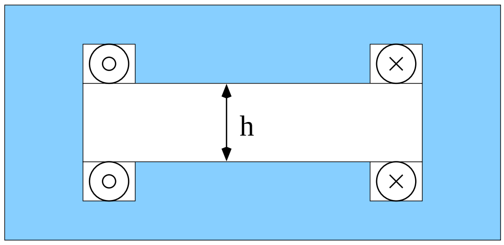
Electromagnetic structures

Exercise 1 (From Edwards and Syphers)



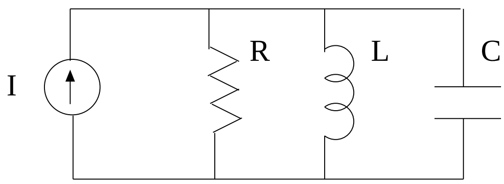
The transverse cross-section of a simple bending magnet is shown in the figure above. There are N turns of conductor carrying current I wound about each pole of the iron magnet. The poles are separated by a distance h. Assuming the permeability of the iron to be infinite, show that the field in the gap of the magnet is given by

$$B = \frac{2\mu_0 NI}{h}$$

Exercise 2

Sketch a cross-section of a focusing and defocusing quadrupole magnet. Label the north and south poles of the magnet, sketch in magnetic field lines, and indicate which magnet is the focusing magnet for positively charged particles. Sketch in arrows that depict the force on a positively charged particle moving directly into the page for the following four locations in the aperture: a horizontal offset to the right of the center of the aperture (no vertical offset); a horizontal offset to the left; a vertical offset above the center of the aperture (no horizontal offset); and a vertical offset below the center of the aperture.

Exercise 3



Accelerating cavities are resonators, and are sometimes modeled by resonant circuits, such as the one shown in the figure above.

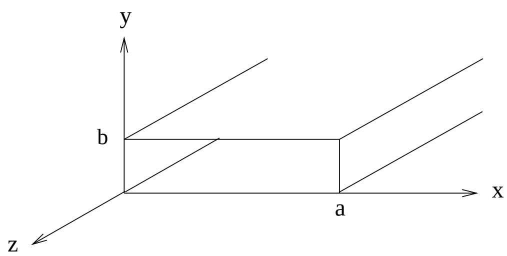
a) Write a circuit equation for a parallel LRC circuit with a current source. The right side of the equation should be the total current flowing into the junction from the source, i, and the left side of the equation should be the sum of the currents flowing out of the junction, $i_R + i_L + i_c$. Write the currents on the left side of the equation in terms of the voltage, v, across the circuit element.

b) The source current is sinusoidal, $i = I \exp(i\omega t)$, and so the voltage response is also sinusoidal, $v = V \exp(i\omega t)$. Plug these into your circuit equation and solve for $\frac{V}{I}$, which will be used to characterize the circuit response.

c) Find the magnitude response $\left|\frac{V}{I}\right|$, and sketch it versus the angular frequency ω. At what angular frequency is the response maximized?

One definition of the Q of the circuit is the fraction of the resonant frequency over the bandwidth, $Q = \omega_{max}/\mathrm{BW}$. One definition of the bandwidth is the frequency range over which the magnitude of the circuit response is not less than $\frac{1}{\sqrt{2}}$ of the maximum magnitude of the response.

Exercise 4



The longitudinal electric field E_z is zero for TE modes in an evacuated waveguide. The longitudinal magnetic field has the form $B_z = B(x,y)\exp i(kz - \omega t)$. The wave equation can be solved using the separation of variables method, i.e. assuming a solution of the form $B_z = X(x)Y(y)\exp i(kz - \omega t)$.

a) Substitute B_z given above into the wave equation,

$$\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right] B_z = 0$$

and then divide all terms by $X(x)Y(y)$. Each term in the equation must be equal to a constant. Write an equation for the $X(x)$ term only, setting it equal to a constant $-k_x^2$. Similarly, write the equation for $Y(y)$ using constant $-k_y^2$.

b) Write the general solution for $X(x)$ and $Y(y)$ for the equations you found in part a).

c) The waveguide cross-section has width $\Delta x = a$ and height $\Delta y = b$. Let the sides of the waveguide be located at $x = 0, x = a, y = 0, y = b$, as shown in the figure. According to Maxwell's equations, the components of the magnetic field are related as follows;

$$
\begin{aligned}
B_x &= \frac{ik}{\left(\frac{\omega}{c}\right)^2 - k^2} \frac{\partial B_z}{\partial x} \\
B_y &= \frac{ik}{\left(\frac{\omega}{c}\right)^2 - k^2} \frac{\partial B_z}{\partial y}
\end{aligned}
$$

The boundary conditions require that the perpendicular component of B be zero at the waveguide walls, i.e. $B_y(x,0) = 0$, $B_y(x,b) = 0$, $B_x(0,y) = 0$, $B_x(a,y) = 0$. Use these boundary conditions, and the equations relating the field components, to find allowed values for k_x and k_y.

d) Show that

$$
k = \sqrt{\left(\frac{\omega}{c}\right)^2 - k_x^2 - k_y^2}
$$

This is the dispersion relation for the waveguide.

e) If the dimensions of the waveguide are $a = 3$ cm and $b = 1$ cm, find the range of drive frequencies for which only one TE mode will propagate down the guide. (Hint: the drive frequency must be above the cut-off frequency for the mode in order for that mode to propagate.)