Lattice Design II: Nonlinear Dynamics

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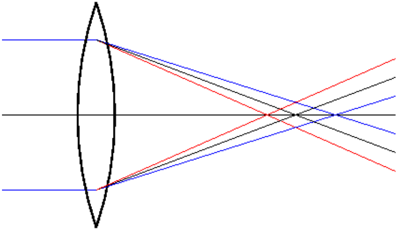
Overview

1. Chromaticity and chromatic corrections
2. Dynamic aperture
3. Energy acceptance

Chromaticity

- Nominal lattice is calculated using nominal momentum p_0.



- Particles with a momentum deviation Δp see a different qudrupole strength

$$k(p) = -\frac{e}{p} g = \frac{e}{p_0 + \Delta p} \approx -\frac{e}{p_0} \left(1 - \frac{\Delta p}{p_0} \right) g = k_0 - \Delta k \qquad (1)$$

- The effect of the momentum deviation can be treated as a quadrupole error

$$\Delta k = \frac{\Delta p}{p} k_0 \qquad (2)$$

Chromaticity (cont'd)

- This leads to a tune change

$$dQ = \frac{\Delta p}{p} \frac{1}{4\pi} k_0 \beta(s) ds \tag{3}$$

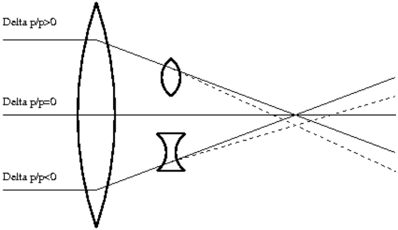
- Integrating over all quadrupoles one gets

$$\xi = \frac{\Delta Q}{\frac{\Delta p}{p}} = \frac{1}{4\pi} \oint k(s) \beta(s) ds \tag{4}$$

- This is the so-called chromaticity
- Most storage rings require chromaticity compensation

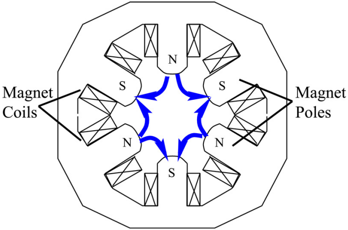
Chromatic Corrections

- Need location where particles are "sorted" by energy, i.e. high dispersion area
$$x_D(s) = D(s)\frac{\Delta p}{p} \tag{5}$$
- Use magnets where focal strength depends on offset, i.e $k \propto x$



Chromatic Corrections (cont'd)

Use sextupole magnets



$$B_x = \frac{\partial \Phi}{\partial x} = g' xy \tag{6}$$

$$B_y = \frac{\partial \Phi}{\partial y} = g'(x^2 - y^2) \tag{7}$$

Gradient along x and y axis:

$$\frac{\partial B_y}{\partial x} = g' x \qquad and \qquad \frac{\partial B_x}{\partial y} = g' x \qquad \Rightarrow \qquad k_{sext} = \frac{e}{p} g' x = mx \tag{8}$$

The effective quadrupole strength depends on the dispersion:

$$k_{sext} = mD \frac{\Delta p}{p} \tag{9}$$

Chromatic Corrections (cont'd)

- To calculate the total chromaticity one needs to integrate over the ring

$$\xi_x = -\frac{1}{4\pi Q_x}\int_0^C \beta_x(s)\left(k(s) - S_0(s)D_x(s)\right)\,ds \qquad (10)$$

$$\xi_y = +\frac{1}{4\pi Q_y}\int_0^C \beta_y(s)\left(k(s) - S_0(s)D_x(s)\right)\,ds \qquad (11)$$

- The natural chromaticity depends only on the quadrupoles

$$\xi_{x0} = -\frac{1}{4\pi Q_x}\int_0^C \beta_x(s)k(s)\,ds \qquad (12)$$

$$\xi_{y0} = +\frac{1}{4\pi Q_y}\int_0^C \beta_y(s)k(s)\,ds \qquad (13)$$

Chromatic Corrections (cont'd)

- So we can express the total chromaticity as

$$\xi_x = \xi_{x0} + \frac{1}{4\pi Q_x} \int_0^C \beta_x(s) S_0(s) D_x(s)\, ds \tag{14}$$

$$\xi_y = \xi_{y0} - \frac{1}{4\pi Q_y} \int_0^C \beta_y(s) S_0(s) D_x(s)\, ds \tag{15}$$

- Using the thin lens approximation, this can be written as

$$\xi_x = \xi_{x0} + \frac{1}{4\pi Q_x} \sum_{i=1}^{N} \beta_{x_i} S_{0_i} D_{x_i} l_{S_i} \tag{16}$$

$$\xi_y = \xi_{y0} + \frac{1}{4\pi Q_y} \sum_{i=1}^{N} \beta_{y_i} S_{0_i} D_{x_i} l_{S_i} \tag{17}$$

Chromatic Corrections (cont'd)

- Assume a correction scheme with two families of sextupoles (with strengths S_{0_1} and S_{0_2} each with length l_s inserted at locations s_1 and s_2 in a cell repeated N times around the ring
- We can then solve the above system of equations and find

$$S_{0_1} = -\frac{4\pi}{Nl_S D_{x_1}} \frac{\beta_{y_2} Q_x \xi_{x_0} + \beta_{x_2} Q_y \xi_{y_0}}{\beta_{x_1}\beta_{y_2} - \beta_{x_2}\beta_{y_1}} \tag{18}$$

$$S_{0_2} = \frac{4\pi}{Nl_S D_{x_2}} \frac{\beta_{y_1} Q_x \xi_{x_0} + \beta_{x_1} Q_y \xi_{y_0}}{\beta_{x_1}\beta_{y_2} - \beta_{x_2}\beta_{y_1}} \tag{19}$$

- Conditions to minimize sextupole strengths:
  - large dispersion
  - large difference bewteen β_x and β_y at the sextupole locations

Chromatic Corrections (cont'd)

- Most storage rings run at slightly positive chromaticity
- Storage rings usually use several families of sextupoles
- Sextupoles introduce non-linear fields which introduce amplitude-dependent betatron oscillations
- trajectories at large amplitude can become chaotic
- preferrably distribute sextupoles

Dynamic Aperture

- Jacques Gareyte: The dynamic aperture is the largest amplitude below which all particles survive for the relevant number of turns.
- Important component of the acceptance of the ring (together with the physical aperture)
- Usually specified in terms of normalised amplitude

$$\frac{A_x}{\gamma} = \gamma_x x^2 + 2\alpha_x x p_x + \beta_x p_x^2 \tag{20}$$

- can be reduced by field errors:
  - quadrupole strength errors
  - nonlinear fields in wigglers
  - systematic higher-order multipoles in magnets (intrinsic)
  - random higher-order multipoles (errors)

Field Errors and Dynamic Aperture

- Multipole field components are typically specified at a reference radius from the magnet axis:

$$\frac{\Delta B_y + i\Delta B_x}{|B(r_0)|} = \sum_n (b_n + ia_n)\left(\frac{x+iy}{r_0}\right)^{n-1} \tag{21}$$

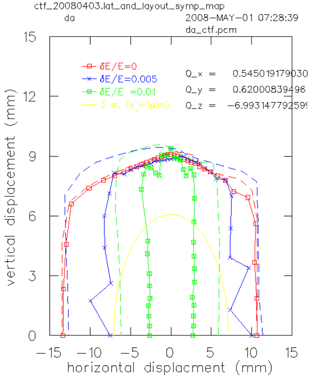
  where b_n are the normal and a_n the skew multipole components.
- $n = 1$ is the dipole component, $n = 2$ the quadrupole and so on
- For systematic errors, the coefficients are fixed values
- For random errors they are usually an rms distribution for each magnet type

Field Errors and Dynamic Aperture

- The values of the coefficients a_n and b_n depend on the magnet design (for systematic and random errors)
- Features that influence the coefficients:
  - shape of pole tips
  - shape of yoke (higher symmetry helps reduce systematic errors)
  - aperture (large aperture reduces multipole errors)
  - length (fringe fields at end)
- Unfortunately the features that result in a good field, also result in an expensive magnet
- Multipole errors can significantly reduce the dynamic aperture
- Robust lattice and minimizing multipole errors can help in achieving only minimal reduction in dynamic aperture

Calculating the Dynamic Aperture

- Simplest method is to set up a grid of particles and track them for the relevant number of turns
- Many tracking codes also provide a command that finds the dynamic aperture
- Calculate frequency map; this provides much more information than just the dynamic aperture
- Start with ideal machine
- For machine with random errors typically a number of different seeds are used

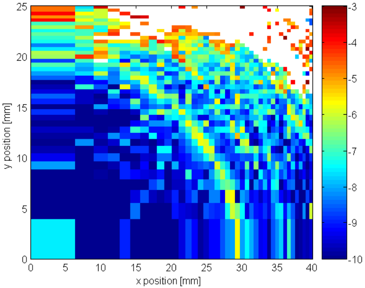


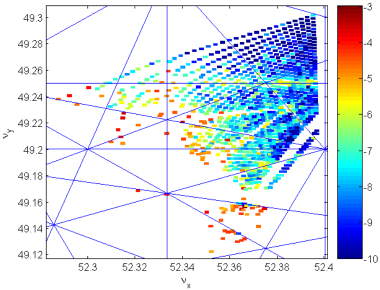
Frequency Map Analysis

- A grid of particles is generated at an arbitrary starting point in the ring. All angles are zero and usually the energy deviations and longitudinal positions inside the bunch as well
- The transverse amplitudes range from zero to a value corresponding to the desired dynamic aperture. The grid is not evenly spaced but the amplitude of the n^{th} particles in one plane is given by $A_n = A_{\max}\sqrt{n/N}$
- This grid of particles is then tracked for a number of turns
- The amplitudes of all particles are recorded each turn.
- This data is then split in two: One set for the first half of turns and one for the second half
- Each set is then used to calculate the tune of each particle.
- The two tunes for each particle are then compared and the difference, called the tune diffusion rate, is saved.
- The lower the diffusion rate, the more stable the particle's trajectory.

Frequency Map Analysis (cont'd)

Example from an early version of the ILC damping rings:





- Provides a lot more information than just the dynamic aperture
- shows which resonances a responsible for limiting dynamic aperture

Measuring Dynamic Aperture

- There are different ways of measuring the dynamic aperture:
- Using fast kicker magnet to kick the beam to large amplitude; Measure at which amplitude half of the beam is lost
- Increase beam emittance (e.g. by changing the rf frequency to change the damping partition numbers) until the beam lifetime is significantly reduced
- Both methods are not very accurate but can give at least some approximation to the dynamic aperture
- It is also possible to measure frequency maps (although only with a much coarser grid than in simulations)

Energy Acceptance

- Energy acceptance determined by
  - Height of RF bucket
  - Off-energy beam dynamics
- The height of the RF bucket can easily be designed to be sufficient, so we will only look at off-energy beam dynamics here.
- Important in damping rings, as injected beam coming from a linac often has fluctuating energy.

Off-energy beam dynamics

- Already discussed chromatic corrections
- Look at chromatic beta-functions (usually only necessary when dealing with large energy spread)
- Dynamic aperture tends to shrink off-energy
- Look at off-energy frequency maps

Off-energy Frequency Maps

Frequency maps for $\frac{\Delta p}{p} = -1\%$ (left), 0% (center) and $+1\%$ (right)