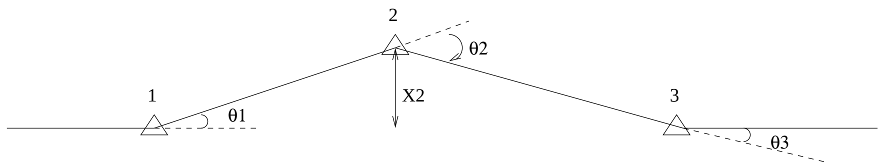
Error Correction

Exercise 1



A localized position excursion ('bump') may be accomplished with three dipoles. When such a three-bump is in a ring with circulating beam, the characteristics of the bump depend on the location of the dipoles and the beta functions at those locations, as well as on the strength of the dipole kicks. Determine the required strength of the dipole kicks given their locations, the beta functions at those locations, and the desired position excursion. The first dipole gives the beam an angular deflection such that it has the desired position offset at the second dipole. The second dipole kicks the beam so that it returns to the position of the design orbit at the third dipole. The third dipole then corrects the remaining angle of the beam with respect to the design orbit.

a) Apply the transfer matrix M_{12} to an initial position error zero, and an angle given by the dipole kick, θ_1. Solve for θ_1 in terms of x_2.

$$\begin{pmatrix} x_2 \\ x_2' \end{pmatrix} = M_{12} \begin{pmatrix} 0 \\ \theta_1 \end{pmatrix}$$

b) Use the condition that $x_3 = 0$ to get an expression for θ_2 in terms of x_2.

c) The less mathematically intensive way to get an expression for θ_3 in terms of x_2 is to use the fact that it takes the same kick strength either to correct a beam with offset x_2 at dipole 2 as it would to make an offset of x_2 at dipole 2 if the beam were going the other direction. Alternatively, but at the cost of much algebra, the angle coordinate portion of the following equation can be solved for θ_3:

$$\begin{pmatrix} 0 \\ 0 \end{pmatrix} = M_{23} \left[\begin{pmatrix} x_2 \\ x_2' \end{pmatrix} + \begin{pmatrix} 0 \\ \theta_2 \end{pmatrix} \right] + \begin{pmatrix} 0 \\ \theta_3 \end{pmatrix}$$

Exercise 2 (From Syphers)
Calculate the dipole corrector strength required to make a local horizontal 3-bump of 5 mm in the Fermilab Tevatron at 900 GeV. The Tevatron has a FODO lattice, with horizontal correctors located near focusing quadrupoles (similarly, vertical correctors are near defocusing locations). Assume the three dipoles used in the bump are located near consecutive focusing quadrupoles, that the phase advance per cell is 70°, and that the beta function at the focusing quadrupoles is 110 meters.

Exercise 3 (From S. Holmes)
An accelerator under construction has been designed with a circumference of 6000 meters and a tune (in each plane) of 20.4. The overwhelming majority of the circumference is populated with 80 FODO cells, each containing two quadrupoles and four dipoles. We would like to assemble this accelerator so that the rms closed orbit distortion is less than 2 mm.

a) What is the typical strength, $\frac{B'l}{B\rho}$, of the quadrupoles required?

b) What is the typical bend angle of the dipoles?

c) What transverse alignment tolerance is required on the quadrupoles to achieve the desired closed orbit distortion?

d) What bending strength uniformity is required in the dipole magnets to achieve the desired closed orbit distortion?

Exercise 4
Determine the tune change in a ring due to a gradient error. A gradient error may be represented by a extra thin quadrupole stuck into the ring with a focal strength which represents the strength of the error.

- Begin with the relation $M = M_0 M_{error}$, where M is the transport matrix for the ring including the gradient error, M_0 is the transport matrix for the ring with no gradient error, and M_{error} is the matrix of a thin quadrupole which represents the gradient error. The matrix elements of M are written in terms of the new perturbed tune, ν. The matrix elements of M_0 are written in terms of the old unperturbed tune, ν_0. The new tune is the old tune plus the tune change, $\nu = \nu_0 + \Delta\nu$.

- One way to relate the tune change to the strength of the error is by equating the traces of the matrices on both sides of the equation above, TR(M)=Tr($M_0 M_{error}$). Write ν as $\nu_0 + \Delta\nu$, with $\Delta\nu$ small, thus the small angle approximation may be used in combination with trigonometric expansions to simplify the expression.

Exercise 5
An accelerator has 100 quadrupole magnets powered in series, so that each has the same magnetic field. The focal length of the quadrupoles is 25 m and the amplitude function at their locations is $\beta = 100m$. If the magnetic gradient of all the quadrupoles is increased by 1%, what tune shift would this generate?