Theory and Practice of Cavity Test Systems

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OVERVIEW

- Voltage controlled oscillator based phase locked loops
- RF system overview for vertical testing.
- RF system overview for Cryomodule testing.
- Coupler conditioning vacuum-RF feedback loop.
- Cavity resonance monitor
- Cable calibrations
- Cable break down in low pressure helium systems.
- Basic RF equations for critically coupled cavities.
- Basic RF equations for over coupled cavities.
- Qo measurements for cryomodules.

Reference 2005 SRF Workshop

SRF Cavity Characteristics

- **Normal conducting cavities**
  - $Q_0$ on the order of 1,000 to 20,000
  - Normally operated near critical coupling where loaded $QL = 2Q_0$
  - Operating BW on the order of 1 MHz

- **SRF cavities**
  - $Q_0$ on the order of $10^8$ to $10^9$
  - Vertical tests, normally operated near critical coupling for
    (BW on the order of 1 Hz to 100 Hz)
  - Accelerator operations configuration, normally operated
    strongly over coupled with a bandwidth between 100 Hz and 10 kHz
• Two fundamental ways to drive a cavity.

  • Fixed frequency systems are used in conjunction with resonance controls like motorized tuners when operating fixed frequency systems in accelerators.

  • Variable frequency systems are used to simplify the system or to test cavities which do not have tuners attached.

• During vertical testing cavity bandwidths on the order of 1 Hz are not uncommon, it would be extremely difficult to maintain the cavity’s frequency while testing.

• At Jefferson Lab we commonly use voltage controlled oscillator based phase locked loops to track the cavity frequency during the test.
• The front end generally makes use of a low noise amplifier and a series of variable attenuators which are used to:
  • Keep the mixer RF level below the maximum level, typically 6 dB below the design LO.
  • Ensure that the mixer and following loop amplifier, crystal detectors, etc. are not power starved.
  • Help to avoid loop oscillations.

• The loop gain is proportional to the cavity gradient. Thus a system that behaves well at 2 MV/m will very likely oscillate at 20 MV/m, unless the loop gain is reduced at higher gradient.

• Although difficult to find. Limiting amplifiers such as Lucent Technologies LG1605 extend the dynamic range of the system while preventing oscillation.
• The mixer can be a simple double balanced mixer. Devices such as a mini circuits ZFM-150 are perfectly adequate.

• The mixer IF output must go to DC.

• Typically you are limited to somewhere between 7 and 13 dBm mixers by the output level of VCO.

• Higher IP3 mixers could be used at the cost of a larger amplifier between the VCO and the LO input. This would provide a better dynamic range.

• Part of the function of the low pass filter is to reject the second harmonic component of the mixer output.
• The secondary function of the low pass filter is to reduce the noise. To that end the bandwidth of the filter is typically 20 kHz.

• The variable gain amplifier circuit is used to adjust the loop gain.

• At the summing junction a center frequency adjustment signal is summed with the output of the loop amplifier, typically we use course and fine ten-turn potentiometers.

• In addition to a custom circuit designs, a Stanford Research SR560 can be used to implement the loop amplifier and filter blocks.

NOTE: If a general purpose frequency source is used for the VCO the loop gain can be measured by manually adjusting the frequency by a fixed amount (i.e. 200 Hz) and measuring the shift in the loop frequency. The gain is the quotient of the two. Gains over 100 are considered acceptable.
Inexpensive broad band devices are available from Mini Circuits for about $50.

- **Minus** – These have very high tuning sensitivities on the order of 3 to 30 MHz/V.
- **Minus** - With a wide frequency range they will be more susceptible to temperature induced drifts.
- **Plus** – They can be used over a broad range without retooling.
- **Plus** – Low cost

Customized VCOs with thermal stabilization an narrow frequency ranges for about $1500

- **Plus** – Low bandwidth crystal based devices are not sensitive to temperature drifts.
- **Plus** – Moderate cost.
- **Minus** – Can not be used to tune to the different pass band frequencies.
- **Minus** – Narrow band required for different cavity frequencies.

**RF source based VCO such as an Agilent E4422B for about $12,000.**

- **Plus** – Low bandwidth to reduce noise issues
- **Plus** – Flexible and stable frequency source
- **Plus** – Has simultaneous AM modulation capabilities which are useful for cavity conditioning, etc.
- **Minus** – High cost device.
• Directional coupler.
  • Coupling dictated by a combination of VCO output level and Mixer LO requirement.
  • LO path may require an amplifier to ensure the proper drive level for the mixer.

• Phase Shifter*
  • Typically mechanical phase shifters are used such as Narda 3752 or Arra D3428B.
  • Ensure that they provide at least 190 degrees of phase shift at the frequency of operation.

• Variable Attenuator*
  • Typically both continuous and step mechanical attenuators are used for manual systems.
  • Narda, Arra manufacture both. Caution should be used if a PIN attenuator is used as it
    will strongly affect the loop phase.

*Vector Modulators are frequently used to supplement the phase shifter and replace the
mechanical attenuators.
  • Analog Devices as well as several other manufactures produce integrated circuits with
    analog controls
  • GT Microwave, Vectronics, and others make connectorized devices with digital controls.
Cavity Phase and Amplitude for VCO/PLL

Phase (Degrees)

Signal Magnitude (Volts)

Frequency (Hz)

Phase x Voltage

Phase
Detector Voltage

Frequency (Hz)
Complete Low Level System Layout
The transmitted power network has a switchable LNA and variable attenuator, both can be controlled by the computer.

The 6 dB pad before the LNA ensures that there is a minimal gap in the continuous gain control settings. Without it there would have been a 12 dB dead band.

The phase shift associated with the PIN attenuator was measured and included as a lookup table in the program. The compensation values are factored into the vector modulator algorithm.

Circulators are used to reduce the mismatch and ensure more stable power meter calibrations. The 10 dB attenuators used in the incident and reflected power path also serve that purpose.

Only low drift fixed devices are used between the power meters and the cavities.
The VCO was moved to an external location so that it could be thermally stabilized. This also allows us to use alternate VCOs.

The 50 dB attenuator was necessary due to the excessive tuning sensitivity of the VCO, which is 5.6 MHz/V.

Crystal detector gain set to reduce the square law errors while maintaining a 1 V amplitude at the DAQ input.

Even though there is a vector modulator, there is a mechanical phase shifter which is used frequently.

Computer controlled vector modulator is used for amplitude and phase control.

All crystal detector signals are buffered and available for observation using an oscilloscope.

Computer controlled and automatic data acquisition ensures repeatable data and methods independent of operator.
Interlocks For Vertical Tests

- During vertical testing medium power amplifiers between 100 W and 500 W are used to drive the cavities.
- No cavity protection interlocks are used during these tests at Jefferson Lab. Each facility and test should be evaluated individually.
- Field emission radiation does present a safety hazard. This is mitigated during vertical testing at Jefferson Lab by using one of 6 shielded vertical dewars.
- High power RF can not be applied to an accelerating structure until the PSS system confirms that the dewar shield lid is closed.
- Low power, less than 1 W, must be applied to the system in order to calibrate the cables.
- A switching system shown here was implemented to perform these functions for “R&D” testing.
- A similar switching system, along with dewar selection switches and permanently installed cables, was implemented for the production system.
Due to the excessive costs to recover from a coupler failure. Full interlocks were implemented for the system, including:

- Arc
- Infrared
- Coupler and cavity Vacuums
- Helium pressure and level
- Coupler cooling water flow

Use of the interlocks was mandatory for all high power operations.

Boonton 4532 pulsed RF power meters were used to acquire waveform records of the pulsed RF power data. A software interlock was added based on HOM coupler power levels.

Circulators added to ensure that user changes to the other outputs would not affect calibrations.

4-Way splitters added so that the RF signals could be used by other systems in parallel with the standard data acquisition process.

Crystal detectors used for operator feedback only.

Computer controlled and automatic data acquisition was necessary for calibration of the field probes using an emitted power technique.
Cryomodule Test System Layout

Personnel safety system provides a permit to allow high voltage operation of the klystron. Operations at less than 1 W allowed without a PSS interface.

Waveguide directional coupler placed in the middle of a 4 m run of waveguide in order to avoid errors due to evanescent modes.

Attenuators were distributed throughout the system in order to reduce the susceptibility to standing wave induced errors.

Polyphaser B50 or MR50 series lightning arrestors were added to the HOM ports after several RF power heads and medium power attenuators (20 W-CW and 500 W-PK) were destroyed. Excessive power was observed on a crystal detector when a cavity had a thermal quench.

At times during the SNS testing a 20 kW CW klystron was substituted for the 1 MW pulsed klystron. Stub tuners and iris plates were used to modify the input coupling of the system. Maximum CW power levels were limited by the coupler power capacity.
VTA TEST SYSTEMS

Chassis from 805 MHz production system.

500 to 1000 MHz VCO-PLL System used for SNS Production

500 to 3,000 MHz VCO-PLL system used for research and development
VERTICAL TEST AREA TESTING PROGRAM

Forward power and phase controlled via an I/Q modulator.

Internal algorithm controls LNA and PIN attenuator to ensure proper signal levels for Mixer and transmitted power crystal detector.

Q₀, incident power, and gradient continuously updated.

Measure and log button updates graph and logs data to a file.

Interactive calibration routine with imbedded instructions.
CRYOMODULE TESTING PROGRAM

“Real time” gradient and forward and reflected power waveforms shown (supplemented with crystal detectors and an oscilloscope)

Loaded Q, Field Probe Q, gradient, and emitted power calculated on each pulse.

Gradient waveform based on an entered value of the field probe Q.

Forward power and phase controlled via an I/Q modulator.

Data continuously logged to a network drive. Waveforms recorded on request.
• System uses analog vacuum signal to control the drive level for a klystron. When the vacuum signal increases the PIN attenuator reduces the RF Drive signal.
• Diode adder ensures that the larger of the two vacuum signals controls the feedback.
• Separate vacuum set point and gain control with analog read back.
• Redundant switching of RF in the event of an interlock fault.
• Ones of millisecond response time achieved. Limited by vacuum gauge controller.
• Phase shift associated with PIN attenuator may cause problems when operating a cavity with a VCO-PLL.
A cavity resonance monitor is a system which provides an output signal which is proportional to the difference in frequency between the input signal and a reference source.

They are useful for making accurate microphonic measurements in time domain.

The front end circuitry requires careful tuning to ensure precise I/Q demodulation.

The limiting amplifier is used to stabilize the gain in the system. Without it a separate power measurement would have to be made in order to calibrate the output signals.
CAVITY RESONANCE MONITOR

Define the input signal to be \( \cos(\omega_0 t + \varphi(t)) \) and the reference signal be \( \cos(\omega_0 t) \).

Passing the input signal through the 90 degree hybrid provides the following:

\[
I = \cos(\omega_1 t + \varphi(t)) \cos(\omega_0 t) = \frac{1}{2} \cos((\omega_1 - \omega_0)t + \varphi(t)) + \frac{1}{2} \cos((\omega_0 + \omega_1)t + \varphi(t))
\]

\[
Q = \sin(\omega_1 t + \varphi(t)) \cos(\omega_0 t) = \frac{1}{2} \sin((\omega_0 + \omega_1)t + \varphi(t)) + \frac{1}{2} \sin((\omega_1 - \omega_0)t + \varphi(t))
\]

Going to base band eliminates the sum frequency terms. To simplify let \( \omega_2 = \omega_1 - \omega_0 = 0 \)

\[
I = \frac{1}{2} \cos(\varphi(t)) \quad \text{and} \quad Q = \frac{1}{2} \sin(\varphi(t))
\]

\[
\frac{dI}{dt} = -\frac{1}{2} \sin(\varphi(t)) \frac{d\varphi(t)}{dt} \quad \text{and} \quad \frac{dQ}{dt} = \frac{1}{2} \cos(\varphi(t)) \frac{d\varphi(t)}{dt}
\]

\[
I \frac{dQ}{dt} - Q \frac{dI}{dt} = \cos^2(\varphi(t)) \frac{d\varphi(t)}{dt} + \sin^2(\varphi(t)) \frac{d\varphi(t)}{dt}
\]

\[
I \frac{dQ}{dt} - Q \frac{dI}{dt} = \left( \cos^2(\varphi(t)) + \sin^2(\varphi(t)) \right) \frac{d\varphi(t)}{dt}
\]

or

\[
I \frac{dQ}{dt} - Q \frac{dI}{dt} = \frac{d\varphi(t)}{dt}
\]

which is equal to the frequency as a function of time.
• A cavity resonance monitor is calibrated by using two stable sources with a common reference signal. The frequency on one source is varied and the difference in output voltage is recorded.

• An alternate method when using a relatively stable cavity is to shift the frequency of the reference sources slightly and measure the subsequent shift in output signal.

• One of the problems with an analog resonance monitor is that mixers are not ideal. At higher frequencies of I and Q, the second harmonic components bleed through to the output giving false frequency content.

• A new DSP based system is currently under development at Jefferson Lab, which shows a great deal of promise. Using a CORDIC algorithm for phase determination and a high resolution analog to digital converter eliminates the need for the difficult to find limiting amplifier; simplifies the calibration process and eliminates gain drifts. A paper will be presented by Tomasz Plawski during the poster session on Thursday I invite you to stop by and visit poster P63.
CRM EXAMPLES

Time domain and frequency domain plots of the background microphonics for a 5-cell CEBAF cavity located in the CEBAF accelerator.
CRM EXAMPLES

Vibrational modes excited by the sudden loss of cavity gradient due to a window discharge on the cavity side of a cold window in the same cavity as the previous slide.
Step response of a cavity excited by a 50 Hz step in the piezo tuner controls. The total range of this tuner was 550 Hz.
CABLE CALIBRATIONS

• Accurate consistent cable calibrations can make or break a test program.

• VSWR mismatches in the RF circuits will cause errors to “appear” when the frequency is shifted or the load mismatch changes.

• Cable calibrations for cavity testing are complicated by the fact that one or more of the cables are only accessible from one end.
  • In a vertical test the incident power cable, the field probe cable, as well as any HOM cables all have sections that are in the helium bath.
  • In cryomodule testing the field probe cable and any HOM cables have sections of cable that are within the cryomodule.

• When possible cables should be calibrated using signal injection and measurement at the other end using either a source and power meter combination; or a network analyzer.

• Cables should be measured at or near the frequency of the test.

• The only way to measure the losses of a cable within a cryostat is to do a two way loss measurement either with a calibrated network analyzer or a source, a circulator and a power meter.
ONE WAY CABLE CALIBRATION

- To calibrate the cable from point A to point C.
- Measure the one way loss of cable B-C.
  - Measure the reference source power level with the reference power meter. (P1)
  - Connect the reference source to point B of cable B-C.
  - Measure the power level with the transmitted power meter. (P2)
  - The one way loss is P1-P2 (dB)
- Measure the two way return loss of cable A-B
  - Connect the reference source to the input terminal of the circulator.
  - Connect the reference power meter to the load port on the circulator.
  - Record the reading on the reference power meter with the output port of the circulator open.* (P3)
  - Connect the output port of the circulator to port B of cable A-B and record the reading on the reference power meter. (P4)
  - The two way return loss is P3-P4 (dB)
- The cable calibration between for the A-C path is $C_{AC} = (P1-P2) + (P3-P4)/2$. 
TWO WAY CABLE CALIBRATION

- To calibrate the cable from point D to F and D to G
  - Measure the forward power calibration from E to F
    - Connect the reference power meter to point E of the cable from the RF drive source.
    - Turn on the RF drive source and increase the power until the power level on the reference power meter is about 2/3 of the maximum allowed.
    - Record the power levels on the reference meter (P5) and the incident meter (P6)
  - Measure the reflected power calibration from E to G
    - Turn off the RF source drive
    - Measure the reference source power level with the reference power meter. (P7)
    - Connect the reference source to point E of the path E-G.
    - Measure the power level with the reflected power meter. (P8)
- Measure the two way loss for the cable D-E with a detuned cavity.
  - Connect the RF drive source to the cavity at point E.
  - Turn on the RF drive source and apply power to the cavity at a frequency about 10 to 20 kHz higher or lower than the cavity’s resonant frequency.
  - Measure the incident (P9) and reflected power (P10) with the respective meters.
- The cable calibration are:
  - Incident $C_{D-F} = (P5 - P6 + P7 - P8 - P9 + P10)/2$ (dB)
  - Reflected $C_{D-G} = (-P5 + P6 + 3*P7 - 3*P8 - P9 + P10)/2$ (dB)
CALIBRATION VERIFICATION

- Two ways to verify calibration procedures are to:
- Calibrate the system using an external cable rather than a cable within the dewar then:
  - For field probe power and reflected power inject a known signal level into the external cable and measure the power using the calibrated meter.
  - For the forward power connect the external cable to a remote power meter and measure the power using the remote power meter and the system power meter.
- In both cases it can be a useful exercise to vary the frequency over a 1 MHz to 2 MHz range and compare the values over the range.
A third way to verify the calibration and look for VSWR problems in the incident power cable is to:

- Use the RF drive source to apply power to either an open test cable that has been calibrated or a detuned cavity.
- Measure the calibrated forward and reflected power. They should be equal.
- Vary the RF frequency by +/- 1MHz in 100 kHz increments.

Variations in the ratio of forward to reflected power indicate a VSWR problem within the cabling system.
Difference between RF readings calibrated at 805 MHz and those taken at nearby frequencies for several different signal paths. The paths with smaller errors had attenuators distributed throughout the signal path.
CABLE BREAKDOWN IN LOW PRESSURE HELIUM

• When vertical testing the incident power cables must pass through the low pressure helium gas in order to get to the fundamental power coupler.

• Both the mating connector space as well as the cable back shell space are susceptible to this phenomena.

• Glow discharges have been produced in un-terminated N-connectors at 20 Torr using as little as 10 Watts.

• Even connectors in 2 K liquid helium have been known to break down at power levels on the order of 150 W, full reflected at the cavity.
CABLE BREAKDOWN IN LOW PRESSURE HELIUM

- Once a breakdown is initiated it will be sustained by the forward power even at levels down to 10 W.
- Such events appear to be Q-switching within the cavity. The gradient will be reduced and the measured Qo will be reduced substantially.
- These discharges destroy connectors and have the potential to cause failures in vacuum feedthroughs.
- To put things in perspective
  - The Paschen minimum is the product of the pressure and distance required for the minimum voltage breakdown in gas.
  - For helium this value is 4 Torr-cm.
  - In other words at 20 Torr the electrode spacing for a minimum voltage breakdown is 2 mm.
- The theory on breakdown in liquid is that:
  - A few watts of heat is produced in the connector, possibly through thermal conduction down the, insulated, center conductor, from the antenna within the cavity, or in the connector pin itself.
  - The liquid helium flashes to gas within the connector
  - A breakdown occurs in the newly produced low pressure gas volume.
CABLE BREAKDOWN IN LOW PRESSURE HELIUM

• To determine if you have a cable discharge, while it is occurring:
  • Detune the frequency of the LLRF system far enough to lose lock in the cavity.
  • Measure the forward and reflected power.
  • Subtract the calibrated forward power from the calibrated reflected power to calculate the lost power.
  • If any significant power is being lost you probably have a glow discharge in the connector.

• On occasion connectors damaged from mechanism this will exhibit this anomalous loss permanently at all power levels.

• Therefore one should turn off the RF power; and repeat the steps above to ensure that the lost power is consistent with the error associated with the measurement.
So what is an engineer to do?

• NEVER make a high power RF connection in low pressure helium gas.

• We use silicon dioxide dielectric, stainless steel jacketed, cables manufactured by Meggitt Safety Systems which have the outer conductor welded into a Conflat flange. This ensures that the high power connections are only made in liquid helium.

• Vent all connector volumes to the helium bath to improve the heat conduction out of the space, especially connector backshells.

• Fill all potential spaces with insulating material. In theory this should work but we have only had limited success at 300 W.

• One option that we have pursued but not fully implemented is to pressurize the cable with helium gas including the connection to the vacuum feed through at the coupler antenna, above the triple point of helium.

• Best of all critically couple the cavities by carefully adjusting the input antenna or by using a variable coupler so that you do not have to use more than 150 W at the cavity.
VERTICAL AND HORIZONTAL TESTING

• During production cavities are generally tested using antenna inserted into the fundamental power couplers or one of the beam pipes. The goal is to have the cavity at or near critical coupling for these tests. In this way a minimum amount of power can be used to reach design gradient. Ideally this means just enough power to overcome the heat losses in the cavity and the power coupled out of the other ports. This has the advantage that the power lost to wall heating can be calculated based on RF measurements.

• In most labs these tests are done in vertical test dewars, hence they are commonly called vertical tests.

• Cavities in a cryomodule are typically tested using the production couplers that are strongly over coupled. This presents a problem as the errors in lost RF power get excessive when 95% to 99.9% of the incident power is reflected back out of the fundamental power coupler.

• During cryomodule tests the RF heat load is measured calorimetrically.
# FUNDAMENTAL TERMS

| Symbol | Definition                          | Unit          | \(|\Omega/m\)|                     |
|--------|-------------------------------------|---------------|----------------|
| \(r/Q\) | Shunt Impedance*                   | \(\Omega\)    |                |
| \(G\)  | Geometry Factor                     | \(\Omega\)    |                |
| \(E\)  | Electric Field                      | V/m           |                |
| \(L\)  | Electrical Length                   | m             |                |
| \(\omega_0\) | Cavity Frequency                | s\(^{-1}\)    |                |
| \(U\)  | Stored Energy                       | J             |                |
| \(r_S\) | Surface Resistance                  | \(\Omega\)    |                |
| \(T_C\) | Critical Temperature                | K             |                |
| \(P_X\) | RF Power at port X                  | W             |                |
| \(P_{emit}\) | Emitted Power                  | W             |                |
| \(R\)  | Shunt impedance                     | \(\Omega\)    |                |

*Operational Temperature | K  
*Residual Surface Resistance | \(\Omega\)  
*Intrinsic Quality Factor | \(Q_0\)  
*FPC Coupling Factor | \(Q_{FPC}\)  
*Field Probe Coupling Factor | \(Q_{FP}, Q_2\)  
*Coupling Impedance | \(\Omega/m\)  
*Beam Current | A  
*Matching Current | A  
*Dissipated Power | W  
*Decay Time | s  
*Shunt Impedance Per Unit L | \(\Omega/m\)  

*Beware that there are different definitions for shunt impedance in use. At Jefferson Lab we use \(R = V^2/P\) that includes transit time factor for \(\beta = 1\).*
FUNDAMENTAL EQUATIONS
(The Doolittle Sheet part 1)

\[ U = \frac{E^2 L}{(r / Q) \omega_0} \]

\[ P = \frac{U \omega_0}{Q} = \frac{E^2 L}{Q (r / Q)} \]

\[ Q_0 = G / rS \parallel Q_{\text{ElectronLo ading}} \]

\[ r_s \approx 10 - 4 (\Omega K / GHz \ 2) \frac{f^2}{T} e^{-1.95T_c / T} + r_{\text{resid}} \]

\[ \frac{1}{Q_L} = \frac{1}{Q_{\text{FUNDIMENTAL L POWER COUPLER}}} + \frac{1}{Q_0} + \frac{1}{Q_{\text{FIELD PROBE}}} + \ldots \]

for over coupled cavities \[ Q_L = Q_0 \parallel Q_{\text{FPC}} \parallel Q_{\text{FP}} \approx Q_{\text{FPC}} \]

\[ R_C = Q_L (r / Q) \]

\[ I_M = E / R_C \]
Power levels for a strongly over coupled cavity, including beam loading but no microphonics:

delivered to beam \( LEI \)

needed from the klystron

\[
\frac{L(E + IR_C)^2}{4R_C} = \frac{L}{4 Q_L(r/Q)} (E + IQ_L(r/Q))^2
\]

reflected to the circulator

\[
\frac{L(E - IR_C)^2}{4R_C} = \frac{L}{4 Q_L(r/Q)} (E - IQ_L(r/Q))^2
\]

Time dependent, complex differential equation where \( \vec{K} \) is the incident wave amplitude in \( \sqrt{\text{Watts}} \), \( \omega_d \) is the (time varying) detune angle, and \( \omega_f = \omega_0 / 2 Q_L \):

\[
\left(1 - j \frac{\omega_d}{\omega_f}\right) \vec{E} + \frac{1}{\omega_f} \frac{d \vec{E}}{dt} = 2 \vec{K} \sqrt{\frac{R_C}{L} - R_C \vec{I}}
\]

One addition to the standards is the equation for the power required for cavity center frequency \( f_0 \) detuned by \( \delta f \) and beam current, \( I_0 \), off crest by \( \psi_B \):

\[
P_{\text{Klystron}} = \frac{L}{R_C} \left( \frac{\beta + 1}{4 \beta} \right) \left( E + I_0 R_C \cos \psi_B \right)^2 + \left( 2 Q_L \frac{\delta f}{f_0} E + I_0 R_C \sin \psi_B \right)^2
\]
For Cavities that are near critical coupled

- Determine if you are overcoupled or under coupled, under coupled or critically coupled.
  - For a SRF cavity use the time domain waveforms.
  - For a lower Q0 cavity use either network analyzer or use a variable coupler moving from under coupled to over coupled.
- Determine the loaded-Q, \( Q_L \) for the Cavity.
  - For a normal conducting cavity you use a network analyzer and measure bandwidth.
  - For a high Q cavity where the bandwidth is << than the IF bandwidth of the network analyzer, one uses a decay measurement.
- Measure the Incident Power, Reflected Power, Transmitted power going out all of the other ports of the cavity.

\[
P_{\text{loss}} = P_{\text{incident}} - P_{\text{reflected}} - P_{\text{transmitted}} - P_{\text{hom a}} - P_{\text{hom b}}
\]

\[
|\Gamma| = \sqrt{\frac{P_{\text{reflected}}}{P_{\text{incident}}}}
\]

\[
\Gamma = C_{\beta} |\Gamma|
\]

\[
\beta^* = \frac{1 - \Gamma}{1 + \Gamma}
\]

\[
Q_L = 2\pi f_0 \tau \text{ or } Q_L \text{ measured with Network Analyzer}
\]
REFLECTED POWER WAVEFORMS

When operating cavities near critical coupling and preparing to make a decay measurement, one of the items that must be determined is the cavity is over coupled or under coupled. Typically a crystal detector is placed on the reflected power signal and the waveform is observed under pulsed conditions.

Signal goes to zero if properly tuned

Initial peak is equal to the reflected power level when cavity detuned in all cases

Over Coupled

Critically Coupled

Under Coupled

Field Probe

Forward Power

\[ \beta > 1 \]

\[ \beta = 1 \]

\[ 1 > \beta > 1/3 \]

\[ \beta < 1/3 \]
Do more math . . . And determine $Q_0$

$$Q^* = \frac{Q_L}{1 + \beta^*}$$

$$Q_1 = \frac{Q_0}{\beta_1}$$

$$\beta_2 = \frac{P_{\text{transmitte } d}}{P_{\text{loss}}}$$

$$\beta_3 = \frac{P_{\text{hom } a}}{P_{\text{loss}}}$$

$$\beta_4 = \frac{P_{\text{hom } b}}{P_{\text{loss}}}$$

$$\beta_1 = \beta^* (1 + \beta_2 + \beta_3 + \beta_4)$$

$$Q_0 = (1 + \beta_1 + \beta_2 + \beta_3 + \beta_4)Q_L$$
• Use $Q_0$ to calculate the other cavity parameters, where $Q_2$ is the field probe-$Q$

$$Q_2 = \frac{Q_0}{\beta_2}$$

$$Q_3 = \frac{Q_0}{\beta_3}$$

$$Q_4 = \frac{Q_0}{\beta_4}$$

$$\frac{E_{acc} \ (V/m \ )}{\frac{Q_0 P_{Loss}}{L} = \sqrt{\frac{(r/Q)}{L}}}$$

$$U \ (Joules \ ) = \frac{Q_0 P_{Loss}}{2 \pi f_0}$$
 CW Measurement

- Once you know the field probe-Q ($Q_2$), you can use it to make CW measurements of $E$, $U$, and $Q_0$

$$P_{\text{loss}} = P_{\text{incident}} - P_{\text{reflected}} - P_{\text{transmitte \ d}} - P_{\text{hom \ a}} - P_{\text{hom \ b}}$$

$$Q_0 = \frac{Q_2 P_{\text{transmitte \ d}}}{P_{\text{loss}}}$$

$$E_{\text{acc}} = \sqrt{Q_2 P_{\text{transmitte \ d}} \left(\frac{r}{Q}\right)}$$

$$U (\text{Joules}) = \frac{Q_2 P_{\text{transmitte \ d}}}{2\pi f_0}$$
The following are basic equations relating to coupling factor, $\beta$.

$$\beta = \frac{1 - C \beta \sqrt{\frac{P_{\text{reflected}}}{P_{\text{incident}}}}}{1 + C \beta \sqrt{\frac{P_{\text{reflected}}}{P_{\text{incident}}}}}$$

where $C \beta$ is 1 for under coupled and -1 for over coupled.

In the case of a strongly over coupled cavity:

In the case $\beta \gg 1$, $Q_L \ll Q_0$, or $Q_{FP}$ thus $Q_L \approx Q_{FPC}$

$$Q_L = 2\pi\tau$$

$$E^2 = \frac{4\beta}{(1 + \beta)} P_{\text{incident}} Q_L \left(\frac{r}{Q}\right)$$

$$E \approx \sqrt{4P_{\text{incident}} Q_L \left(\frac{r}{Q}\right)}$$

Although using the forward power to calculate gradient is a reasonable technique, practical experience says that there can easily be as much as 25% difference between the gradient measured using this technique as compared to the that measured using the emitted power technique or using a well calibrated field probe measurement. This difference can be reduced by properly tuning the phase locked loop, for a variable frequency system or the cavity for a fixed frequency system.
EMITTED POWER MEASUREMENT
THE REFERENCE MEASUREMENT FOR STRONGLY OVER COUPLED CAVITIES

Consider what happens when you suddenly remove the incident RF power from a cavity that has the stored energy $U$. This stored energy leaves the system through dissipation due to wall losses, i.e. $Q_0$ losses, and as RF power that is emitted from all of the RF ports in the system. Since $Q_L \ll Q_{FP}$ and $Q_L \ll Q_0$ in a strongly over coupled superconducting cavity the stored energy can be calculated as:

$$U = \int_{t_0}^{\infty} P_{\text{emitted}}(t) \, dt \approx \int_{t_0}^{\infty} P_{\text{reflected}}(t) \, dt$$

Historically value of $U$ was measured using a gating circuit and an RMS power meter. In a sampled system, such as can be done with a Boonton 4532 pulsed power meter, the stored energy can be approximated by:

$$U \approx \sum_{m}^{N} \left( P_{\text{reflected}_i} \right) \Delta t$$

Where $m$ is the sample point where the incident power is removed and $N$ is the total number of sample points. In addition to the errors associated with the power measurement, there are errors in this measurement which are introduced by the sampling system that can be reduced by proper choice of system parameters.
EMITTED POWER MEASUREMENT UNCERTAINTY

The uncertainty in the stored energy is given by the following:

\[
\Delta U = U \sqrt{\Delta C_R^2 + \Delta P_{CAL}^2} + \Delta t(N - m)C_R P_{min} + (\Delta P_{emitted})_m \Delta t + \tau(P_{emitted})_N
\]

Where:

- \( \Delta C_R \) is the percentage error in the power reading due to the cable calibration
- \( \Delta P_{CAL} \) is the error in the power meter calibration
- \( \Delta t(N - m)C_R P_{min} \) is the contribution of the power meter noise floor during the integration
- \( (\Delta P_{emitted})_m \Delta t \) is due to the jitter in the start of the integration and the peak of the emitted transient
- \( \tau(P_{emitted})_N \) is the error introduced because you only summed the series to \( N \) and not to \( \infty \)

The last two errors can be minimized by sampling the system at a high sample rate compared to the decay time and insuring that that \( (m-N)\Delta t \) is greater than 4 decay time constants.
FIELD PROBE CALIBRATION

Once the stored energy has been determined the gradient can be calculated by using the following:

\[ E_{\text{Emitted}} = \sqrt{2\pi f_0 \ast U \ast \frac{r}{Q}} \]

Where the emitted subscript is just an indicator of method used to determine the value. The filed probe coupling factor, \( Q_{FP} \) can be calculated using:

\[ Q_{FP} = \frac{E_{\text{Emitted}}^2}{(P_{\text{Transmitted}} \ast d)_{m-1}} \ast \frac{L}{r / Q} \]

Where \( P_{\text{Transmitted}} \) is sampled just prior to removal of the incident power signal. Normally an average of several points just prior to \( m \) is used for this value.

With good calibrations and proper sample rates the gradient, \( E \), can be measured with an accuracy of 5% to 7% and \( Q \) of the field probe to about 10% to 12%.
When making a CW measurement of $Q_0$ on a cavity that is near critical coupling the dissipated power is calculated as the forward power minus the sum RF power leaving the system either as reflected power as it exits the field probe port, HOM port, etc. The error stack up is given as the following:

$$\frac{\Delta P_{Disp}}{P_{Disp}} = \sqrt{\left(\frac{\Delta P_{incident}}{P_{incident}}\right)^2 + \left(\frac{\Delta P_{reflected}}{P_{reflected}}\right)^2 + \left(\frac{\Delta P_{transmitte}}{P_{transmitte}}\right)^2}$$

As was stated earlier, $\beta$ is a measure of the magnitude of the reflected power as compared to incident power. At $\beta = 1$ the error in the dissipated power, as measured using the RF signals, is minimized and approximately the value of the error in the incident power. As $\beta$ gets much above 8 and much below 0.15 the errors in dissipated power, and subsequently calculated values of $Q_0$ start to become excessive.
The typical gradient errors are on the order of 5% to 7% with the same constraints. This assumes that:

- The errors in the power* measurements are less than 6%
- The error in tau is less than 3%.
- The system has as a low VSWR

*Note that Power measurement errors include nonlinearity of the power meters (typically 1% to 2%) as well as absolute accuracy of the instruments (typically 2% to 4%).

Thus for a 40 MV/m peak gradient measurement the actual value is probably between 38 and 42 MV/m.
Qo MEASUREMENTS STRONGLY OVER COUPLED

When making a $Q_0$ measurement on a cavity that is strongly over coupled the dissipated power must be measured calorimetrically. To do this:

- The inlet and outlet values on the helium vessel are closed
- The rate of rise of the helium pressure is measured under static heat load.
- The rate of rise of the helium pressure is measured under a heat load of static plus known resistive power.
- The rate of rise of the helium pressure is measured under a heat load of static plus unknown cavity dissipated power.
- The following equation is used to calculate the unknown cavity dissipated power.

$$\begin{align*}
P_{\text{DISIPATED}} &= \left\{ \frac{dP}{dt} \right\}_{RF-ON} - \left\{ \frac{dP}{dt} \right\}_{\text{STATIC}} \\
&
\left\{ \frac{dP}{dt} \right\}_{\text{HEATER-ON}} - \left\{ \frac{dP}{dt} \right\}_{\text{STATIC}} \right\} P_{\text{HEATER}}
\end{align*}$$

where $\left( \frac{dP}{dt} \right)$ is the rate of rise of the pressure under the different conditions.
CONCLUSION

- Quality measurements necessary to qualify superconducting cavities require quality equipment designs, careful measurement techniques and well characterized calibrations processes.

- Errors for the standard measurements are calculable. However, they are a function of the measurement equipment, the quality of the calibration and the specific conditions of each data point. As such they should be included in the measurement system not as an afterthought.

- In addition to the slides presented, I have included a handout of the equations for both the cavity measurements and the associated errors.