# Fundamentals of Detector Physics and Measurements Lab - I

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# Outline

- Lecture I
  - Constants, atoms, cross sections
  - Photoelectric, TOF
  - PMT, SiPM Scint, Cerenkov
- Lecture II
  - Collisions, cross sections
  - Multiple scattering, radiation length
  - dE/dx, MIP, Range
  - Critical Energy

# Outline II

- Lecture III
  - B fields, trajectories
  - Quadrupoles, focal length
  - Drift and Diffusion
  - Pulse formation in unity gain and gas gain
- Lecture IV
  - Radiation NR, Thompson, Compton
  - Relativistic radiation
  - Bremm, Pair Production

# Sizes, Forces…References

- High Energy Physics is summarized in "The Review of Particle Properties" at
- http://pdg.lbl.gov/2011/reviews/contents_sports.html
- All the basic Physics of detectors is summarized in the Particle Data Group site. References are given there also.
- Other References:


  - [1] Detectors for Particle Radiation, K. Kleinknecht, Cambridge University Press (1987).
  - [2] Experimental Techniques in High Energy Physics, T. Ferbel, Addison-Wesley Publishing Co., Inc. (1987).
  - [3] Instrumentation in High Energy Physics, Ed. F. Sauli, World Scientific (1992).
  - [4] Instrumentation in Elementary Particle Physics, J.C. Anjos, D. Hartill, F. Sauli, M. Sheaf, Rio de Janeiro, 1990, World Scientific Publishing Co. (1992).
  - [5] Instrumentation in Elementary Particle Physics, C.W. Fabjan, J.E. Pilcher, Trieste 1987, World Scientific Publishing Co. (1988).
  - [6] "Particle Detectors" C.W. Fabjan, H.F. Fisher, Repts. Progr. Phys. 43, 1003 (1980)

# Constants to Remember

$\hbar c = 0.2\,GeV\ fm,\ 1\ GeV\ = 10^9\ eV$

$= 2000\,eV\ \overset{o}{A}$

$1\overset{o}{A} = 10^{-8}\,cm,\ \ 1\ fm = 10^{-13}\ cm$

$= 0.1\,nm.$

Atoms are about 1 A in size. Light from atoms is ~ 2000 A

$\alpha = e^2 / \hbar c \sim 1/137$

$\alpha_s = g_s^2 / \hbar c \sim (1/10 - 1)$

The EM force is characterized by a coupling constant of magnitude ~ 1/137

$E_o = -mc^2\alpha^2 / 2$

$m_e c^2 = 0.51 MeV, E_o = 13.6 eV$

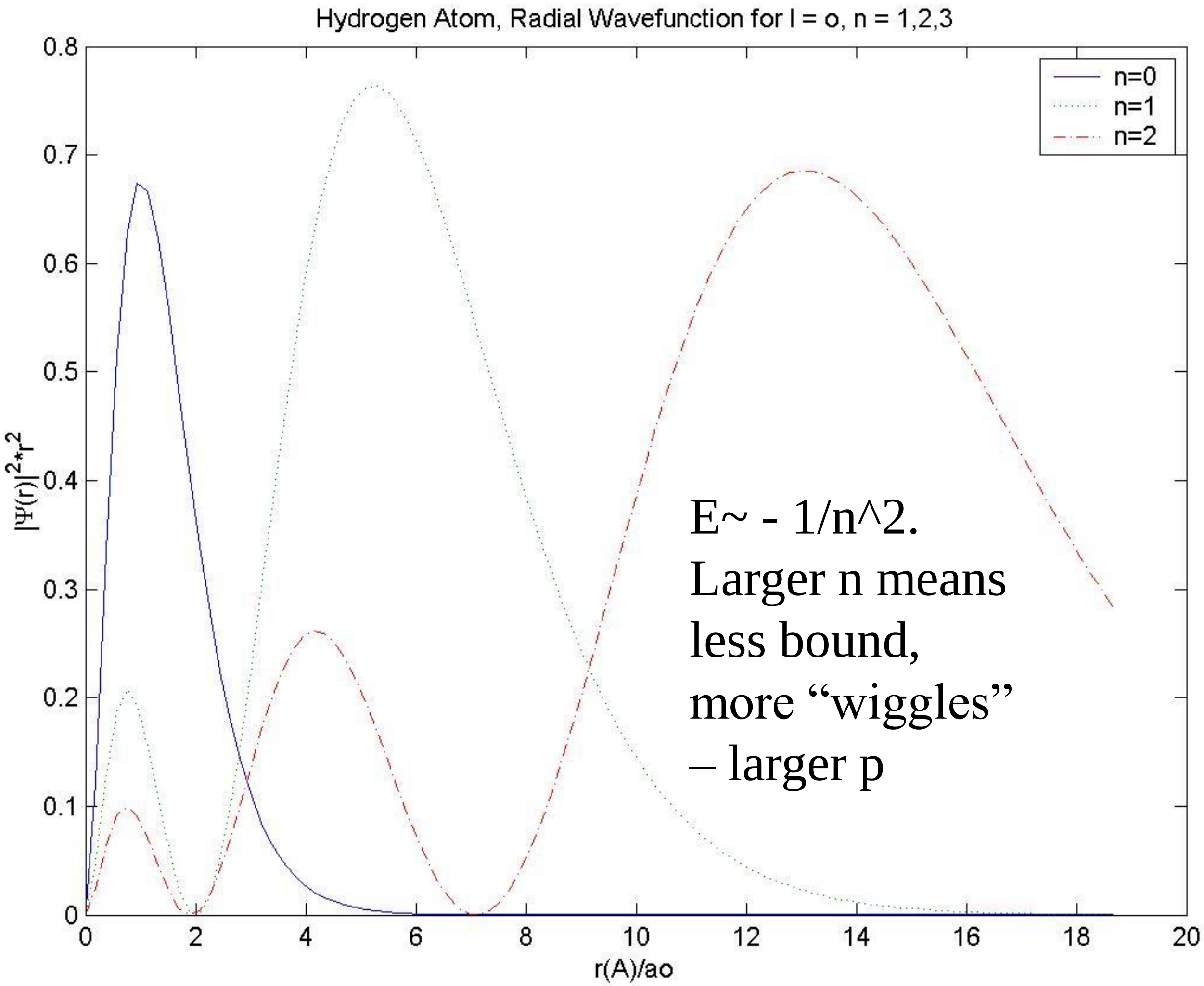
$a_o = \lambda\!\!\!^{-} / \alpha = 0.54\overset{o}{A}, E = E_o = -e^2 / 2a_o$

$\lambda\!\!\!^{-} = \hbar / mc = 0.004\overset{o}{A}$

$\beta = \alpha, \beta = v / c$

Electrons are bound into atoms with energies ~ 1 eV and their motion is non-relativistic.

# Demo – H Atom




E~ - 1/n^2.
Larger n means less bound, more “wiggles” – larger p

There will be a series of “demonstrations” which use simulation and display/computation to make the equations more “real” and where parameters can be varied and their effects observed.

# Atomic, Nuclear Sizes

$$\sigma_{atom} \sim \pi a_o^2 \sim 3x10^8 b, a_o \sim 1 \overset{o}{A}$$

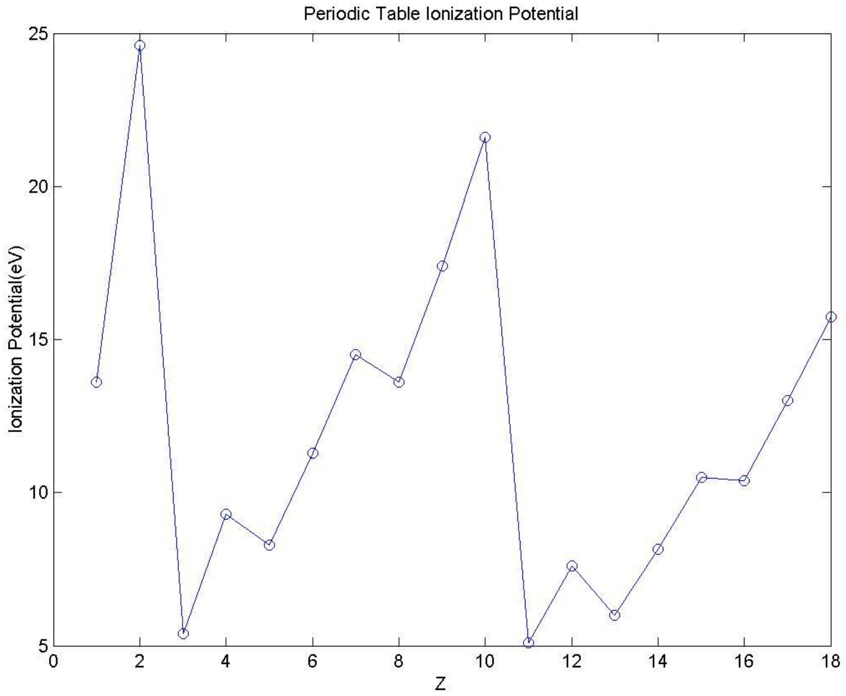
$$\sigma_{nuc} \sim \pi a_N^2 \sim 31 mb, a_N \sim 1 fm$$

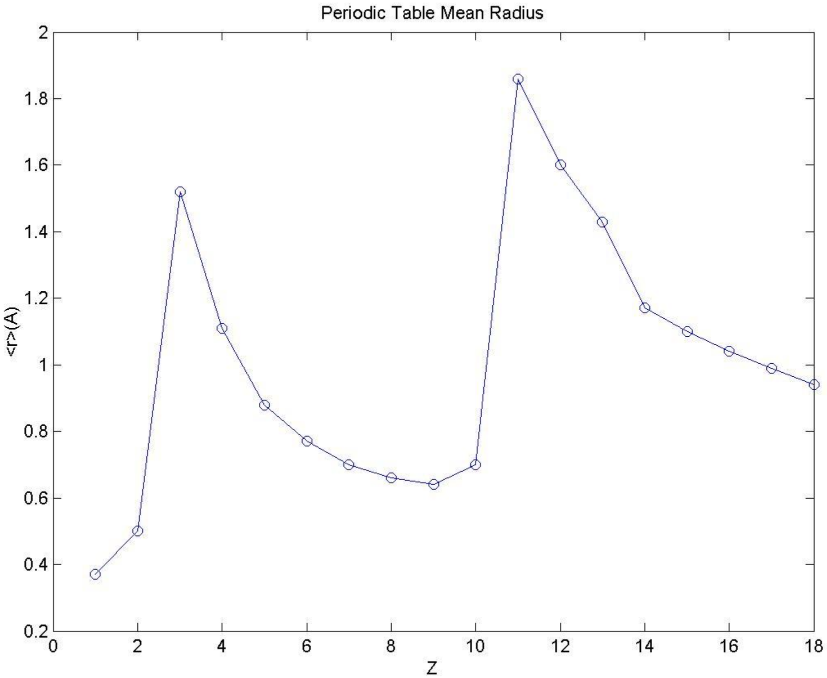
$$(\hbar c)^2 = 0.4 GeV^2 mb$$

$$1mb = 10^{-27} cm^2, 1b = 10^{-24} cm^2$$

Atomic cross sections are ~ mega - barns. Nuclei are ~ 1/100,000 times smaller than atoms and the binding is ~ 100,000 times stronger – eV for atoms and MeV for nuclei. The binding energy /nucleon is ~ 8 MeV Nuclear cross sections are ~ mb.

# Demo – Periodic Table




The first ionization potential is ~ 10 eV and the charge radius is ~ 1 A. The structure in these quantities is understood with Fermi exclusion and the energy states. The "filling sequence" is 2(2l+1) = 2 S states, 6 P states, 10 D states. A shell starts with a metal , goes to a base, and ends with a noble gas. Nuclei have a similar "shell structure".

# Mean Free path

$$N(\mathrm{x}) = N(0)\exp(-N_o \rho x \sigma / A)$$

$$<L>^{-1} \quad = [N_o \rho \sigma / A](cm)^{-1}$$

$$<L\rho>^{-1} = [N_o \sigma / A](gm/cm^2)^{-1}$$

The cross section is fundamental. The mean free path, or interaction length, in material depends on the state of the material – gas, liquid or solid.

The cross section in Ar is ~ $10^{-16} cm^2$ – a typical atomic cross section.

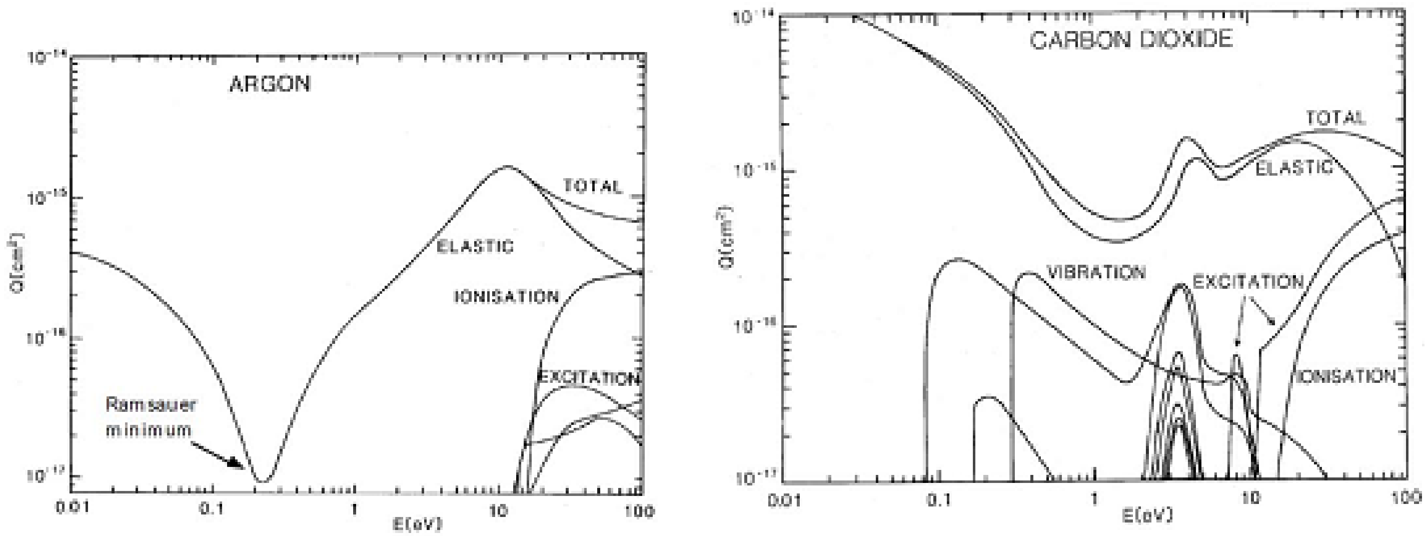



Figure 50: Electron collision cross sections in Argon (a) and Carbon Dioxide (b).

# Nuclear Cross Sections

$$\underline{V} = \frac{4\pi}{3} a_N^3 = A \frac{4\pi}{3} \lambdabar_P^3$$

$$a_N \sim \lambdabar_P A^{1/3}$$

Picture a nucleus as a bag of nucleons (n and p) each of size = their Compton wavelength.

$$\sigma_N \sim \pi a_N^2$$

$$\sim A^{2/3}$$

$$< L > \sim A^{1/3}$$

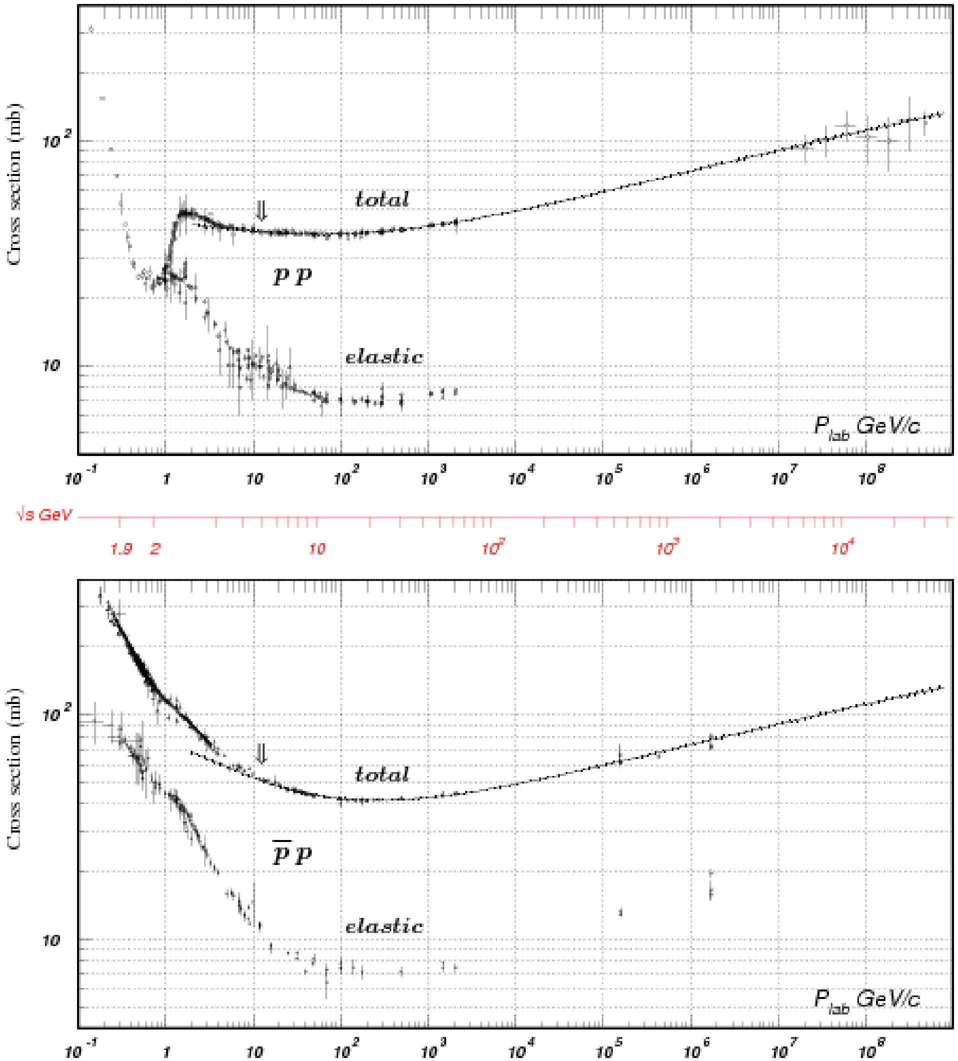
$$\lambda_I \sim (35\, gm / cm^2) A^{1/3}$$

The size of the nucleus grows slowly with A, as does the mean free path for collision.

As a rough rule of thump the mean free path for an inelastic collision, with the density dependence removed is ~ a constant time A^1/3.

A p is of size ~ 1 fm. The cross section is then ~ 10 mb. The cross section grows slowly with A.

# Proton Cross Sections

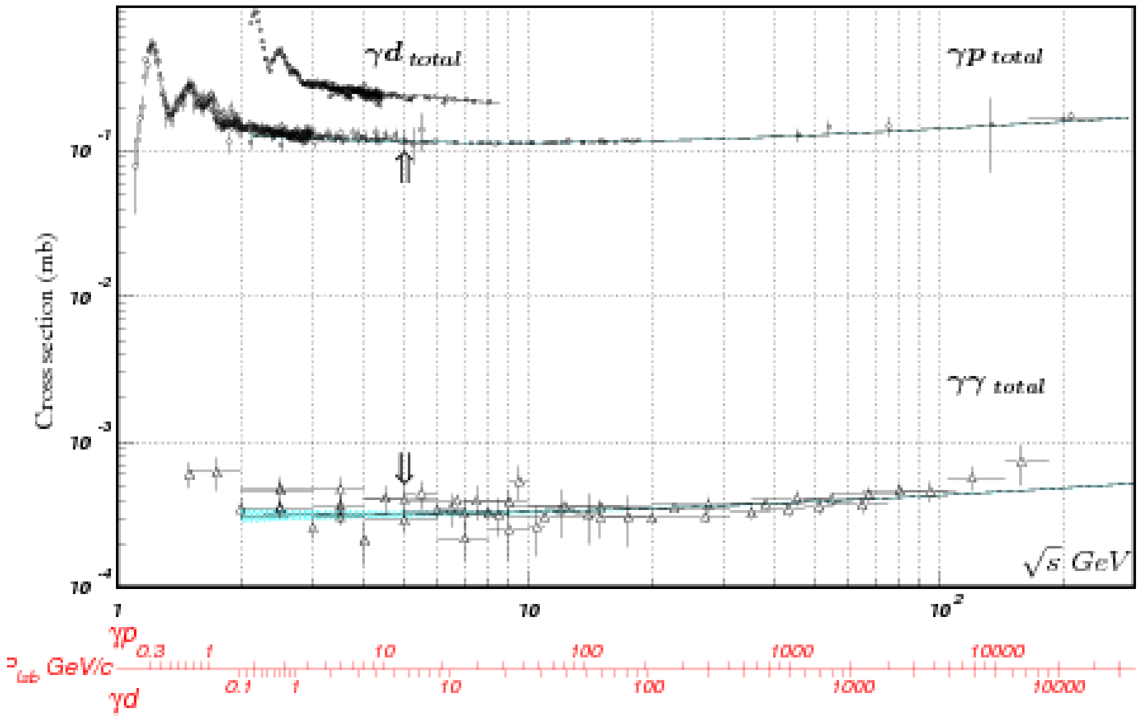



Plots of the energy dependence of the total and elastic cross sections for proton and anti-protons.

At low energies there is a strong energy dependence and protons have a smaller cross section than anti-protons.

Above ~ 10 GeV the cross section is largely inelastic and largely energy independent. It acts like a “grey sphere”.

# Photon Cross Sections




The photon-proton cross section above a few GeV is ~ constant. The size is roughly the p-p cross section times the squared ratio of the strong to EM coupling constants or ~ 0.18 mb.

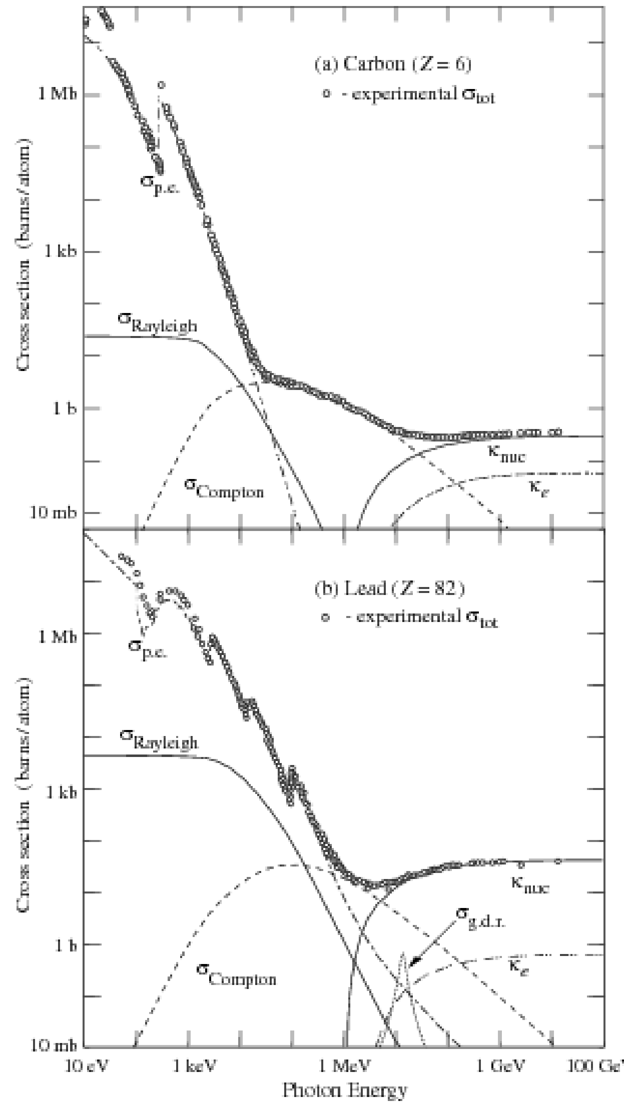
High Energy cross sections

How can light scatter off light? No charge!

# Photon Cross Section – C, Pb

The cross sections for a photon on Pb may be mega barns! This will be discussed later in looking at the photoelectric effect.

Photon scattering is very energy dependent. At low energies there is photoelectric absorption – with sharp edges in energy which mirror the atomic state energies. At energies ~ 1 MeV the largest process is elastic photon-nucleus Compton scattering. At higher energies electron-positron pair production becomes energetically possible.



# Thompson/Raleigh Scattering

$$\frac{d\sigma_T}{d\Omega} = \left(e^2 / mc^2\right)^2 \sin^2\theta \equiv \left\langle d\underline{P} / d\Omega \right\rangle / \left\langle \left| \vec{S} \right| \right\rangle$$

$$\sigma_T = \frac{8\pi}{3}\left(\alpha \lambda\!\!\!^{-}\right)^2 \qquad = 0.66 \text{ b}$$

$$\sigma_T = \frac{8}{3}\left[\pi a_0^2\right]\alpha^4$$

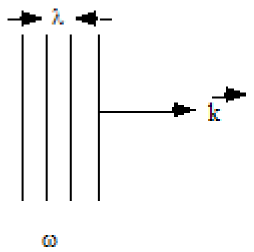
$$a_o = \lambda\!\!\!^{-} / \alpha \; (Section\ 1)$$

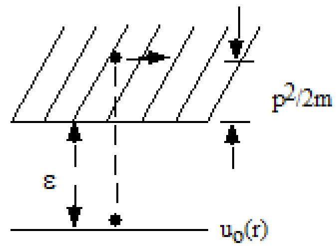
$$\sigma_T / \pi a_0^2 \sim \alpha^4 \sim 10^{-8}$$

Thompson scattering is low energy scattering of photons off electrons. The electric field of the photon accelerates the electrons and they radiate in a dipole angular distribution at the same frequency as the incident photon – elastic scattering.

The effective size of the scattering is alpha times the electron Compton wavelength. Compared to an atomic cross section it is alpha^4 times smaller.

# Photoelectric Effect




$$A =< f \mid H_I \mid i >$$

$$\sim \frac{e}{m} \int e^{i\vec{p}\cdot\vec{r}/\hbar} (\vec{A}\cdot\vec{p}) u_o(r)\, d\vec{r}$$

Sharp energy edges when the photon matches an electron bound state energy

$$\sigma_{PE} \sim \alpha ƛ^2 \left[\frac{mc^2}{\hbar\omega}\right] [\lambda_{DB}/a]^5$$

$$\sim 1/\omega^{7/2}$$

$$\hbar\omega \sim p^2/2m$$

$$\sigma_{PE} \sim \frac{32\pi}{3}\sqrt{2}(Z\alpha)^4 Z \left(\frac{m}{\hbar\omega}\right)^{7/2} (\alpha ƛ)^2$$

$$\sigma_T \sim \frac{8\pi}{3} Z(\alpha ƛ)^2$$

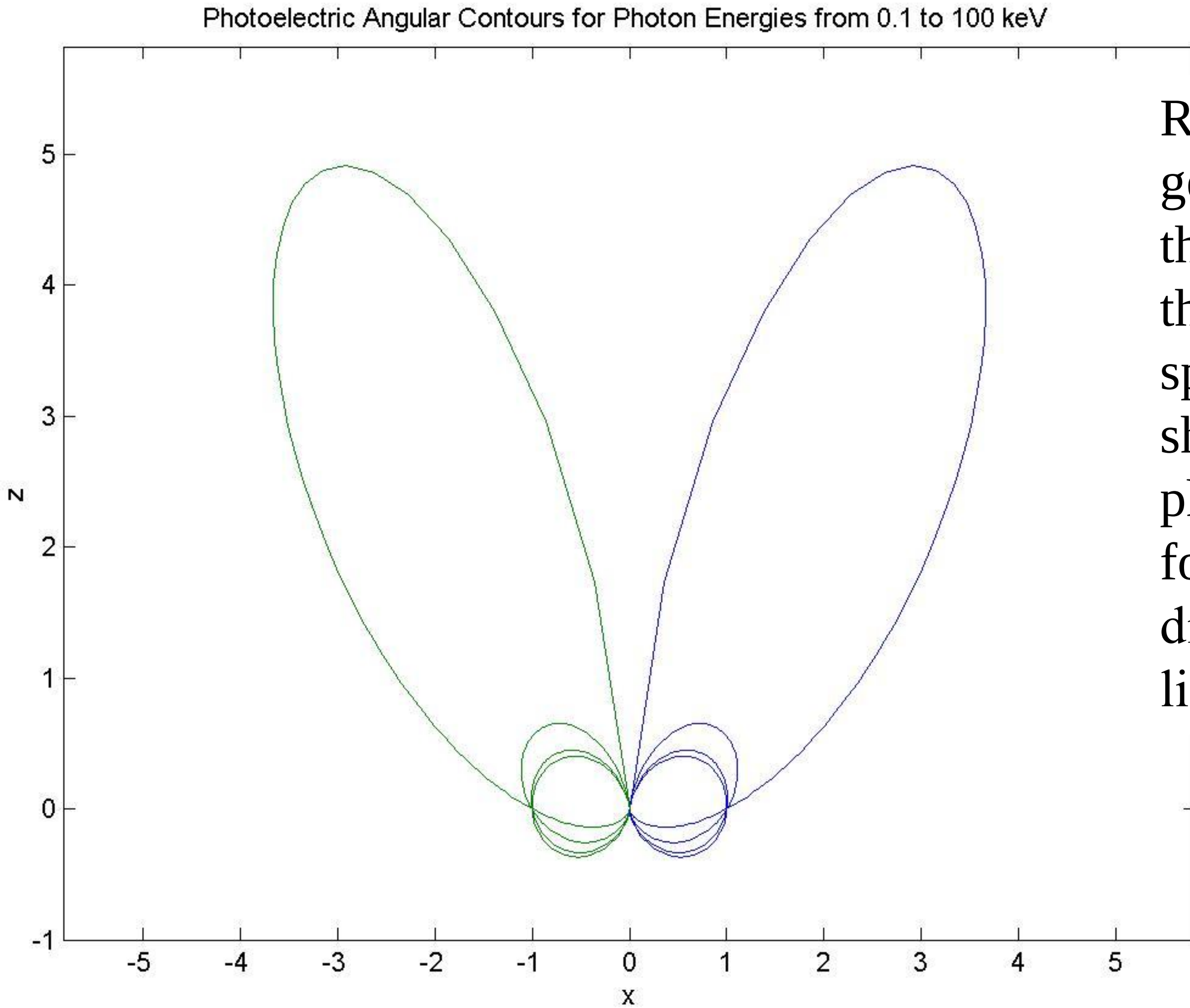
$$\sigma_{PE}/\sigma_T \sim 4\sqrt{2}(Z\alpha)^4 \left(m_e c^2/\hbar\omega\right)^{7/2}.$$

For energies < 0.511 MeV and for heavy atoms, the PE dominates. At higher energies, elastic scattering takes over.

deBroglie wavelength is

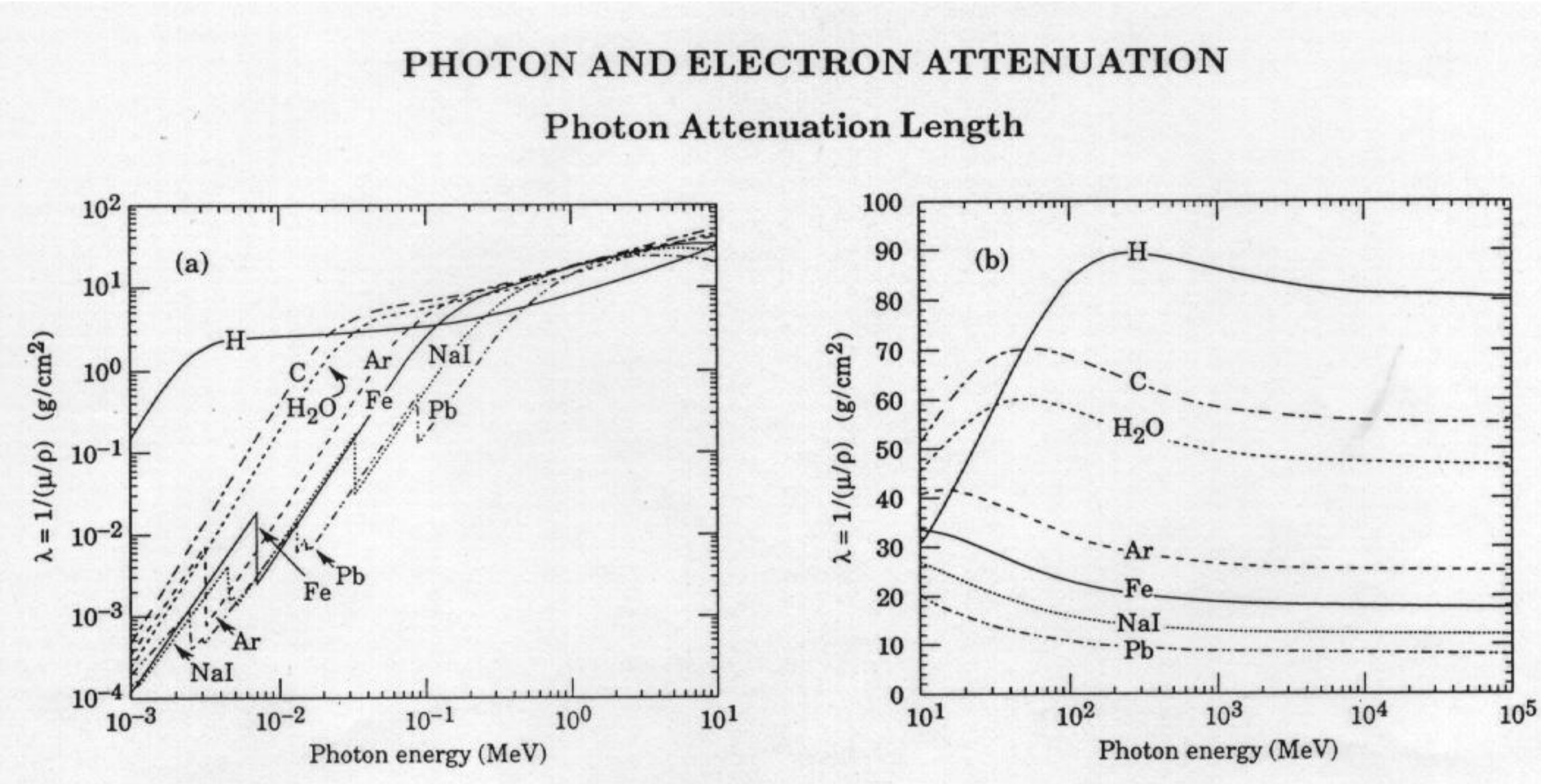
$$ƛ_{dB} = \hbar / p$$

# Demo - Photoelectric



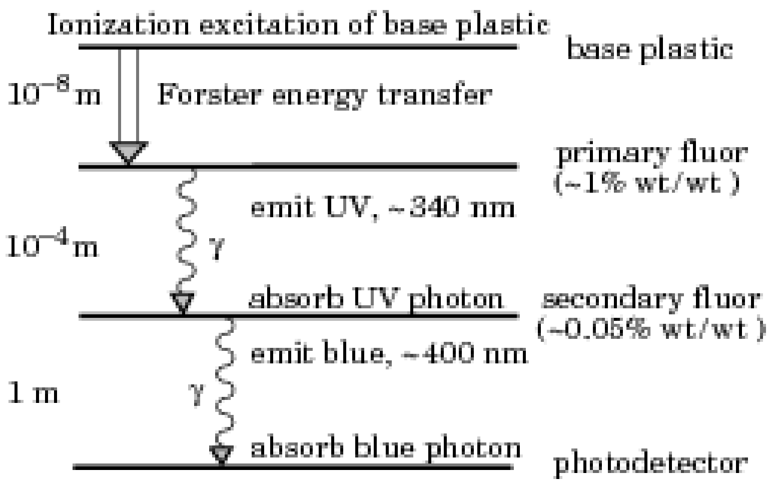

Relativity tends, in general, to push the radiation from the roughly spherical ( dipole) shape at low photon energy to a forward (in photon direction) “search light” pattern.

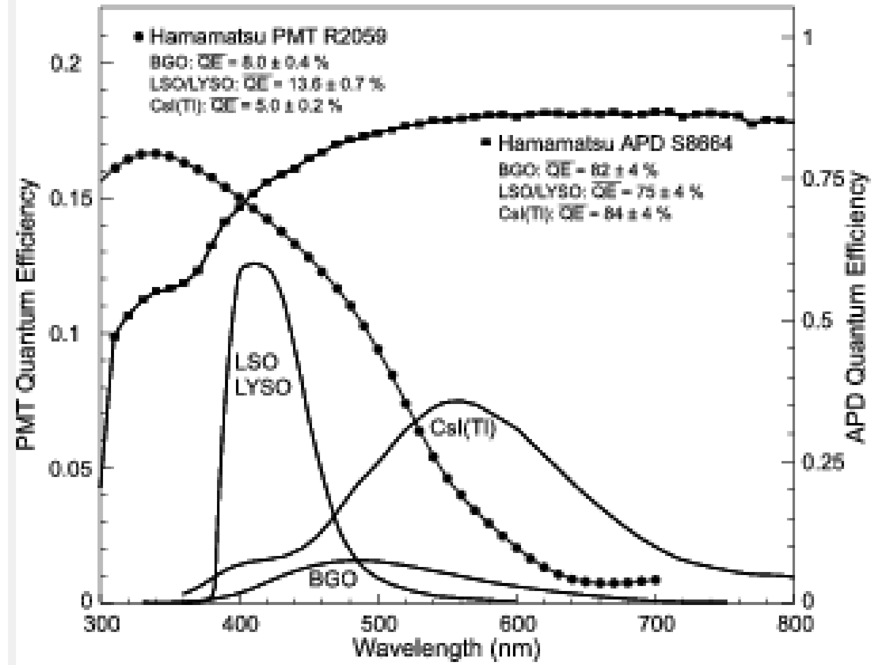
# Photon Interaction Length




Low energy, high cross section, IL ~ 0.01 gm/cm^2. At higher energies, min cross section ~ Compton scattering, IL ~ 50 gm/cm^2 (Z dependent). Above ~ 1 GeV IL~ constant (inelastic pair production) ~ 20 gm/cm^2.

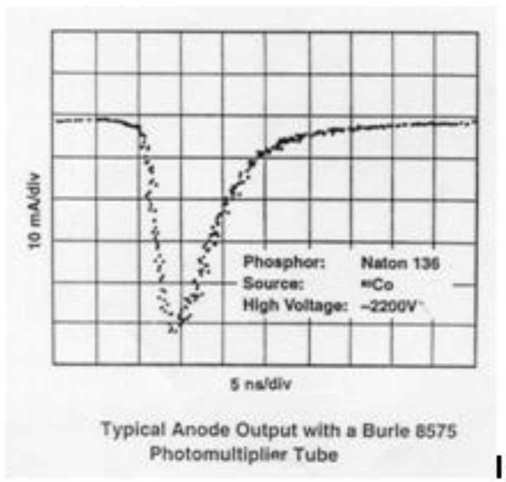
# Scintillation - Fluors




Scintillator responds to ionization of atoms by charged particle collisions. The energy is transferred to a primary fluor and thence to a secondary by way of a UV photon which then de-excites by emission of a visible photon. PMT have a cathode of material which absorbs the visible photon by photoelectric effect yielding a detectable electron. The efficiency is ~ 15%.

# PMT – Pulse and Dynodes



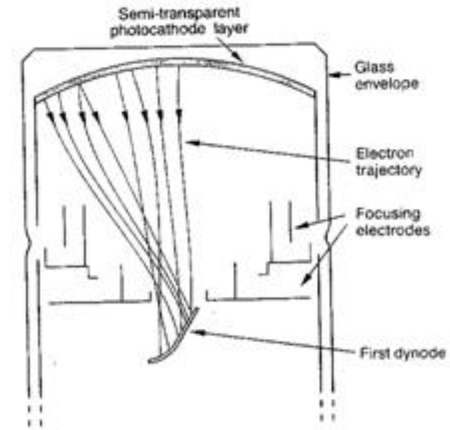




Fig. 2.11: Schematic of the electrode structure of a typical photomultiplier. (From Ref. 2.10.)

Fig. 2.13: Oscilloscope trace of a PMT current pulse output. Note the rise time of ~ 2 nsec and the pulse width of ~ 10 nsec full width at half maximum, FWHM. The peak current is ~ 50 mA and the signal pulse height spread is ~ 10%. (From Ref. 2.10.)

The photoelectrons impact on “dynodes” where the electron yield is multiplied by a factor ~ 4 . With n stages, the gain is ~ 4^n. The current pulse is delivered within a few nsec. For 1 pe delivered in 5 nsec with a 12 stage PMT, dynode gain 4, we have a current pulse of 0.54 mA or 27 mV into 50 Ohms.

# Time of Flight – PMT + Scint

$1\gamma / 100eV$

$500\gamma / cm$

Rule of thumb in scintillator

$vt = (\beta c)t = L$

$\gamma = 1/\sqrt{1-\beta^2} = \varepsilon / m \sim p / m$

$\beta = \sqrt{1 - \frac{1}{\gamma^2}} \sim 1 - \frac{1}{2\gamma^2}, \beta \to 1$

$t \sim \frac{L}{c}\left(1 + \frac{1}{2\gamma^2}\right)$

$\Delta t \sim \frac{L}{c}\left(\frac{m_1^2 - m_2^2}{2p^2}\right), \beta \to 1$

$\tau \sim 1.6\, n\sec$

$\lambda_{\max} \sim 4250\,\mathring{A}(\sim 2.1eV)$

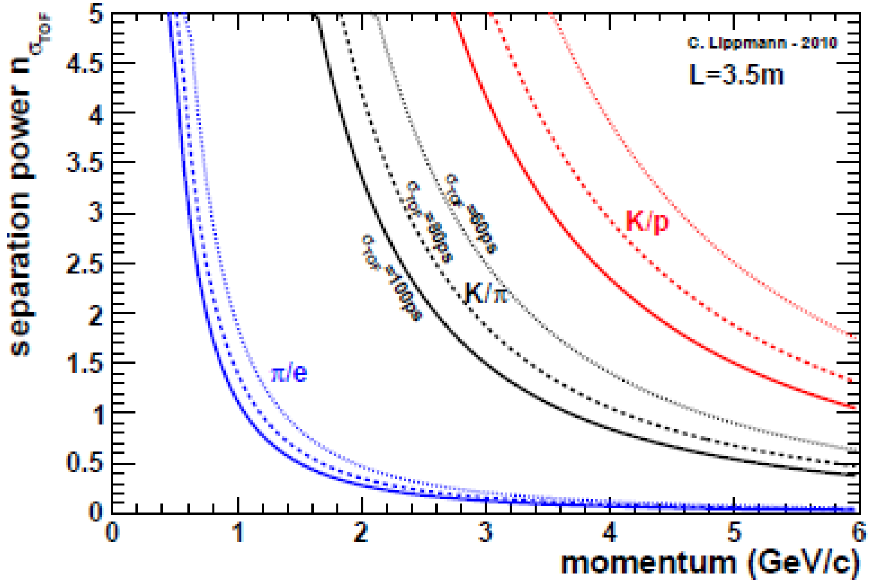
Rough estimates of PMT rise time and gain. Scintillator emission spectrum.

$\delta \sim 3 = gain / dynode$

$G \sim (\delta)^{N_d} = PMT\, gain$

TOF between 2 fixed points. Needed resolution is ~ the TOF of light times the mass to momentum ratio squared

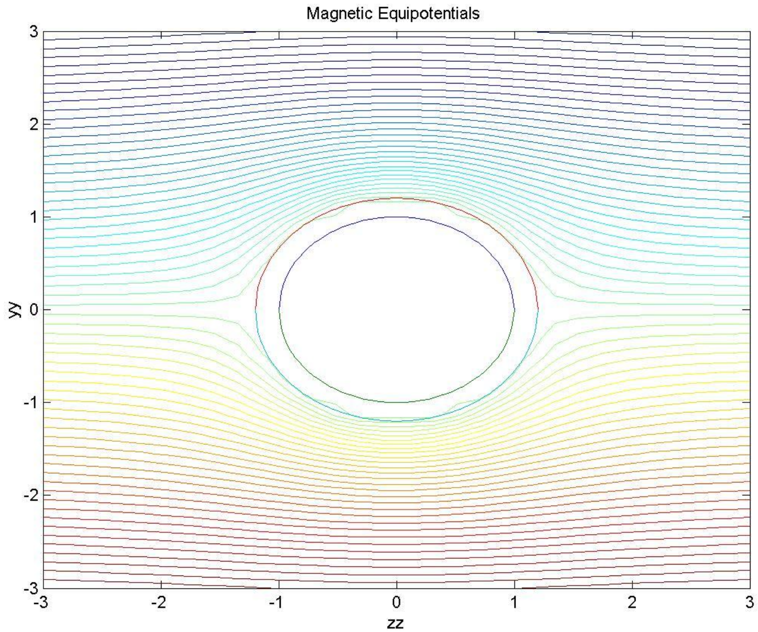
# TOF Separation

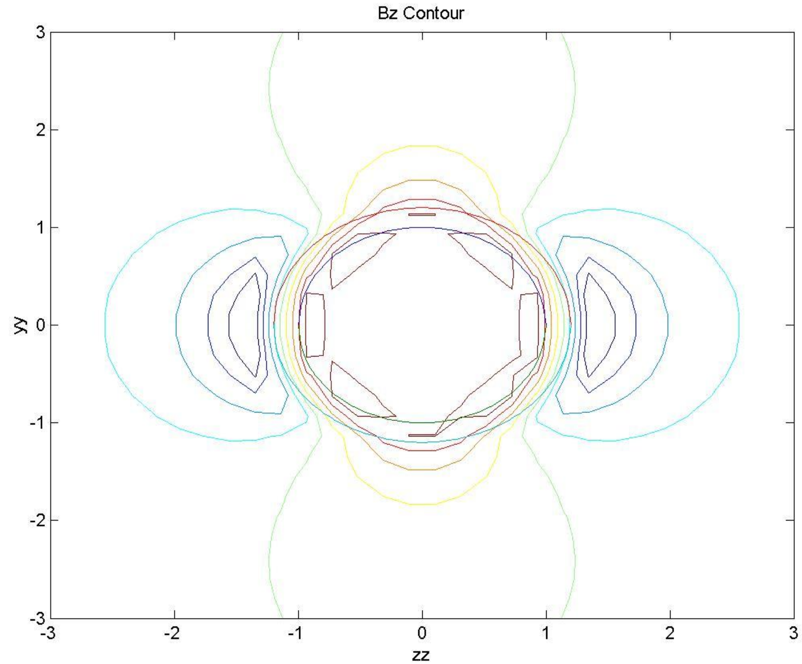



One sigma for pi/e at ~ 1 GeV, K/pi at ~ 4 GeV and p/K at ~ 8 GeV. The time resolution needed for a reasonable flight path is ~ (10,100) ps.

Figure 15: Particle separation with TOF measurements for three different system time resolutions ($\sigma_{TOF}$ = 60, 80 and 100 ps) and for a track length $L$ = 3.5 m. Infinitely good precisions on momentum and track length measurements are assumed.

# Demo - Magnetic Shield

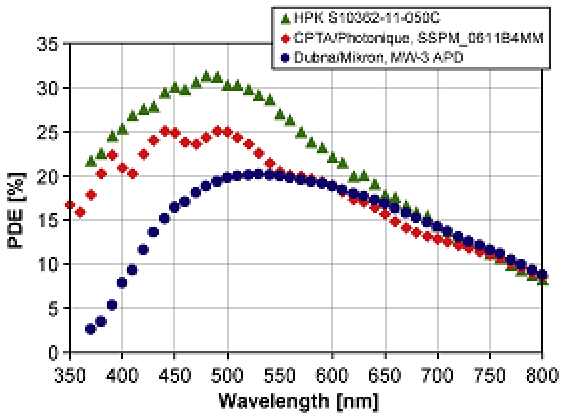


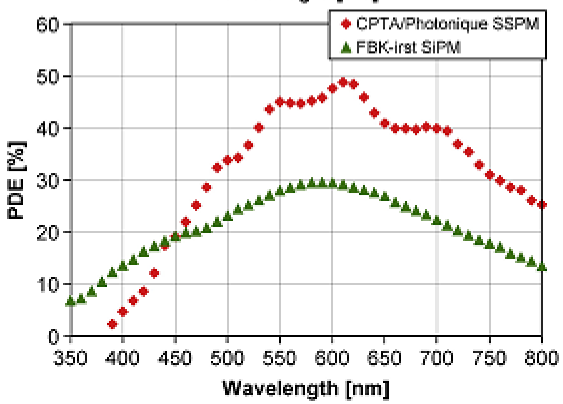




b/a = 1.5, mu = 1000

Because the photo-effect emits ~ eV electrons, the PMT is sensitive to B fields as the e may be swept off the first dynode. A standard approach is to shield the PMT using “mu metal” wrappings which reduces the B field by the permiability.

# SiPM




High quantum efficiency

Geiger mode -> non-linear response at high "occupancy"

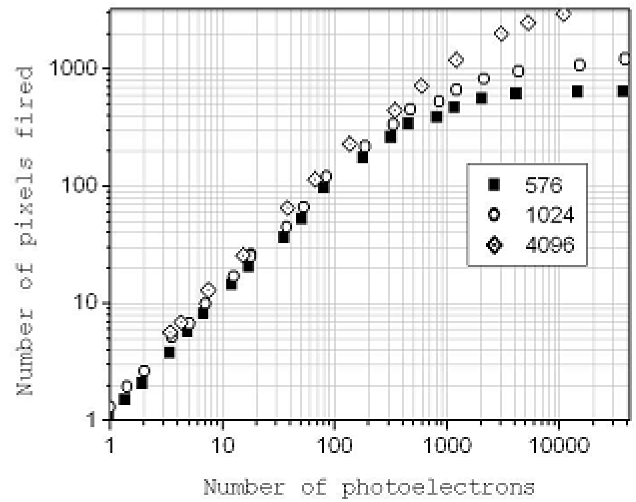



Figure 5: *Non-linear response of MEPhI/PULSAR SiPMs with different number of pixels. The light signal is produced by a fast laser (40 ps). Plot from [16].*

SiPM are solid state avalanche photo-diode arrays which are small, cheap and which have high gain, good quantum efficiency, high speed and are insensitive to magnetic fields. Silicon diodes will be covered later.

# EM Relationships

Electromagnetic Relations

| Quantity | Gaussian CGS | SI |
|---|---|---|
| Charge:<br>Electron charge $e$:<br>Potential:<br>Magnetic field: | 2.997 924 58 x $10^9$ esu<br>4.803 2068 x $10^{-10}$ esu<br>(1/299.792 458) statvolt (ergs/esu)<br>$10^4$ gauss = $10^6$ dyne/esu | = 1 C – 1 A s<br>= 1.602 17733 x $10^{-19}$ C<br>= 1 V = 1 J $c^{-1}$<br>= 1 T = 1 N $A^{-1}m^{-1}$ |
| Lorentz force: | $F = q(E + \frac{v}{c} \times B)$ | $F = q(E + v \times B)$ |
| Maxwell equations: | $\nabla \cdot D = 4\pi\rho$<br>$\nabla \times H - \frac{1}{c}\frac{\partial D}{\partial t} = \frac{4\pi}{c}J$<br>$\nabla \cdot B = 0$<br>$\nabla \times E + \frac{1}{c}\frac{\partial B}{\partial t} = 0$ | $\nabla \cdot D = \rho$<br>$\nabla \times H - \frac{\partial D}{\partial t} = J$<br>$\nabla \cdot B = 0$<br>$\nabla \times E + \frac{\partial B}{\partial t} = 0$ |
| Materials:<br>Permitivity of free space:<br>Permeability of free space: | $D = \varepsilon E,\ H = B/\mu$<br>1<br>1 | $D = \varepsilon E,\ H = B/\mu$<br>$\varepsilon_0 = 8.854\,187... \times 10^{-12}\ F\,m^{-1}$<br>$\mu_0 = 4\pi \times 10^{-7}\ N A^{-2}$ |
| Fields from potentials: | $E = -\nabla V - \frac{1}{c}\frac{\partial A}{\partial t}$<br>$B = \nabla \times A$ | $E = -\nabla V - \frac{1}{c}\frac{\partial A}{\partial t}$<br>$B = \nabla \times A$ |
| Static potentials<br>(coulomb gauge) | $V = \sum_{charges} \frac{q_i}{r_i} = \int \frac{\rho(r')}{\lvert r - r' \rvert} d^3x'$<br>$A = \frac{1}{c} \sum_{currents} \frac{I_i}{r_i} = \frac{1}{c} \int \frac{J(r')}{\lvert r - r' \rvert} d^3x'$ | $V = \frac{1}{4\pi\varepsilon_0} \sum_{charges} \frac{q_i}{r_i} = \frac{1}{4\pi\varepsilon_0} \int \frac{\rho(r')}{\lvert r - r' \rvert} d^3x'$<br>$A = \frac{\mu_0}{4\pi} \sum_{currents} \frac{I_i}{r_i} = \frac{\mu_0}{4\pi} \int \frac{J(r')}{\lvert r - r' \rvert} d^3x'$ |
| Relativistic transformations<br>(v is the velocity of the primed frame as seen in the unprimed frame) | $E'_{\parallel} = E_{\parallel}$<br>$E'_{\perp} = \gamma\left(E_{\perp} + \frac{1}{c} v \times B\right)$<br>$B'_{\parallel} = B_{\parallel}$<br>$B'_{\perp} = \gamma\left(B_{\perp} - \frac{1}{c} v \times B\right)$ | $E'_{\parallel} = E_{\parallel}$<br>$E'_{\perp} = \gamma\left(E_{\perp} + v \times B\right)$<br>$B'_{\parallel} = B_{\parallel}$<br>$B'_{\perp} = \gamma\left(B_{\perp} - \frac{1}{c^2} v \times E\right)$ |

$$\frac{1}{4\pi\varepsilon_0} = c^2 \times 10^{-7}\ N\,A^{-2} = 8.987\,55... \times 10^9\ m\,F^{-1}; \quad \frac{\mu_0}{4} = 10^{-7}\ N\,A^{-2}; \quad c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 2.997\,924$$

Be prepared to convert between CGS and MKS units. Textbooks are not uniform in the system of units which are used.

# Skin Depth

$$\vec{\nabla} \cdot (\mu \vec{H}) = 0$$

$$\vec{\nabla} \cdot (\varepsilon \vec{E}) = 0$$

$$\vec{\nabla} \times \vec{E} + \frac{\mu}{c} \frac{\partial \vec{H}}{\partial t} = 0$$

$$\vec{\nabla} \times \vec{H} - \frac{\varepsilon}{c} \frac{\partial \vec{E}}{\partial t} - \frac{4\pi\sigma}{c} \vec{E} = 0$$

$$(\vec{j} = \sigma \vec{E})$$

Maxwell's Eqs. in material

$$\left[ k^2 - (\mu\varepsilon \frac{\omega^2}{c^2} + 4\pi i \frac{\mu\omega\sigma}{c^2}) \right] \vec{E} = 0$$

$$k^2 = \mu\varepsilon \left( \frac{\omega}{c} \right)^2 \left[ 1 + i \left( \frac{4\pi\sigma}{\omega\varepsilon} \right) \right]$$

$$\vec{H} = \left( \vec{k} \times \vec{E} \right) c / \mu\omega$$

$$i\left( \vec{k} \times \vec{H} \right) + i\varepsilon \frac{\omega}{c} \vec{E} - \frac{4\pi\sigma}{c} \vec{E} = 0$$

$$\left| \vec{E} \right| \sim e^{-x/\delta} \sim e^{-\mathrm{Im}(k)x}$$

Im(k) means attenuation

$$\delta = 1/\,\mathrm{Im}(k)$$

$$\delta / c \sim 1/\sqrt{\sigma\omega\mu}$$

A conductor is a perfect DC shield against electric fields. What about higher frequencies? The "skin depth" is the distance a field penetrates into a conductor and it depends on the frequency, permitivity and conductivity. RF fields stay on the surface.

# Cerenkov Angle

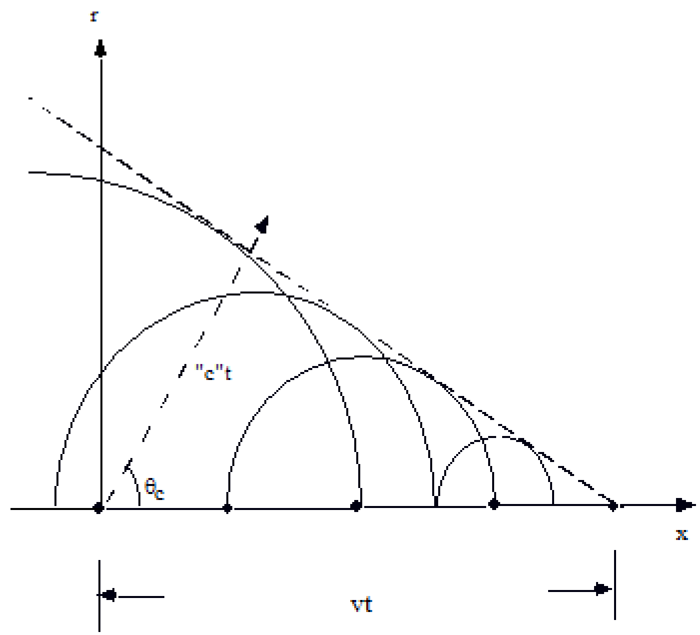


Fig. 3.1:Cerenkov cone construction using Huygen's principle.

$$"c" = c/n$$

$$\cos\theta_c = "c"/v = "c"t/vt$$

$$= c/nv = 1/\beta n$$

$$v > "c" \;\; and \; \cos\theta_c < 1 \; if \;\; \beta > 1/n$$

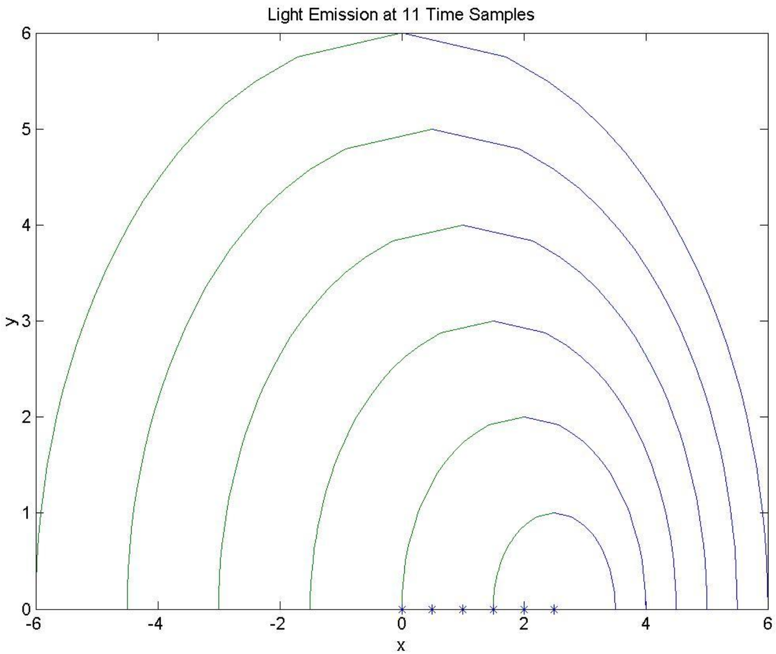
A moving charge which exceeds "c' in material will radiate photons (UV) - shock wave.

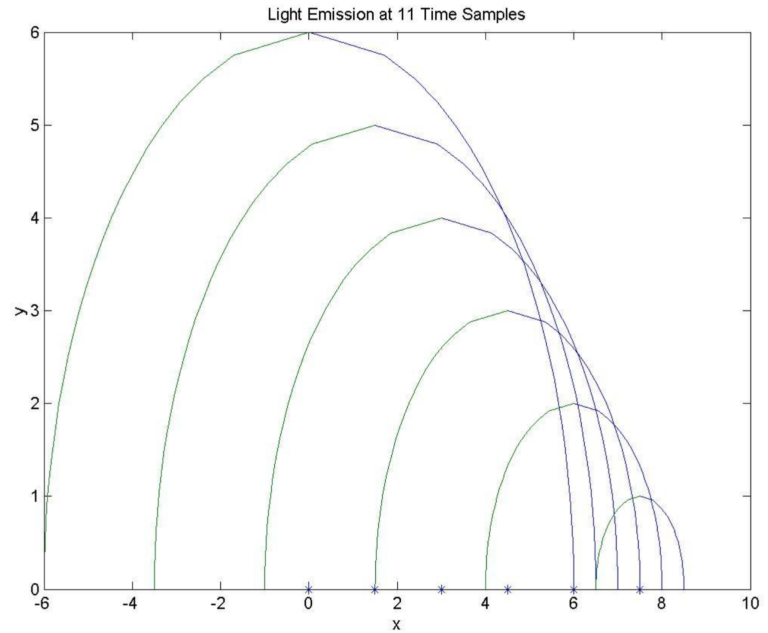
$$v = c/\sqrt{\mu\varepsilon}$$

$$k = \sqrt{\mu\varepsilon}\,(\omega/c)\;,\;\left[k^2 - \mu\varepsilon\left(\omega/c\right)^2\right]\vec{E} = 0$$

$$n = \sqrt{\mu\varepsilon}, k = \omega/(c/n) = \omega/v$$

# Demo – Doppler, Cerenkov




v/c = 0.5

v/c = 1.5

Note the red shift, blue shift, and lack of a transverse Doppler frequency shift.

Once v > "c" the shock wave front builds up at the characteristic Cerenkov angle.

# Particle ID and Cerenkov

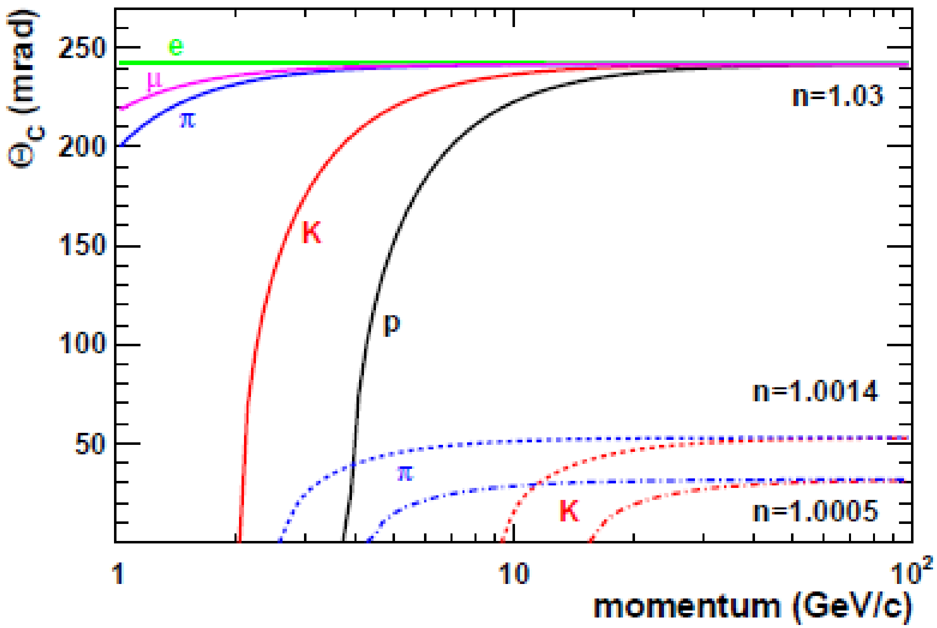
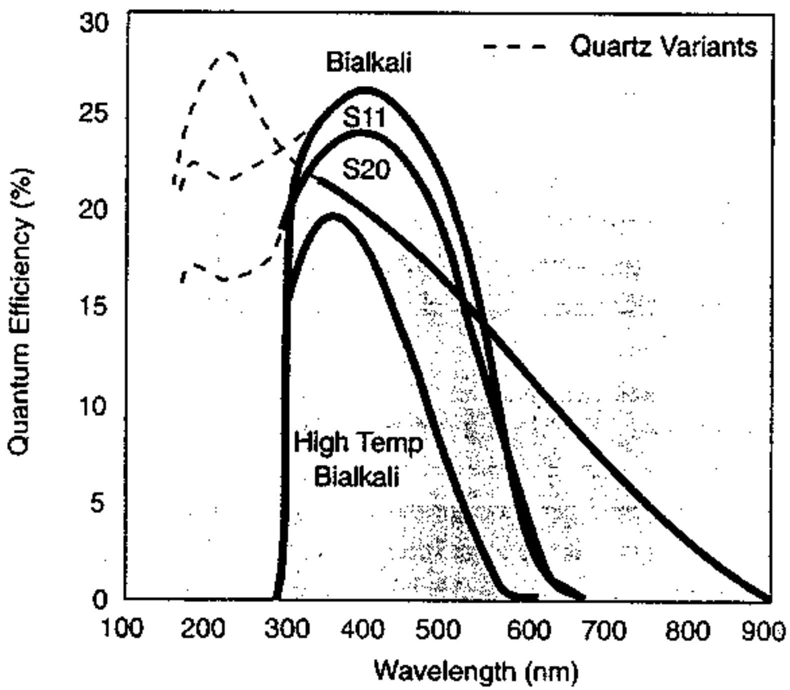



Figure 25: Cherenkov angles as a function of momentum for different particle species and for the three different values of the refractive index $n$ corresponding to the three radiator materials used in the LHCb RICH setup.

Since v depends on the particle mass, the existence of Cerenkov radiation and the angle coupled to a measurement of particle momentum give particle ID - RICH.

# Cerenkov Spectrum




$$\frac{d^2N_c}{d\omega dx} = \left(\frac{\alpha}{c}\right)\sin^2\theta_c = \frac{\alpha}{c}\left(1-\frac{1}{\beta^2\varepsilon}\right)$$

$$N_c = \frac{\alpha}{c}\sin^2\theta_c\,\Delta x\,\Delta\omega.$$

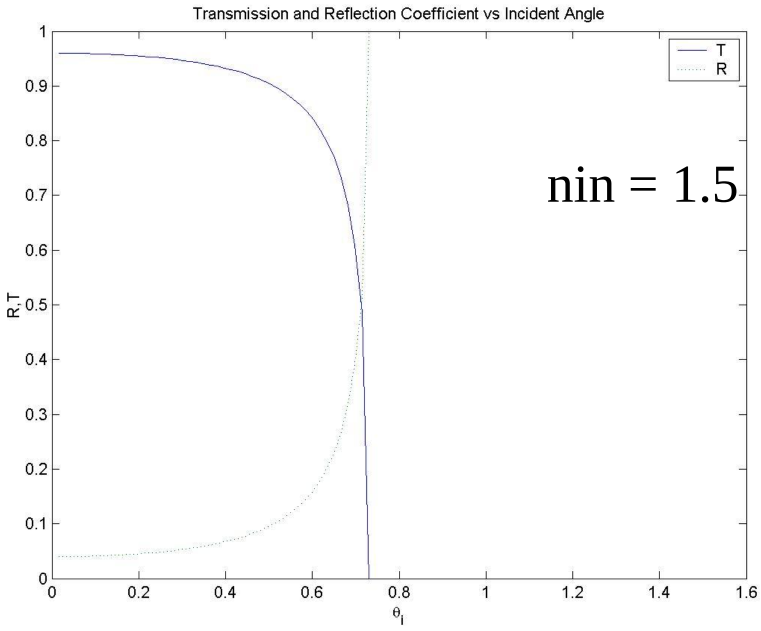
$$\hbar c = 2\times 10^{-5}\,eV\,cm$$

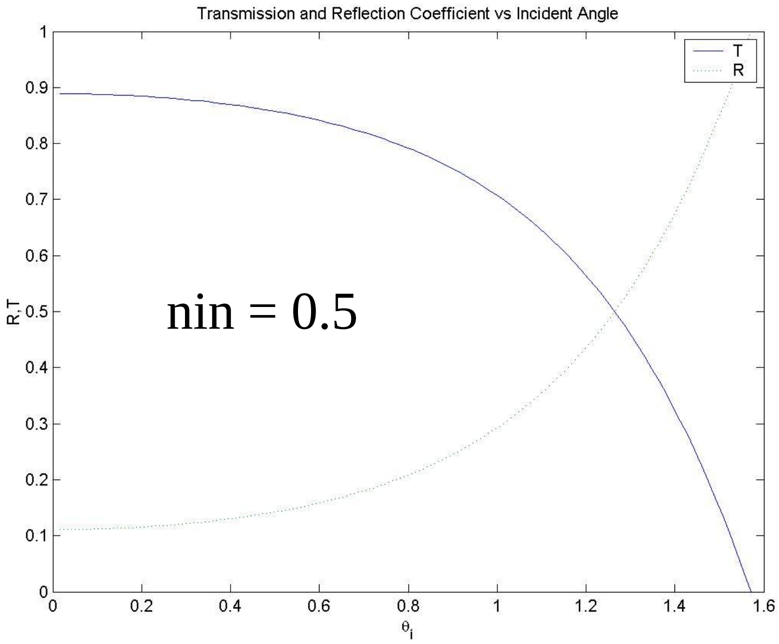
$$\frac{d^2N_c}{d(\hbar\omega)dx} \sim 365\sin^2\theta_c/(eV\cdot cm)$$

Since the Cerenkov photons extend deep into the UV it is important to transmit the light to a PMT – particularly the “window”.

The spectrum for Cerenkov light is simple. It depends on the charge coupling constant, and is flat in frequency. We want all the frequency range we can get.

# Demo Reflect/Transmit




Total internal reflection? At an interface where the indices of refraction differs a lot, transmission if not very efficient. Normal incidence gives maximal transmission.

# Cerenkov Parameters

$$n \equiv 1+\delta,\ 1-\beta \sim 1/2\gamma^2\ ,\ \gamma = 1/\sqrt{1-\beta^2}$$

$$\theta_c^2 \sim \frac{1}{\gamma_{TH}^2} - \frac{1}{\gamma^2}$$

$$\gamma_{TH}^2 \sim 1/2\delta$$

Assume gas, n ~ 1. There is a threshold for photon emission.

$$\theta_c^{max} = 1/\gamma_{TH}$$

$$N_c^{max} \sim \delta \sim 1/2\gamma_{TH}^2$$

There is a maximum number of photons

$$\lambda = \frac{\hbar c(2\pi)}{\hbar\omega} = \frac{12,400\, eV\, \overset{o}{A}}{E_\gamma(eV)}$$

$$C_aF_2,\ E_\gamma < 9eV,\ \lambda > 1380\,\overset{o}{A}$$

Windows of CaF2 give a large yield