Day 5

Limiting phenomena in rings: coherent & incoherent effects

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Beams have internal (self-forces)

- Space charge forces
  - Like charges repel
  - Like currents attract
- For a long thin beam

$$E_{sp}(V/cm) = \frac{60\ I_{beam}(A)}{R_{beam}(cm)}$$

$$B_{\theta}(gauss) = \frac{I_{beam}(A)}{5\ R_{beam}(cm)}$$

Net force due to transverse self-fields

In vacuum:

Beam's transverse self-force scale as $1/\gamma^2$

➙ Space charge repulsion: $E_{sp,\perp} \sim N_{beam}$

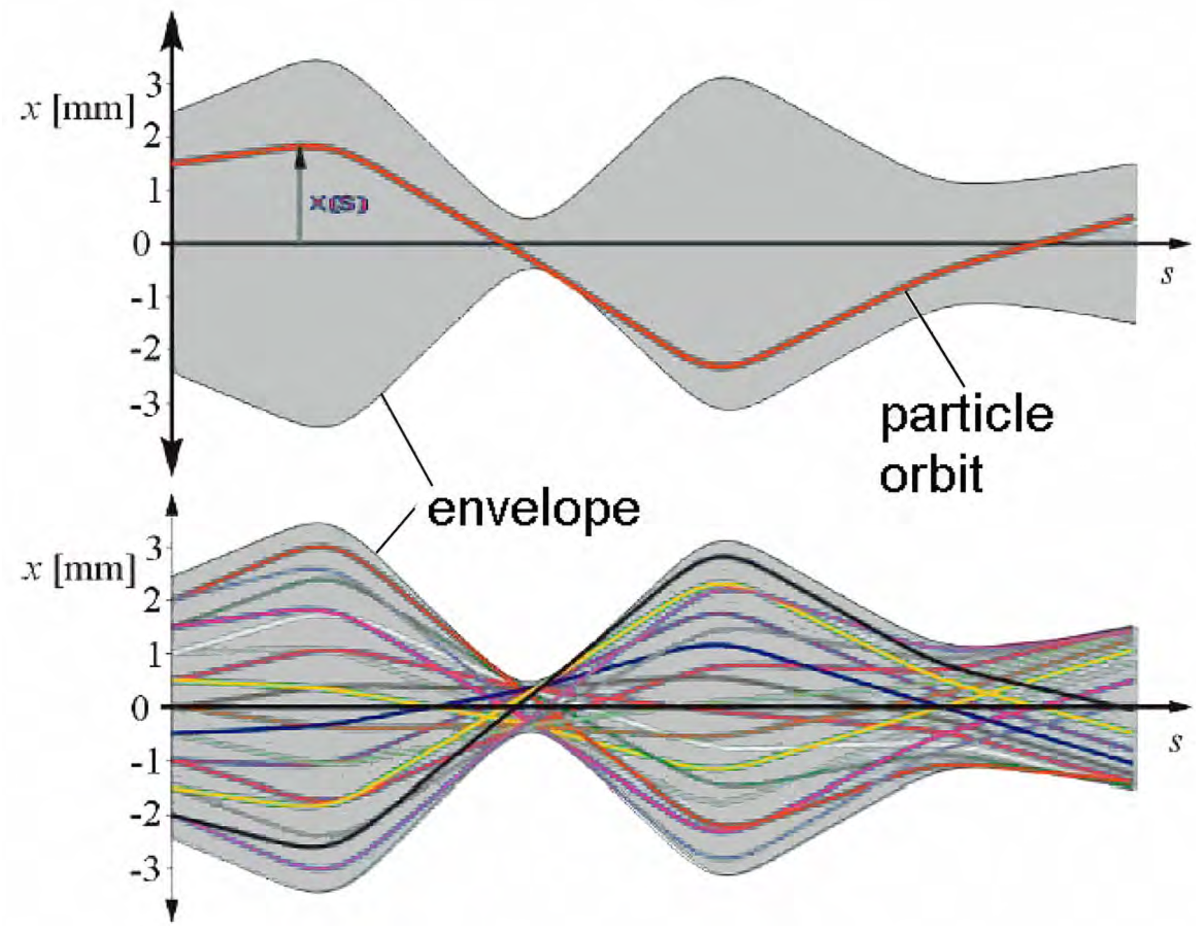
➙ Pinch field: $B_\theta \sim I_{beam} \sim v_z N_{beam} \sim v_z E_{sp}$

$$\therefore F_{sp,\perp} = q\,(E_{sp,\perp} + v_z \times B_\theta) \sim (1-v^2)\, N_{beam} \sim N_{beam}/\gamma^2$$

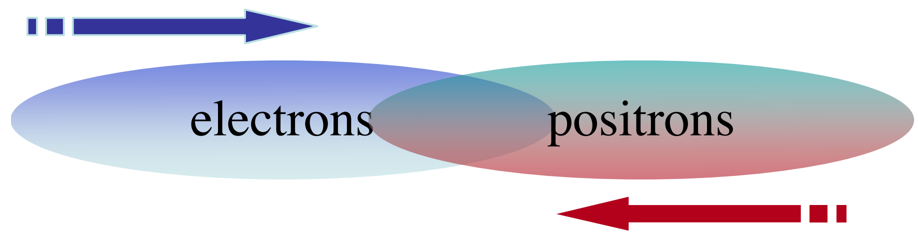
Beams in collision are ***not*** in vacuum (beam-beam effects)

We see that ε characterizes the beam while β(s) characterizes the machine optics

- β(s) sets the physical aperture of the accelerator because the beam size scales as $\sigma_x(s) = \sqrt{\varepsilon_x \beta_x(s)}$



Example: Megagauss fields in linear collider



At Interaction Point space charge cancels; currents add

==> strong beam-beam focus

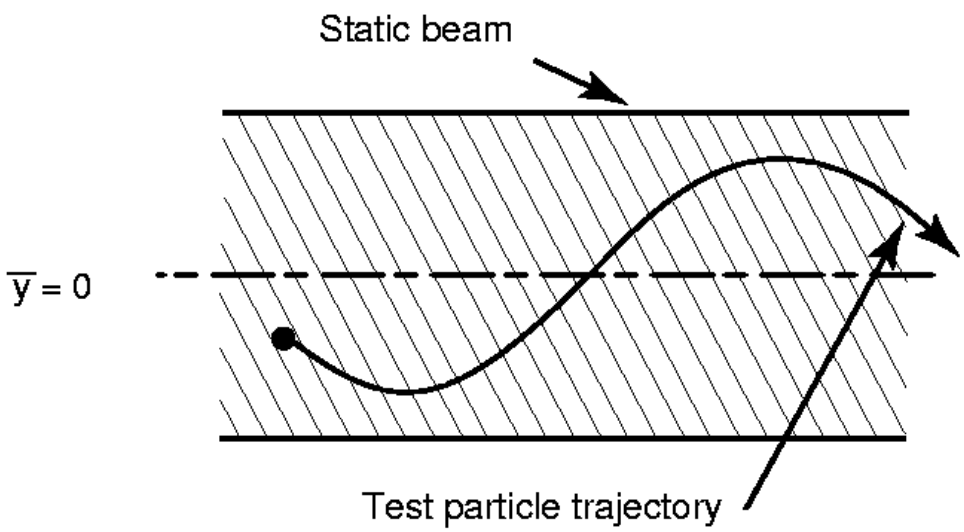
--> Luminosity enhancement

--> Strong synchrotron radiation

Consider 250 GeV beams with 1 kA focused to 100 nm

$$B_{peak} \sim 40 \text{ Mgauss}$$

Types of tune shifts: Incoherent motion



- Center of mass does not move
- Beam environment does not "see" any motion
- Each particle is characterized by an individual amplitude & phase

From: E. Wilson Adams lectures

Incoherent collective effects

- Beam-gas scattering
  - → Elastic scattering on nuclei => leave physical aperture
  - → Bremsstrahlung
  - → Elastic scattering on electrons
  - → Inelastic scattering on electrons

  leave rf-aperture

  ========> reduce beam lifetime
- Ion trapping (also electron cloud) - scenario
  - → Beam losses + synchrotron radiation => gas in vacuum chamber
  - → Beam ionizes gas
  - → Beam fields trap ions
  - → Pressure increases linearly with time
  - → Beam -gas scattering increases
- Intra-beam scattering

Intensity dependent effects

- Types of effects
  - Space charge forces in individual beams
  - Wakefield effects
  - Beam-beam effects
- General approach: solve

$$\boxed{x'' + K(s)x = \frac{1}{\gamma m \beta^2 c^2} F_{non-linear}}$$

- For example, a Gaussian beam has

$$F_{SC} = \frac{e^2 N}{2\pi\varepsilon_o \gamma^2 r}\left(1 - e^{-r^2/2\sigma^2}\right) \quad where\ N = \ charge/unit\ length$$

- For r < σ

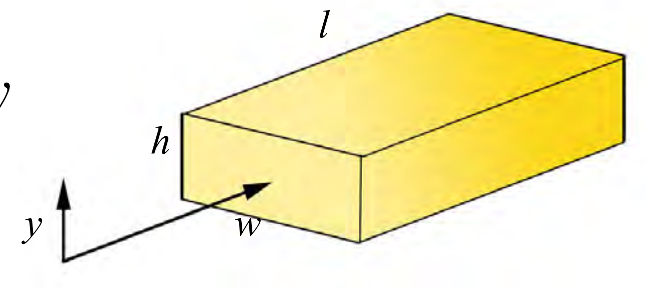
$$F_{SC} \approx \frac{e^2 N}{4\pi\varepsilon_o \gamma^2} r$$

Beam-beam tune shift

$$\Delta p_y(y) \sim \left(F_{elec} + F_{mag}\right)\frac{l}{2c} \quad \textit{(once the particle passes l/2 the other bunch has passed)}$$

- For $\gamma >> 1$, $F_{elec} \approx F_{mag}$

$$\mathscr{E} = \frac{N_{beam}e}{lwh}y$$



$$\therefore \Delta p_y(y) \approx 2\frac{e\mathscr{E}}{2c} = \frac{e^2 N_B}{\varepsilon_o cwh}y$$

$$\Rightarrow \frac{\Delta p_y}{p_o} = \Delta y' \sim y \quad \text{similar to gradient error } k_y\Delta s \text{ with } k_y\Delta s = \frac{\Delta y'}{y}$$

- Therefore the tune shift is

$$\Delta Q = -\frac{\beta^*}{4\pi}k_y\Delta s \approx \frac{r_e\beta^* N}{\gamma wh} \quad \text{where } r_e = \frac{e^2}{4\pi\varepsilon_o mc^2}$$

- For a Gaussian beam

$$\boxed{\Delta Q \approx \frac{r_e}{2}\frac{\beta^* N}{\gamma A_{\text{int}}}}$$

Effect of tune shift on luminosity

- The luminosity is

$$\mathcal{L} = \frac{f_{coll} N_1 N_2}{4 A_{\text{int}}}$$

- Write the area in terms of emittance & β at the IR

$$A_{\text{int}} = \sigma_x \sigma_y = \sqrt{\beta_x^* \varepsilon_x} \circ \sqrt{\beta_y^* \varepsilon_y}$$

- For simplicity assume that

$$\frac{\beta_x^*}{\beta_y^*} = \frac{\varepsilon_x}{\varepsilon_y} \Rightarrow \beta_x^* = \frac{\varepsilon_x}{\varepsilon_y} \beta_y^* \Rightarrow \beta_x^* \varepsilon_x = \frac{\varepsilon_x^2}{\varepsilon_y} \beta_y^*$$

- In that case

$$A_{\text{int}} = \varepsilon_x \beta_y^*$$

- And

$$\mathcal{L} = \frac{f_{coll} N_1 N_2}{4 \varepsilon_x \beta_y^*} \sim \frac{I_{beam}^2}{\varepsilon_x \beta_y^*}$$

Increase N to the tune shift limit

- We saw that

$$\boxed{\Delta Q_y \approx \frac{r_e}{2}\frac{\beta^* N}{\gamma A_{\text{int}}}}$$

or

$$N = \Delta Q_y \frac{2\gamma A_{\text{int}}}{r_e \beta^*} = \Delta Q_y \frac{2\gamma \varepsilon_x \beta^*}{r_e \beta^*} = \frac{2}{r_e}\gamma \varepsilon_x \Delta Q_y$$

Therefore the tune shift limited luminosity is

$$\boxed{\mathcal{L} = \frac{2}{r_e}\Delta Q_y \frac{f_{coll} N_1 \gamma \varepsilon_x}{4\varepsilon_x \beta_y^*} \sim \Delta Q_y \left(\frac{IE}{\beta_y^*}\right)}$$

Incoherent tune shift for in a synchrotron

Assume: 1) an unbunched beam (no acceleration), & 2) uniform density in a circular x-y cross section (not very realistic)

$$x'' + \left(K(s) + \underline{K_{SC}(s)}\right)x = 0$$ ➔ Q_{x0} (external) + ΔQ_x (space charge)

For small "gradient errors" k_x

$$\underline{\Delta Q_x} = \frac{1}{4\pi}\int_0^{2R\pi} k_x(s)\beta_x(s)ds = \frac{1}{4\pi}\int_0^{2R\pi} \underline{K_{SC}(s)}\beta_x(s)ds$$

where

$$K_{SC} = -\frac{2r_0 I}{ea^2\beta^3\gamma^3 c}$$

$$\Delta Q_x = -\frac{1}{4\pi}\int_0^{2\pi R}\frac{2r_0 I}{e\beta^3\gamma^3 c}\frac{\beta_x(s)}{a^2}ds = -\frac{r_0 R I}{e\beta^3\gamma^3 c}\left\langle\frac{\beta_x(s)}{a^2(s)}\right\rangle = -\frac{r_0 R I}{e\beta^3\gamma^3 c\,\varepsilon_x}$$

From: E. Wilson Adams lectures

Incoherent tune shift limits current at injection

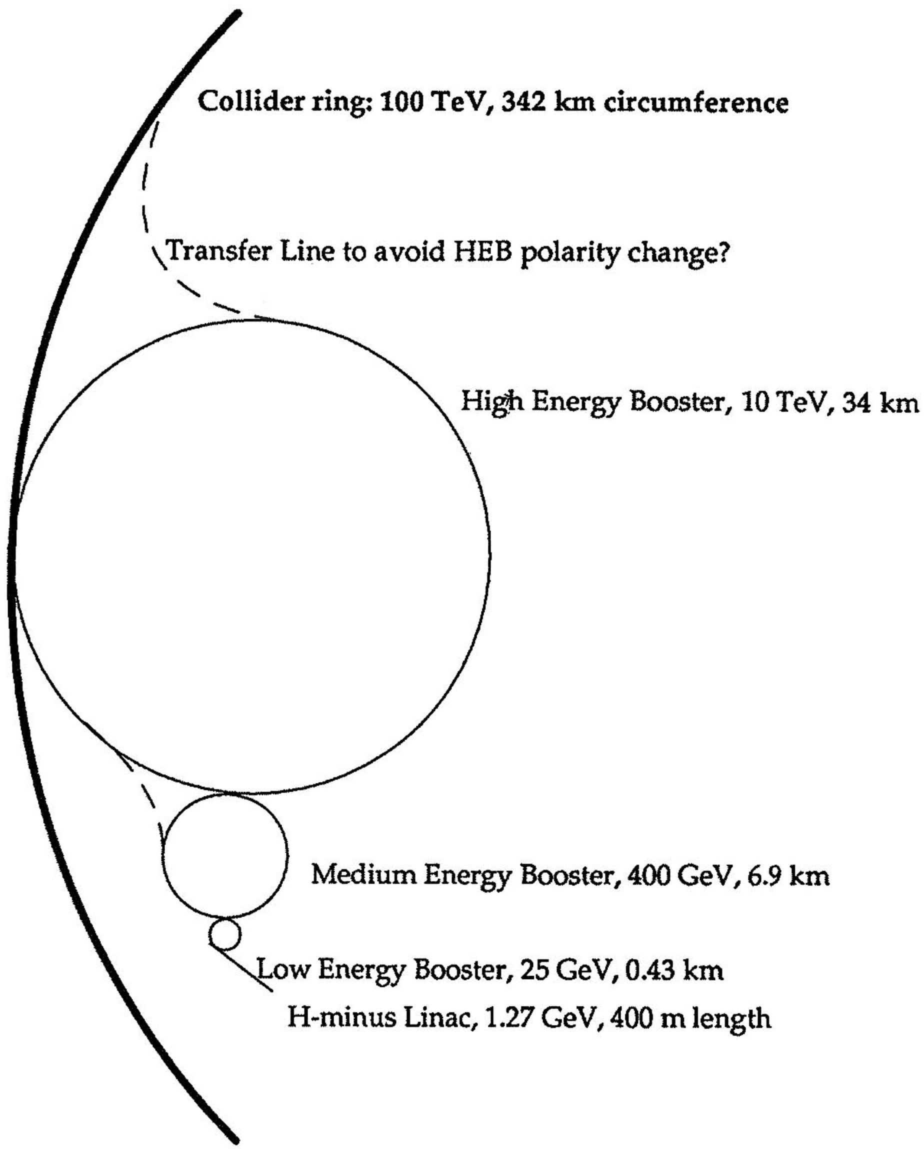
$$\Delta Q_{x,y} = -\frac{r_0}{2\pi\beta^2\gamma^3}\frac{N}{\varepsilon_{x,y}}$$

using I = (Neβc)/(2πR) with
N…number of particles in ring
$\varepsilon_{x,y}$….emittance containing 100% of particles

- ❖ "Direct" space charge, unbunched beam in a synchrotron
- ❖ Vanishes for γ » 1
- ❖ Important for low-energy hadron machines
- ❖ *Independent of machine size* 2πR for a *given N*
- ❖ *Overcome by higher energy injection ==> cost*

From: E. Wilson Adams lectures

Injection chain for a 200 TeV Collider



Beam lifetime

Based on F. Sannibale USPAS Lecture

Finite aperture of accelerator ==> loss of beam particles

- Many processes can excite particles on orbits larger than the nominal.
  - ➝ If new orbit displacement exceed the aperture, the particle is lost

- The limiting aperture in accelerators can be either *physical* or *dynamic*.
  - ➝ Vacuum chamber defines the physical aperture
  - ➝ Momentum acceptance defines the dynamical aperture

From: Sannibale USPAS Lecture

Important processes in particle loss

- Gas scattering, scattering with the other particles in the beam, quantum lifetime, tune resonances, &collisions

- Radiation damping plays a major role for electron/positron rings
  - ➙For ions, lifetime is usually much longer
    - Perturbations progressively build-up & generate losses

- Most applications require storing the beam as long as possible

==> limiting the effects of the residual gas scattering

==> ultra high vacuum technology

From: Sannibale USPAS Lecture

What do we mean by lifetime?

- Number of particles lost at time t is proportional to the number of particles present in the beam at time t

$$dN = -\alpha N(t)\, dt \qquad with \quad \alpha \equiv constant$$

- Define the lifetime $\tau = 1/\alpha$; then $\boxed{N = N_0 e^{-t/\tau}}$

- Lifetime is the time to reduce the number of beam particles to $1/e$ of the initial value

- Calculate the lifetime due to the individual effects (gas, Touschek, …)

$$\frac{1}{\tau_{total}} = \frac{1}{\tau_1} + \frac{1}{\tau_2} + \frac{1}{\tau_3} + ...$$

From: Sannibale USPAS Lecture

Is the lifetime really constant?

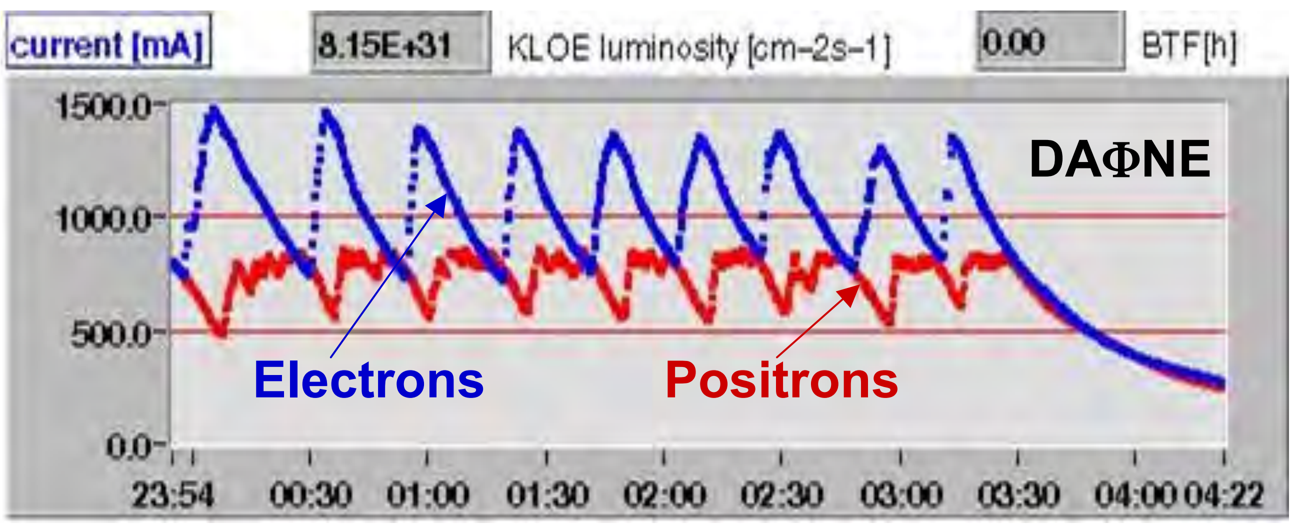
- In typical electron storage rings, lifetime depends on beam current
- Example: the *Touschek effect* losses depend on current.
  - When the stored current decreases, the losses due to Touschek decrease ==> lifetime increases
- Example: Synchrotron radiation radiated by the beam desorbs gas molecules trapped in the vacuum chamber
  - The higher the stored current, the higher the synchrotron radiation intensity and the higher the desorption from the wall.
  - Pressure in the vacuum chamber increases with current

  ==> increased scattering between the beam and the residual gas

  ==> reduction of the beam lifetime

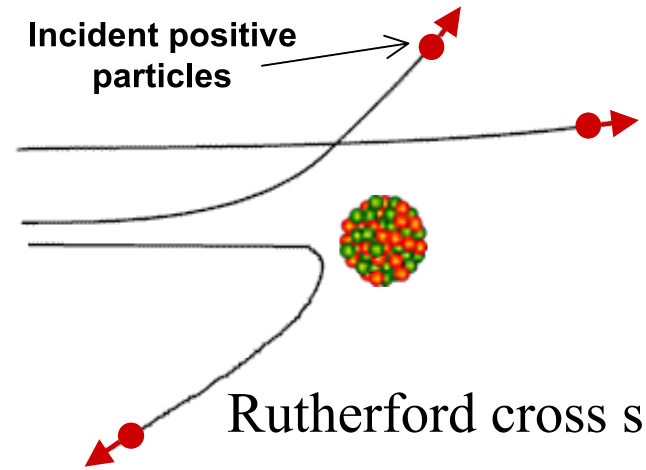
From: Sannibale USPAS Lecture

Examples of beam lifetime measurements



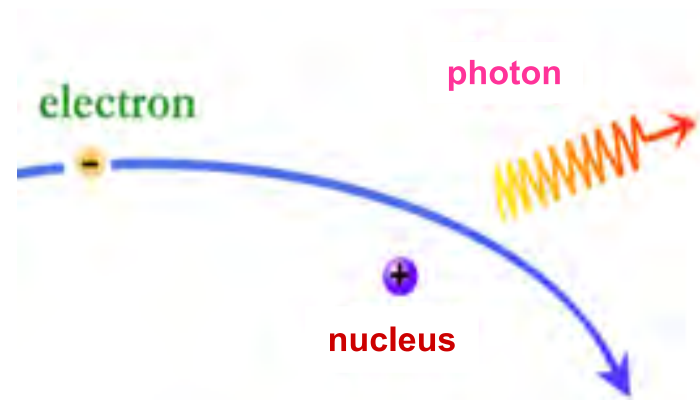
Beam loss by scattering

- Elastic (Coulomb scattering) from residual background gas
  - Scattered beam particle undergoes transverse (betatron) oscillations.
  - If the oscillation amplitude exceeds ring acceptance the particle is lost



Rutherford cross section

- Inelastic scattering causes particles to *lose energy*
  - Bremsstrahlung or atomic excitation
  - If energy loss exceeds the momentum acceptance the particle is lost



Bremsstrahlung cross section

Elastic scattering loss process

- Loss rate is $$\left.\frac{dN}{dt}\right|_{Gas} = -\phi_{beam\ particles} N_{molecules} \sigma^*_R$$

$$\phi_{beam\ particles} = \frac{N}{A_{beam} T_{rev}} = \frac{N}{A_{beam}} \frac{\beta c}{L_{ring}}$$

$$N_{molecules} = n A_{beam} L_{ring}$$

$$\sigma^*_R = \int_{Lost} \frac{d\sigma_{Rutherford}}{d\Omega} d\Omega = \int_0^{2\pi} d\varphi \int_{\theta_{MAX}}^{\pi} \frac{d\sigma_{Rutherford}}{d\Omega} \sin\theta \, d\theta$$

$$\frac{d\sigma_R}{d\Omega} = \frac{1}{(4\pi\varepsilon_0)^2} \left(\frac{Z_{beam} Z_{gas} e^2}{2\beta c\, p} \right)^2 \frac{1}{\sin^4(\theta/2)} \quad [MKS]$$

Gas scattering lifetime

- Integrating yields

$$\left.\frac{dN}{dt}\right|_{Gas} = -\frac{\pi\, n\, N\beta\, c}{(4\pi\,\varepsilon_0)^2}\left(\frac{Z_{Inc}Ze^2}{\beta\, c\, p}\right)^2 \frac{1}{\tan^2(\theta_{MAX}/2)}$$

Loss rate for gas elastic scattering [*MKS*]

- For M-atomic molecules of gas $n = M\, n_0 \frac{P_{[Torr]}}{760}$

- For a ring with acceptance ε_A & for small θ $\langle\theta_{MAX}\rangle = \sqrt{\frac{\varepsilon_A}{\langle\beta_n\rangle}}$

==>

$$\tau_{Gas} \cong \frac{760}{P_{[Torr]}} \frac{4\pi\varepsilon_0^2}{\beta\, c\, M\, n_0}\left(\frac{\beta\, c\, p}{Z_{Inc}Ze^2}\right)^2 \frac{\varepsilon_A}{\langle\beta_T\rangle} \quad [MKS]$$

Inelastic scattering lifetimes

- Beam-gas bremsstrahlung: if E_A is the energy acceptance

$$\tau_{Brem[hours]} \cong -\frac{153.14}{\ln\left(\Delta E_A / E_0\right)} \frac{1}{P_{[nTorr]}}$$

- Inelastic excitation: For an average β_n

$$\tau_{Gas[hours]} \cong 10.25 \frac{E^2_{0[GeV]}}{P_{[nTorr]}} \frac{\varepsilon_{A[\mu m]}}{\left\langle \beta_n \right\rangle_{[m]}}$$

Touschek effect: Intra-beam Coulomb scattering

- Coulomb scattering between beam particles can transfer transverse momentum to the longitudinal plane
  - If the $p_{||}+\Delta p_{||}$ of the scattered particles is outside the momentum acceptance, the particles are lost
  - First observation by Touschek at ADA e^+e^- ring
- Computation is best done in the beam frame where the relative motion of the particles is non-relativistic
  - Then boost the result to the lab frame

$$\boxed{\frac{1}{\tau_{Tousch.}} \propto \frac{1}{\gamma^3}\frac{N_{beam}}{\sigma_x \sigma_y \sigma_S}\frac{1}{\left(\Delta p_A / p_0\right)^2} \propto \frac{1}{\gamma^3}\frac{N_{beam}}{A_{beam}\sigma_S}\frac{1}{\hat{V}_{RF}}}$$

Transverse quantum lifetime

- At a fixed s, transverse particle motion is purely sinusoidal

$$x_T = a\sqrt{\beta_n}\sin\left(\omega_{\beta_n} t + \varphi\right) \qquad T = x, y$$

- Tunes are chosen in order to avoid resonances.
  - → At a fixed azimuthal position, a particle turn after turn sweeps all possible positions between the envelope
- Photon emission randomly changes the "invariant" *a* & consequently changes the trajectory envelope as well.
- Cumulative photon emission can bring the particle envelope beyond acceptance in some azimuthal point
  - → The particle is lost

Quantum lifetime was first estimated by Bruck and Sands

$$\tau_{Q_T} \cong \tau_{D_T} \frac{\sigma_T^2}{A_T^2} \exp\left(A_T^2 / 2\sigma_T^2\right) \quad T = x, y$$

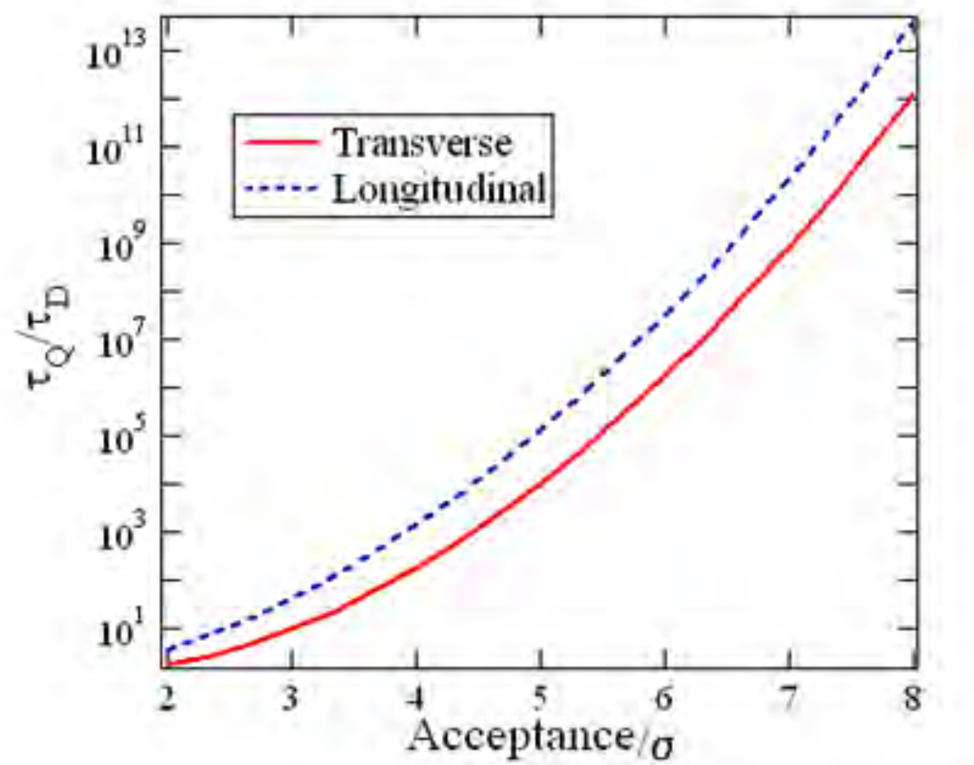
Transverse quantum lifetime

$$where \quad \sigma_T^2 = \beta_T \varepsilon_T + \left(D_T \frac{\sigma_E}{E_0} \right)^2 \quad T = x, y$$

$\tau_{D_T} \equiv$ *transverse damping time*

$$\tau_{Q_L} \cong \tau_{D_L} \exp\left(\Delta E_A^2 / 2\sigma_E^2\right)$$

Longitudinal quantum lifetime



✺ Quantum lifetime varies very strongly with the ratio between acceptance & rms size.

Values for this ratio ≥6 are usually required

Lifetime summary

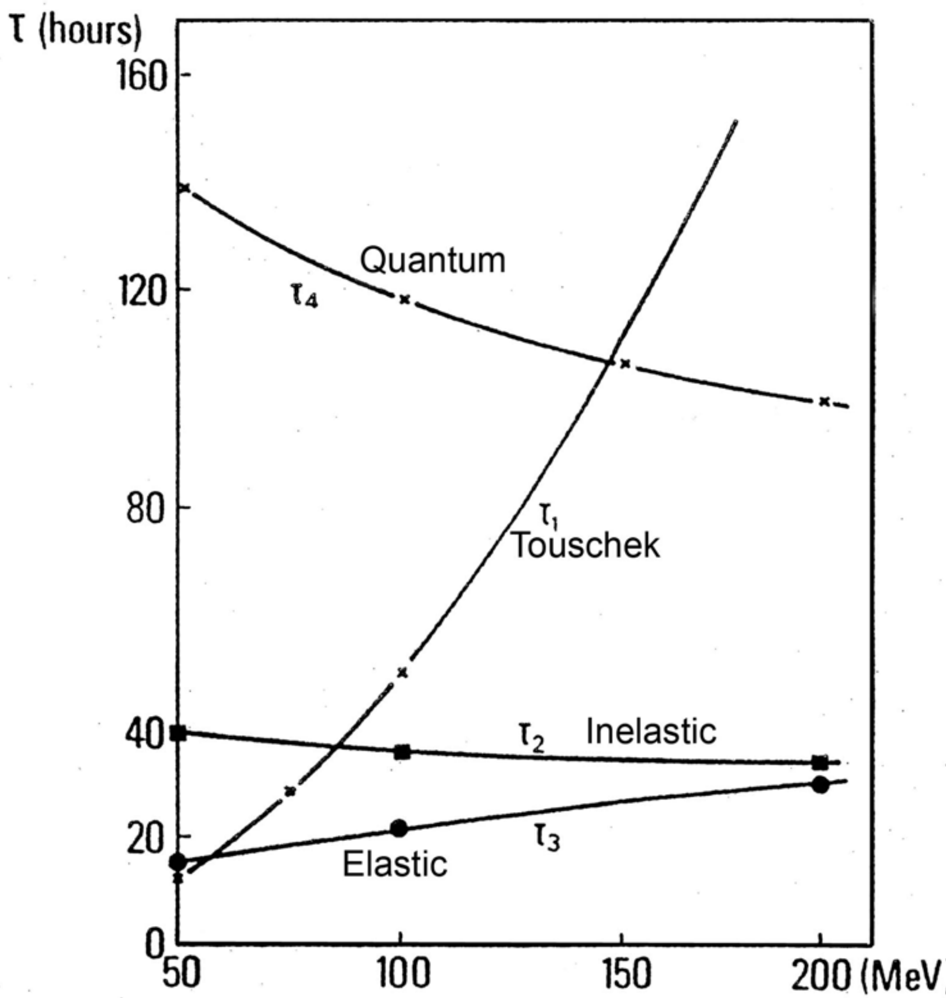


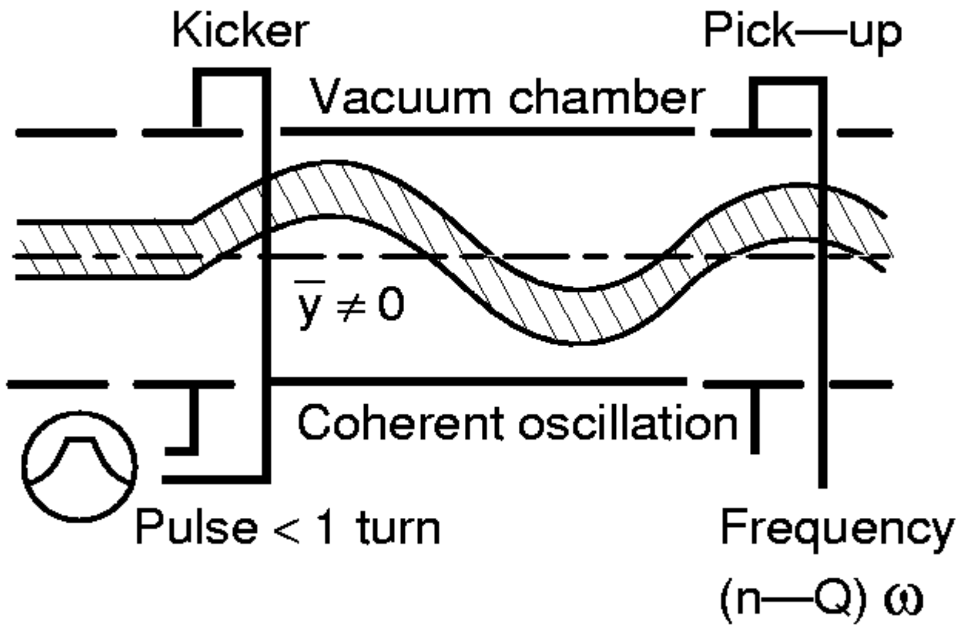
Fig. 1. Lifetime resulting from the different types of beam–gas interaction

Coherent limitations on beams in storage rings

Coherent effects are characterized by impedances

- Collective effects on the bunch as a whole driven by the collective forces generated by the beam
- Low Q diseases limit impedance for the ring
  - Transverse - "fast" head-tail effect
  - Longitudinal - Bunch lengthening "microwave" effect
- High Q diseases:
  - Multi-bunch instabilities
- Cures:
  - Vacuum chamber design
  - Lower current if possible
  - Landau damping

Types of tune shifts: Coherent motion



Center of mass moves with a betatron oscillation

Beam environment (e.g. a position monitor) “sees” a “collective motion”

Within the coherent motion, each particles has its individual motion

From: E. Wilson Adams lectures

Reminder about impedances

- Describe interaction of beam with the vacuum chamber by impedances

- Example: Longitudinal motion is described by a current I.
  - ➞ Modification of the motion is described by ΔI

- Radiated field, $\Delta E \propto$ displacement => driving voltage

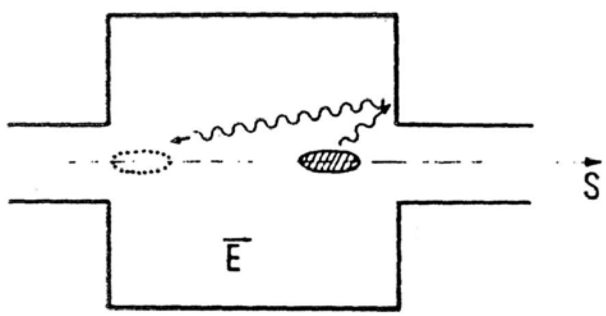
$$\Delta V = Z \, \Delta I$$

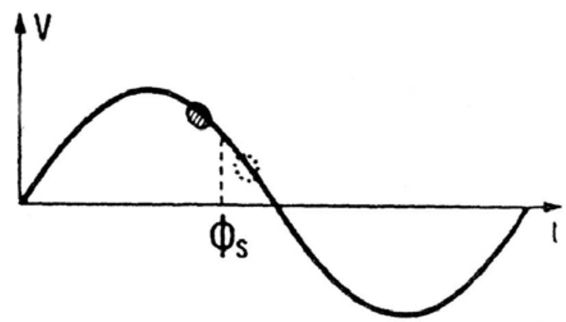
- Z is the Fourier transform of the wake-field
  - ➞ Longitudinal impedances are given in Ohms
  - ➞ Transverse impedances are given in Ohms / meter

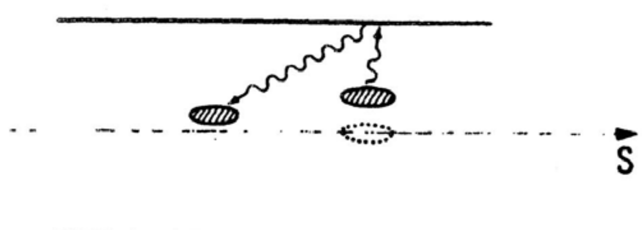
Impedance relations

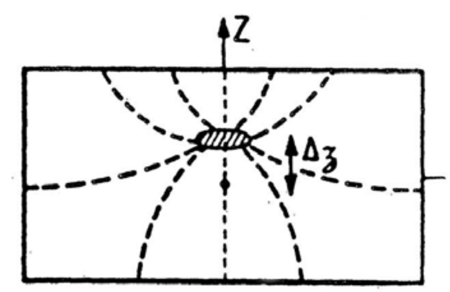
- For a cylindrical vacuum chamber of radius b

$$Z_\perp = \frac{2c}{b^2}\frac{Z_{||}}{\omega}$$









Example of coherent bunch lengthening

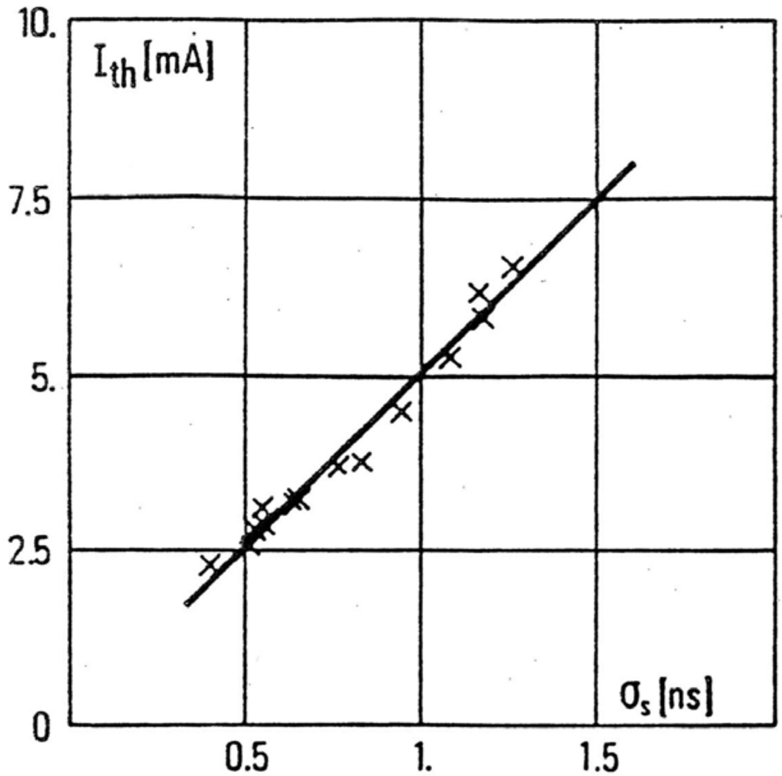


Fig. 9. Transverse instability threshold vs bunch length measured on ACO.

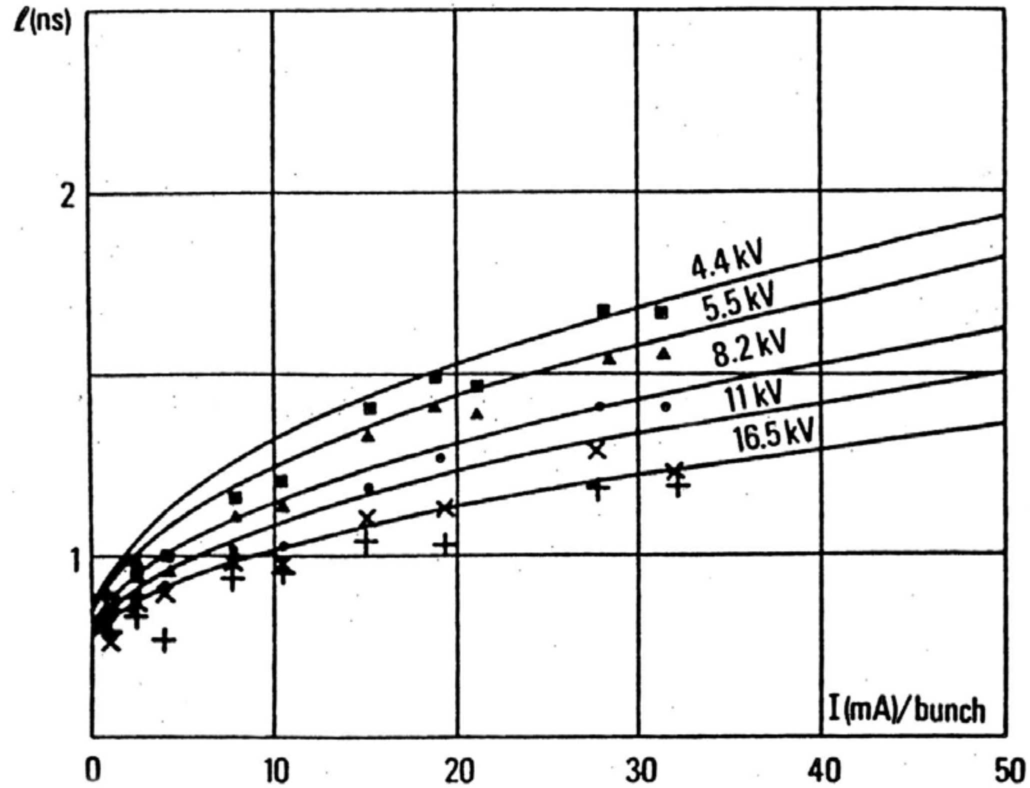


Fig. 13. Bunch length measurements on ACO.

Phase mix or Landau damping

(Model from Barletta & Briggs)

Very high order multipole: 1-D model

Motion in coordinate space

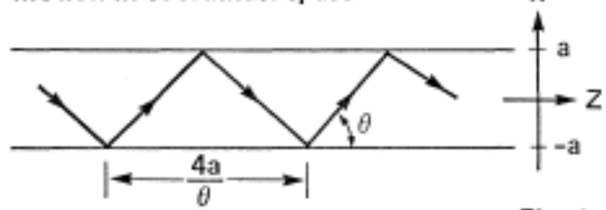


Fig. 1a

Motion in phase space:

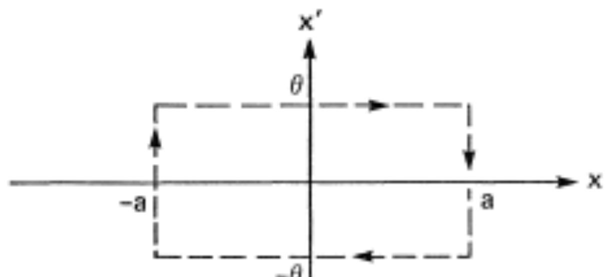


Fig. 1b

Motion in phase space for "real" multipole channel

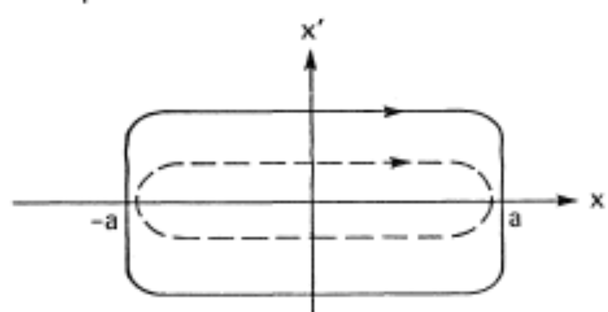
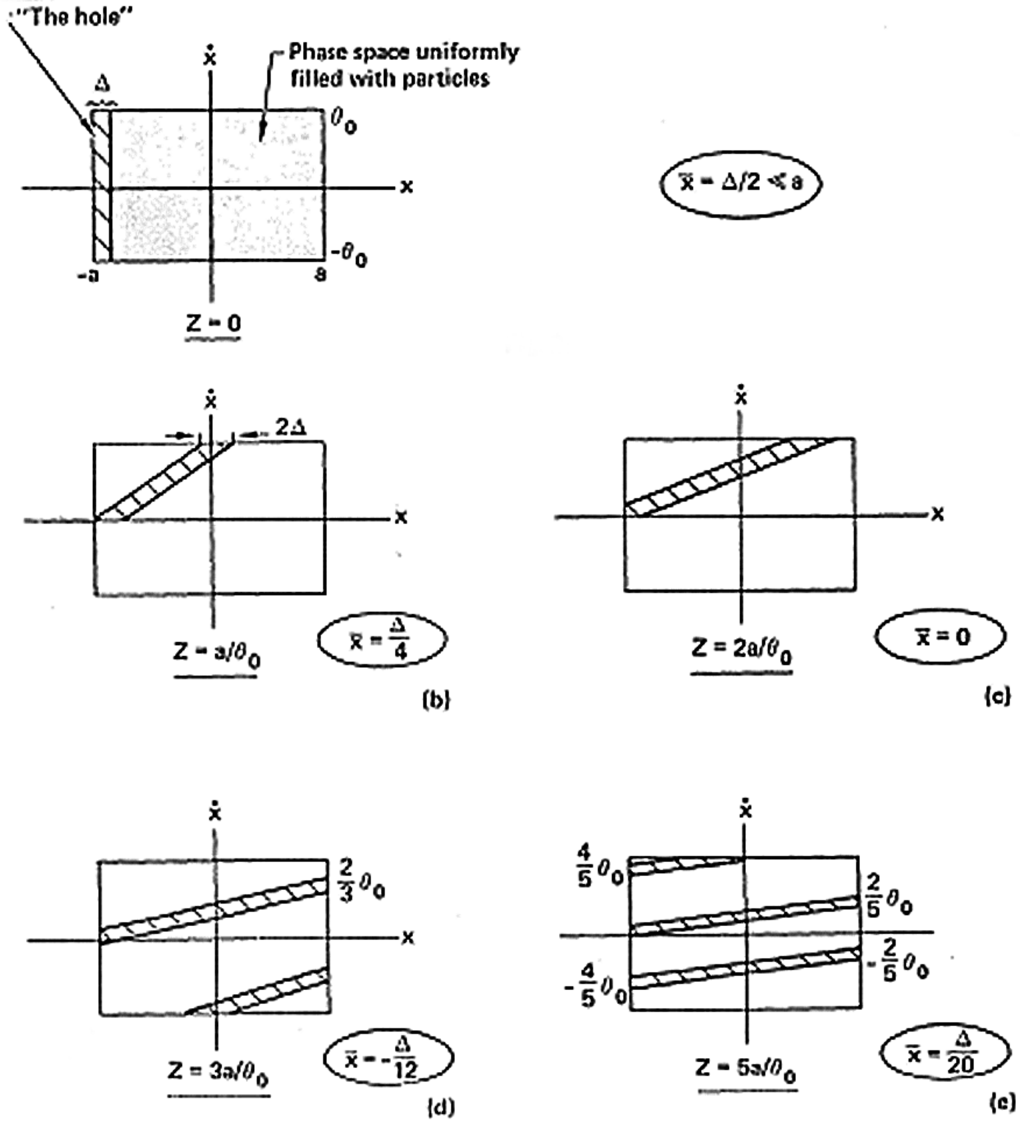
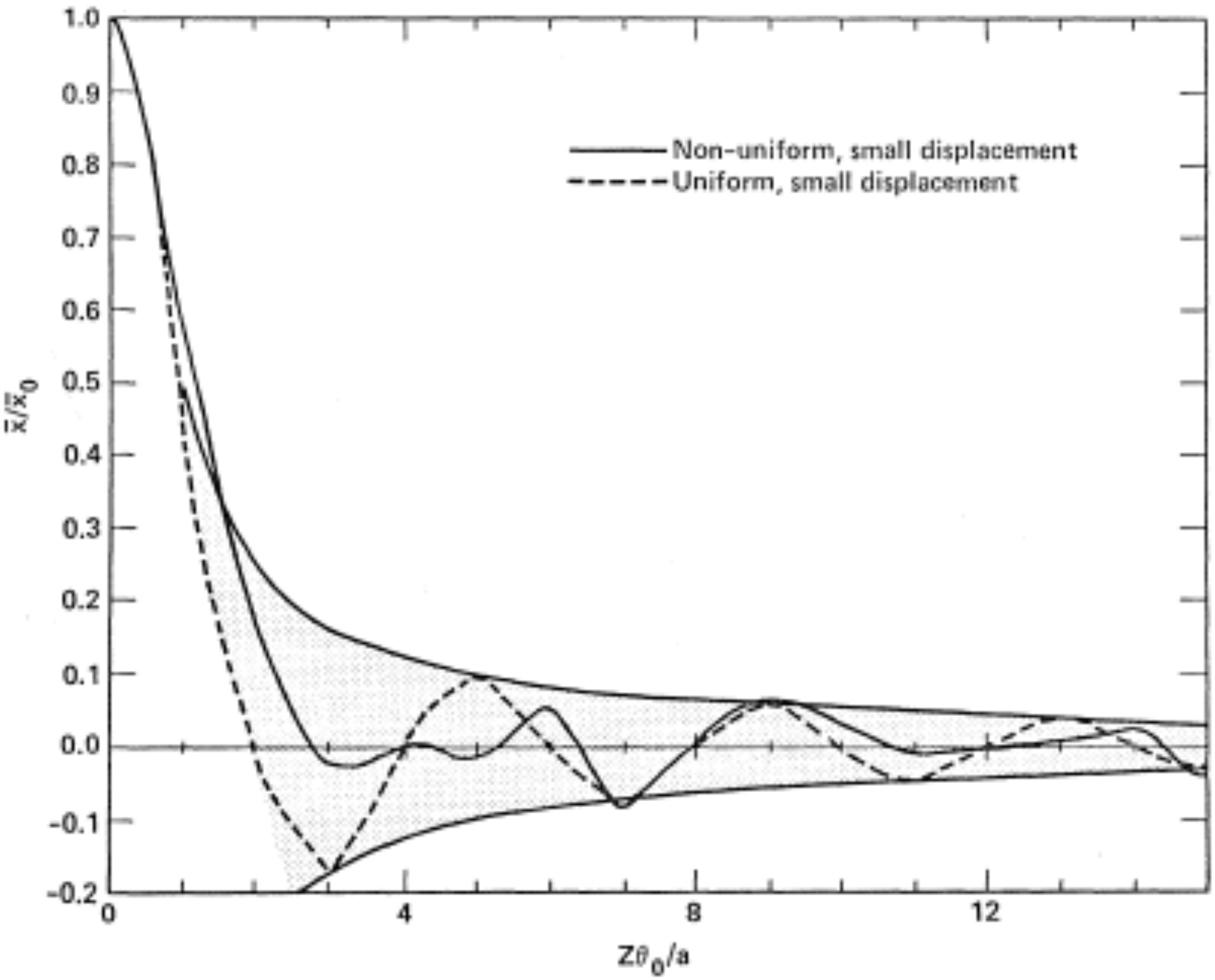


Fig. 1c

Phase mix damping of small uniform displacement

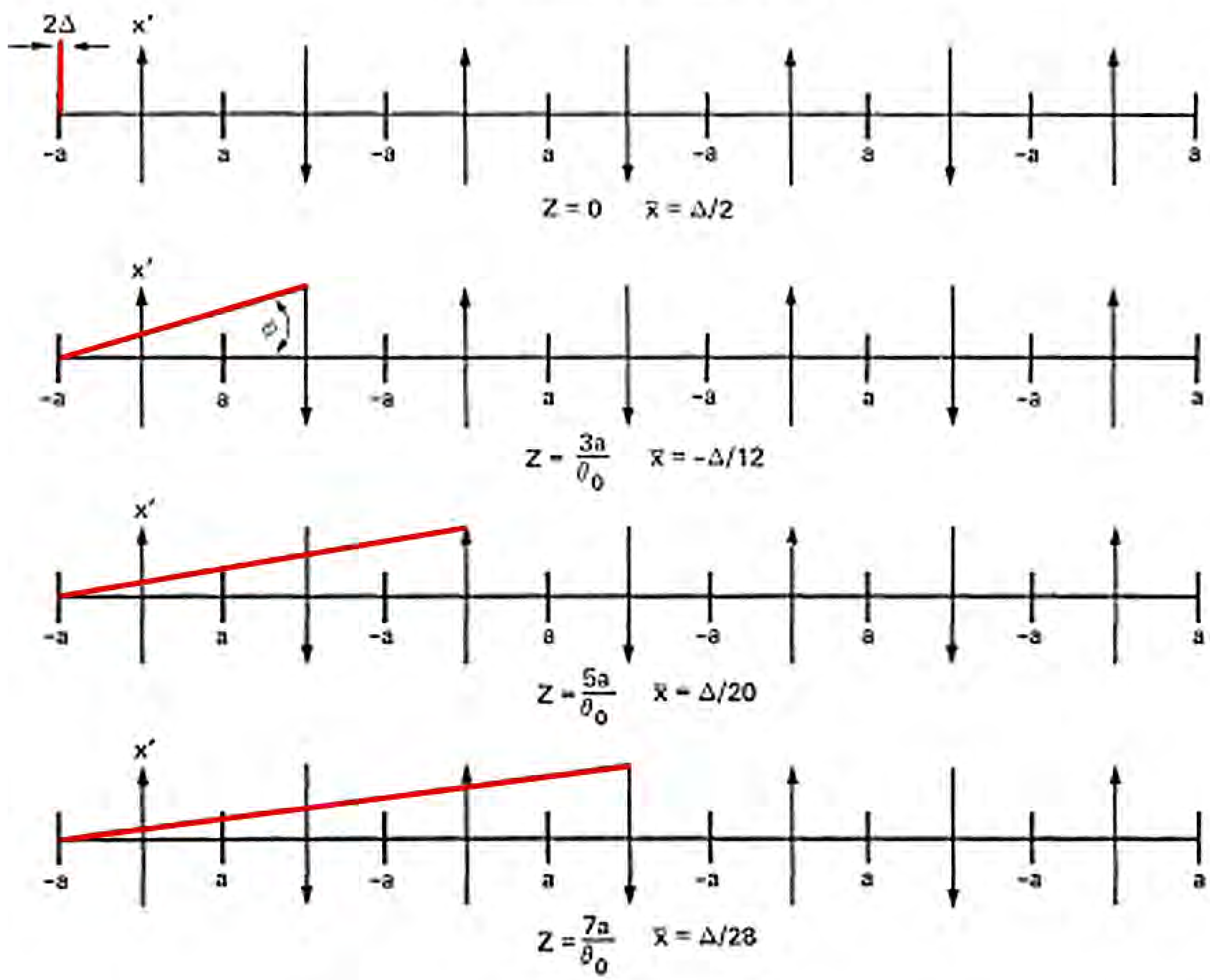


Small uniform displacements damp as 1/z

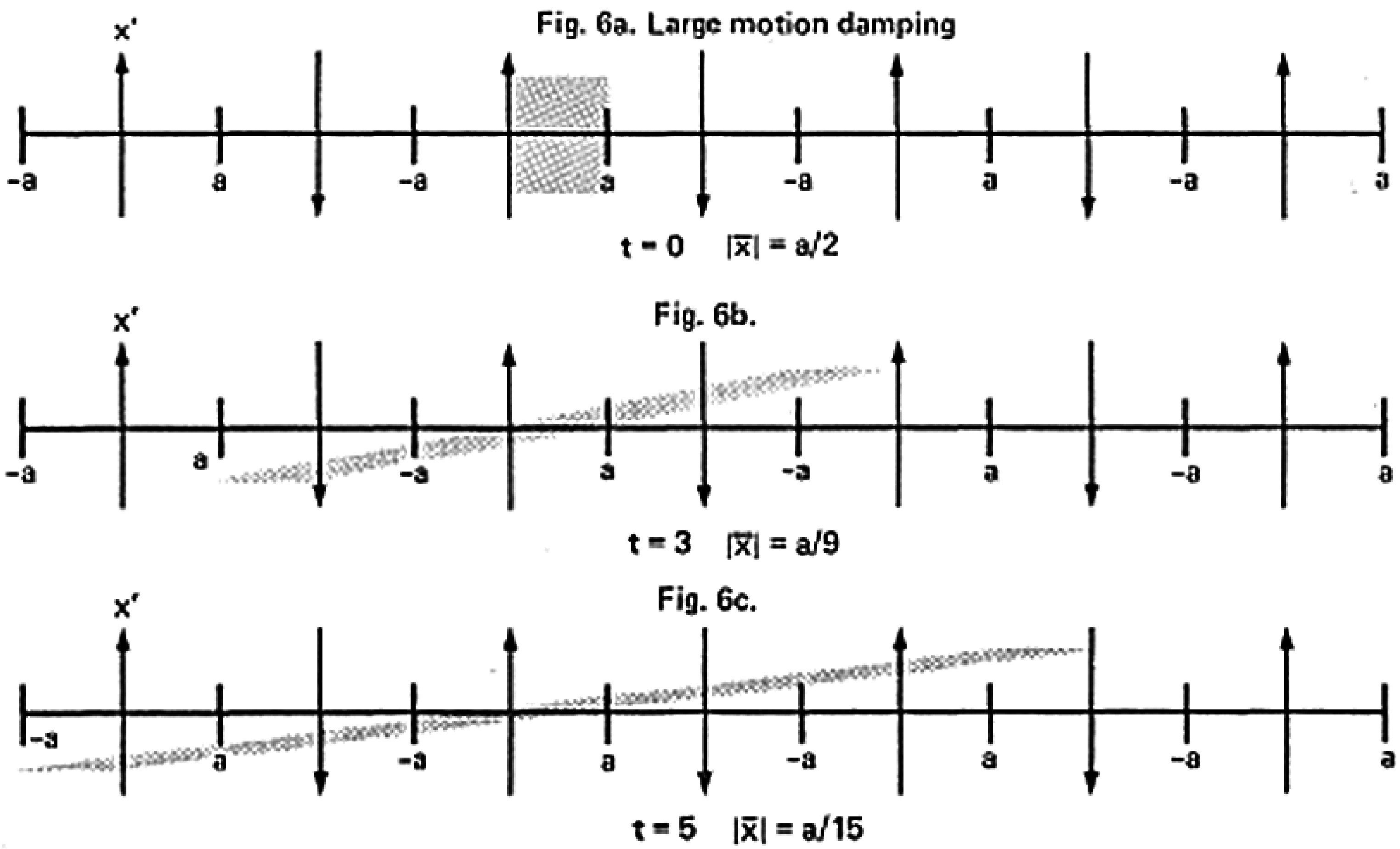


Phase mix damping of small uniform displacement

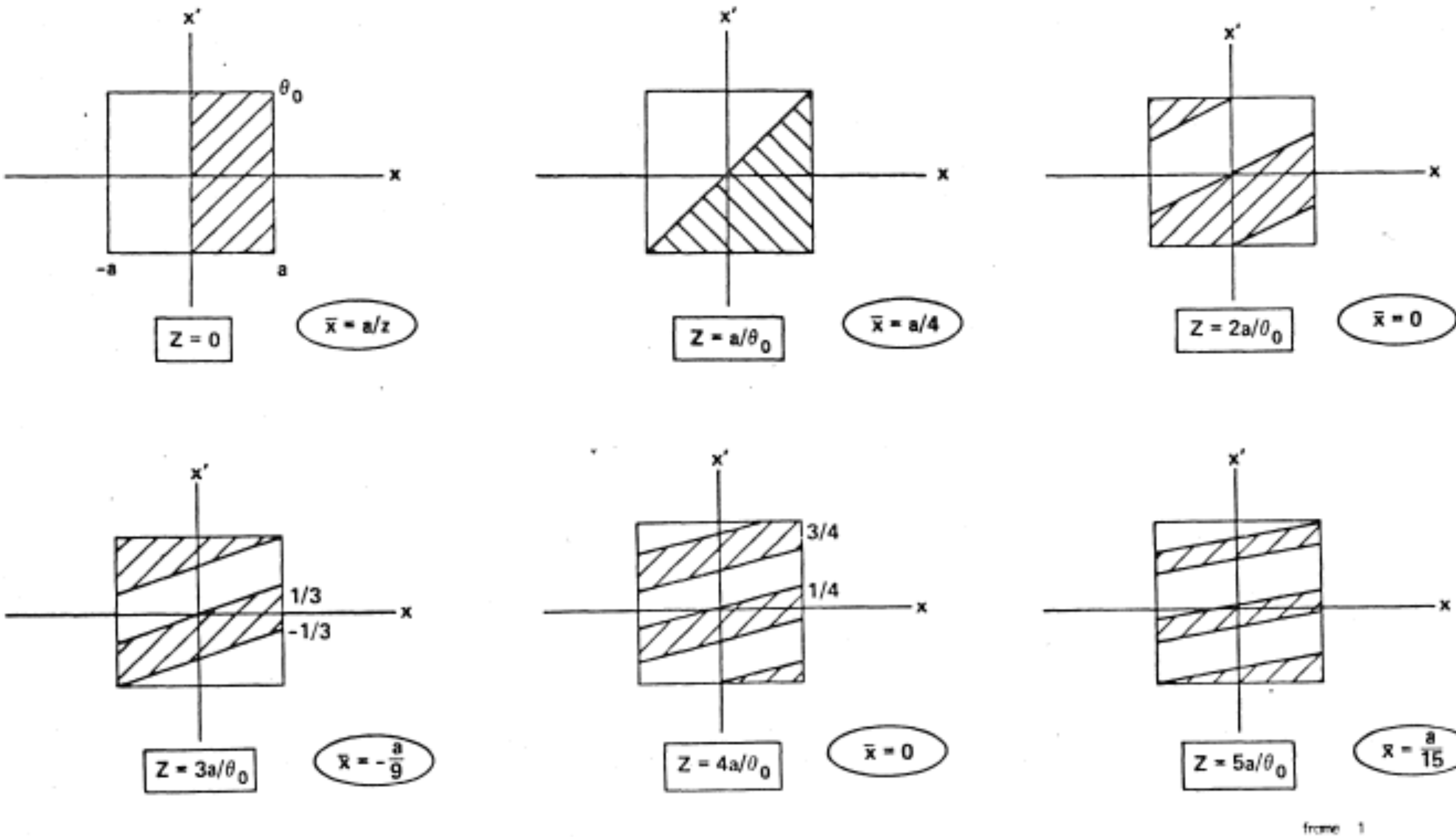
Fig. 5 Motion of phase space sliver



Large displacements damp slower than *1/z*



Phase space rapidly assumes asymptotic form



Scattering reduces the damping rate

Fig. 10a

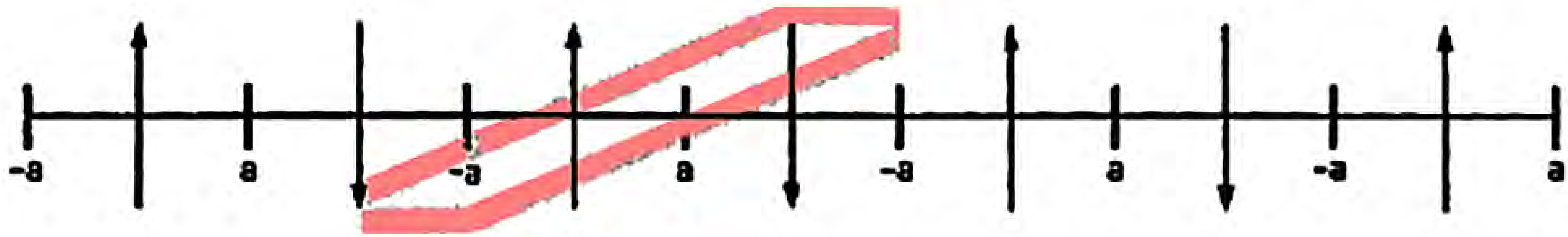


Fig. 10b

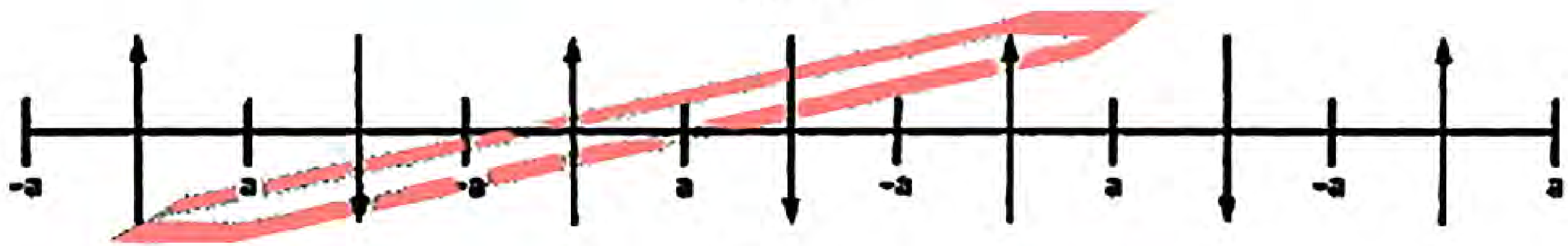
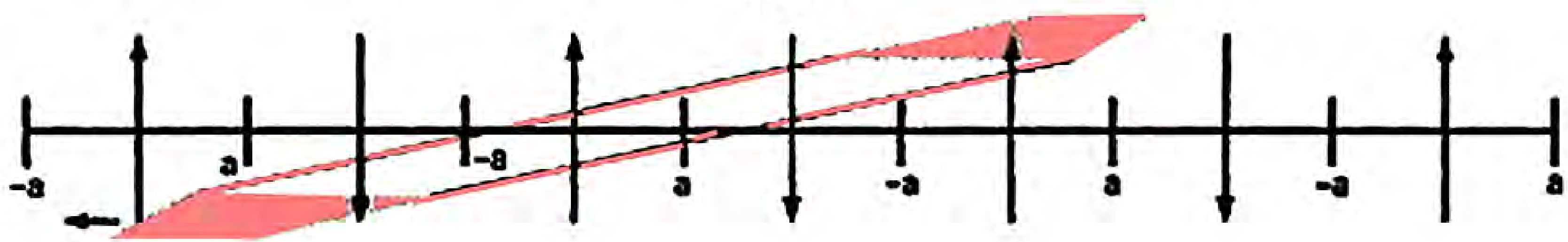
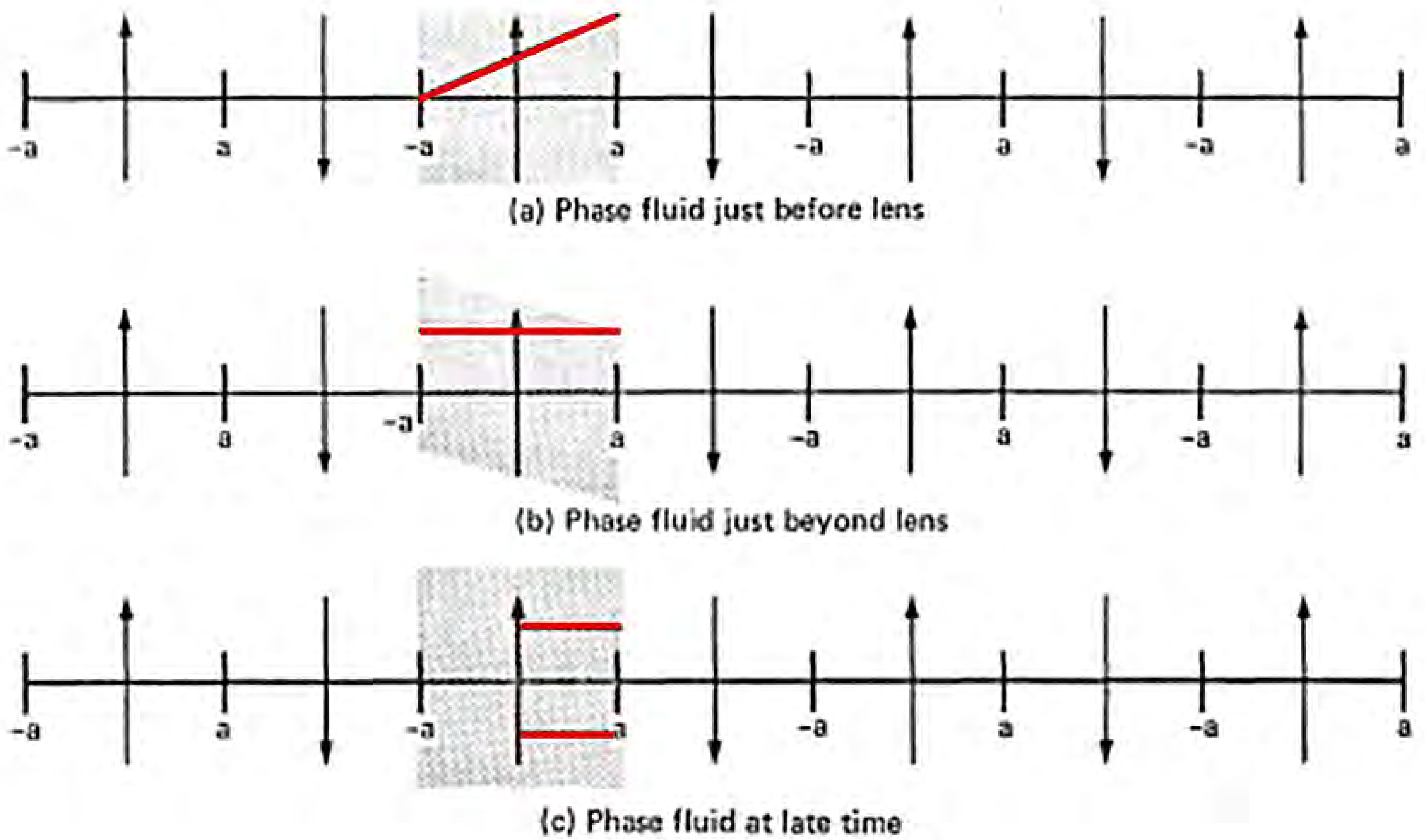


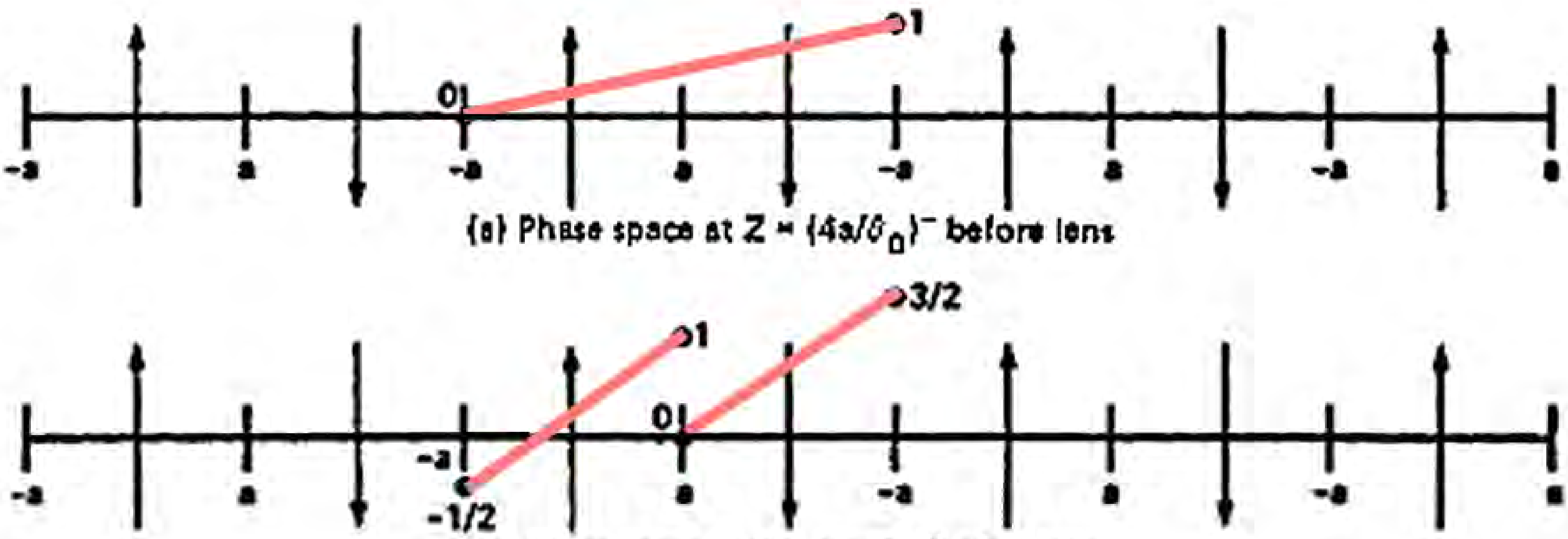
Fig. 10c



Passage of beam through a convergent lens slows damping



Evaluation of effect of divergent lens

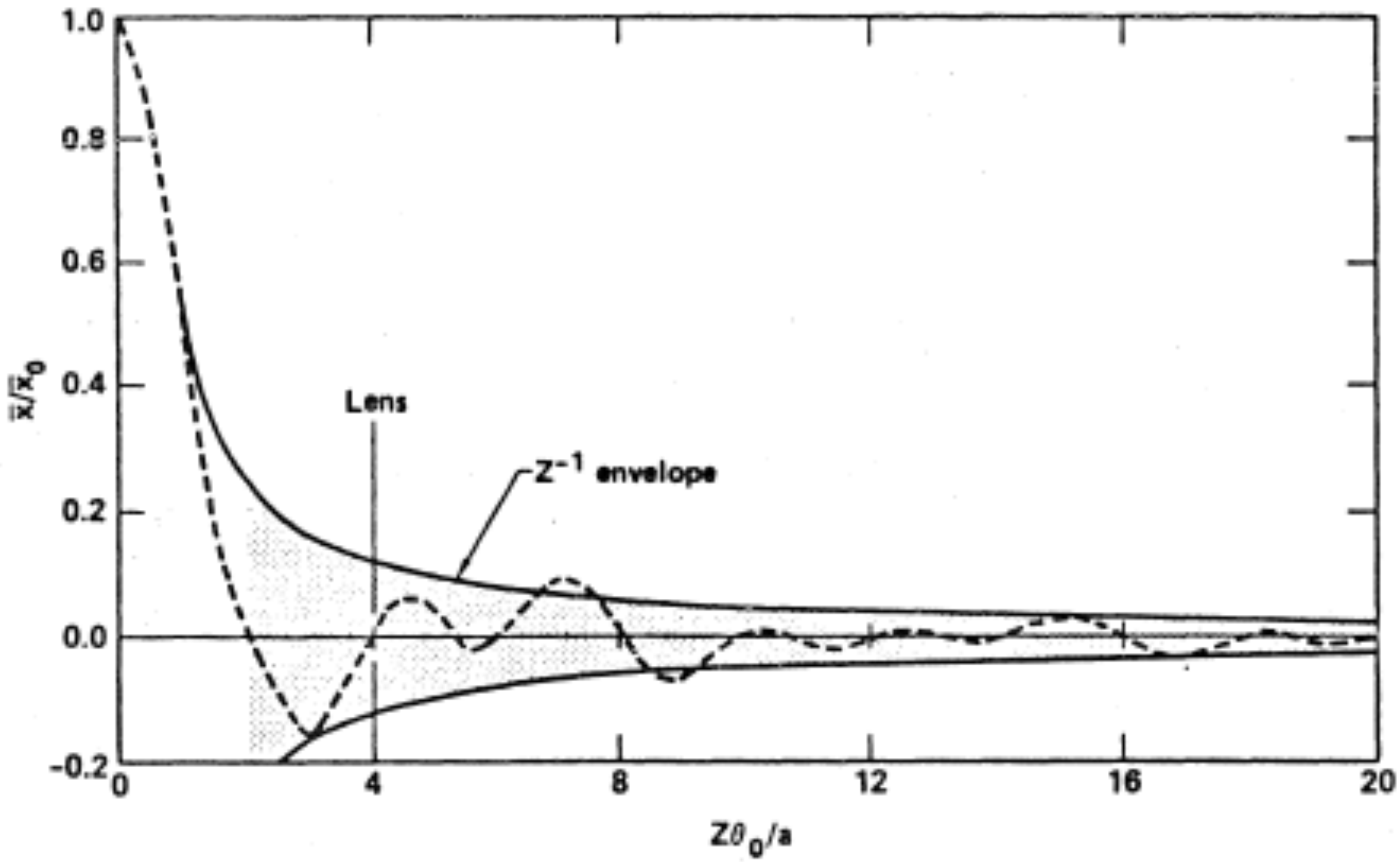


(a) Phase space at $Z = (4a/\theta_0)^-$ before lens

(b) Phase space at $Z = (4a/\theta_0)^+$ after lens

$$\frac{\bar{x}}{x_0} = -\left\{ -(-1)^{I(t/4)}\left(2 - \frac{t_3}{2}\right)\left(\frac{t_3}{2} \bmod 2\right) + (-1)^{I(t/2)}(2 - t_2)(t_2 \bmod 2) - (-1)^{I(3t/4)}\left(2 - \frac{3t_1}{2}\right)\left(\frac{3t_1}{2} \bmod 2\right) \right\} \Big/ (8 + 6t) \qquad (4)$$

Divergent lens speeds damping slightly



Simple damping model - conclusions

- Mean displacement can damp rapidly upon entering an anharmonic transport channel
- Once phase fluid assumes asymptotic form damping proceeds very slowly
- If initial state matches asymptotic form, damping is slow from the start
- Damping rate depends on microscopic form of phase fluid
  - Beams with the same mean displacement can damp at quite different rates
  - Scattering is likely to slow damping
- The price of changing the damping rate is increasing the emittance of the beam