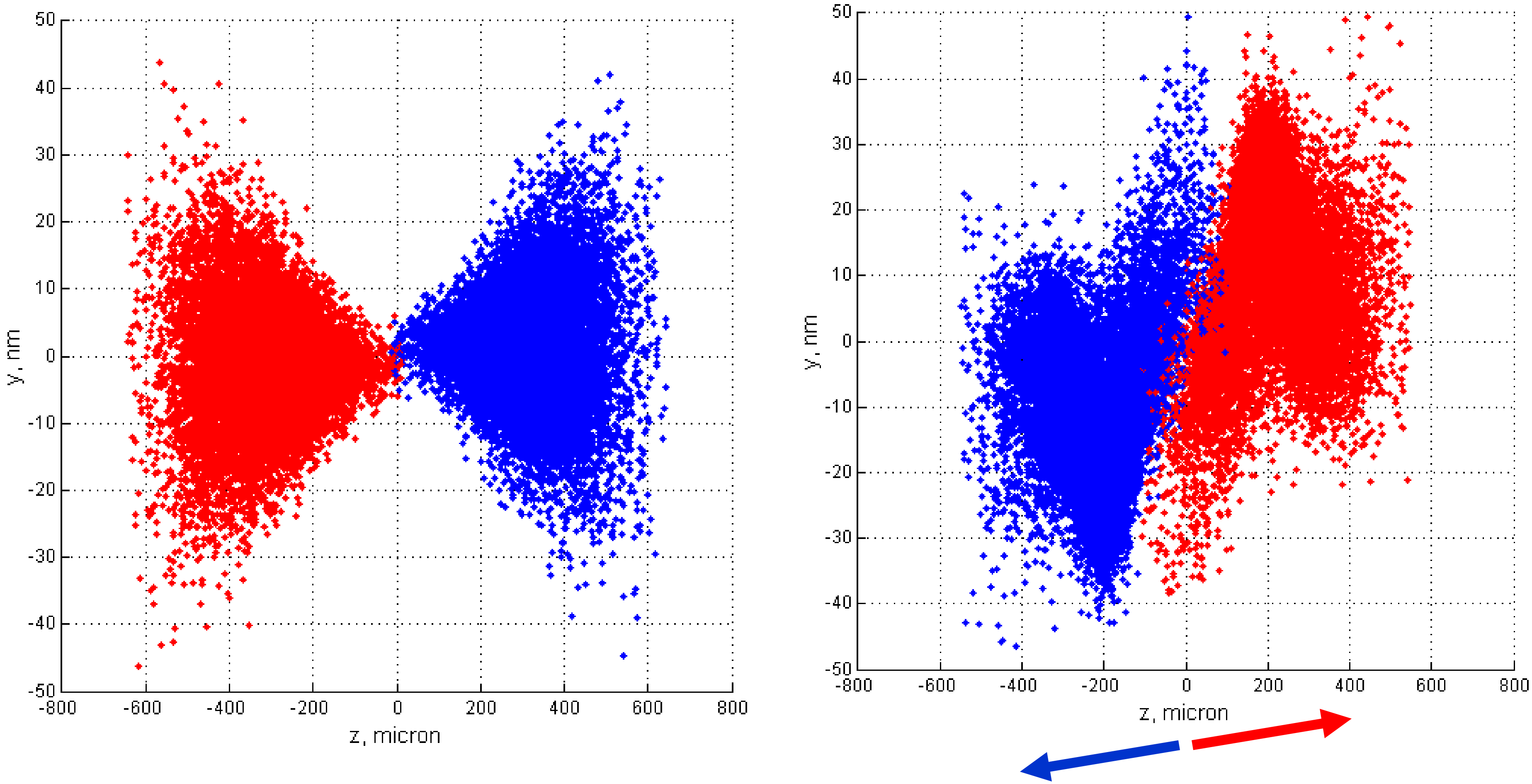


Nx2
D_y~24

Beam-beam instability is clearly pronounced

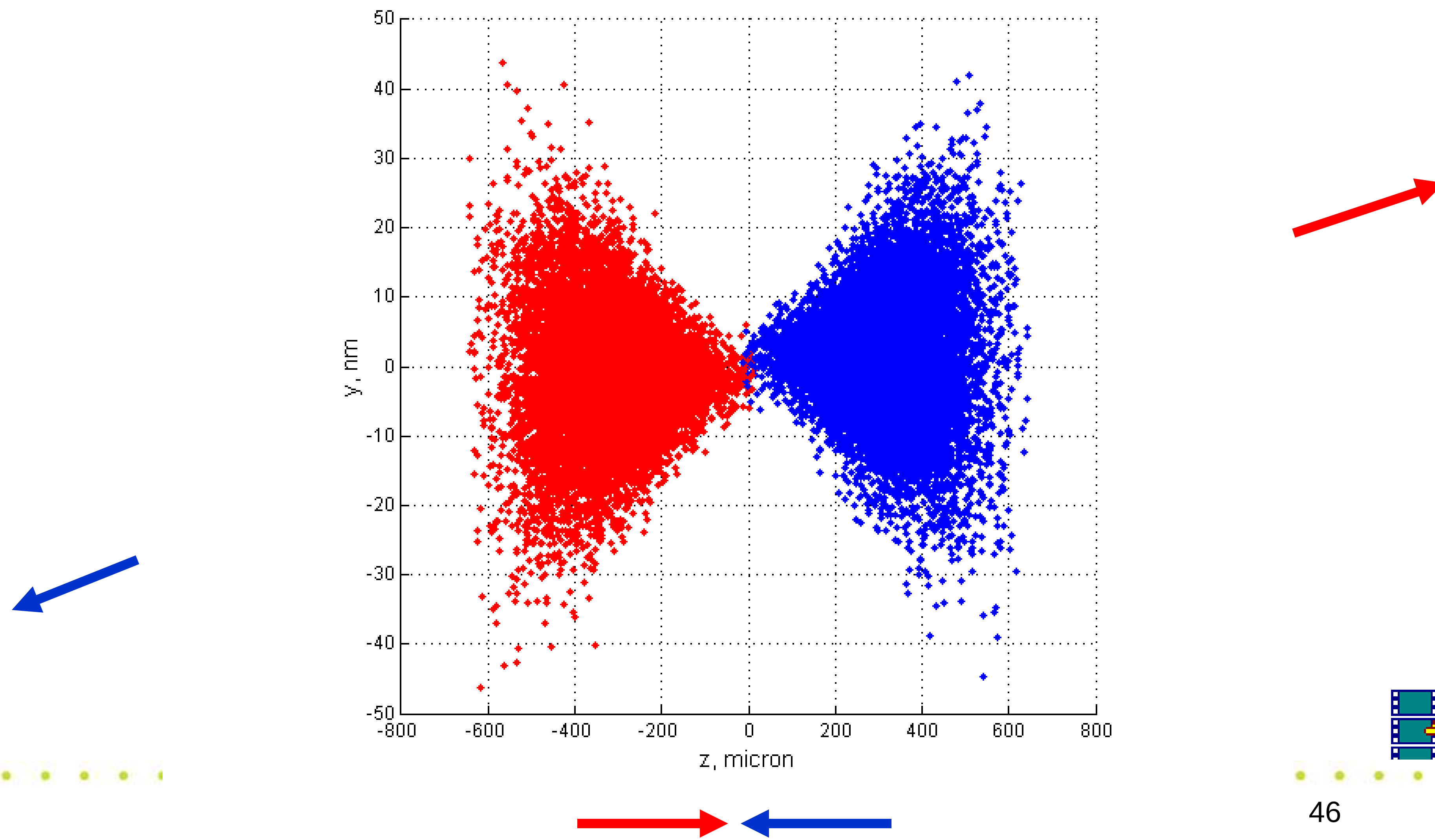
Luminosity enhancement is compromised by higher sensitivity to initial offsets

Beam-beam deflection

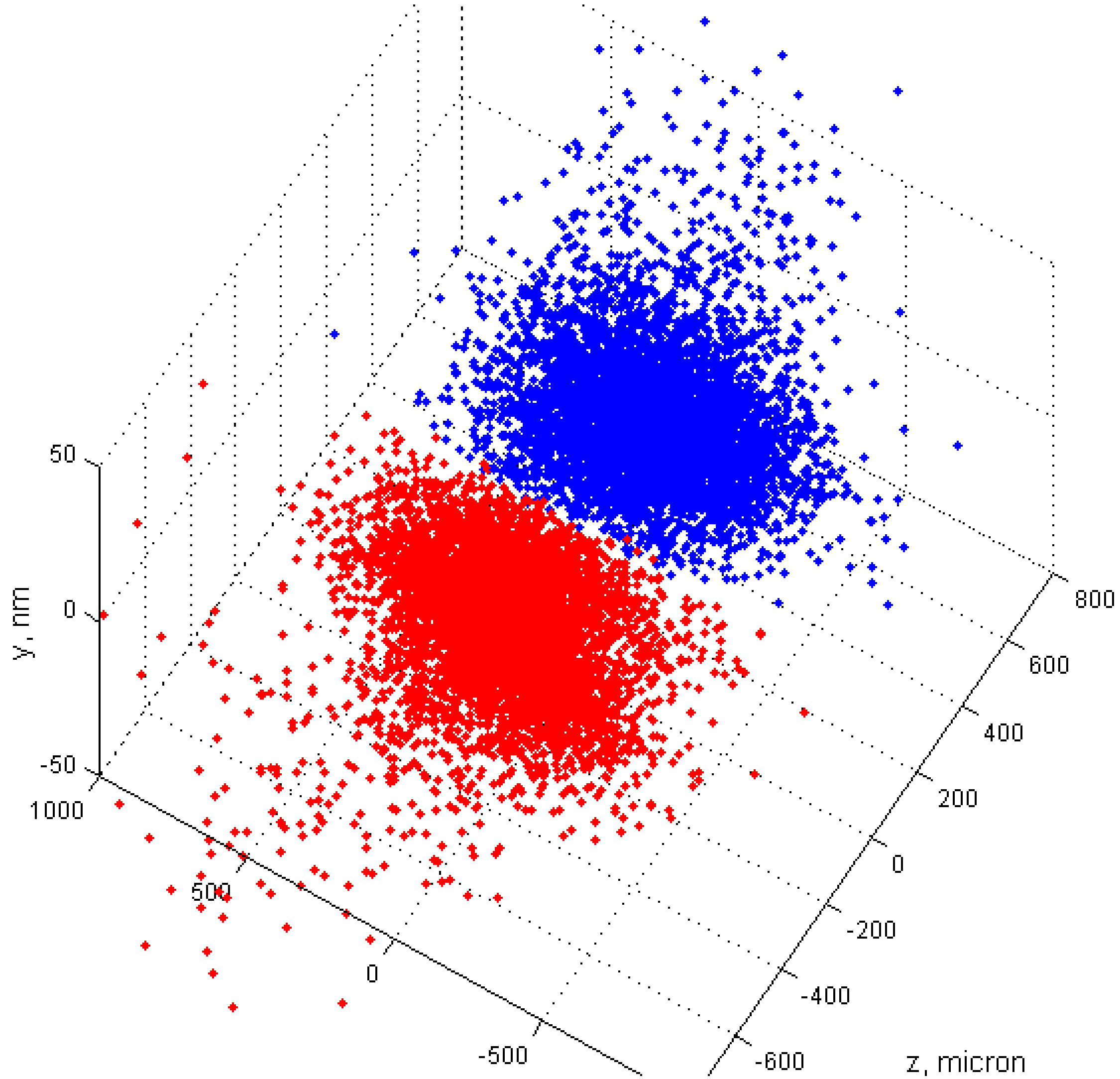


Sub nm offsets at IP cause large well detectable offsets (micron scale) of the beam a few meters downstream

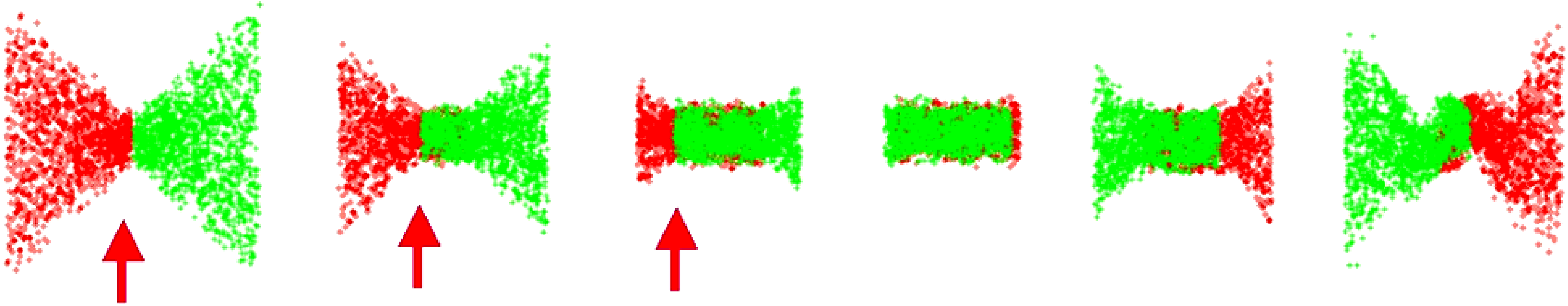
Beam-beam deflection allow to control collisions



Beam-beam collisions calculated by Guinea-Pig [D. Schulte]



Beam-beam: Travelling focus



- Suggested by V.Balakin – idea is to use beam-beam forces for additional focusing of the beam – allows some gain of luminosity or overcome somewhat the hour-glass effect
- Figure shows simulation of traveling focus. The arrows show the position of the focus point during collision

END OF FIRST BEAM BEAM LECTURE

Pinch and Luminosity enhancement

During collision, the bunches focus each other (self-focusing or pinching) leading to an increase in luminosity

Luminosity enhancement factor $\Rightarrow$ $H_D = \frac{L}{L_0} = \frac{\sigma_{x0}\sigma_{y0}}{\sigma_x \sigma_y}$

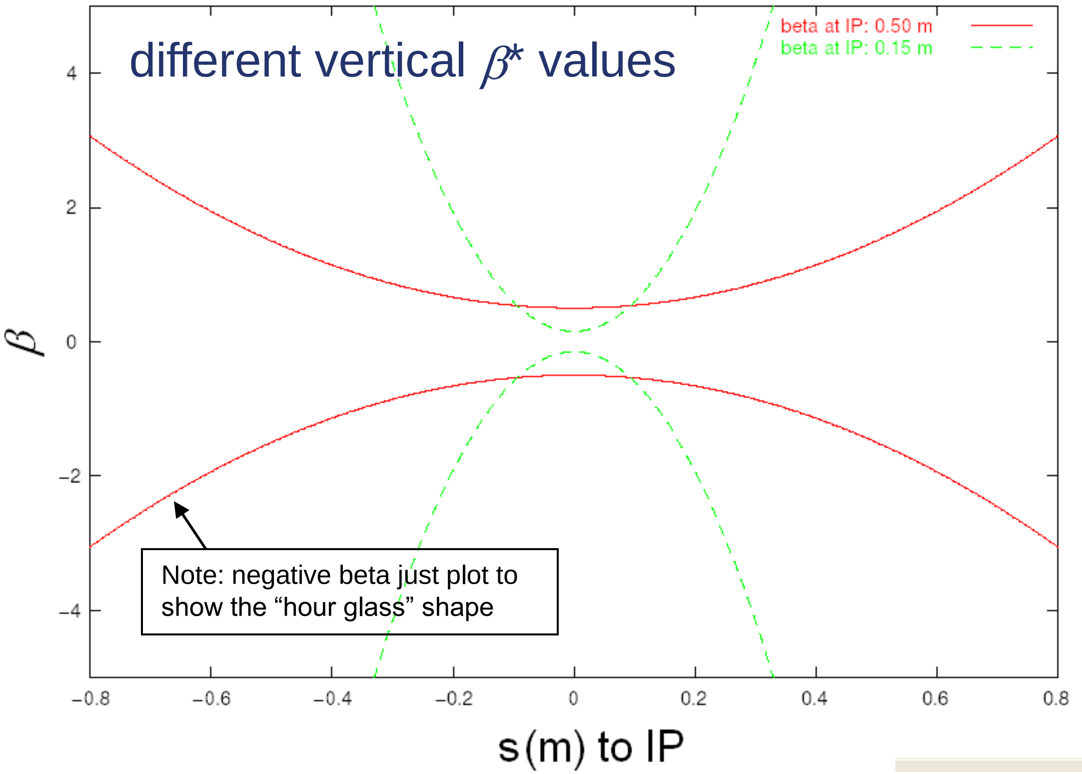
very few analytical results on this parameter. Insight gained with extensive simulations.

Hour-glass term

$$H_{Dx,y} = 1 + D_{x,y}^{1/4}\left(\frac{D_{x,y}^3}{1 + D_{x,y}^3}\right)\left[\ln\left(\sqrt{D_{x,y}} + 1\right) + 2\ln\left(\frac{0.8\beta_{x,y}}{\sigma_z}\right)\right]$$

Fit to simulation results for head-on collision

Hour-Glass effect



Transverse beam sizes cannot be considered constant but vary with β near IP. Beta has quadratic dependence with distance s

$$\beta(s) = \beta^{*}\left(1 + \left(\frac{s}{\beta^{*}}\right)^{2}\right)$$

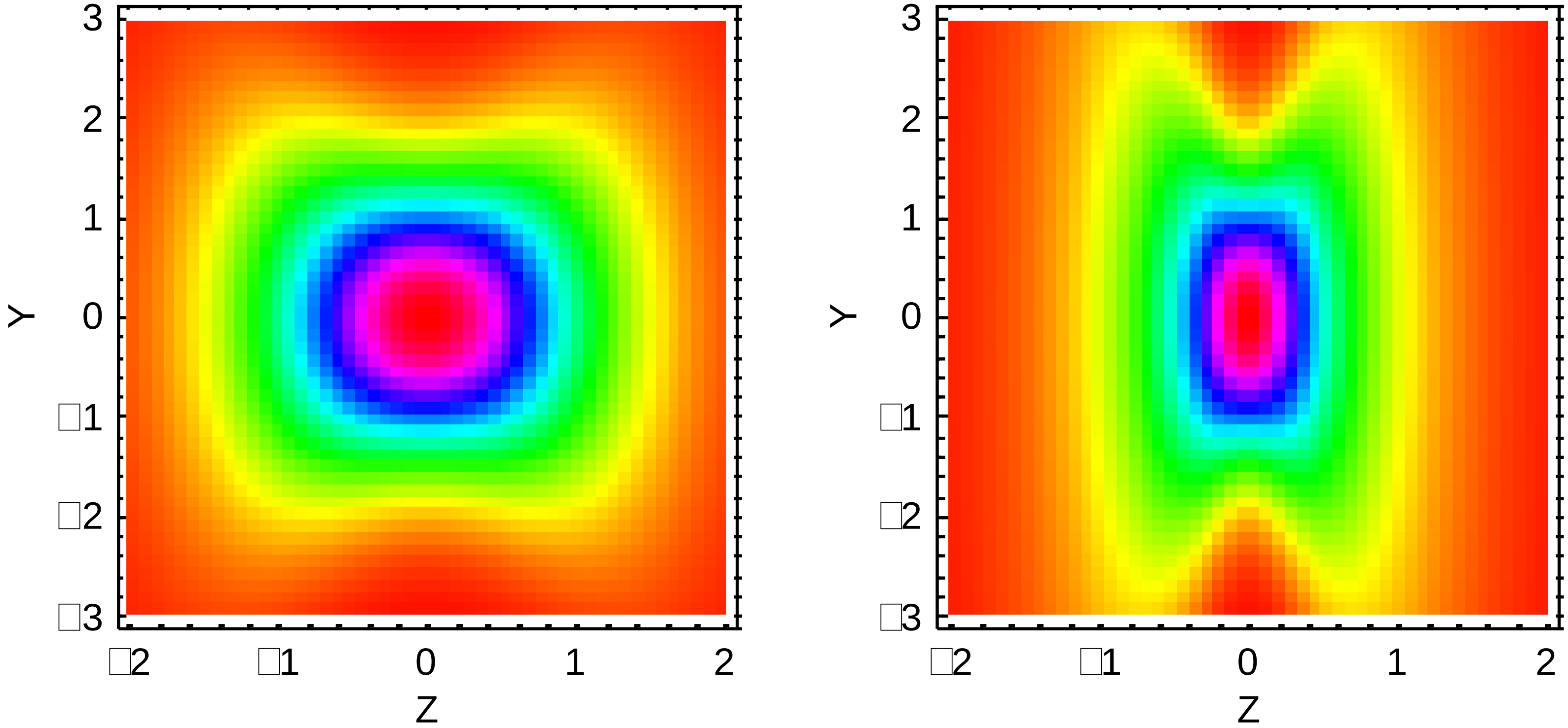
Beam sizes $\sigma_y(s) = \sqrt{\beta_y(s) \cdot \varepsilon_y}$ vary linearly with s at IP

❾ important when β_y ❍ σ_z since not all particles collide at minimum of transverse beam size → reducing luminosity.

Rule: $\sigma_z \leq \beta_y$

❾ "hour-glass" effect from shape of β

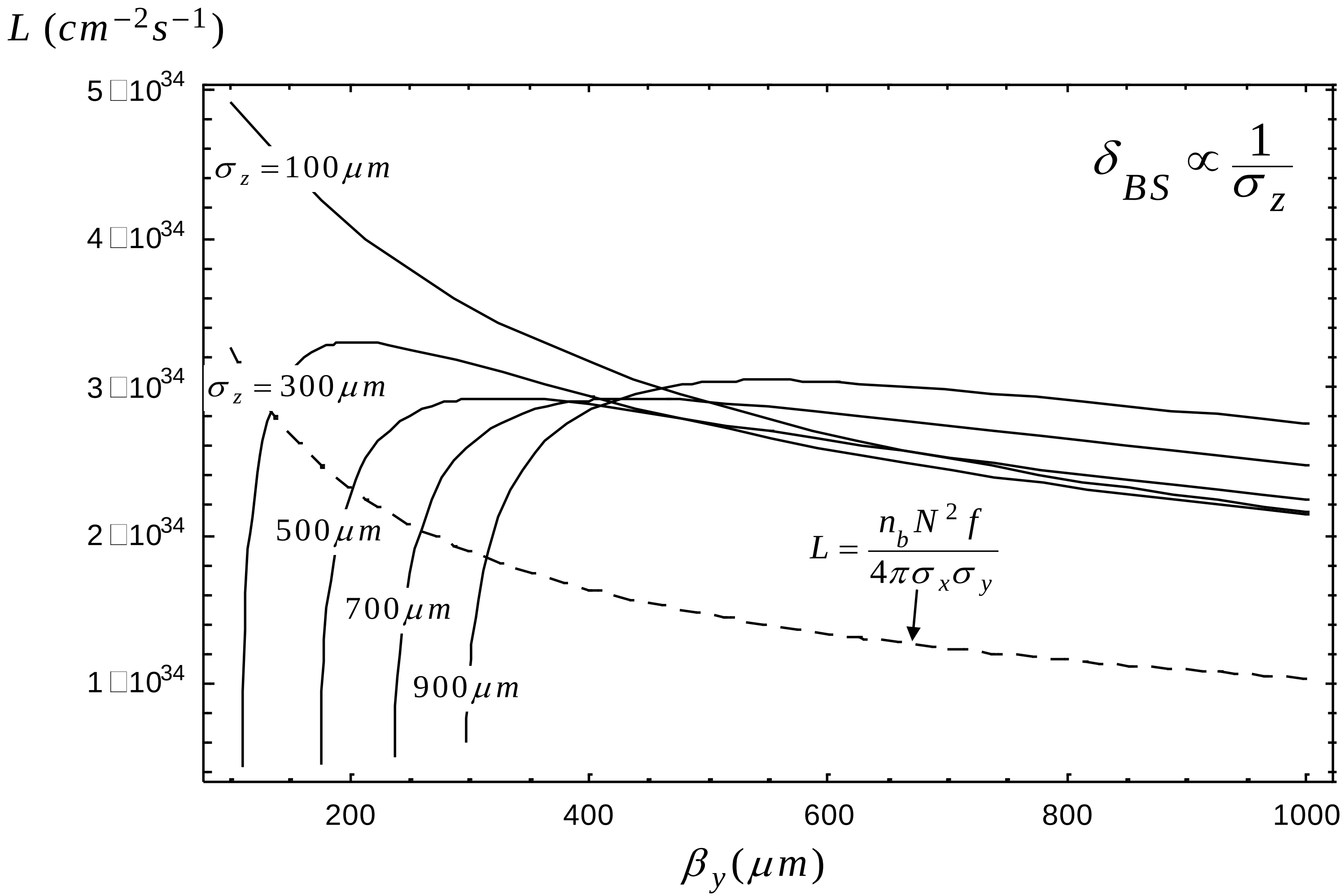
The *Luminosity* Issue: Hour-Glass



β = "depth of focus"

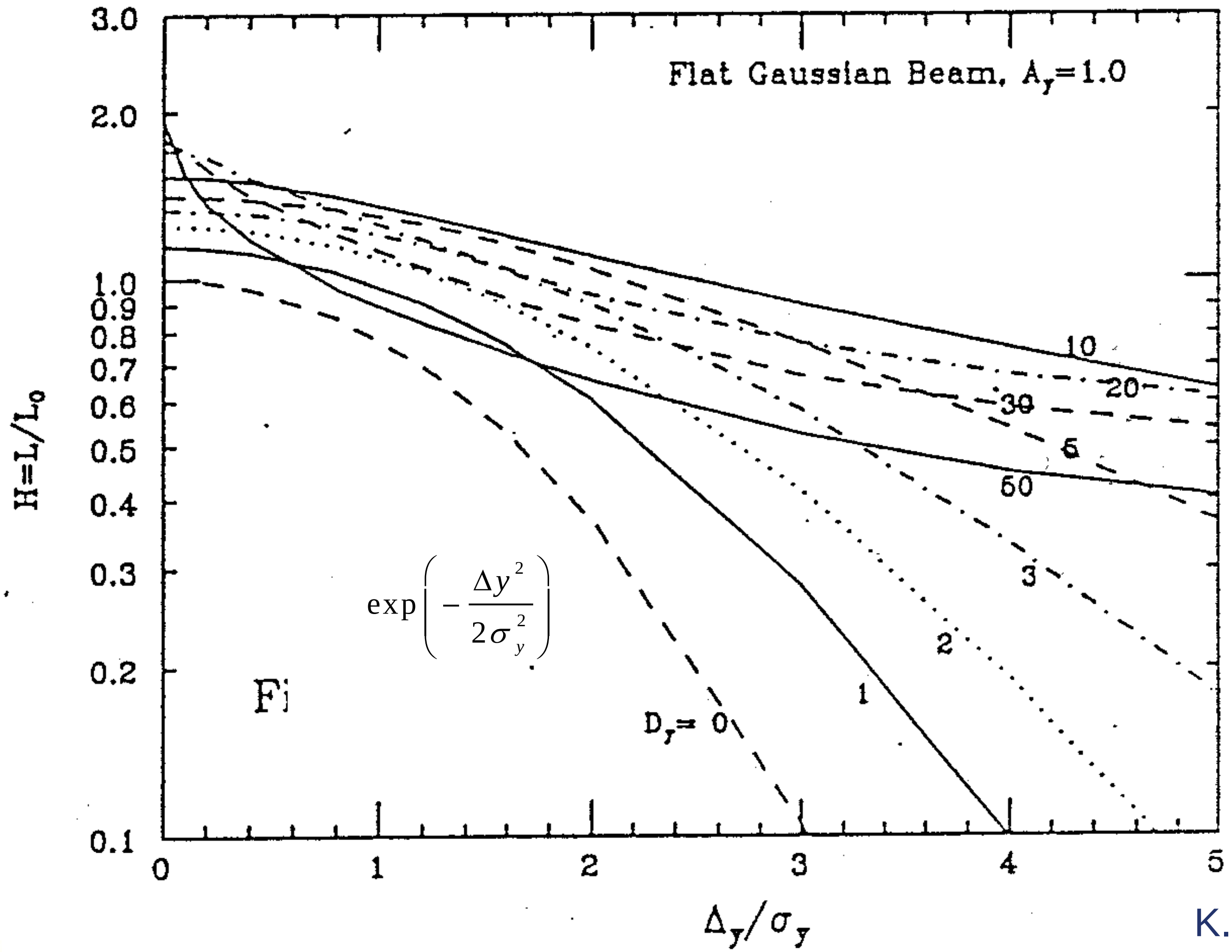
reasonable lower limit for β is bunch length σ_z

Luminosity as a function of β_y



Luminosity with beam offset and pinch enhancement

Beam-beam simulations

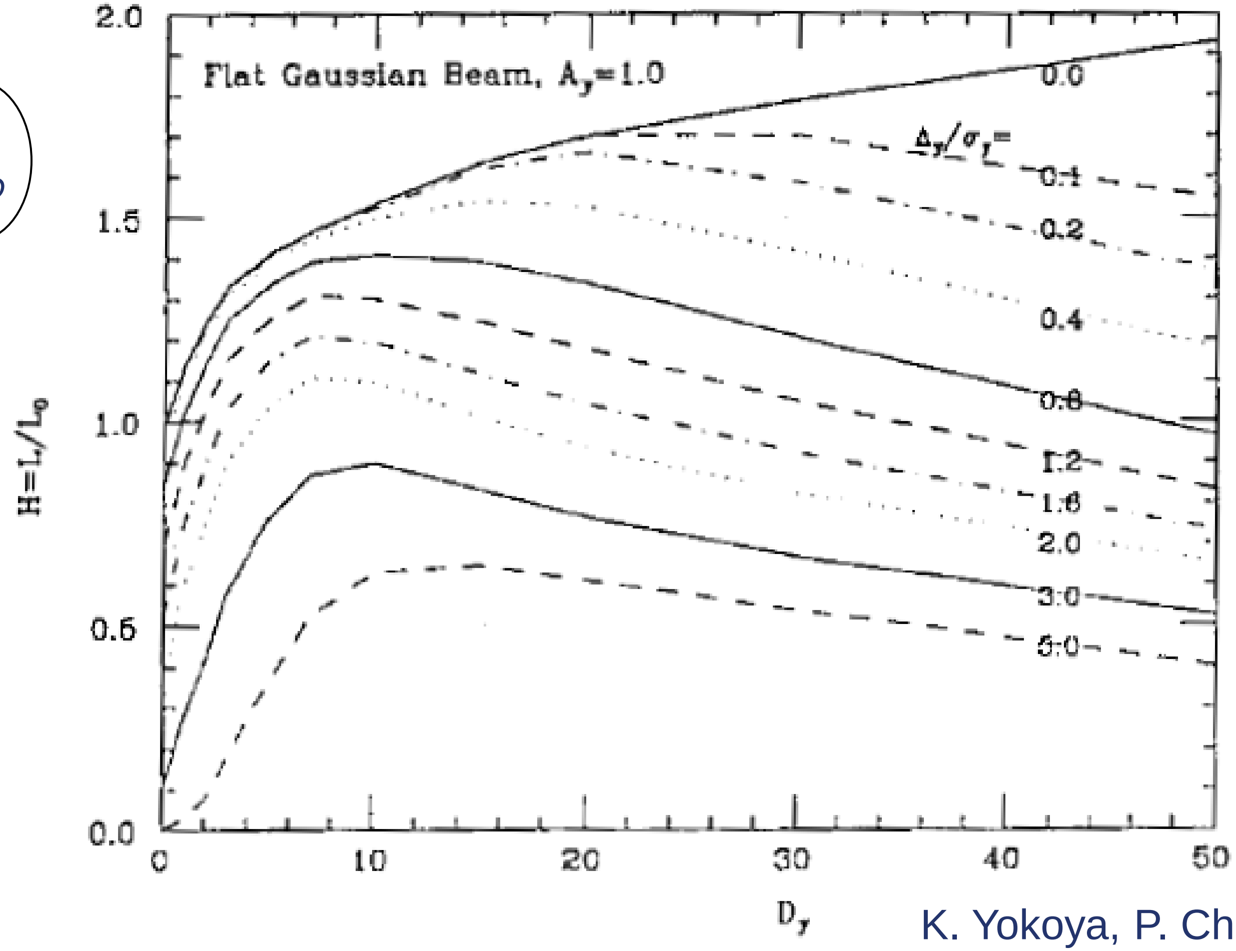


K. Yokoya, P. Chen

Luminosity and disruption

$$L = \frac{N_1 N_2}{4\pi\sigma_x\sigma_y} n_b f \, H_D$$

results simulations:



K. Yokoya, P. Chen

Luminosity with D and $\Delta y/\sigma_y$. Note at large D >10, tiny offsets degrade sensibly the luminosity. Typically H_D~2 for linear colliders.

Quantum Beamstrahlung

Particles accelerated transversely by the magnetic field of the opposite bunch. In linear colliders:

Exercise (3):
compute B_{max} for CLIC beam

- Magnetic fields reach kilo-Tesla!
- Longitudinal extent of the field is short σ_z~10^{-4}, but emitted SR plays crucial role in linear colliders

"Beamstrahlung" physics effects

- Spread in collision energy of e^+e^- → Luminosity spectrum dilution
- Radiation interacts with beam fields to produce background e^+e^- and $\mu^+\mu^-$ pairs

1 beneficial effect → Beamstrahlung used for diagnostics to keep beams in collision

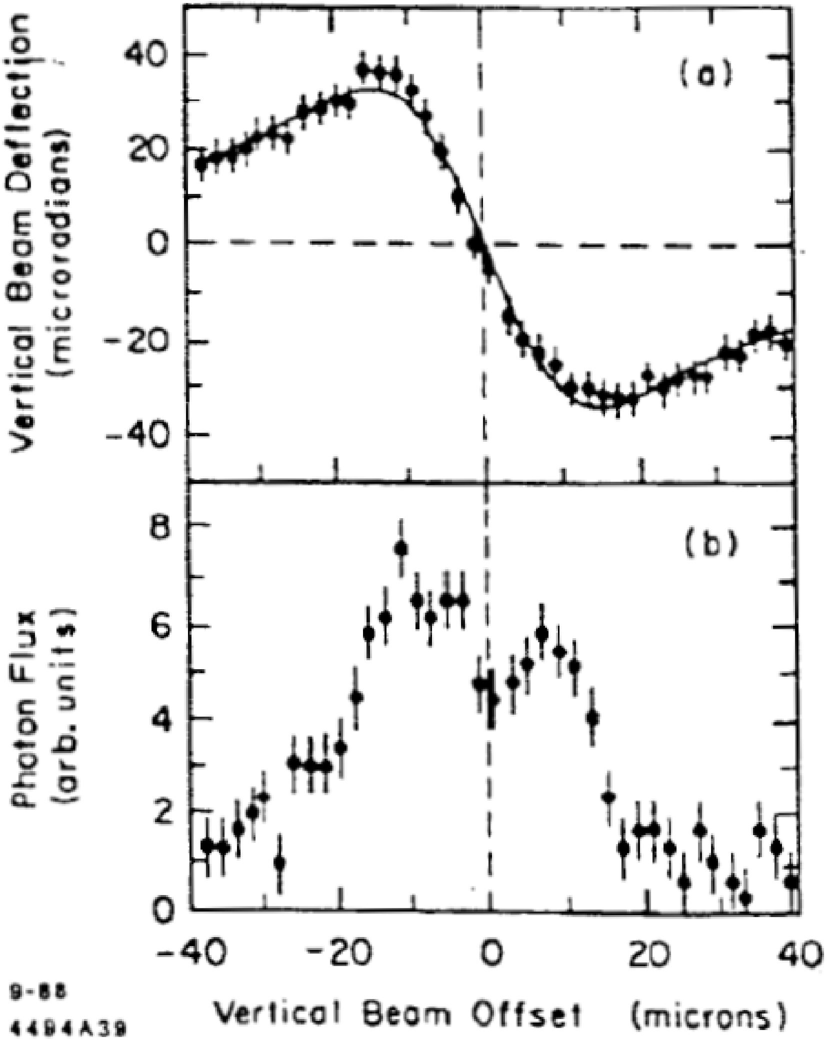
Beamstrahlung diagnostics

First observed at SLC and used as a tool to optimize luminosity.

Changed vertical beam offset and measured beamstrahlung photons downstream.
Maximum photon flux when

$$\Delta y \sim \sigma_y$$

(max opposite bunch fields)



Beamstrahlung not proportional to luminosity.

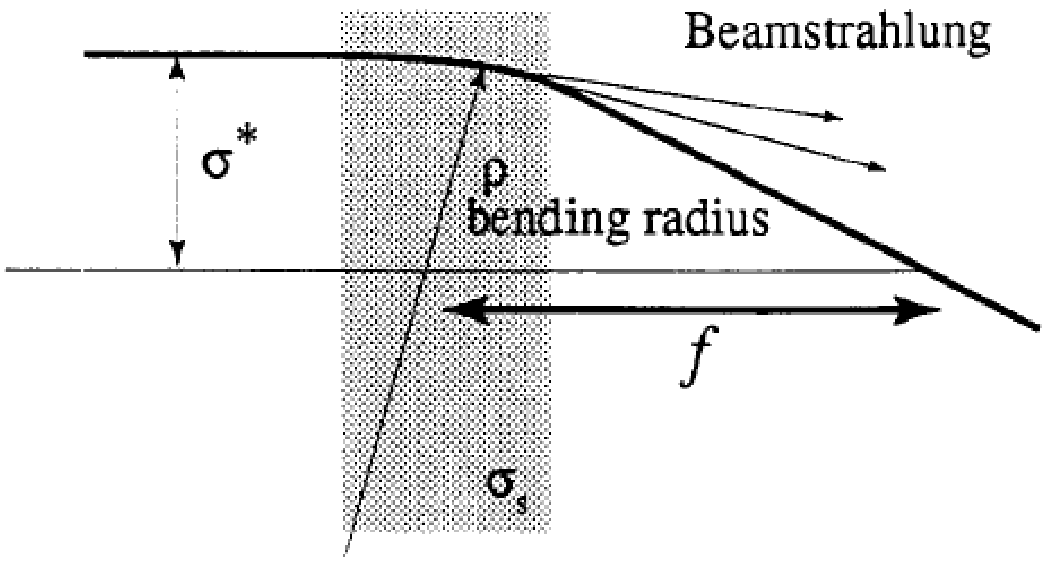
Emission radiation picture

Opening cone angle is approximately $\theta_c \sim 1/\gamma$

Observer sees radiation emitted for a radiation length $l_R = \rho/\gamma$

Critical energy defined at half power spectrum (A. Seryi lecture)

$$\varepsilon_c = \hbar\omega_c = \frac{3}{2}\frac{\hbar\gamma^3 c}{\rho} \qquad \varepsilon_c[GeV] = 0.664\ E^2[TeV]\ B[Tesla]$$



beamstrahlung

Beamstrahlung is fully characterized by the parameter

$$\Upsilon = \frac{2}{3}\frac{\varepsilon_c}{E} = \gamma \frac{\langle B \rangle}{B_c}$$

B_c Shwinger critical field $B_c = m^2 c^2 / e\hbar = 4.42\ \mathrm{GTesla}$

For Gaussian beams the Υ average and maximum is computed

$$\Upsilon_{avg} \approx \frac{5}{6}\frac{N r_e^2 \gamma}{\alpha \sigma_z (\sigma_x + \sigma_y)} \qquad \Upsilon_{\max} \approx \frac{12}{5}\Upsilon_{avg}$$

Υ not constant during collision

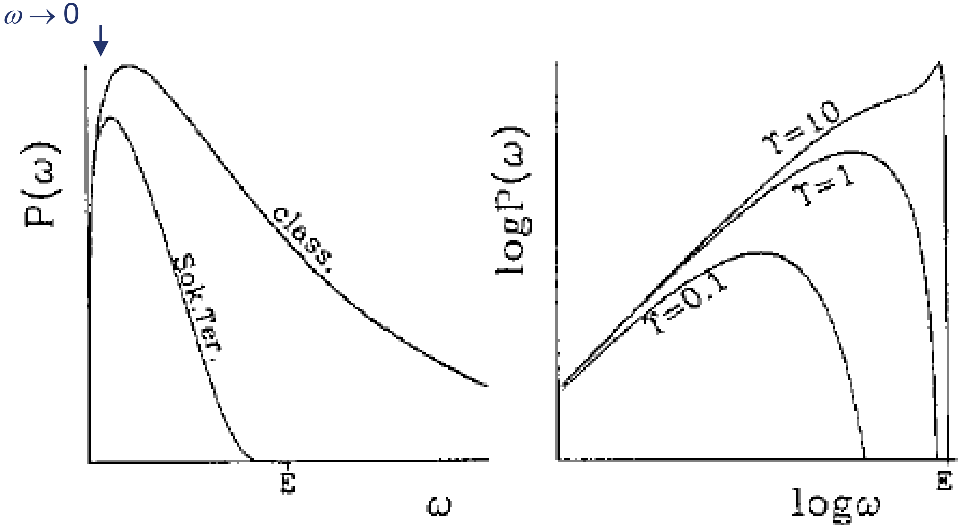
In linear colliders Υ~0.1 to ¤ 1, meaning that emitted photon energy is comparable or may exceed initial electron energy: recoil of electrons, quantum nature and breakdown of “classical” synchrotron radiation spectrum.

power spectrum

Beamstrahlung radiation formula for arbitrary Υ was first derived by Sokolov-Ternov by using Dirac equation in a uniform magnetic field and computing the transition rates. The photon emission spectrum is

$$\frac{dW_\gamma}{d\omega} = \frac{\alpha}{\sqrt{3}\pi\gamma^2} F_{BS} = -\frac{\alpha}{\sqrt{3}\pi\gamma^2}\left[\int_\xi^\infty K_{1/3}(\xi')\,d\xi' + \underbrace{\frac{y^2}{1-y}K_{2/3}(\xi)}_{\text{quantum beamstrahlung}}\right]$$

$$\lim_{\omega\to 0} P(\omega) \approx \omega^{1/3}$$



log log plot

with $\xi = \frac{2}{3\Upsilon}\frac{y}{1-y} \quad (y = \hbar\omega/E)$

In the limit $\Upsilon \rightarrow 0$ the spectrum reduces to the classical formula. The high energy part is truncated at beam energy.

Luminosity and beamstrahlung

Beamstrahlung emission characteristic parameters:

approx. number of emitted photons $$n_\gamma \approx 2.54 \left[\frac{\alpha \sigma_z}{\lambda_e \gamma} \right] \Upsilon_{avg} U_0 \left(\Upsilon_{avg} \right)$$

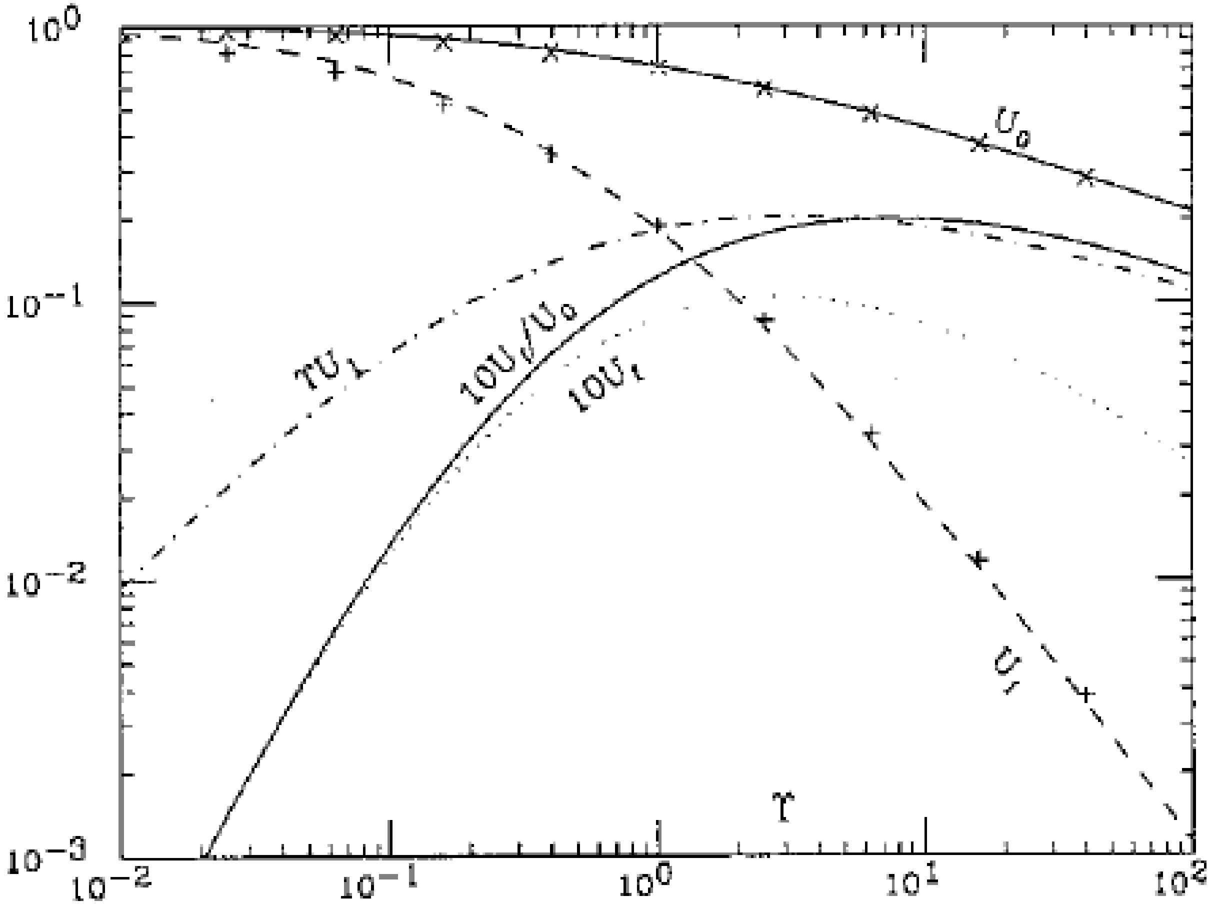
and the relative beam particles energy loss $$\delta_{BS} = \left\langle -\frac{\Delta E}{E} \right\rangle \approx 1.24 \left[\frac{\alpha \sigma_z}{\lambda_e \gamma} \right] \Upsilon^2_{avg} U_1 \left(\Upsilon_{avg} \right)$$

where $U_0(\Upsilon) \approx \dfrac{1}{\sqrt{1 + \Upsilon^{2/3}}}$ and $U_1(\Upsilon) \approx \dfrac{1}{\left(1 + 1.5\Upsilon^{2/3}\right)^2}$

$$\lambda_e = r_e / \alpha$$

	CLIC	ILC 500
Υ_{avg}	5.0	0.04
n_γ	1.8	1.1
δ_{BS} [%]	29	2

beamstrahlung parameters



ΥU_1 ~ constant over wide range $0.1<\Upsilon<100$. Thus, to keep δ_{BS} < 10% most linear collider designs choose $\alpha\sigma_z\Upsilon/\lambda_e\gamma \doteq 1$; n_γ is also ~unity.

Luminosity and beamstrahlung

Beamstrahlung causes a spread in the center of mass energy of $e^- e^+$. This effect is characterized by the parameter δ_{BS}. Although $\delta_{BS} < 1\%$ for the SLC, it can be a severe limiting factor for the performances of any future linear collider. Limiting the beamstrahlung emission is of great concern for the design of the interaction region:

<u>Flat beams</u>

flat beams: reduce beamstrahlung without sacrificing luminosity

Beamstrahlung parameters depend on the inverse sum while Luminosity depend on the product of beam sizes

$$L = \frac{N_1 N_2}{4\pi\sigma_x^* \sigma_y^*} n_b f \; H_D$$

low energy regime $\Upsilon \rightarrow 0$

$$\delta_{BS} \approx 0.86\gamma \frac{r_e^3}{\sigma_z} \frac{N^2}{\left(\sigma_x + \sigma_y\right)^2}$$

also re-call

$$\Upsilon_{avg} \propto \frac{N}{\sigma_z \left(\sigma_x + \sigma_y\right)}$$

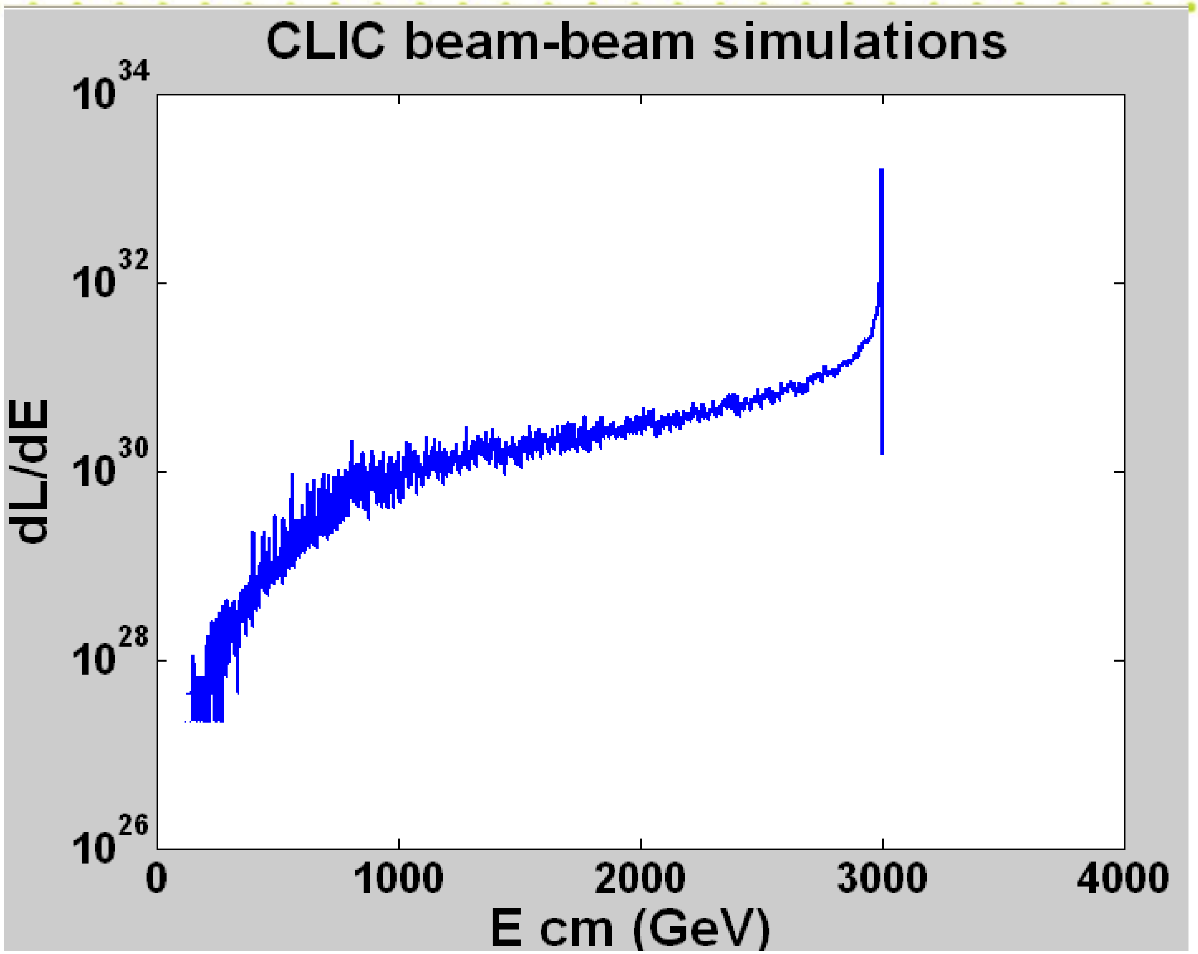
Most practical cure is to produce <u>very flat beams</u> R= σ_x/σ_y → large, by increasing σ_x and squizing σ_y to scale δ_{BS}✪ $1/\sigma_x^2$ and Υ ✪ $1/\sigma_x$ without sacrificing Luminosity ✪ $1/(\sigma_x\sigma_y)$. R = 85 CLIC and R = 115 ILC.

Luminosity spectrum

Peaked at nominal c.m. energy but long low energy tail

INPUT EXAMPLE
energy = 1500 GeV;
particles = 0.256 e10
sigma_x = 60 nm
sigma_y = 0.7 nm
sigma_z = 30.8 um
beta_x = 16.0 mm
beta_y = 0.07 mm

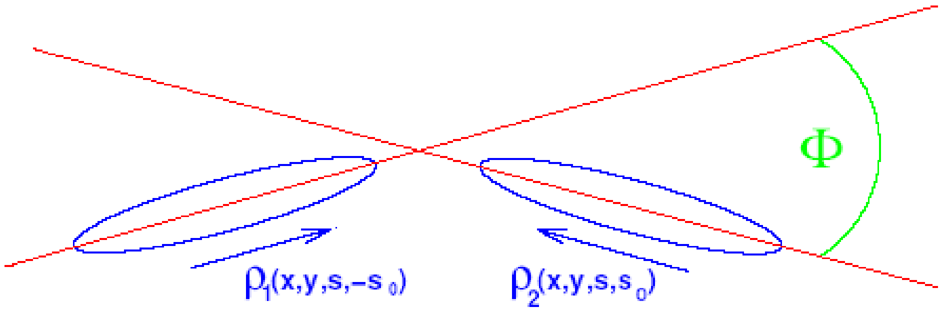
OUTPUT:
Υmax=9.68447;
δ_E=17%



Crossing angle

To avoid unwanted parassitic collisions between closely spaced bunches, a crossing angle is used in either LCs. The luminosity overlap integral gives an additional purely geometrical factor *S*

$$L = \frac{N_1 N_2 n_b f}{4\pi\sigma_x\sigma_y} \cdot S$$



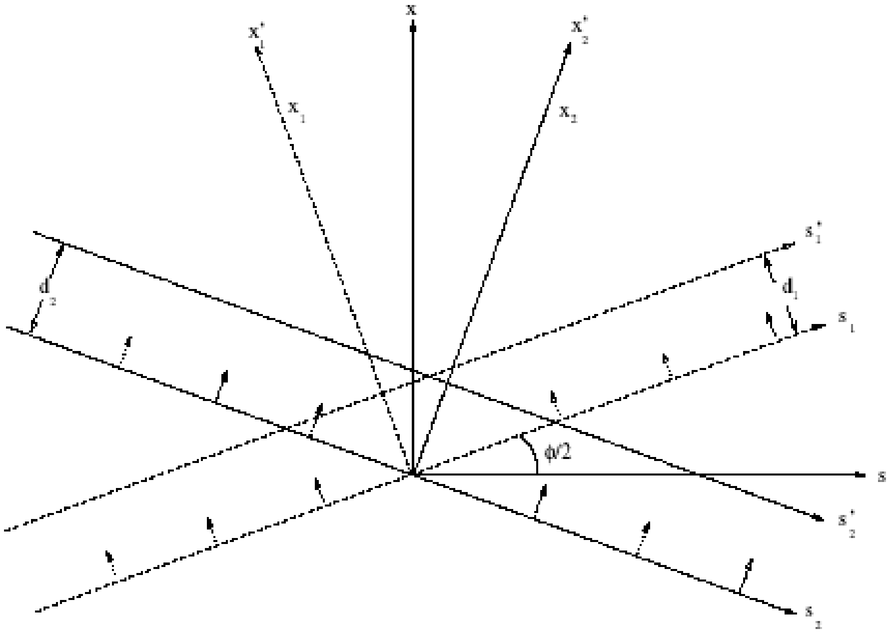
$$S = \frac{1}{\sqrt{1+(\sigma_x/\sigma_z\cdot\tan(\phi/2))^2}}\frac{1}{\sqrt{1+(\sigma_z/\sigma_x\cdot\tan(\phi/2))^2}}$$

for small ϕ and $\sigma_z \gg \sigma_x$ the reduction factor is $S \approx \left(1+(\frac{\sigma_z}{\sigma_x}\cdot\frac{\phi}{2})^2\right)^{-1/2}$

typically, LC crossing angles $\phi \rightarrow$ 0-20 mrad.

Crossing angle and offset

If small transverse offsets are also considered



$$L = \frac{N_1 N_2 n_b f}{4\pi\sigma_x\sigma_y} W \cdot e^{\frac{B^2}{A}} S$$

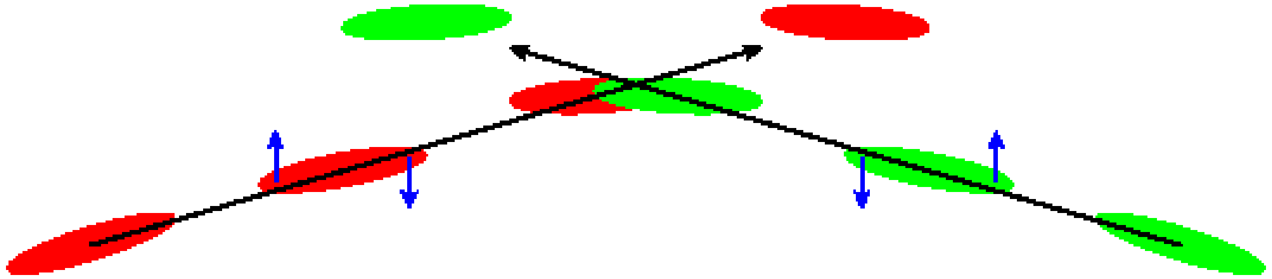
where *S* is the crossing angle reduction factor, *W* is the reduction factor in presence of beam offsets and *exp[B²/A]* in presence of both angle and offset, and

$$W = e^{-\frac{1}{4\sigma_x^2}(d_2-d_1)^2} \quad B = (d_2 - d_1)\sin(\phi/2)\big/2\sigma_x^2 \quad A = \frac{\sin^2\phi/2}{\sigma_x^2} + \frac{\cos^2\phi/2}{\sigma_z^2}$$

Crab crossing

Advantage of crossing angle is that allows many closely spaced bunches and make extraction of the beam easier. Disadvantage is a reduced luminosity and the fact that particles in a bunch experience different forces, since they pass the opposite bunch at different times.

Crab crossing is obtained with a rotation or “tilt” in the z-x planes by means of an upstream “crab cavity”, to allow head-on collision



and fully restore luminosity. Crab cavity R&D is ongoing for the LC.

Pair production

Production of e^+e^- pairs (see N. Mokhov lectures) is source of detector background:

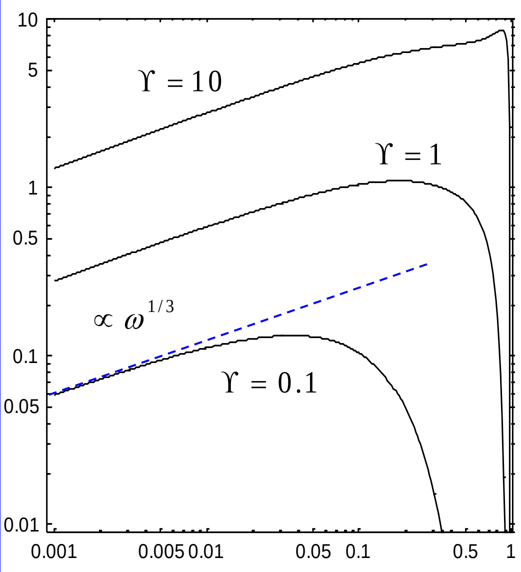
Incoherent process → beamstrahlung "real" photons interact with oncoming electrons or positrons. Incoherent processes occur at low energies.

Coherent process → photons propagating through the transverse electromagnetic field of the oncoming beam has a probability of turning into e+e- pairs. Contributed either by real or virtual photons.

Pair Production

- Incoherent e^+e^- pairs $\Upsilon<0.6$
  - Breit-Wheeler: $\gamma\gamma \rightarrow e^+e^-$
  - Bethe-Heitler: $e^\pm\gamma \rightarrow e^\pm e^+e^-$
  - Landau-Lifshitz: $e^+e^- \rightarrow e^+e^-e^+e^-$
- Coherent e^+e^- pairs $0.6<\Upsilon<100$
  - threshold defined by

$$\chi \equiv \frac{\hbar\omega}{m_0c^2}\frac{2B}{B_s} = \frac{\hbar\omega}{E}\Upsilon \geq 1$$



$y = \hbar\omega / E$

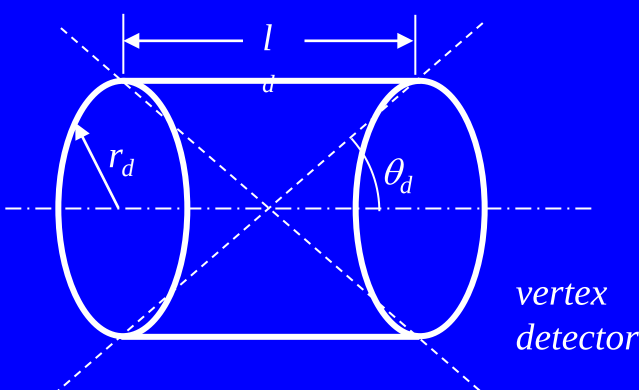
for $\Upsilon>1$ $\quad \hbar\omega/E \sim O(1) \quad \Rightarrow \chi \geq 1$

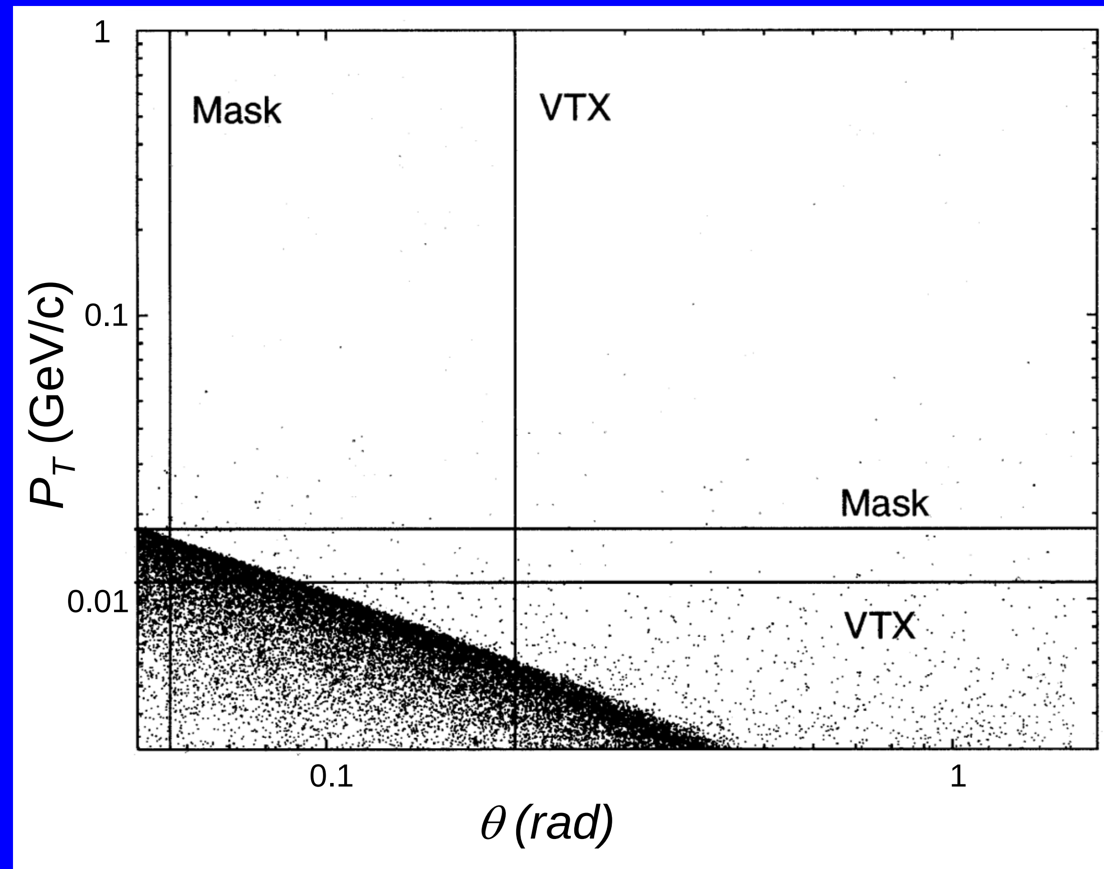
for intermediate colliders (E_{cm}<1TeV), incoherent pairs dominate

Pair Production

e^+e^- pairs are a potential major source of background

Most important: angle with beam axis (θ) and transverse momentum P_T.





pairs curl-up (spiral) in solenoid field of detector

$$r = \frac{P_T}{cB_z} < r_d$$

Many of these slides are based on previous contributions to the field of beam-beam and beam delivery system

In particular, Thanks to:

N. Walker, D. Shulte, A. Seryi, K. Yokoya, P. Chen, K. Brown and to many other colleagues …

References

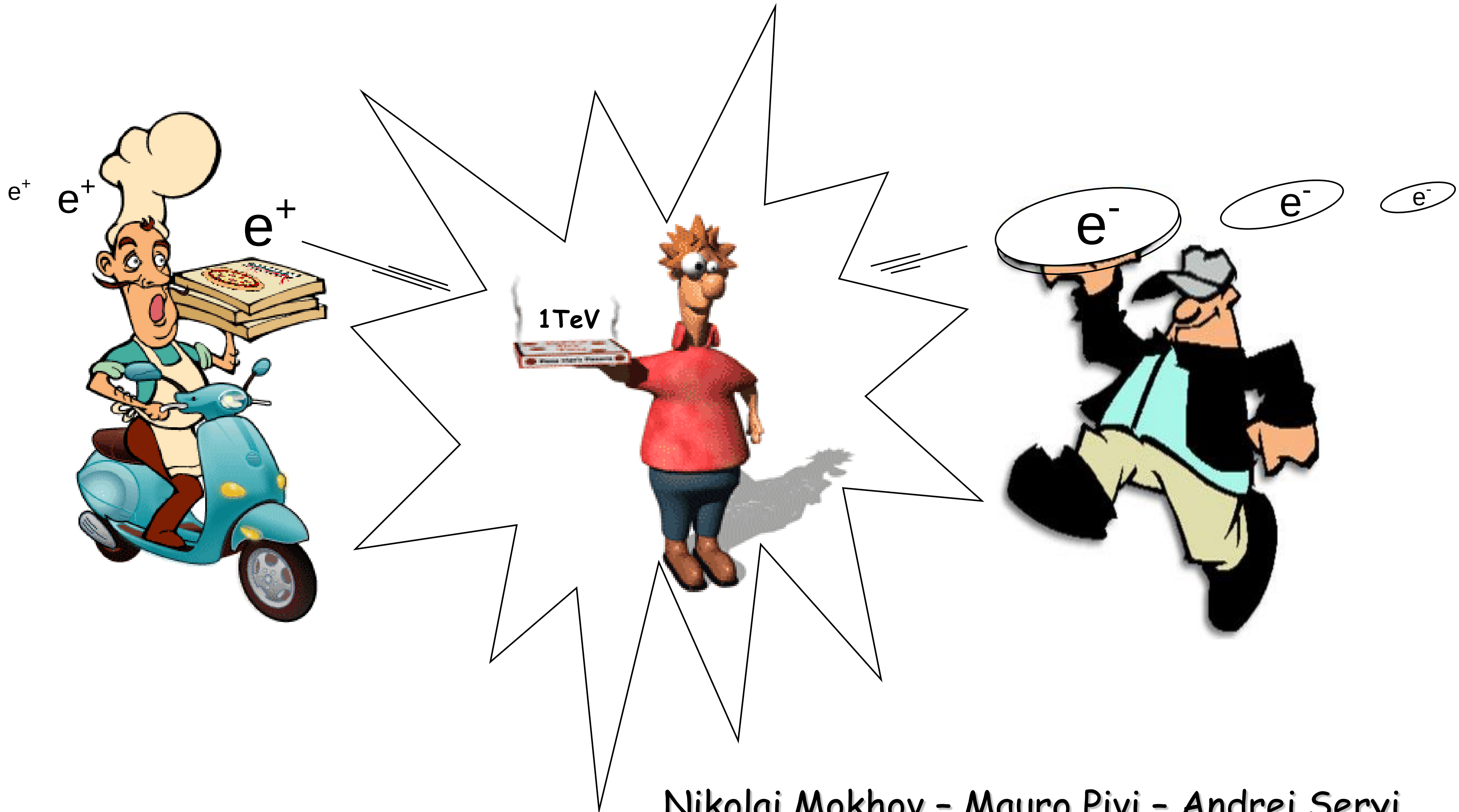
Some references on beam-beam effects and final focus system:

[1] K. Yokoya and P. Chen, KEK Preprint 91-2
[2] N. Walker USPAS03 Lect. 1 and 7, Santa Barbara, CA
[3] D. Shulte ILC school Sokendai, Hayama, Japan May (2006)
[4] A. Seryi USPAS03 Santa Barbara
[5] L. Evans CAS84
[6] A. Chao, M. Tigner *Handbook of Accel. Phys. And Eng.*
[7] W. Herr and B. Muratori CAS03 CERN-2006-02
[8] E. Keil CAS95
[9] L. Rivkin CAS95
[10] G. Dugan USPAS02
[11] B. Siemann *Luminosity and Beam Beam effects,* Lect. 5, (1998)
[12] K. Oide Phys. Rev. Letter 61, 15, (1998)
[13] K. Brown SLAC-PUB-4811
[14] K. Brown, R. Sevranckx SLAC-PUB-3381
[15] P. Raimondi, A. Seryi Phys Rev. Letter86,3779 (2001)

Supplemental material

Feedback (keeping beam in collision)
Kink instability
Beam Beam Kick
Long Range Kink
banana beam
spent beam and exit angle
luminosity monitoring

beam delivered



Nikolai Mokhov – Mauro Pivi – Andrei Seryi