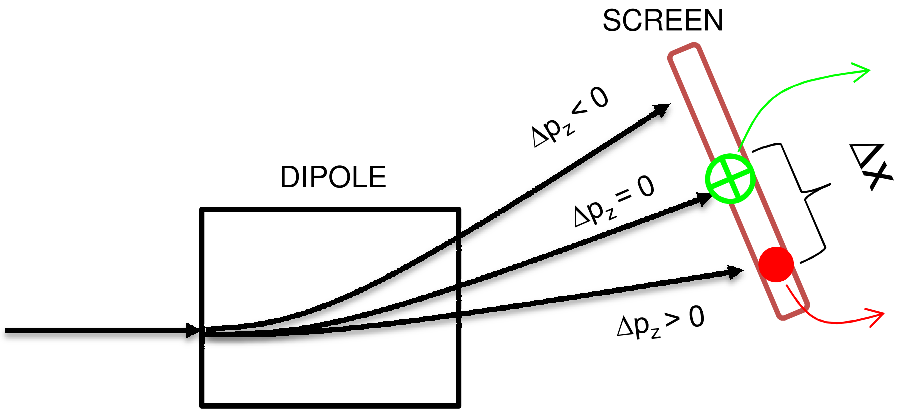
Layout definition for beam diagnostics

S. Di Mitri (1.5 hr.)

Mean energy (Spectrometer)



"on-energy" particle centers the screen: $\langle p_z \rangle \left[\frac{GeV}{c}\right] = 0.3 \cdot B_{y,0}[T] \cdot R[m]$

"off-energy" particle follows a different (dispersive) path: $p_z = \langle p_z \rangle \left(1 + \frac{\Delta x}{\eta_x}\right)$

- **How to proceed:**
  1. geometry of the ref. trajectory (θ, R) is fixed by the mechanical assembly,
  2. B_y is chosen to center the *beam* onto the detector (screen or BPM),
  3. calculate $\langle p_z \rangle$.

- **Measurement errors:**
  - trajectory distortion before / after the dipole magnet,
  - dipole field calibration errors (vs. the supplying current),
  - misalignment of the dipole / detector.

  Typical error is of the order of 1 MeV at energies higher than tens of MeV.

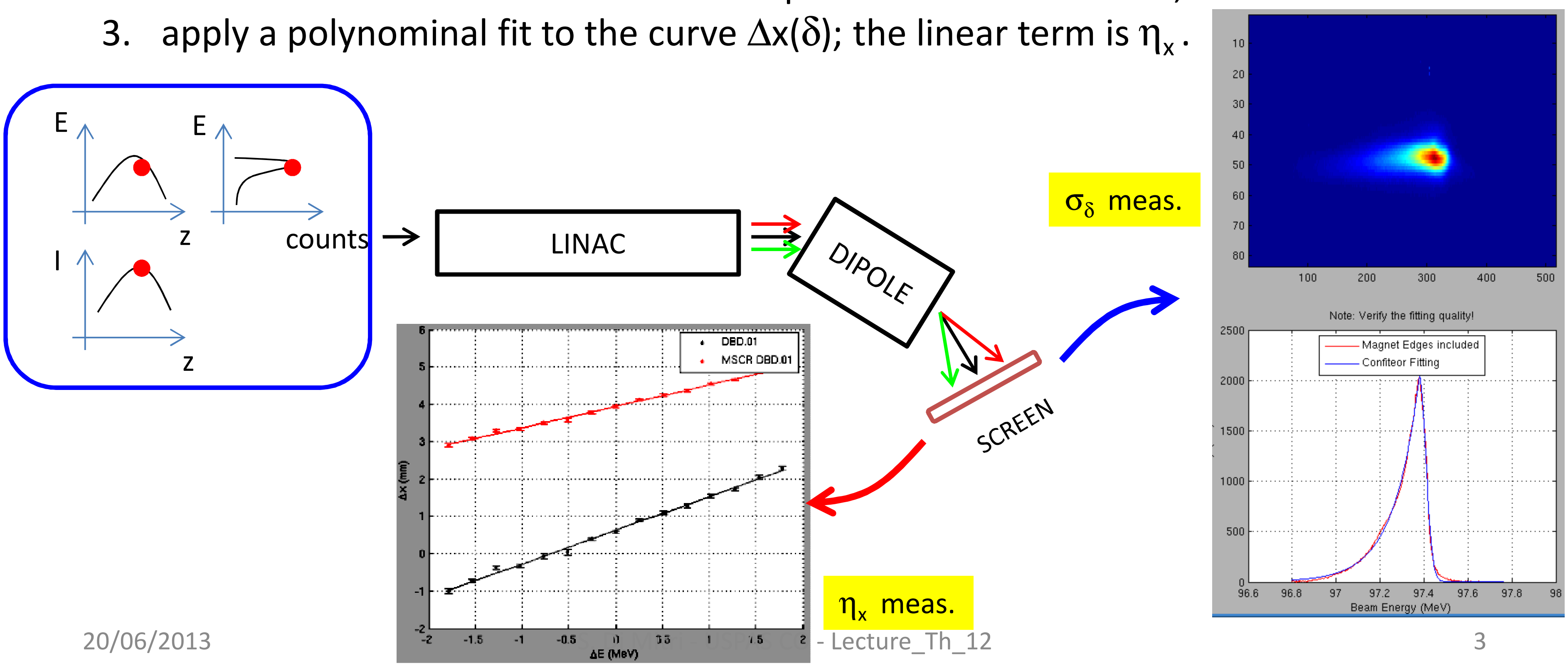
Energy spread (Spectrometer)

❖ Average over the beam particle ensemble:

$$\langle p_z^2 \rangle = \langle p_z \rangle^2 + \langle p_z \rangle^2 \frac{\langle \Delta x^2 \rangle}{\eta_x^2}; \Rightarrow \boxed{\sigma_E = \langle E \rangle \frac{\sigma_x}{\eta_x}}$$

❖ The energy spread measurement depends on the value of η_x. This can be measured in turn as follows:

1. change the beam *mean* energy (by a few %),
2. look at the variation of beam centroid position on the detector,
3. apply a polynominal fit to the curve $\Delta x(\delta)$; the linear term is η_x .



Energy resolution

❖ The **beam spot size** at the screen is the sum of the geometric (betatron) and chromatic (dispersive) particle motion:

$$\sigma_x = \sqrt{\varepsilon_x \beta_x + \eta_x^2 \sigma_\delta^2}$$

❖ The ***energy resolution*** due to the beam ***geometric optics*** (for an *infinite screen resolution)* is:

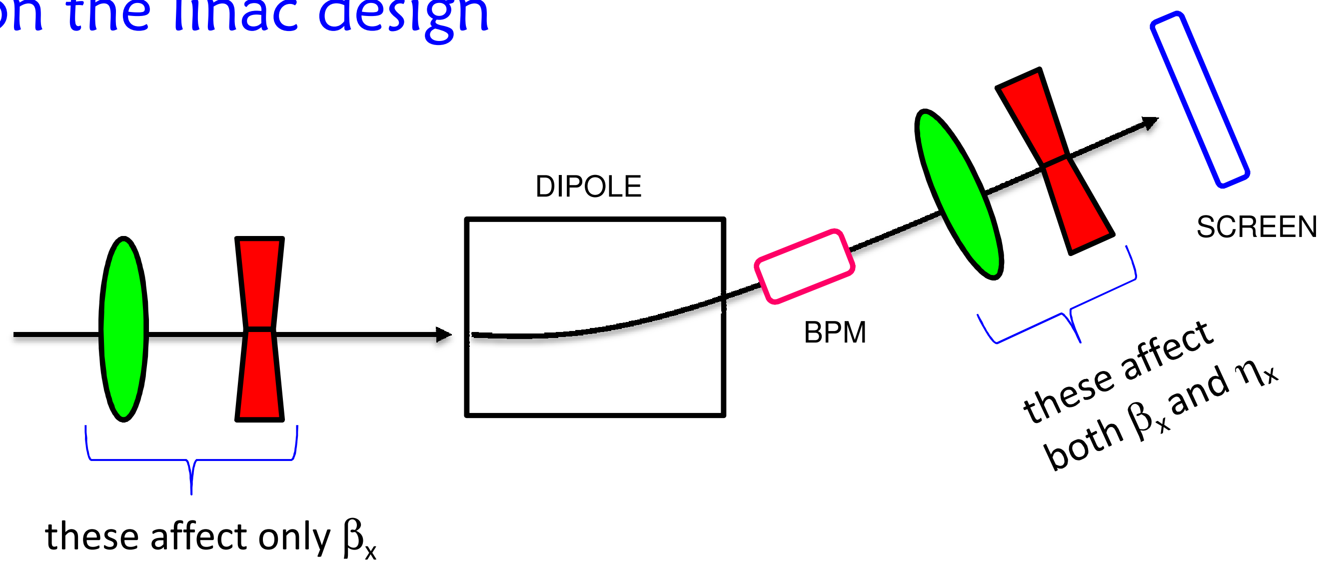
$$\boxed{\sigma_{E,res} = \frac{\sqrt{\varepsilon_x \beta_x}}{\eta_x} \langle E \rangle} \quad \Rightarrow \quad \text{minimize } \beta_x / \sqrt{\eta_x} \text{ by design}$$

❖ In reality, the finite ***screen resolution*** (millimeter per pixel) can be neglected if it smaller than the geometric beam size:

$$\sigma_{E,res} = \frac{\sqrt{\varepsilon_x \beta_x}}{\eta_x} \langle E \rangle >> \frac{\sigma_{screen,res}}{\eta_x} \langle E \rangle \quad \Rightarrow \quad \sigma_{screen,res} << \sqrt{\varepsilon_x \beta_x}$$

▪EXERCISE: assume $\sigma_{screen,res}$ = 30 μm/pixel, $\gamma\varepsilon_x$ = 1 mm mrad at 100 MeV, η_x = 1.5m. Evaluate β_x at the screen to ensure $\sigma_{E,res}$ = 10keV. Is it large enough to dominate over $\sigma_{screen,res}$?

Impact on the linac design

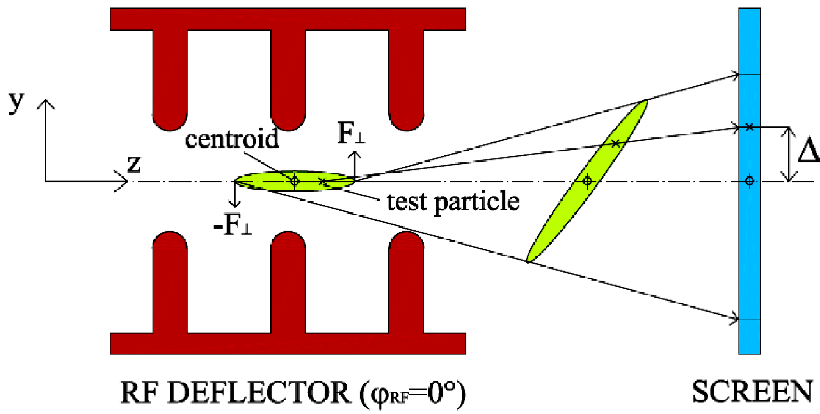


- Make η_x **large** to minimize $\sigma_{E,res}$ $\Rightarrow$ large bending angle (typically > 20deg)
  $\Rightarrow$ add quadrupoles *after* the dipole
- Make β_x **small** to minimize $\sigma_{E,res}$ $\Rightarrow$ use quadrupoles *after* the dipole
  $\Rightarrow$ use quadrupoles *before* the dipole
- For any given screen resolution, make β_x larger to increase the beam size
  $\Rightarrow$ tune the **ratio** $\sqrt{\beta_x}/\eta_x$ to keep $\sigma_{E,res}$ fixed

- A **BPM** soon **after the dipole** can be easily used for the mean energy measurement, being less sensitive to trajectory distortion occurring between the dipole and the screen.
- A **SCREEN** at the **line end** takes advantage of the maximum dispersion and optics tuning. It is devoted to the energy spread measurement.

Bunch length (RF deflector)

Pictures courtesy of P. Craievich



$$\Delta y_S(z) = R_{34}\Delta y'_D$$

- RF deflector transverse kick:

$$\Delta y'_D \approx \frac{eV_\perp}{E}\Big[\underbrace{z\frac{\omega_{RF}}{c}\cos\varphi_{RF}}_{\text{this applies to the entire bunch length}} + \underbrace{\sin\varphi_{RF}}_{\text{this applies to the CM}}\Big]$$

- Transport matrix element:

$$R_{34} = \sqrt{\beta_D\beta_S}\sin(\Delta\psi_{DS})$$

For $\varphi_{RF} \approx 0^o$, the **RMS spot size** on the screen is: $\boxed{\sigma_{y,S} = \frac{eV_\perp}{E}\sigma_z\left[\frac{\omega_{RF}}{c}\cos\varphi_{RF}\right]\sqrt{\beta_D\beta_S}\sin(\Delta\psi_{DS})}$

For ***$\varphi_{RF} \approx 90^o$***, the ***CM deviation*** onto the screen is: $\langle\Delta y_S\rangle = \frac{eV_\perp}{E}\sqrt{\beta_D\beta_S}\sin(\Delta\psi_{DS})\sin\varphi_{RF}$

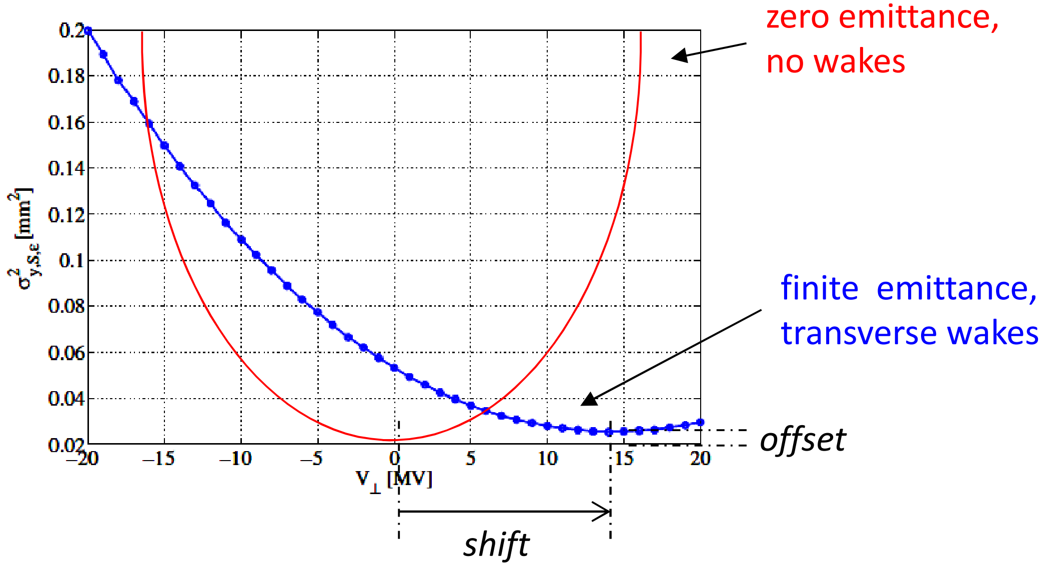
How to proceed:

- vary φ_{RF} and measure $<\Delta y_S>$, then fit the curve to compute the "**optics calibration factor**" $B = eV_\perp R_{34}/E$ [mm/rad];
- now measure $\sigma_{y,S}$ and evaluate the **bunch length** as $\sigma_z = \sigma_{y,S} / B$;
- if you know $V_\perp/E$, B provides a way to benchmark the theoretical machine optics (the viceversa is also true).

Bunch length resolution

Pictures courtesy of P. Craievich

❖ The beam **finite transverse emittance** and residual z-y correlations (e.g., banana shape from **transverse wakefield**) affect the ***calibration curve***, eventually the bunch length resolution.

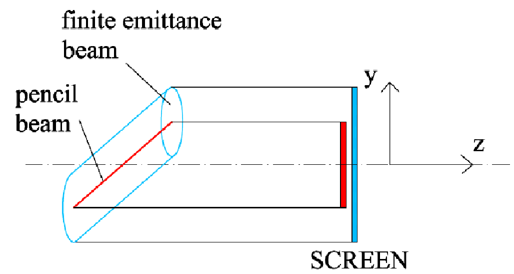


- **Shift of the minimum** reveals an incoming **z-y correlation**, which is removed by a nonzero $V_\perp$.

- **Vertical offset of the minimum** reveals a nonzero vertical **emittance** (finite spot size for $V_\perp$=0).

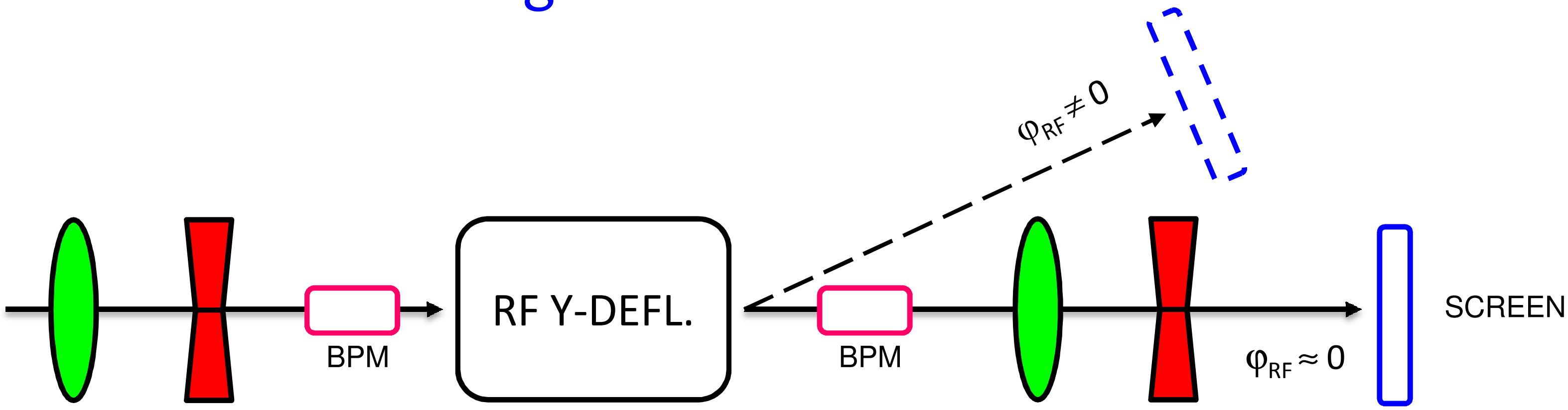
❖ If we assume no net wakefield effect (assume our emittance bumps are working well), the **bunch length resolution** due to the beam ***geometric optics*** (for an *infinite screen resolution)* at the zero-crossing RF phase is:

$$\sigma_{z,res} = \frac{\sigma_{y,S}}{T_{RF} R_{34}} = \frac{p_z c \sqrt{\varepsilon_y}}{e V_\perp k_{RF} \sqrt{\beta_D} \left|\sin \Delta\psi_{DS}\right|}$$



❖ For a finite screen resolution, $V_\perp$ and $\beta_{y,S}$ should be large enough to ensure $\sigma_{y,S} \gg \sigma_{screen,res}$.

Impact on the linac design

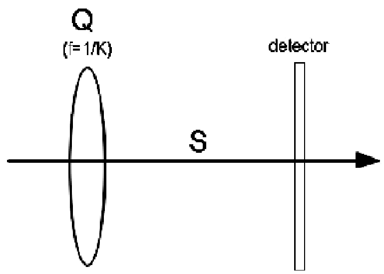


- Make $\beta_{y,D}$ **large** to minimize $\sigma_{z,res}$ $\Rightarrow$ use quadrupoles *before* the deflector
- Make $\Delta\psi_{S,D}$ **close to $\pi/2$** to minimize $\sigma_{z,res}$ $\Rightarrow$ use quadrupoles *before* and, if necessary, *after* the deflector
- Make $\beta_{y,S}$ **large** to dominate over $\sigma_{screen,res}$ $\Rightarrow$ use quadrupoles *before* and, if necessary, *after* the deflector

- Two **BPMs at the deflector edges** can be used for beam steering onto the deflector electric axis, thus for a correct setting of the RF phase.
- A **SCREEN at the end** of the diagnostic line is devoted to the bunch length measurement

Projected emittance (Quadrupole scan)

Courtesy of Minty and Zimmermann



$$Q \cong \begin{pmatrix} 1 & 0 \\ K & 1 \end{pmatrix} \quad S = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix}$$

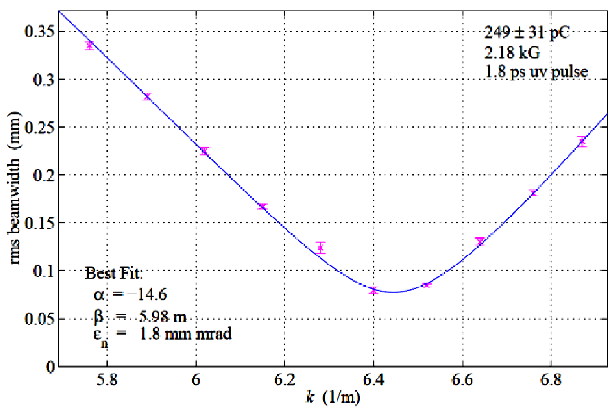
- The **transport matrix** from the quadrupole (thin lens approximation) to the detector (screen) is assumed to be **known:**

$$T = QS = \begin{pmatrix} S_{11} + KS_{12} & S_{12} \\ S_{21} + KS_{22} & S_{22} \end{pmatrix}$$

- The beam matrix transforms according to: $\Sigma_f = T\Sigma_0 T^t$

- The beam matrix element $\langle x^2 \rangle$ is quadratic in the quadrupole integrated strength K = kl:

$$\begin{aligned} \Sigma_{11}(=\langle x^2\rangle) = & ({S_{11}}^2 \Sigma_{11_0} + 2 S_{11} S_{12} \Sigma_{12_0} + {S_{12}}^2 \Sigma_{22_0}) \\ & + (2 S_{11} S_{12} \Sigma_{11_0} + 2 {S_{12}}^2 \Sigma_{12_0}) K + {S_{12}}^2 \Sigma_{11_0} K^2 \end{aligned}$$



- In practice: **vary the quad strength k and measure σ_x** at the screen. Then **fit with a parabola:**

$$\begin{aligned} \Sigma_{11} &= A(K-B)^2 + C \\ &= AK^2 - 2ABK + (C + AB^2) \end{aligned}$$

- Evaluate the emittance and the Twiss parameters from the fitting coefficients:

$$\boxed{\epsilon_x = \sqrt{AC}/S_{12}^2 \qquad \beta_x = \frac{\Sigma_{11}}{\epsilon} = \sqrt{\frac{A}{C}} \qquad \alpha_x = -\frac{\Sigma_{12}}{\epsilon} = \sqrt{\frac{A}{C}}\left(B + \frac{S_{11}}{S_{12}}\right)}$$

Projected emittance (Multiple screens with fixed optics)

- **Multiple beam size mesurements** are now done with fixed quadrupole strength, but at **different locations**.

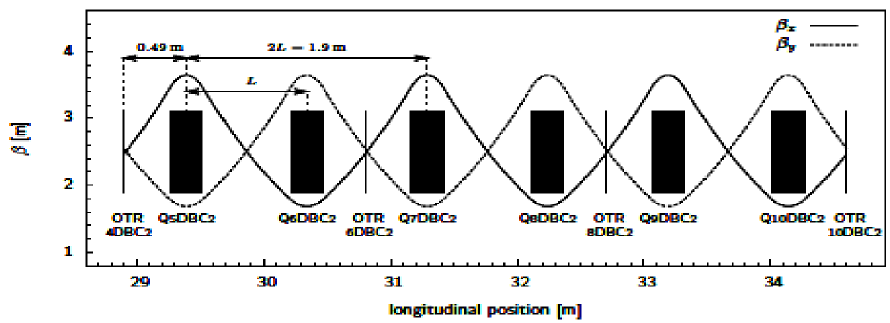
- **The beam size** at screen *1...n-th* **transforms** according to:

$$\begin{pmatrix} (\sigma_x{}^{(1)})^2 \\ (\sigma_x{}^{(2)})^2 \\ (\sigma_x{}^{(3)})^2 \\ \cdots \\ (\sigma_x{}^{n})^2 \end{pmatrix} = \begin{pmatrix} (R_{11}{}^{(1)})^2 & 2R_{11}^{(1)}R_{12}^{(1)} & (R_{12}{}^{(1)})^2 \\ (R_{11}{}^{(2)})^2 & 2R_{11}^{(2)}R_{12}^{(2)} & (R_{12}{}^{(2)})^2 \\ (R_{11}{}^{(3)})^2 & 2R_{11}^{(3)}R_{12}^{(3)} & (R_{12}{}^{(3)})^2 \\ & \cdots & \\ (R_{11}{}^{(n)})^2 & R_{11}^{(n)}R_{12}^{(n)} & (R_{12}{}^{(n)})^2 \end{pmatrix} \begin{pmatrix} \beta(s_0)\epsilon \\ -\alpha(s_0)\epsilon \\ \gamma(s_0)\epsilon \end{pmatrix}$$

Beam size measured at screen *1.....n.* *Vector Σ.*

Transport elements from the source point to the nth-screen. *Matrix B.*

Initial ellipse in phase space. *Vector o.*



- Determine vector *o* by **minimizing the sum** (least square fit):

$$\chi^2 = \sum_{l=1}^{n} \frac{1}{\sigma^2_{\Sigma_x^{(l)}}} \left(\Sigma_x^{(l)} - \sum_{i=1}^{3} B_{li} o_i \right)^2$$

Beam size measurement error

Beam size measured

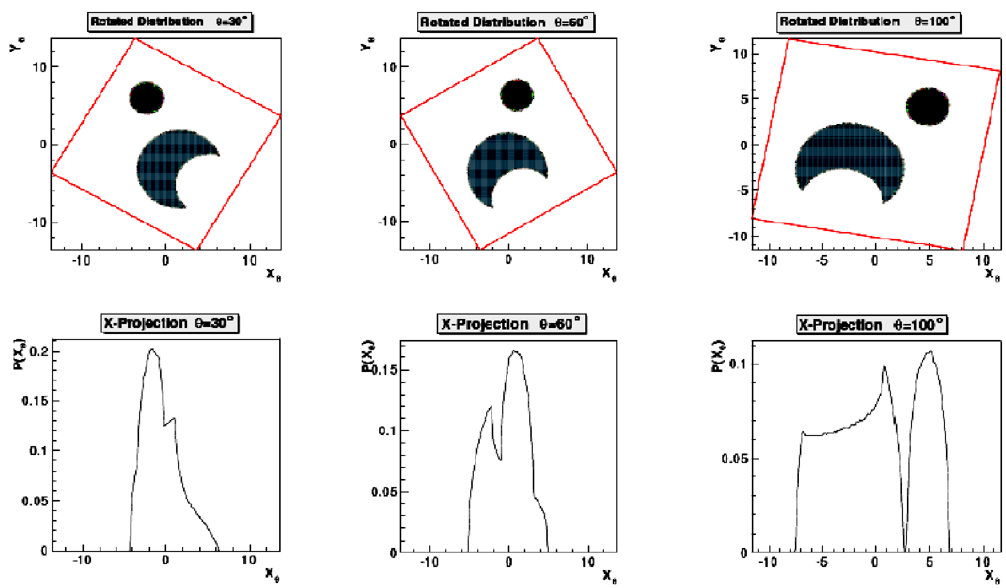
Transport matrix (quads setting)

The system has solution for *at least 3 screens*, for 3 independent parameters to be determined:

$$\epsilon = \sqrt{o_1 o_3 - o_2^2}\,, \qquad \beta = o_1/\epsilon\,, \qquad \alpha = -o_2/\epsilon\,.$$

Projected emittance (Tomographic reconstruction of phase space)

- Make the beam **phase space rotating** along the line. Look at **different screens** (fixed optics) or by applying different quadrupole strengths (quad scan).
- Collect all **phase space projections** onto the spatial coordinate (projected beam size). Then apply the ***MENT algorithm*** to reconstruct, for the given transport matrix, the phase space at the source point.
- In general, tomographic reconstruction is as accurate as many experimental points are used. But, redundancy is avoided as the phase space is sampled at different rotation angles. **Minimum reconstruction error is for 45° phase advance** between samples.

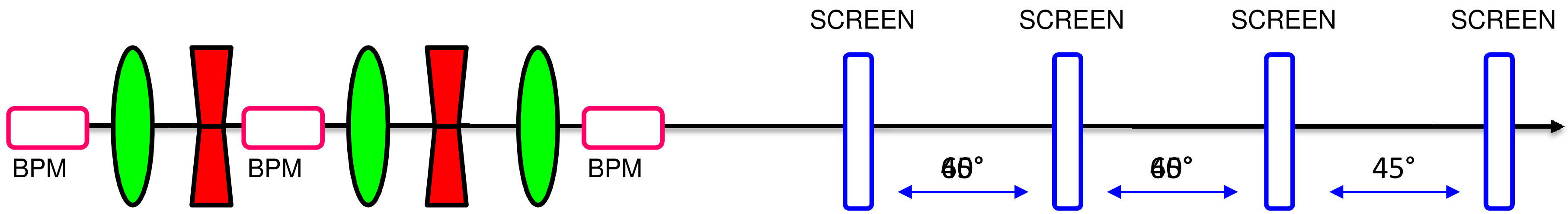


~ phase space at the detector(s)

~ projections

Courtesy of S. Ferry

Impact on the linac design



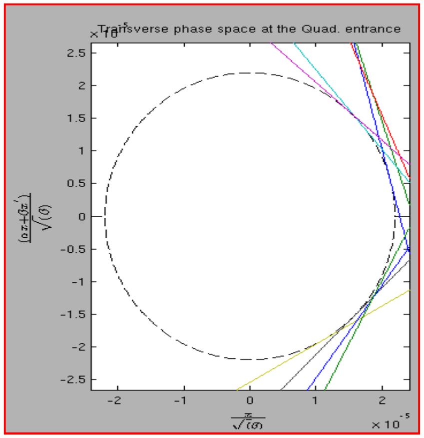
- At leats one quadrupole and one detector for the "**quad-scan**".
- **More quads** can be tuned *simultaneously* to make the scan in one plane while the **beam size** is mantained **fixed** in the other ("multiple quadrupoles scan").
- By adding at least **two more screens**, *e.g.* at 60° phase advance $\Rightarrow$ "multiple screens – fixed optics", **without screwing up** the optics for **beam production**.
- With "**mutiple screens**", the emittance measurement is less sensitive to erros with (at least) 4 screens at 45° phase advance. This would also permit "phase space **tomography**" with four points, at fixed optics.

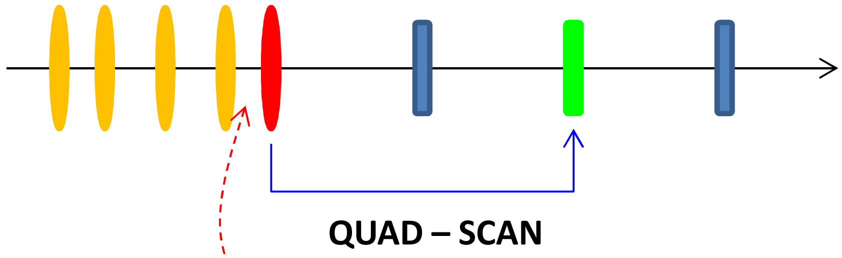
- **BPMs near quadrupoles** allow the beam to be centered into the magnets, so avoiding trajectory steering when the strengths are varied.
- **Beam size detectors** can be either *metallic targets* (*e.g.*, ~100µm thick Yttrium Aluminum Garnet target producing visible fluorescence, or ~10µm thick Aluminum foil producing Optical transition Radiation). Alternatively, *wire scanners* ed to intercept the beam.

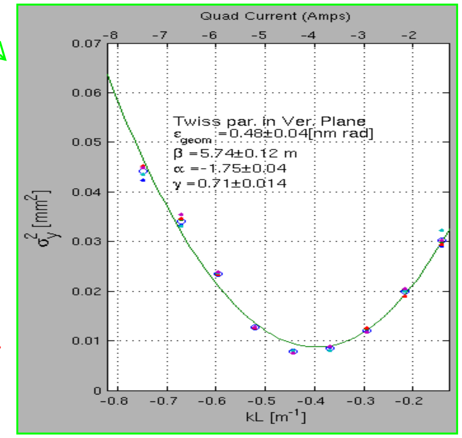
Which phase space?

Pictures courtesy of G. Penco and S. Ferry

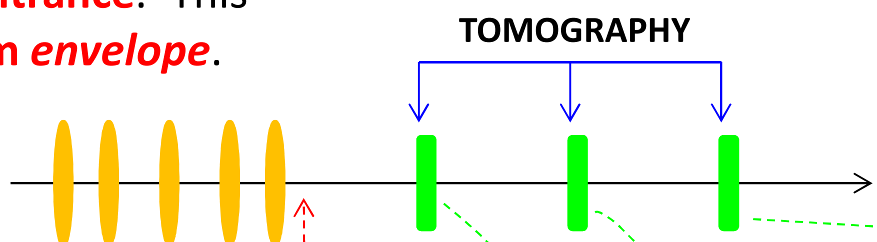
Measure the beam size **at the screen**



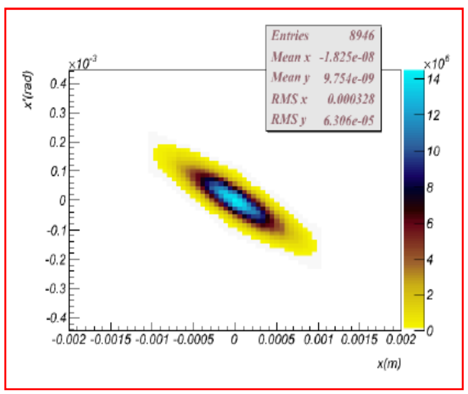


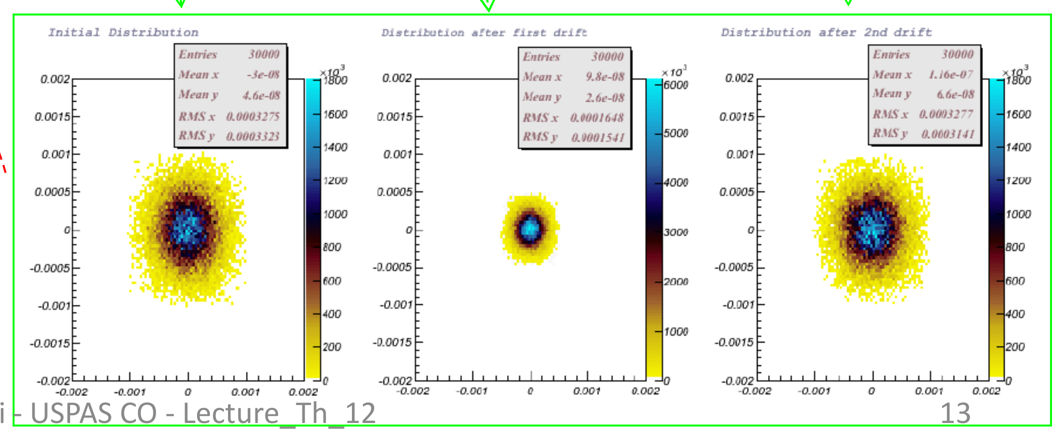


Trace the phase *space ellipse* back **at the quadrupole entrance**. This *approximates* the **beam *envelope***.



Measure the beam size **at the screens**

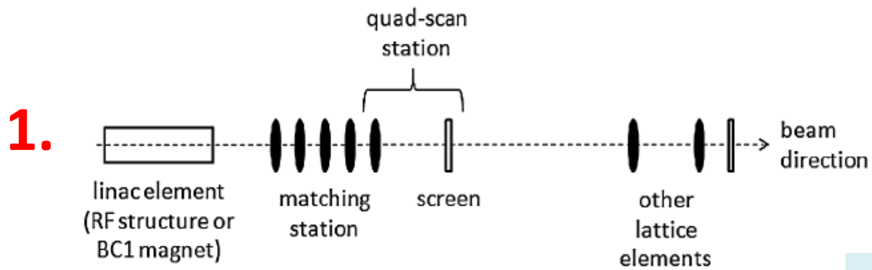




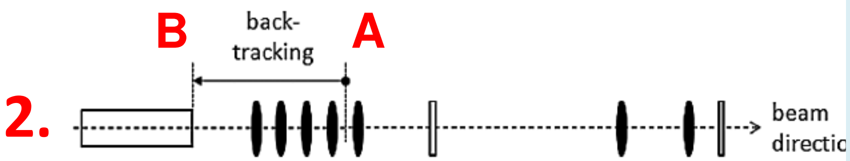
Trace the *phase space density* back at a **certain point of the line**. This is the initial ***charge distribution in the phase space***.

Emittance measurement for optics matching

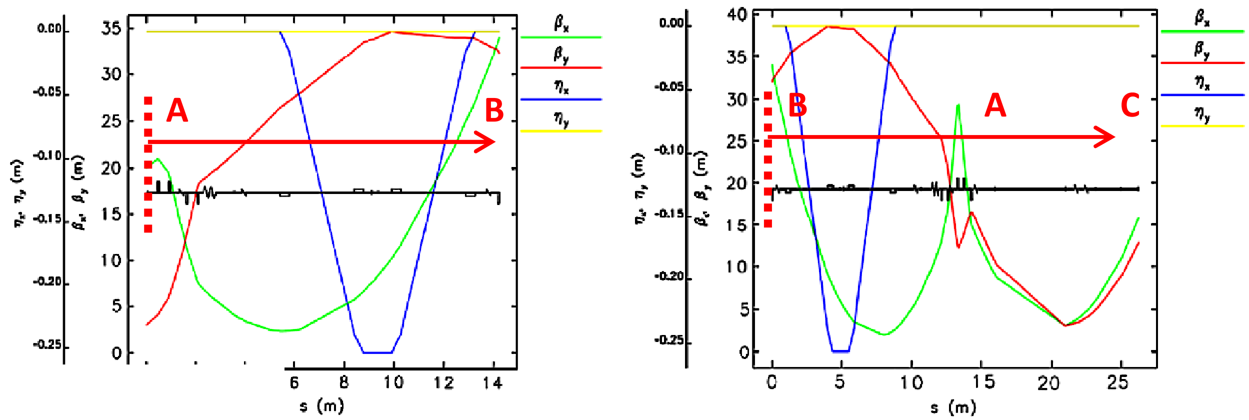
S. Di Mitri et al., PRST-AB 012802 (2012)

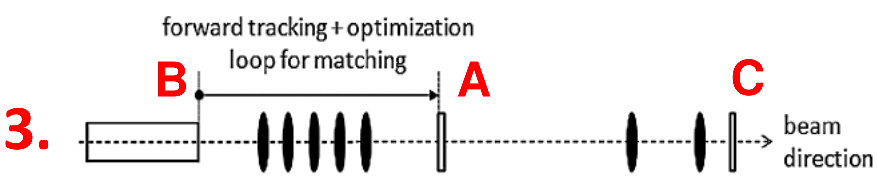


Quad-scan provides the beam Twiss parameters at the entrance of the last quadrupole magnet of the matching station.

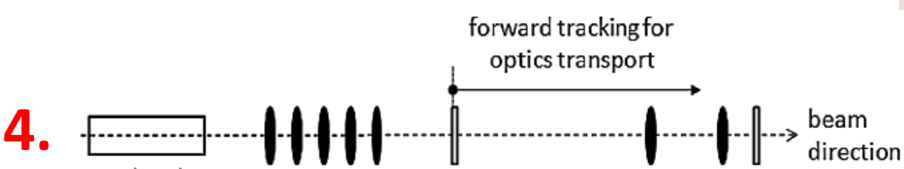


The machine is read by ELEGANT code (MATLAB + Tango-server interfaced) and the measured Twiss parameters are back-tracked to a point upstream of the matching station.





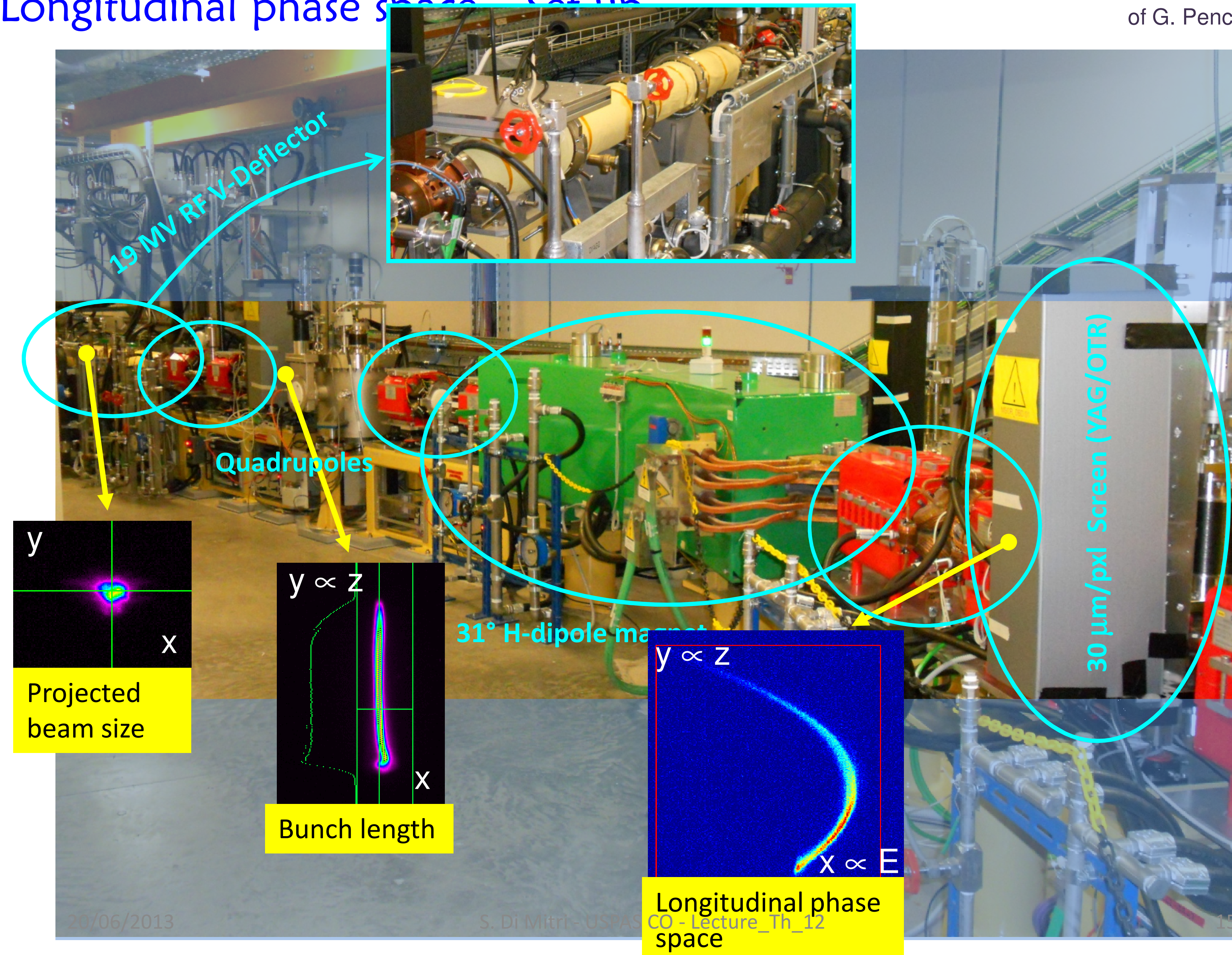
ELEGANT optimizes the quad strengths to match the beam Twiss parameters to the design values.



The solution is applied to the machine and the beam is transported downstream.

Longitudinal phase space – Set up

Pictures courtesy of G. Penco et al.



Longitudinal phase space – Post-processing

Pictures courtesy of G. Penco

This panel inputs the dispersion for energy spread measurement

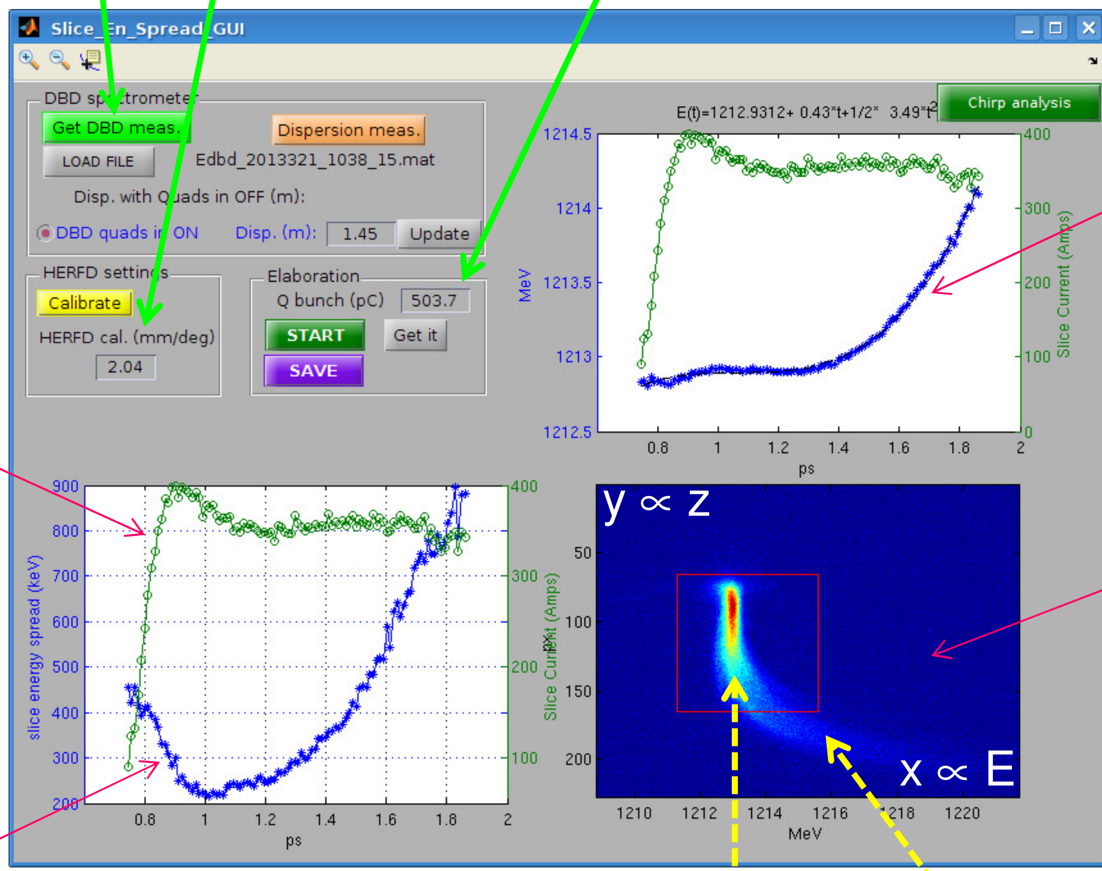
This panel inputs the RF deflector calibration for bunch length measurement.

This panel inputs the beam charge for peak current measurement.

X-position of each slice centroid provides the **slice mean energy**.

Charge density of each slice provides the **slice peak current**.

X-width of each slice provides the **slice energy spread**.



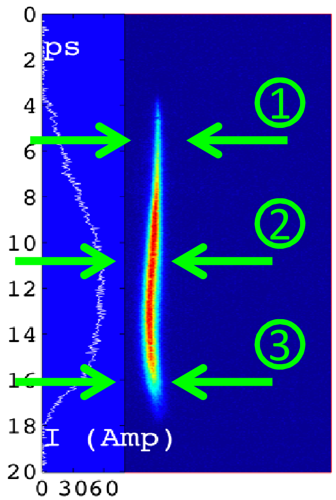
The **screen image** is post-processed for slice analysis. Optics was tuned for energy and bunch length max. resolution.

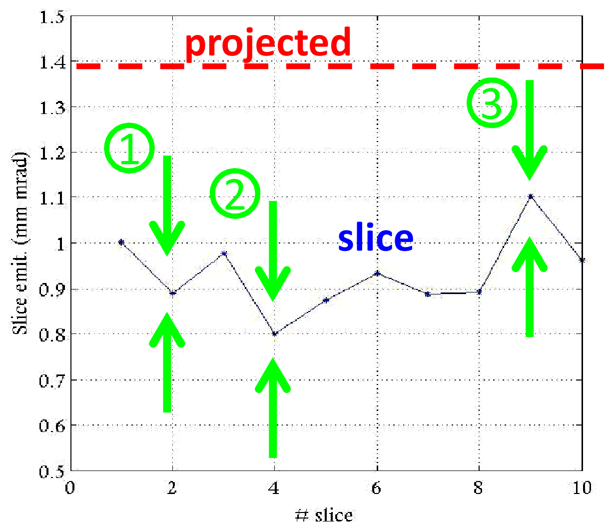
Notice: the *slice* energy spread is the sum of the *uncorrelated* and *correlated* energy spread.

Slice emittance and slice energy spread

Pictures courtesy
of G. Penco, S. Spampinati et al.

❖ RF **V-deflector +** beam goes **straight**:

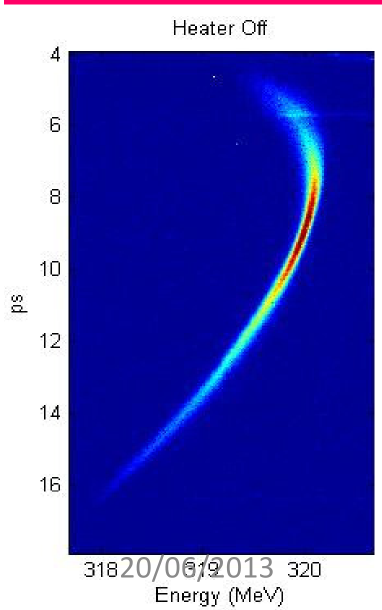


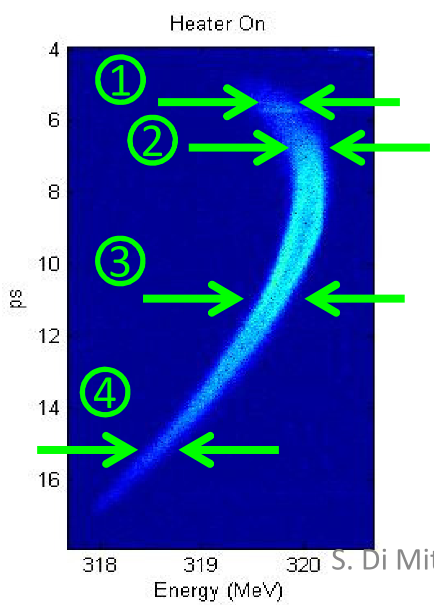


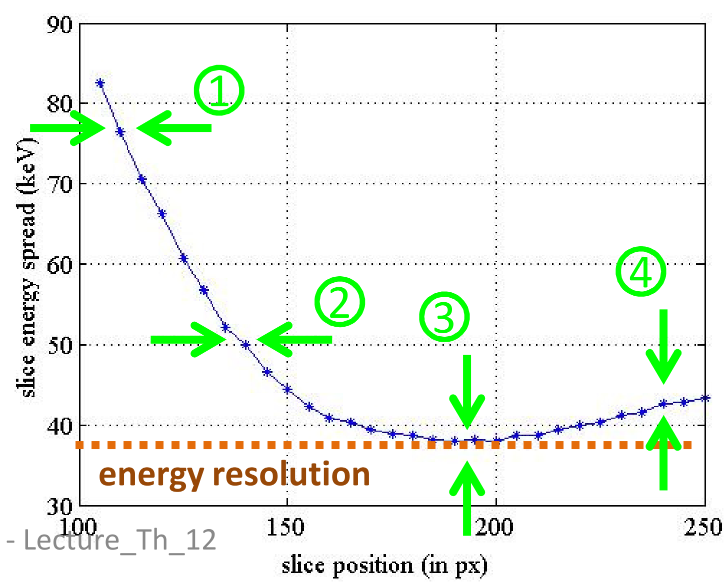
- **Quad-scan** is applied to the stretched beam. The image is then sampled and emittance is computed for each slice.

- The **slice energy spread** is dominated by the RF curvature (x-E correlation) at the bunch edges, and by the optics resolution in the bunch core.

❖ RF **V-deflector + H-bent** beam:



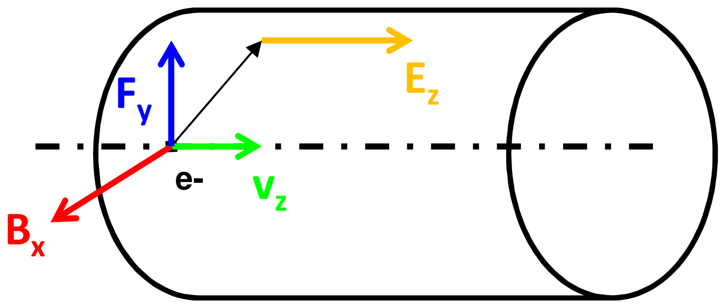




RF deflector-induced energy spread

- ❑ The existence of a **time-varying *transverse* magnetic field (B_x)** for deflection implies an ***off-axis longitudinal* electric field (E_z)**, linear with *r* (Panofsky-Wenzel theorem).

RF DEFLECTOR (MAGNETIC)



- ❑ Owing to the particles longitudinal motion, **E_z(r)** changes the longitudfinal momentum → **RF deflector induced energy spread, δ.**

$$\delta \cong \frac{1}{L}\int_0^L ds \frac{\Delta p_z(s)}{p_z} = \frac{1}{L}\int_0^L ds \int_0^{s'} \left(-\frac{e}{p_z c}\right) E_z(y,s')ds' = -\frac{e}{p_z c}\frac{1}{L}\int_0^L ds \int_0^{s'} E_{z,0} k_{RF}\, y(s) \cos\varphi_{RF}\, ds' = \ldots$$

- ❑ Why do particles sample E_z(y) *off-axis* ?
  - They travel off-axis due to the finite **beam size** (σ_β).
  - They are (vertically) displaced by ***deflection*** inside the cavity, for any finite cavity length.

Practical case

- For a <u>finite RMS beam size</u> $\sigma_y = \sqrt{\beta_{y,D}\varepsilon_y}$ we obtain from the previous equation:

$$\sigma_{\delta,\beta} \cong \ldots = -\frac{eE_{z,0}k_{RF}\cos\varphi_{RF}}{p_z c}\frac{1}{L}\int_0^L ds\int_0^{s'} ds'\sqrt{\beta_{y,D}\varepsilon_y} \cong -\frac{eE_{z,0}k_{RF}\cos\varphi_{RF}}{p_z c}\frac{1}{L}\cdot\frac{L^2}{2} \cong \left(\frac{eV_{RF}k_{RF}\cos\varphi_{RF}}{p_z c}\right)\frac{\sqrt{\beta_{y,D}\varepsilon_y}}{2}$$

- For a <u>finite cavity length</u> L we obtain from the previous equation:

$$\sigma_{\delta,L} \cong \ldots = -\frac{eE_{z,0}k_{RF}\cos\varphi_{RF}}{p_z c}\frac{1}{L}\int_0^L ds\int_0^{s'} ds'\int_0^{s''} y'(s)ds'' = -\frac{eE_{z,0}k_{RF}\cos\varphi_{RF}}{p_z c}\frac{1}{L}\cdot\frac{eV_{RF}k_{RF}\cos\varphi_{RF}}{p_z c}\frac{L^3}{6}z \cong$$

$$\cong \left(\frac{eV_{RF}k_{RF}\cos\varphi_{RF}}{p_z c}\right)^2\frac{L}{6}z$$

At zero-crossing

$$\boxed{\sigma_\delta \approx \frac{eV_{RF}k_{RF}}{2p_z c}\sqrt{\beta_{y,D}\varepsilon_y + \sigma_z^2\left(\frac{eV_{RF}k_{RF}}{p_z c}\right)^2\frac{L^2}{9}}}$$

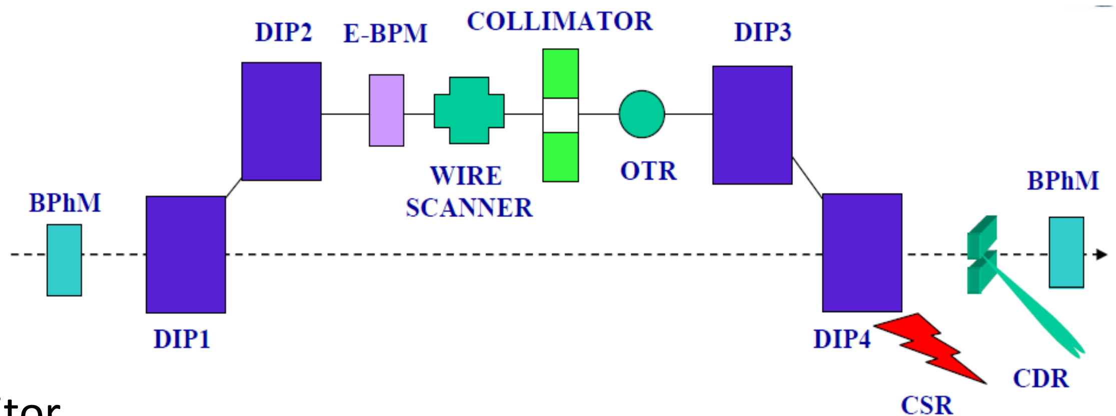
This is *uncorrelated* with z ("energy spread")

This is a *correlated* value ("energy chirp") averaged over *z*.

More and more diagnostics (mention)

Pictures courtesy of M. Veronese

Presence of a dispersive region (*e.g.*, bunch compressor) encourages physicists to ask more diagnostics for time, energy and transverse beam characterization, both on-line and destructive.



- ✓ **BPhM** = Bunch PHase (Arrival) Monitor,
  → e-beam arrival time respect to the machine clock. On-line → *<u>arrival time jitter feedback</u>*.
  → When used before and after a chicane, it gives the beam time-delay across it.

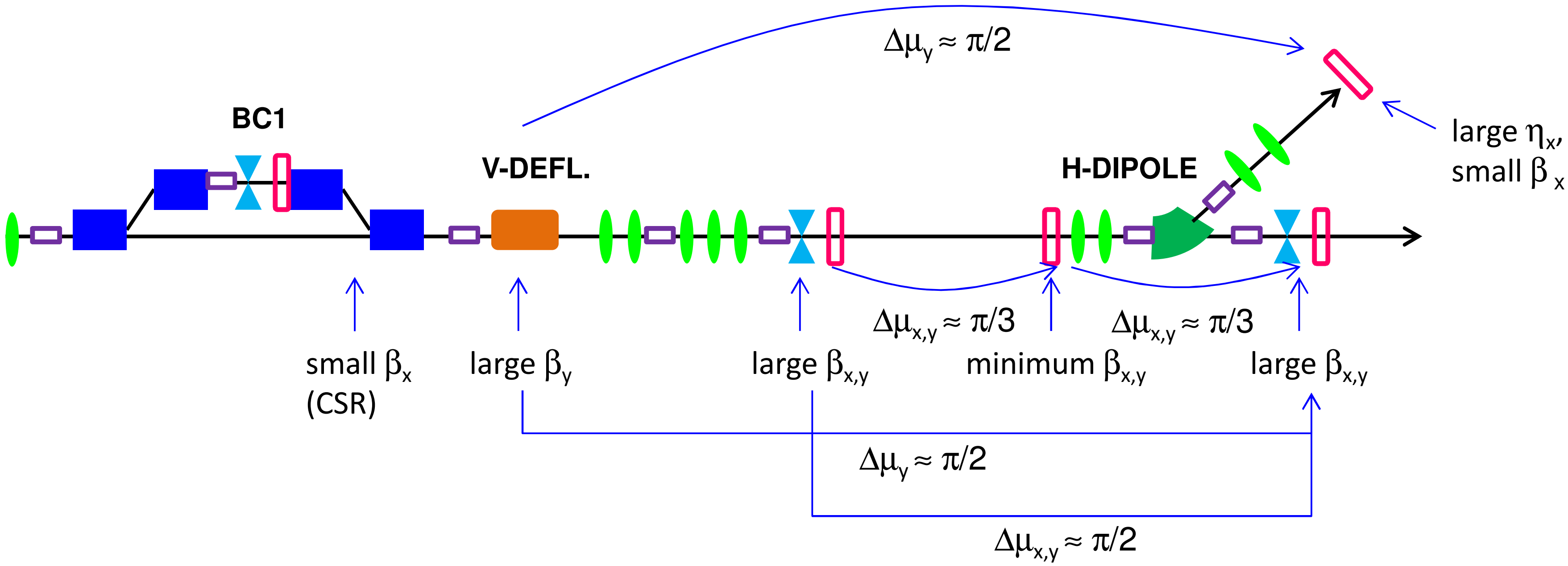
- ✓ **E-BPM** = Energy – Beam Position Monitor,
  → e-beam mean energy. On-line → *<u>energy-feedback</u>*.

- ✓ **WIRE SCANNER** (alternatively, YAG/OTR SCREEN),
  → beam energy spread. Destructing.

- ✓ **COLLIMATOR** (or SCRAPER),
  → in the presence of a linear t-E correlation (energy chirp), it selects *<u>longitudinal bunch slices</u>*, to be characterized downstream with *<u>no need of a deflector</u>*.

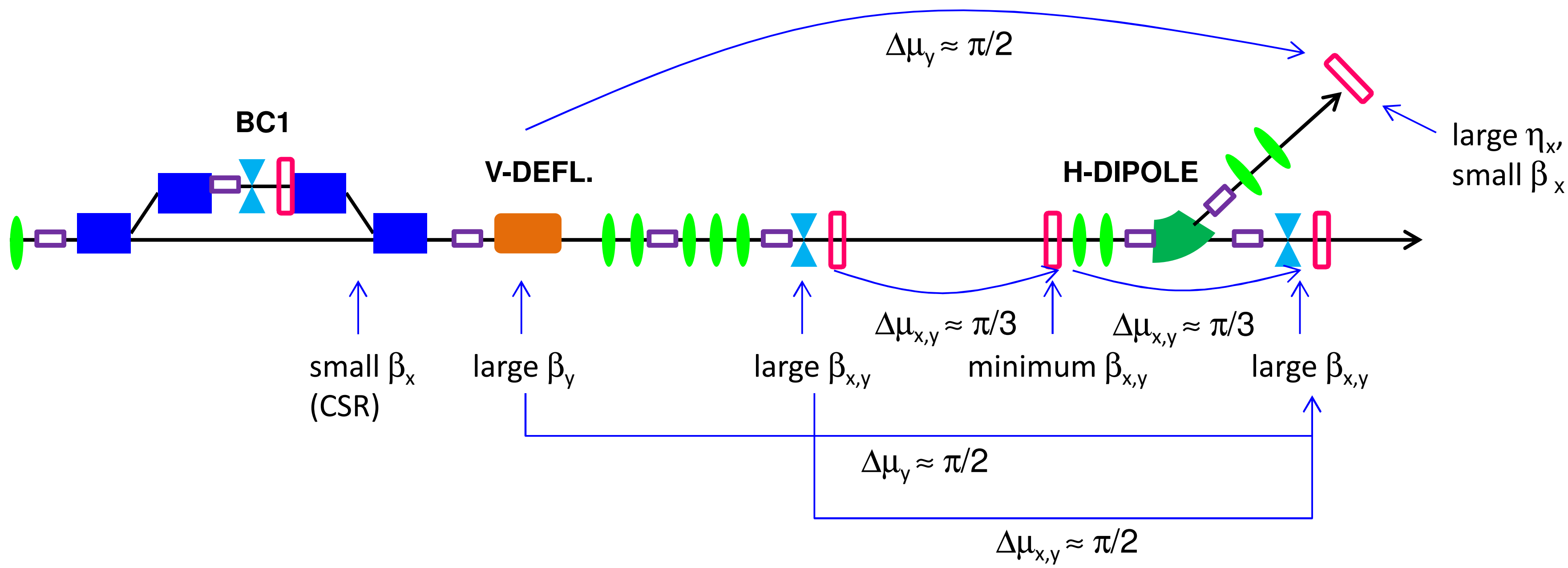
- ✓ **CSR + CDR** = Coherent Synchrotron Radiation + Coherent Diffraction Radiation Monitors,
  → bunch length variation. On-line → *<u>bunch length (peak current) feedback</u>*.

Summing up

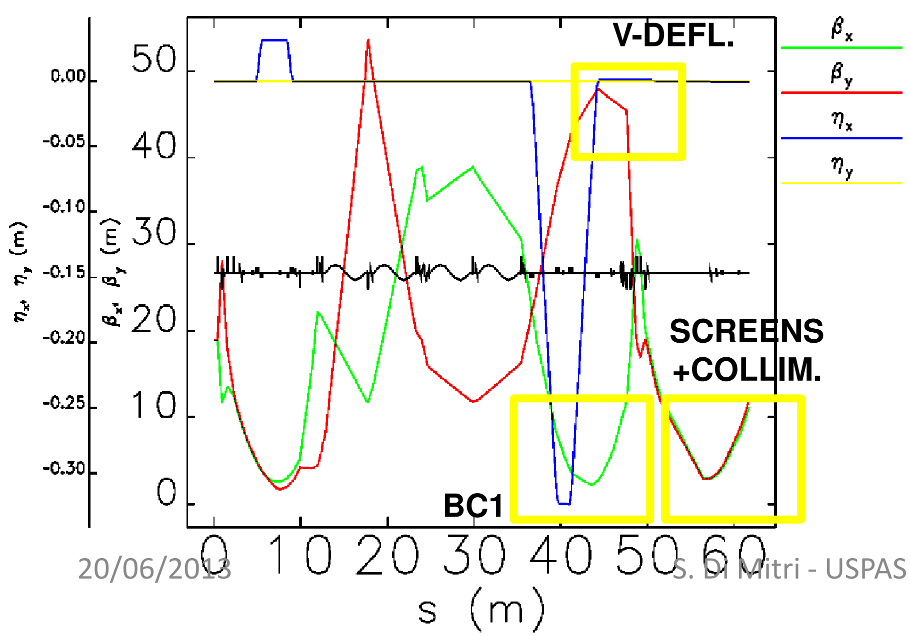


1. **Mean energy**
2. **Energy spread**
3. **Quadrupole scan**
4. **Transverse tomography**
5. **Bunch length**
6. **Slice transverse emittance**
7. **Slice energy spread**
8. **Geometric collimation**
9. **Slicing**

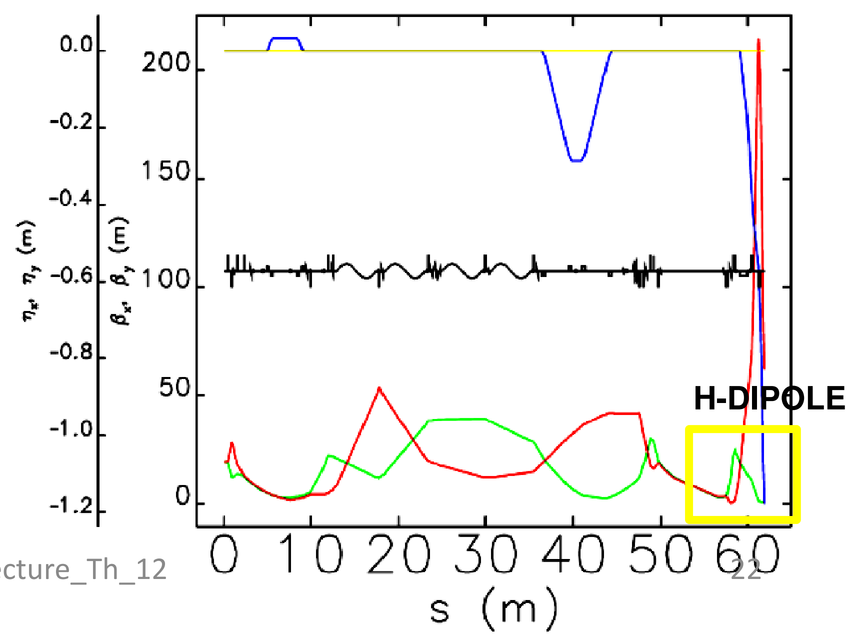
Summing up



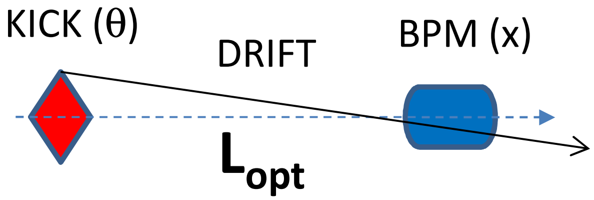
Straight line



Spectrometer line



Trajectory steering (1-to-1)

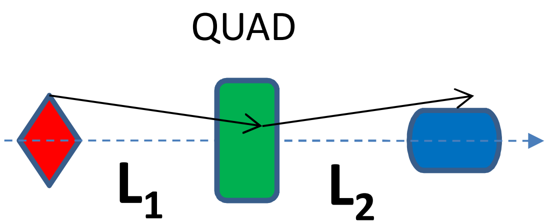


$$x \sim \vartheta\sqrt{\beta_k\beta_b}\,\sin(\Delta\mu_{kb})$$

- ❑ Look for an optimum distance $\mathbf{L_{opt}}$ i.e. phase advance $\mathbf{\Delta\mu_{k\beta}=\pi/2}$ that *minimizes the steerer's kick*, for any given displacement x at the end:

$$\Delta\mu_{kb} \approx \frac{s_k - s_b}{\bar{\beta}} \quad \Rightarrow \quad l_{opt} = \frac{\pi}{2}\bar{\beta}$$

- ❑ Now consider a quadrupole magnet in between:

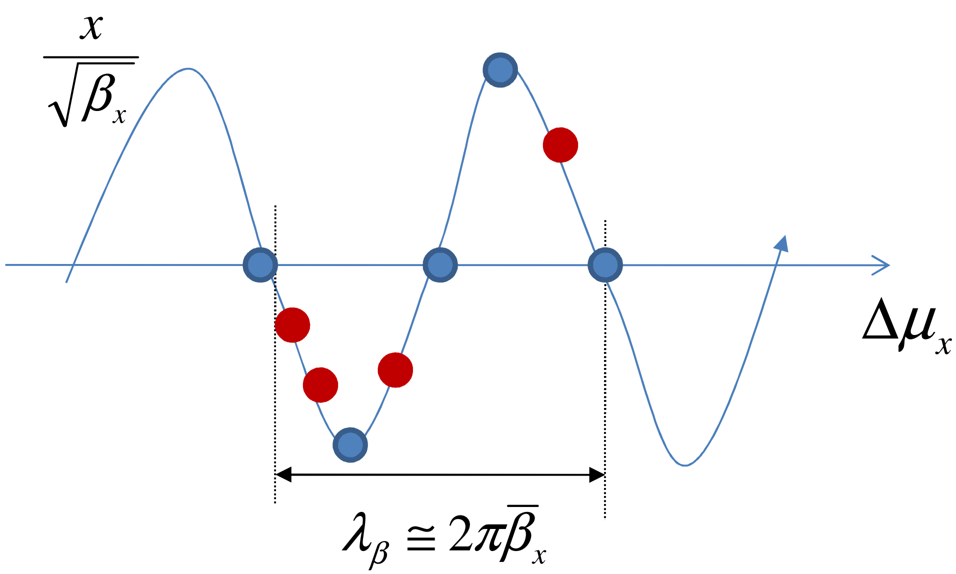


$$M_{kb} = M_{BPM}M_DM_QM_DM_{KICK} = \begin{pmatrix} 1-\frac{L_2}{f_q} & L_1+L_2-\frac{L_1L_2}{f_q} \\ -\frac{1}{f_q} & 1-\frac{L_1}{f_q} \end{pmatrix}, \qquad \frac{1}{f_q} = kl_q$$

$$\frac{1}{2}Tr(M_{kb}) \equiv \cos\Delta\mu_x = 1 - \frac{L_1+L_2}{2f_q} \equiv 0 \quad \Rightarrow \quad L_{opt} = L_1 + L_2 = 2f_q$$

- ❑ In real life spatial constraints force to compromises.
  - A BPM ***too close*** to the steerer makes the steering effect largely inefficient – poor sensitivity.
  - A BPM ***too far*** from the steerer, leads to poor reconstruction of the beam trajectory – poor correction.

Impact on the linac design



4 BPMs per β-period can sample and reconstruct the trajectory.

Over sampling (more BPMs and correctors) may lead to:

- improvement of the correction sensitivity,
  → weaker corrector strengths
- redundancy,
  → stronger corrector strengths

Optimize the correctors and BPMs layout with *codes*.