Fundamentals of Accelerators - 2012

Lecture - Day 8

William A. Barletta

Director, US Particle Accelerator School

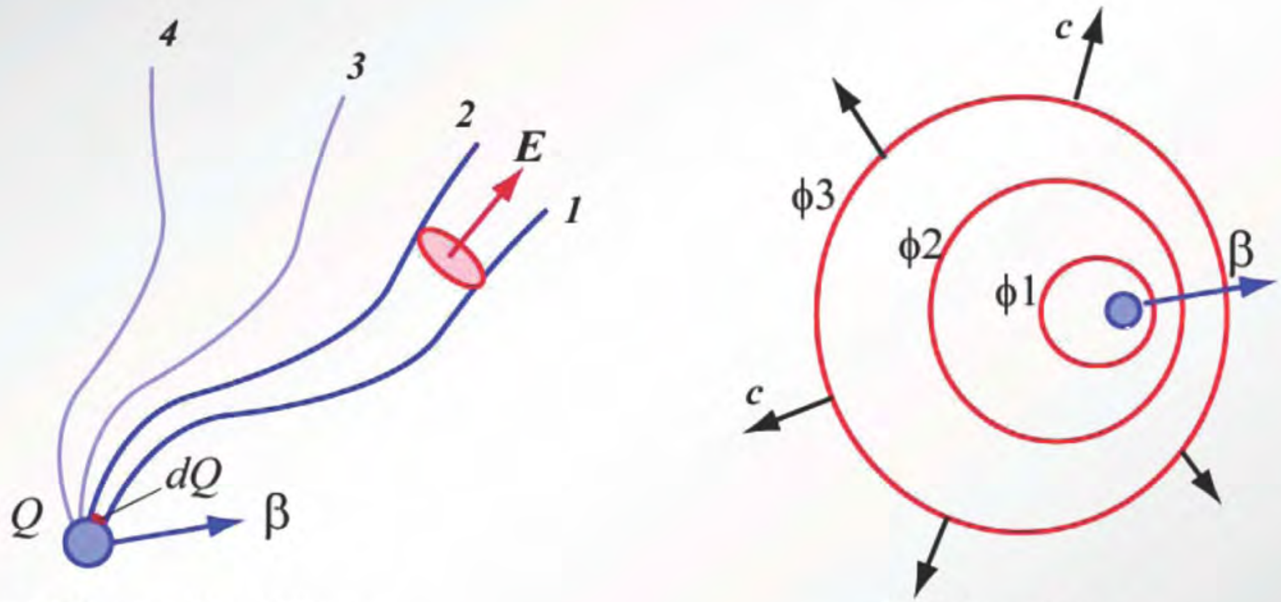
Dept. of Physics, MIT

Economics Faculty, University of Ljubljana

What do we mean by radiation?

- Energy is transmitted by the electromagnetic field to infinity
  - Applies in all inertial frames
  - Carried by an electromagnetic wave
- Source of the energy
  - Motion of charges

Schematic of electric field

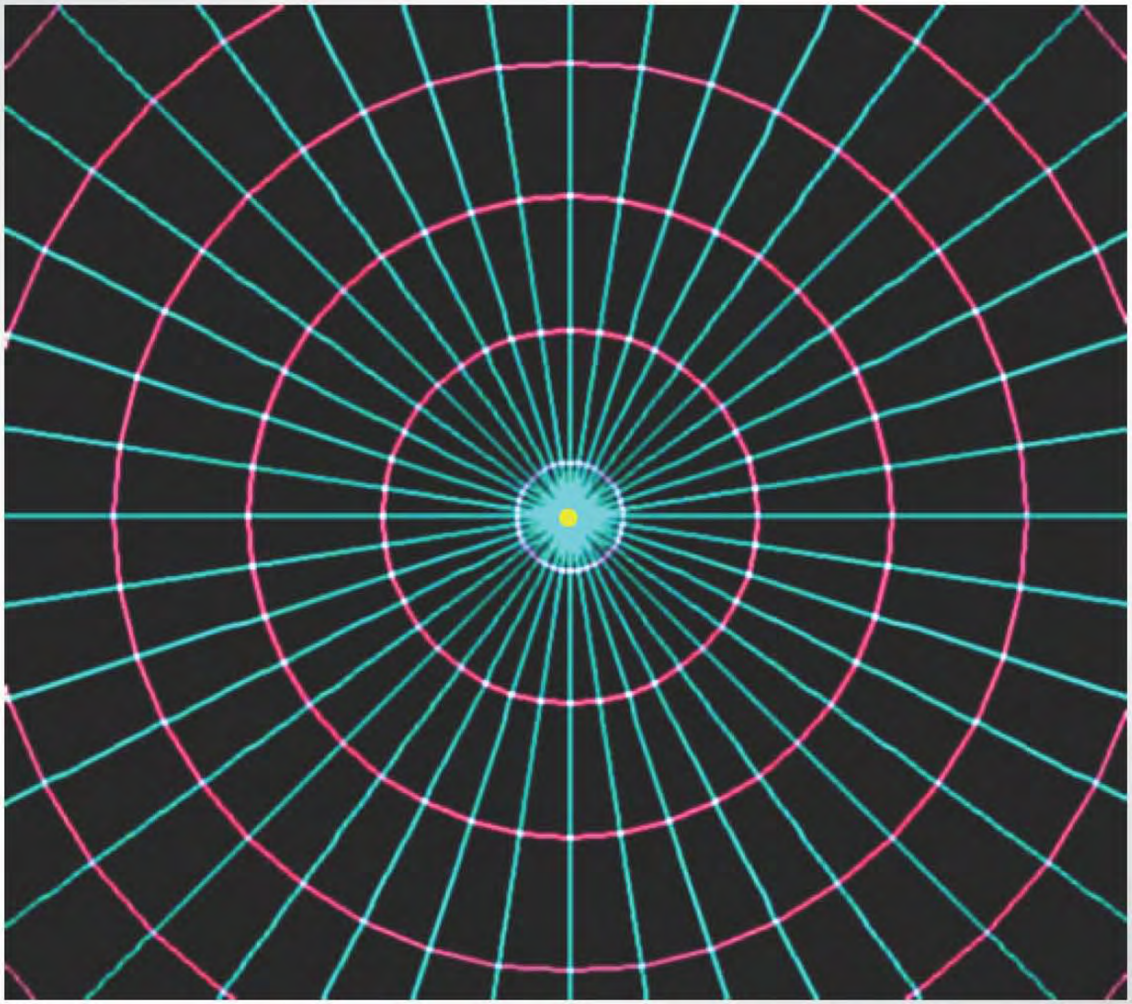


(a) Electric Field Lines

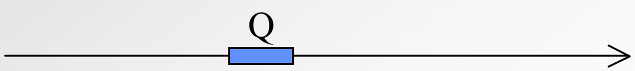
(b) Wavefronts

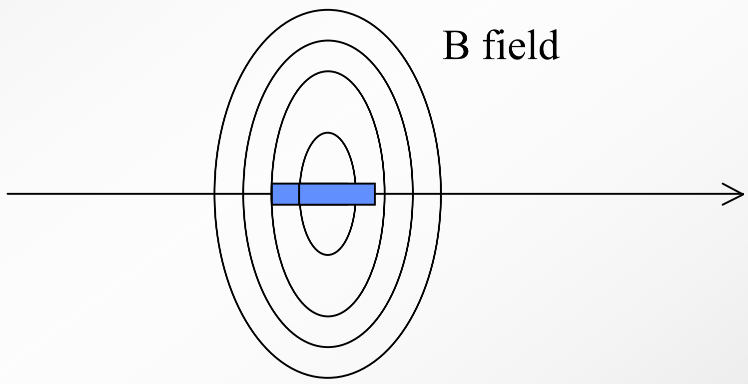
From: T. Shintake, New Real-time Simulation Technique for Synchrotron and Undulator Radiations, Proc. LINAC 2002, Gyeongju, Korea

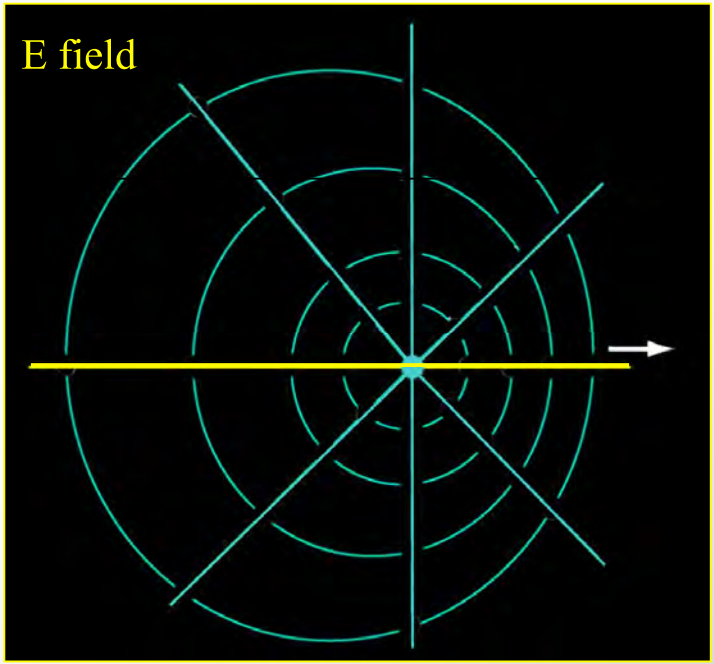
Static charge



Particle moving in a straight line with constant velocity

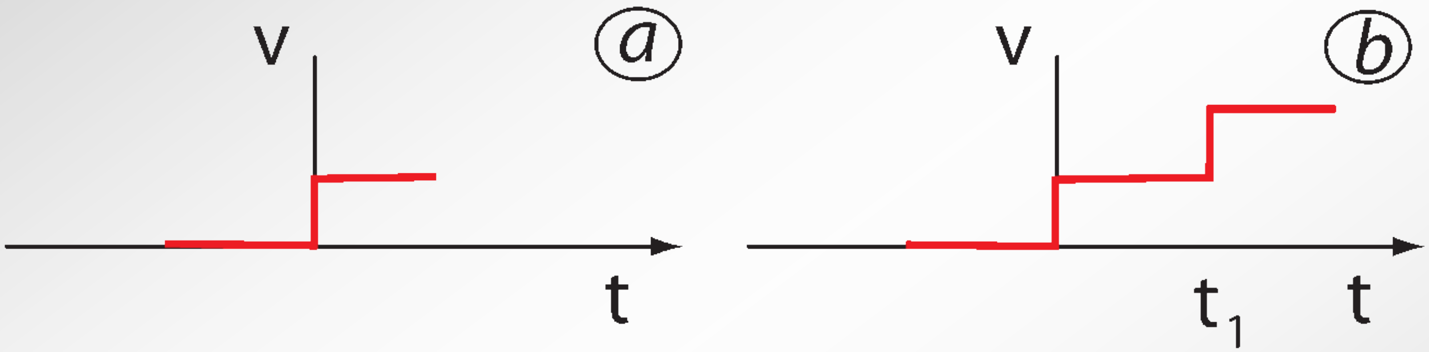


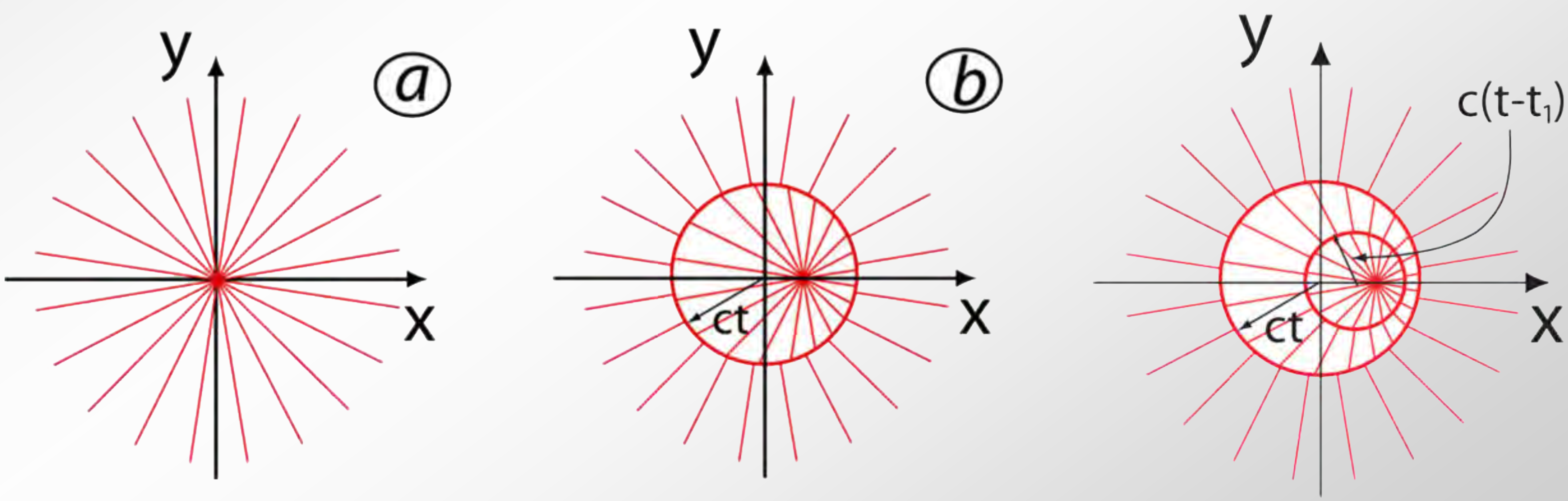




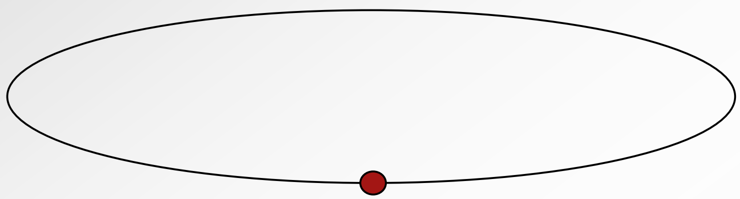
Consider the fields from an electron
with abrupt accelerations

- At r = ct, ∃ a transition region from one field to the other. At large r, the field in this layer becomes the radiation field.

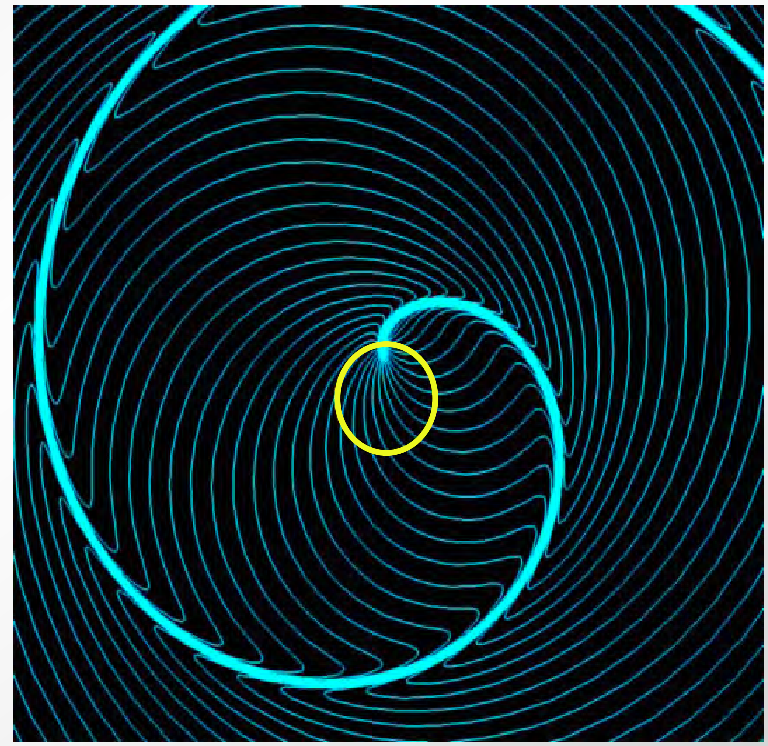




Particle moving in a circle at constant speed

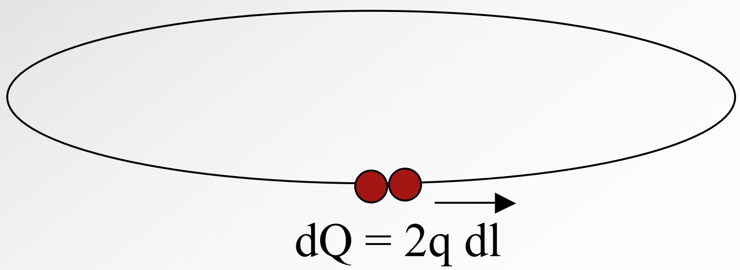


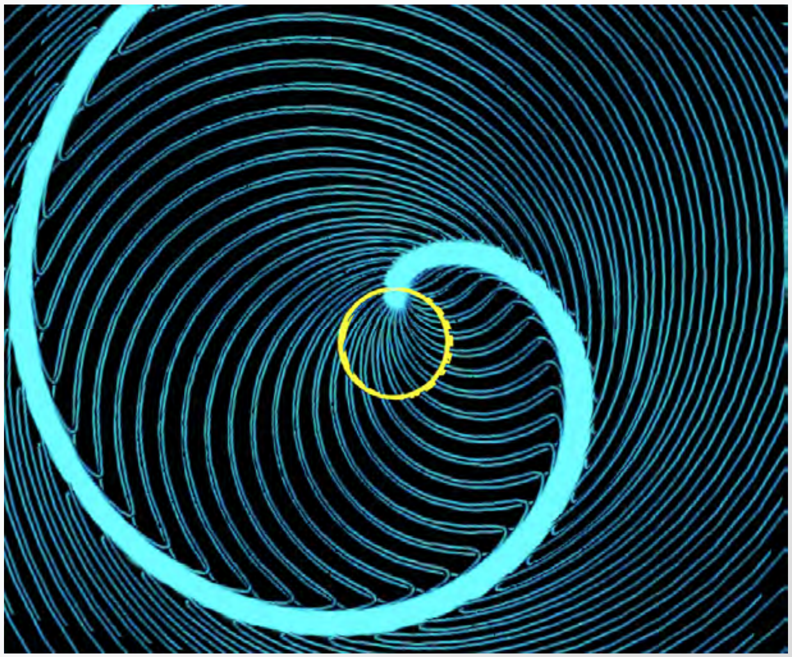
dQ = q dl



Field energy flows to infinity

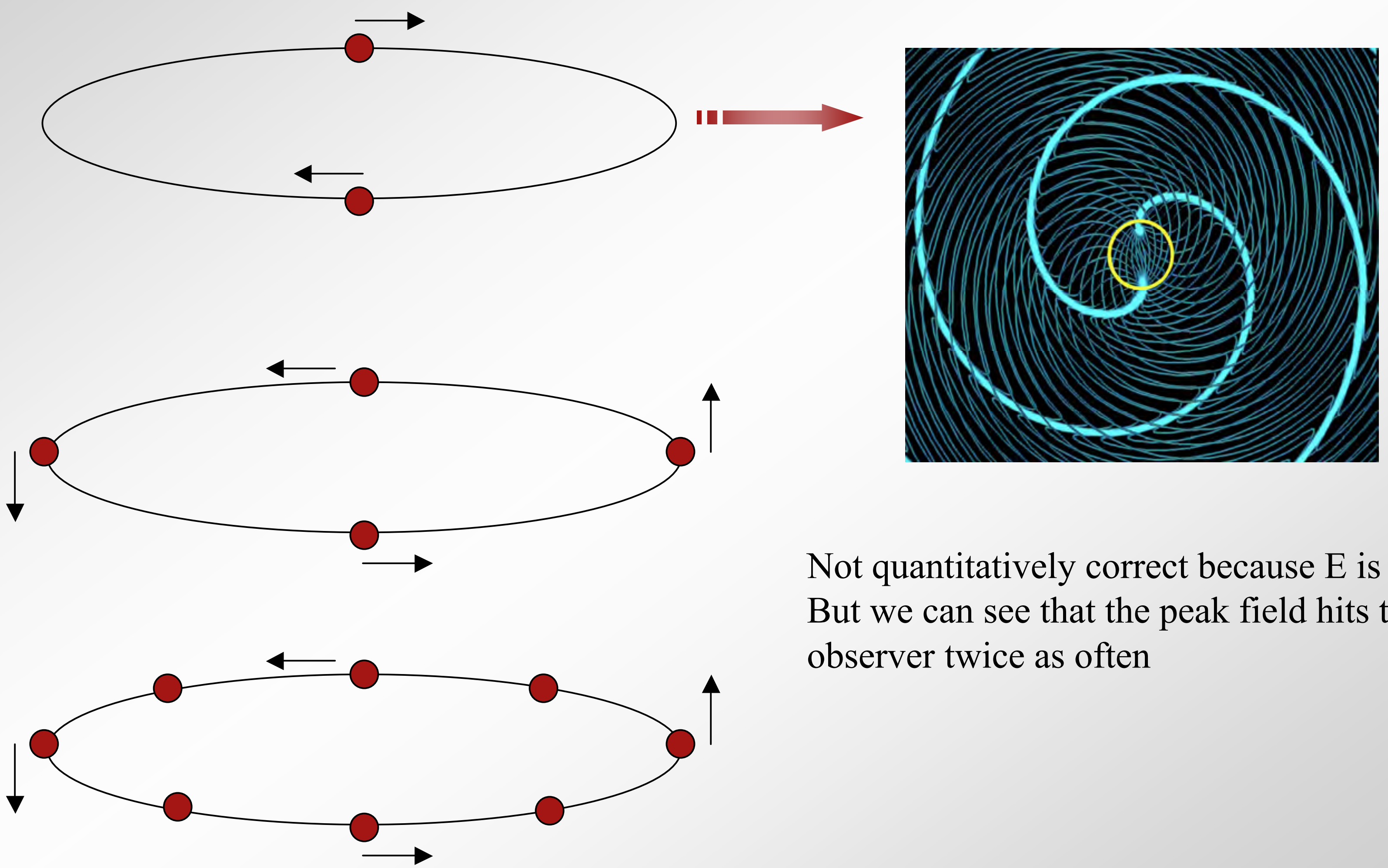
Remember that fields add, we can compute radiation from a charge twice as long





The wavelength of the radiation doubles

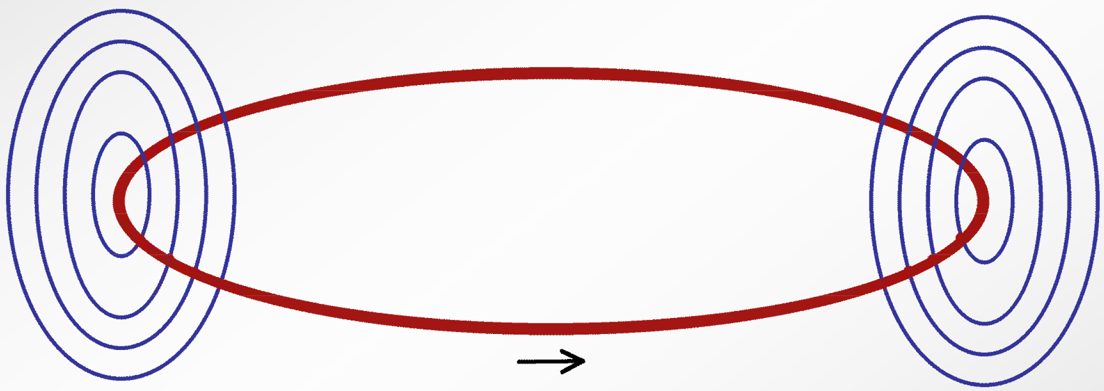
All these radiate



Not quantitatively correct because E is a vector;
But we can see that the peak field hits the
observer twice as often

Current loop: No radiation

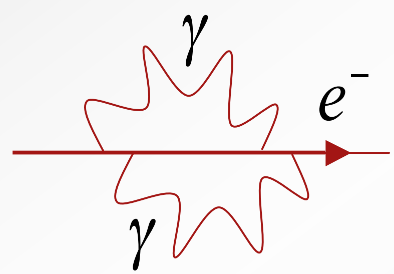
Field is static



B field

QED approach: Why do particles radiate when accelerated?

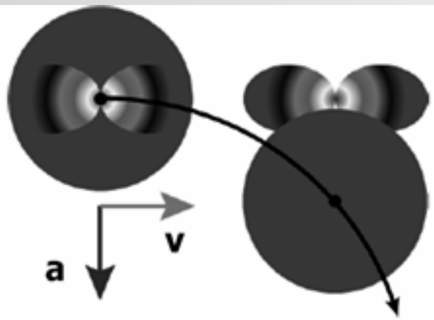
- Charged particles in free space are "surrounded" by *virtual photons*
  - Appear & disappear & travel with the particles.



- Acceleration separates the charge from the photons & "kicks" photons onto the "mass shell"
- Lighter particles have less inertia & radiate photons more efficiently
- In the field of the dipoles in a synchrotron, charged particles move on a curved trajectory.
  - Transverse acceleration generates the *synchrotron radiation*

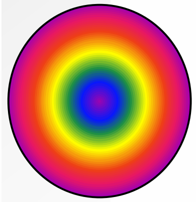
Electrons radiate ~αγ photons per radian of turning

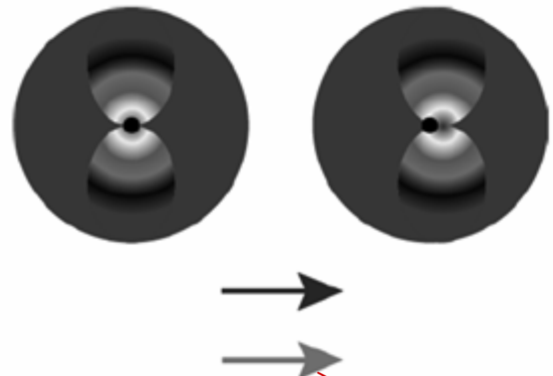
Longitudinal vs. transverse acceleration



Radiation field quickly separates itself from the Coulomb field

$$P_{\perp} = \frac{q^2}{6\pi\varepsilon_0 m_0^2 c^3}\gamma^2\left(\frac{d\mathbf{p}_{\perp}}{dt}\right)^2$$





Radiation field cannot separate itself from the Coulomb field

$$\cancel{P_{\parallel} = \frac{q^2}{6\pi\varepsilon_0 m_0^2 c^3}\left(\frac{d\mathbf{p}_{\parallel}}{dt}\right)^2}$$

negligible!

$$\boxed{P_{\perp} = \frac{c}{6\pi\varepsilon_0}q^2\frac{(\beta\gamma)^4}{\rho^2} \qquad \rho = curvature\ \ radius}$$

Radiated power for transverse acceleration increases dramatically with energy

Limits the maximum energy obtainable with a storage ring

Energy lost per turn by electrons

$$\frac{dU}{dt} = -P_{SR} = -\frac{2c\,r_e}{3\left(m_0c^2\right)^3}\frac{E^4}{\rho^2} \quad \Rightarrow \quad U_0 = \int_{finite\ \rho} P_{SR}\,dt \quad energy\ lost\ per\ turn$$

For relativistic electrons:

$$s = \beta ct \cong ct \Rightarrow dt = \frac{ds}{c} \qquad U_0 = \frac{1}{c}\int_{finite\ \rho} P_{SR}\,ds = \frac{2r_eE_0^4}{3\left(m_0c^2\right)^3}\int_{finite\ \rho}\frac{ds}{\rho^2}$$

For dipole magnets with constant radius *r* (*iso-magnetic* case):

$$\boxed{U_0 = \frac{4\pi\,r_e}{3\left(m_0c^2\right)^3}\frac{E_0^4}{\rho} = \frac{e^2}{3\varepsilon_o}\frac{\gamma^4}{\rho}}$$

The average radiated power is given by:

$$\boxed{\left\langle P_{SR}\right\rangle = \frac{U_0}{T_0} = \frac{4\pi\,c\,r_e}{3\left(m_0c^2\right)^3}\frac{E_0^4}{\rho L} \qquad where\ L \equiv\ ring\ circumference}$$

Energy loss to synchrotron radiation (practical units)

Energy Loss per turn (per particle)

$$\boxed{U_{o,electron}(keV) = \frac{e^2\gamma^4}{3\varepsilon_0\rho} = 88.46\frac{E(GeV)^4}{\rho(m)}}$$

$$\boxed{U_{o,proton}(keV) = \frac{e^2\gamma^4}{3\varepsilon_0\rho} = 6.03\frac{E(TeV)^4}{\rho(m)}}$$

Power radiated by a beam of average current I_b: to be restored by RF system

$$N_{tot} = \frac{I_b \cdot T_{rev}}{e}$$

$$\boxed{P_{electron}(kW) = \frac{e\gamma^4}{3\varepsilon_0\rho} I_b = 88.46\frac{E(GeV)^4 I(A)}{\rho(m)}}$$

$$\boxed{P_{proton}(kW) = \frac{e\gamma^4}{3\varepsilon_0\rho} I_b = 6.03\frac{E(TeV)^4 I(A)}{\rho(m)}}$$

Power radiated by a beam of average current I_b in a dipole of length L (energy loss per second)

$$P_e(kW) = \frac{e\gamma^4}{6\pi\varepsilon_0\rho^2} L I_b = 14.08\frac{L(m)I(A)E(GeV)^4}{\rho(m)^2}$$

Frequency spectrum

- Radiation is emitted in a cone of angle $1/\gamma$
- Therefore the radiation that sweeps the observer is emitted by the particle during the retarded time period

$$\Delta t_{ret} \approx \rho / \gamma c$$

- Assume that γ and ρ do not change appreciably during Δt.

- At the observer

$$\Delta t_{obs} = \Delta t_{ret} \frac{dt_{obs}}{dt_{ret}} = \frac{1}{\gamma^2} \Delta t_{ret}$$

- Therefore the observer sees $\Delta\omega \sim 1/\ \Delta t_{obs}$

$$\Delta\omega \sim \frac{c}{\rho} \gamma^3$$

Critical frequency and critical angle

$$\frac{d^3 I}{d\Omega\, d\omega} = \frac{e^2}{16\pi^3 \varepsilon_0 c}\left(\frac{2\omega\rho}{3c\gamma^2}\right)^2 \left(1+\gamma^2\theta^2\right)^2 \left[K_{2/3}^2(\xi) + \frac{\gamma^2\theta^2}{1+\gamma^2\theta^2} K_{1/3}^2(\xi)\right]$$

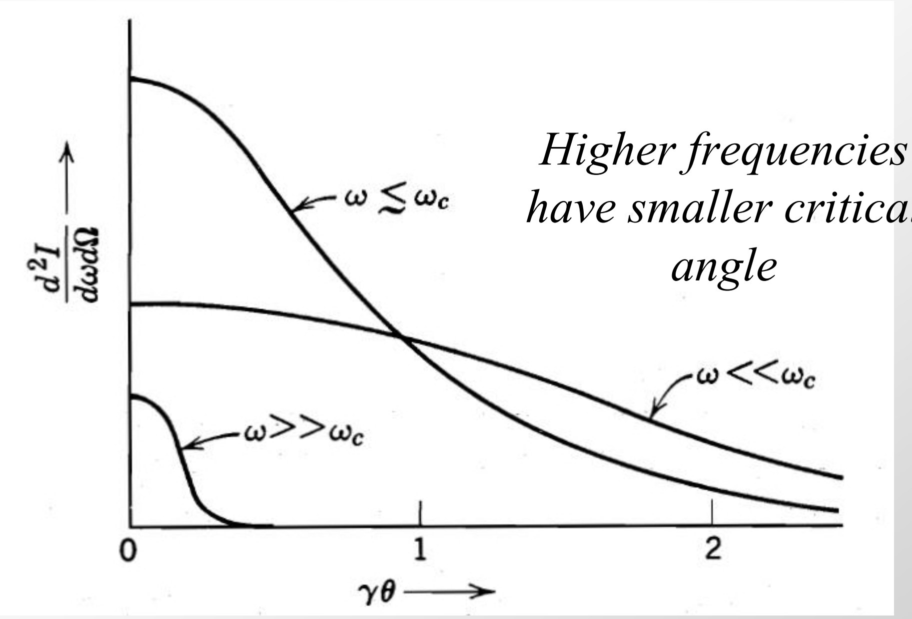
Properties of the modified Bessel function ==> radiation intensity is negligible for x >> 1

$$\xi = \frac{\omega\rho}{3c\gamma^3}\left(1+\gamma^2\theta^2\right)^{3/2} >> 1$$

Critical frequency $\omega_c = \frac{3}{2}\frac{c}{\rho}\gamma^3$

$$\approx \omega_{rev}\gamma^3$$

Critical angle $\theta_c = \frac{1}{\gamma}\left(\frac{\omega_c}{\omega}\right)^{1/3}$



Higher frequencies have smaller critical angle

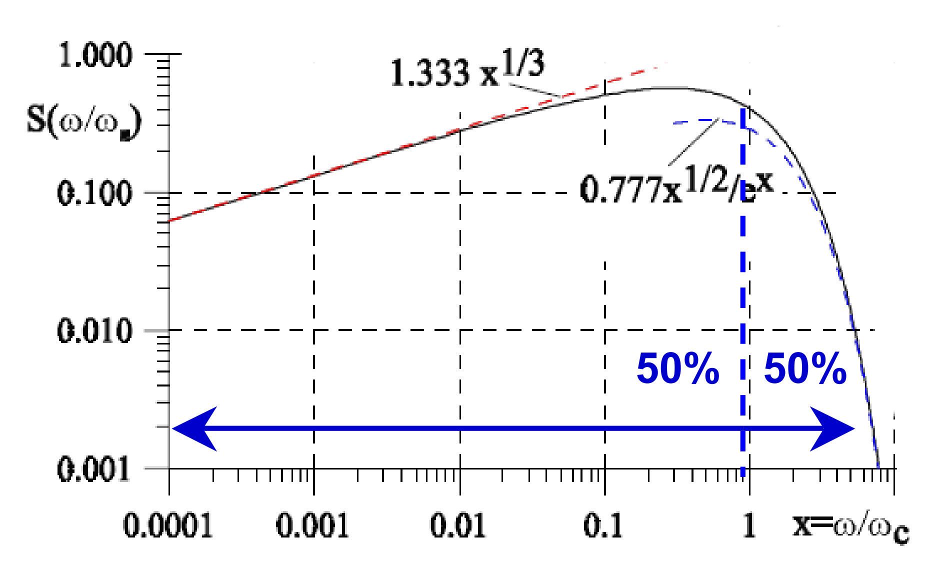
For frequencies much larger than the critical frequency and angles much larger than the critical angle the synchrotron radiation emission is negligible

Integrate over all angles ==>
Frequency distribution of radiation

The integrated spectral density up to the critical frequency contains half of the total energy radiated, the peak occurs approximately at $0.3\omega_c$

where the critical photon energy is

$$\varepsilon_c = \hbar\omega_c = \frac{3}{2}\frac{\hbar c}{\rho}\gamma^3$$



For *electrons*, the critical energy in practical units is

$$\varepsilon_c[keV] = 2.218\frac{E[GeV]^3}{\rho[m]} = 0.665 \cdot E[GeV]^2 \cdot B[T]$$

Number of photons emitted

- Since the energy lost per turn is

$$U_0 \sim \frac{e^2\gamma^4}{\rho}$$

- And average energy per photon is the

$$\langle \varepsilon_\gamma \rangle \approx \frac{1}{3}\varepsilon_c = \frac{\hbar\omega_c}{3} = \frac{1}{2}\frac{\hbar c}{\rho}\gamma^3$$

- The average number of photons emitted per revolution is

$$\langle n_\gamma \rangle \approx 2\pi\alpha_{fine}\gamma$$

Comparison of S.R. Characteristics

		LEP200	LHC	SSC	HERA	VLHC
Beam particle		e+ e-	p	p	p	p
Circumference	km	26.7	26.7	82.9	6.45	95
Beam energy	TeV	0.1	7	20	0.82	50
Beam current	A	0.006	0.54	0.072	0.05	0.125
Critical energy of SR	eV	$7\ 10^{5}$	44	284	0.34	3000
SR power (total)	kW	$1.7\ 10^{4}$	7.5	8.8	$3\ 10^{-4}$	800
Linear power density	W/m	882	0.22	0.14	$8\ 10^{-5}$	4
Desorbing photons	$s^{-1}\ m^{-1}$	$2.4\ 10^{16}$	$1\ 10^{17}$	$6.6\ 10^{15}$	none	$3\ 10^{16}$

From: O. Grobner CERN-LHC/VAC VLHC Workshop Sept. 2008

Synchrotron radiation plays a major role in electron storage ring dynamics

- Charged particles radiate when accelerated
- Transverse acceleration induces significant radiation (synchrotron radiation) while longitudinal acceleration generates negligible radiation ($1/\gamma^2$).

$$\frac{dU}{dt} = -P_{SR} = -\frac{2 c\, r_e}{3\left(m_0 c^2\right)^3} \frac{E^4}{\rho^2}$$

$r_e \equiv$ *classical electron radius*

$\rho \equiv$ *trajectory curvature*

$$U_0 = \int_{finite\ \rho} P_{SR}\, dt \quad \textit{energy lost per turn}$$

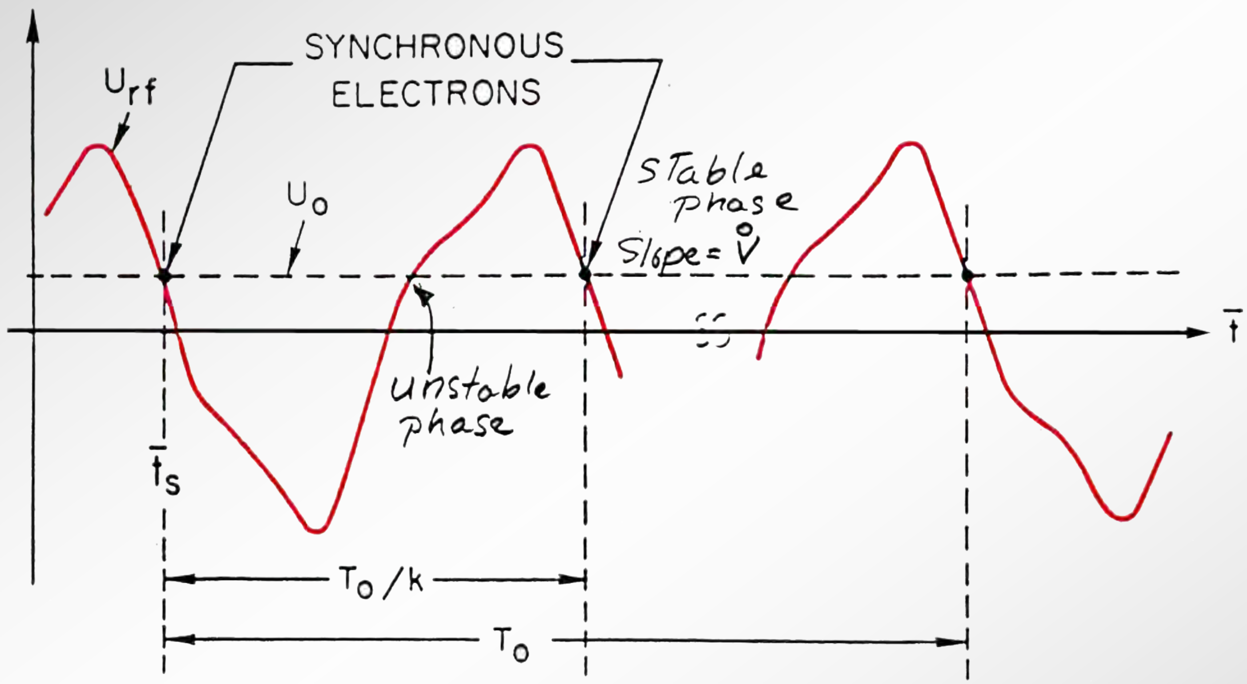
$$\alpha_D = -\frac{1}{2T_0} \frac{dU}{dE}\bigg|_{E_0} = \frac{1}{2T_0} \frac{d}{dE}\left[\oint P_{SR}(E_0)\, dt\right]$$

α_{DX}, α_{DY} *damping in all planes*

$\dfrac{\sigma_p}{p_0}$ *equilibrium momentum spread and emittances*

$\varepsilon_X, \varepsilon_Y$

RF system restores energy loss



Particles change energy according to the phase of the field in the RF cavity

$$\Delta E = eV(t) = eV_o \sin(\omega_{RF} t)$$

For the synchronous particle

$$\boxed{\Delta E = U_0 = eV_0 \sin(\varphi_s)}$$

Energy loss + dispersion ==>
Longitudinal (synchrotron) oscillations

Longitudinal dynamics are described by

1) **ε**, energy deviation, w.r.t the synchronous particle

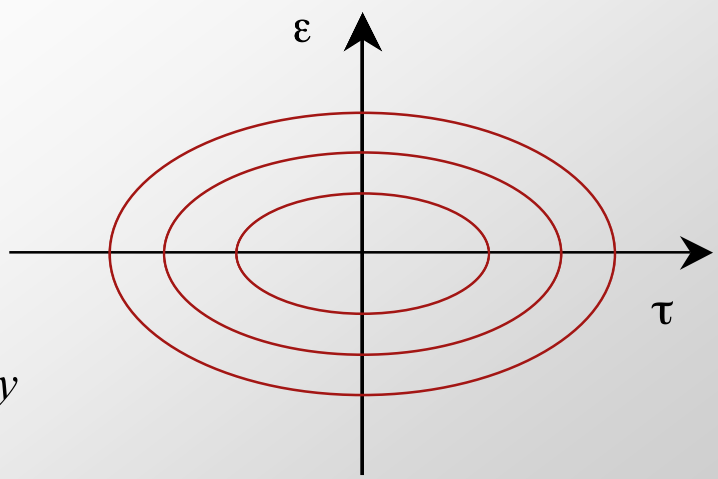
2) **τ**, time delay w.r.t. the synchronous particle

$$\varepsilon' = \frac{qV_0}{L}\left[\sin(\phi_s + \omega\tau) - \sin\phi_s\right] \quad \text{and} \quad \tau' = -\frac{\alpha_c}{E_s}\varepsilon$$

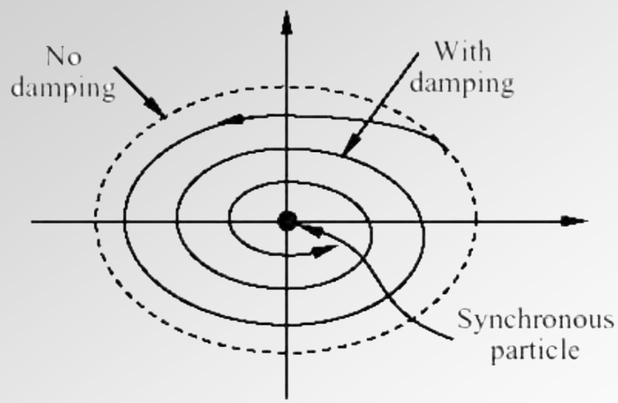
Linearized equations describe elliptical phase space trajectories

$$\varepsilon' = \frac{e}{T_0}\frac{dV}{dt}\tau \qquad \tau' = -\frac{\alpha_c}{E_s}\varepsilon$$

$$\boxed{\omega_s^2 = \frac{\alpha_c e\dot{V}}{T_0 E_0}}$$ *angular synchrotron frequency*



Radiation damping of energy fluctuations



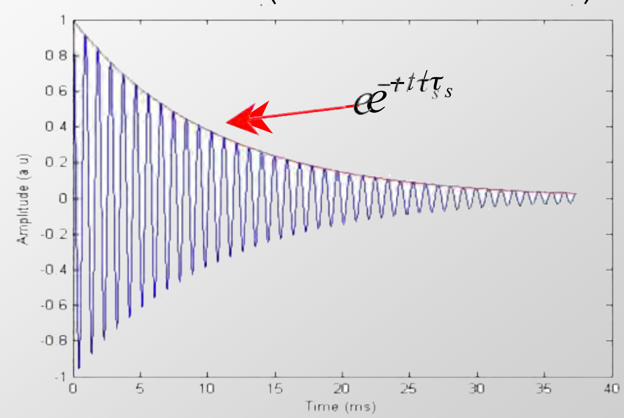
The derivative $\dfrac{dU_0}{dE}$ (> 0)
is responsible for the damping of the longitudinal oscillations

Combine the two equations for (ε, τ) in a single 2nd order differential equation

$$\frac{d^2\varepsilon}{dt^2} + \frac{2}{\tau_s}\frac{d\varepsilon}{dt} + \omega_s^2\varepsilon = 0 \longrightarrow \varepsilon = Ae^{-t/\tau_s}\sin\left(\sqrt{\omega_s^2 - \frac{4}{\tau_s^2}}\,t + \varphi\right)$$

$$\omega_s^2 = \frac{\alpha e\dot{V}}{T_0 E_0}$$ **angular synchrotron frequency**

$$\frac{1}{\tau_s} = \frac{1}{2T_0}\frac{dU_0}{dE}$$ **longitudinal damping time**



Damping times

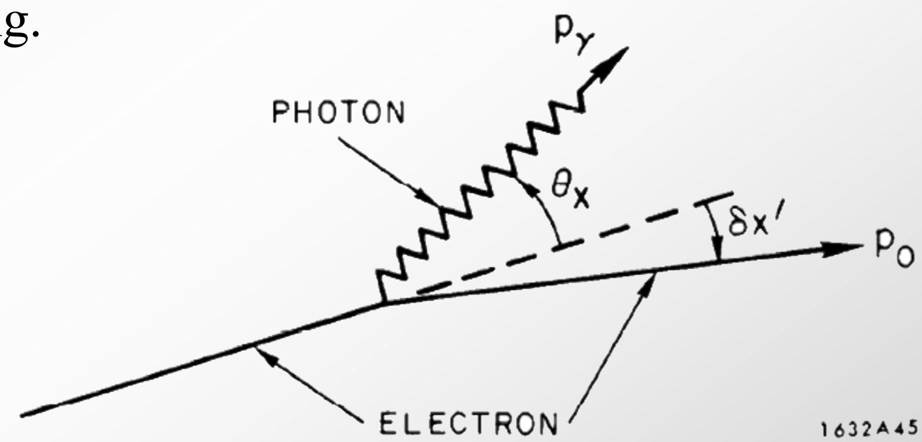
- The energy damping time ~ the time for beam to radiate its original energy
- Typically

$$T_i = \frac{4\pi}{C_\gamma} \frac{R\rho}{J_i E_o^3}$$

- Where $J_e \approx 2$, $J_x \approx 1$, $J_y \approx 1$ and $C_\gamma = 8.9 \times 10^{-5}\, meter - GeV^{-3}$
- Note $\Sigma J_i = 4$ (partition theorem)

Quantum Nature of Synchrotron Radiation

- Synchrotron radiation induces damping in all planes.
  - Collapse of beam to a single point is prevented by the *quantum nature of synchrotron radiation*
- Photons are randomly emitted in quanta of discrete energy
  - Every time a photon is emitted the parent electron “jumps” in energy and angle
- Radiation perturbs excites oscillations in all the planes.
  - Oscillations grow until reaching *equilibrium* balanced by radiation damping.



Energy fluctuations

- Expected $\Delta E_{quantum}$ comes from the deviation of $<\mathcal{N}_\gamma>$ emitted in one damping time, τ_E

- $<\mathcal{N}_\gamma> = n_\gamma \, \tau_E$

$$\Longrightarrow \Delta<\mathcal{N}_\gamma> = (n_\gamma \, \tau_E)^{1/2}$$

- The mean energy of each quantum $\sim \varepsilon_{crit}$

- $\Longrightarrow \sigma_\varepsilon = \varepsilon_{crit}(n_\gamma \, \tau_E)^{1/2}$

- Note that $n_\gamma = P_\gamma / \varepsilon_{crit}$ and $\tau_E = E_o / P_\gamma$

Therefore, ...

- The quantum nature of synchrotron radiation emission generates energy fluctuations

$$\frac{\Delta E}{E} \approx \frac{\langle E_{crit} E_o \rangle^{1/2}}{E_o} \approx \frac{C_q \gamma_o^2}{J_\varepsilon \rho_{curv} E_o} \sim \frac{\gamma}{\rho}$$

where C_q is the Compton wavelength of the electron

$$C_q = 3.8 \text{ x } 10^{-13} \text{ m}$$

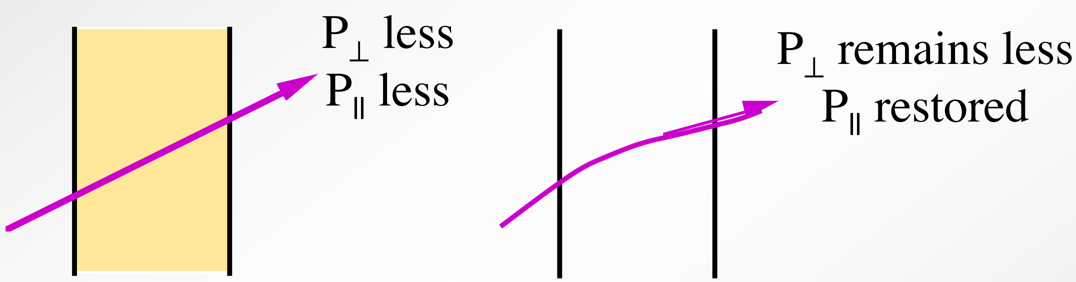
- Bunch length is set by the momentum compaction & V_{rf}

$$\sigma_z^2 = 2\pi \left(\frac{\Delta E}{E} \right) \frac{\alpha_c R E_o}{e \dot{V}}$$

- Using a harmonic rf-cavity can produce shorter bunches

Schematic of radiation cooling

Transverse cooling:



Passage through dipoles

Acceleration in RF cavity

Limited by quantum excitation

Emittance & momentum spread
are set by beam energy & lattice functions

- At equilibrium the momentum spread is given by:

$$\left(\frac{\sigma_p}{p_0}\right)^2 = \frac{C_q \gamma_0^2}{J_S} \frac{\oint 1/\rho^3\, ds}{\oint 1/\rho^2\, ds} \qquad where \;\; C_q = 3.84 \times 10^{-13}\, m$$

$$\left(\frac{\sigma_p}{p_0}\right)^2 = \frac{C_q \gamma_0^2}{J_S \rho}$$

iso – magnetic case

- For the horizontal emittance at equilibrium:

$$\varepsilon = C_q \frac{\gamma_0^2}{J_X} \frac{\oint H/\rho^3 ds}{\oint 1/\rho^2 ds}$$

where: $H(s) = \beta_T D'^2 + \gamma_T D^2 + 2\alpha_T D D'$

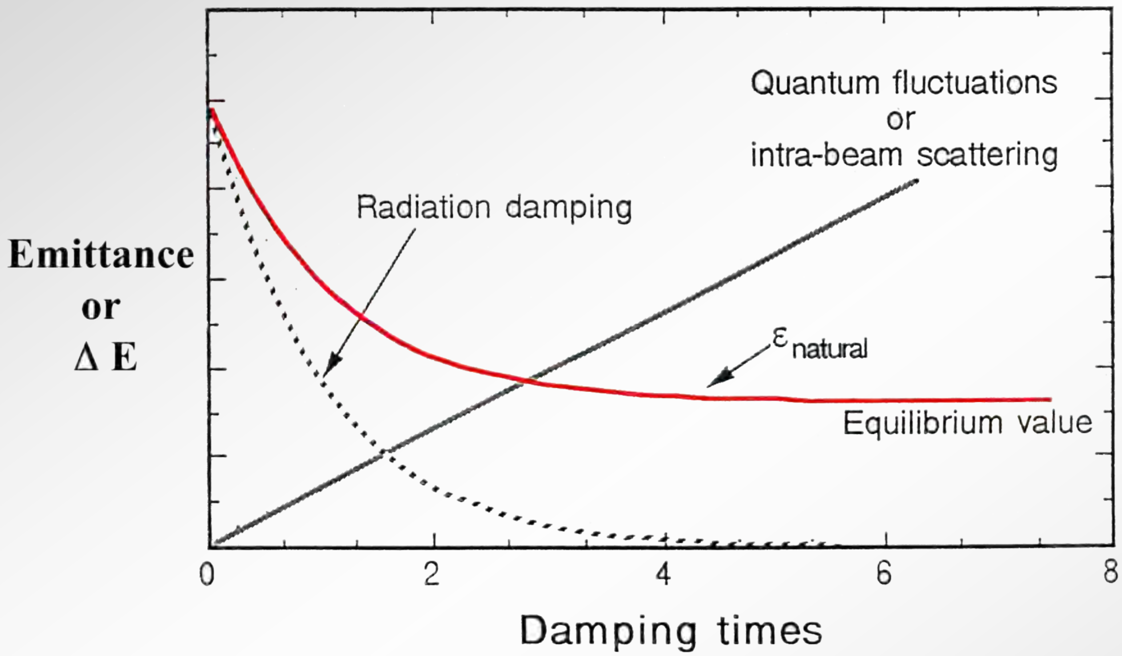
- In the vertical plane, when no vertical bend is present, the synchrotron radiation contribution to the equilibrium vertical emittance is very small

- Vertical emittance is defined by machine imperfections & nonlinearities that couple the horizontal & vertical planes:

$$\varepsilon_Y = \frac{\kappa}{\kappa + 1}\varepsilon \quad and \quad \varepsilon_X = \frac{1}{\kappa + 1}\varepsilon$$

with $\kappa \equiv$ *coupling factor*

Equilibrium emittance & ΔE



- Set

Growth rate due to fluctuations (linear) = exponential damping rate due to radiation

==> equilibrium value of emittance or ΔE ~ $\gamma^2\,\theta^3$

$$\varepsilon_{natural} = \varepsilon_1 e^{-2t/\tau_d} + \varepsilon_{eq}\left(1 - e^{-2t/\tau_d}\right)$$

Quantum lifetime

- At a fixed observation point, transverse particle motion looks sinusoidal

$$x_T = a\sqrt{\beta_n}\,\sin\left(\omega_{\beta_n} t + \varphi\right) \qquad T = x, y$$

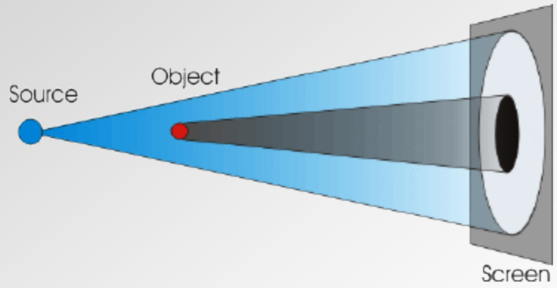
- Tunes are chosen in order to avoid resonances.
  - At a fixed azimuth, turn-after-turn a particle sweeps all possible positions within the envelope
- Photon emission randomly changes the "invariant" *a*
  - Consequently changes the trajectory envelope as well.
- Cumulative photon emission can bring the envelope beyond acceptance at some azimuth
  - The particle is lost

This mechanism is called the transverse quantum lifetime

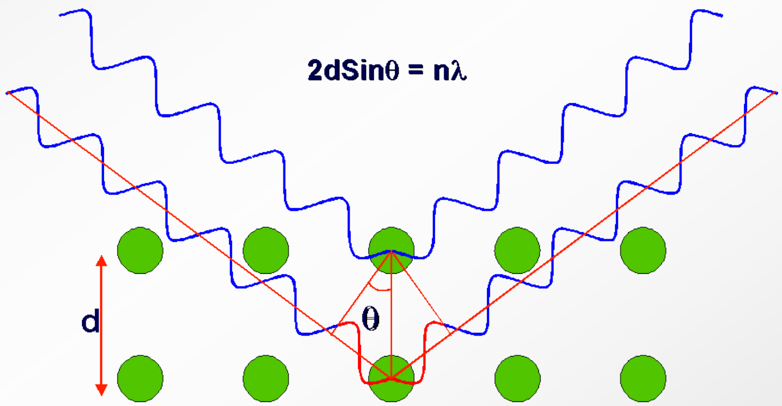
Several time scales govern particle dynamics in storage rings

- Damping: several ms for electrons, ~ infinity for heavier particles
- Synchrotron oscillations: ~ tens of ms
- Revolution period: ~ hundreds of ns to ms
- Betatron oscillations: ~ tens of ns

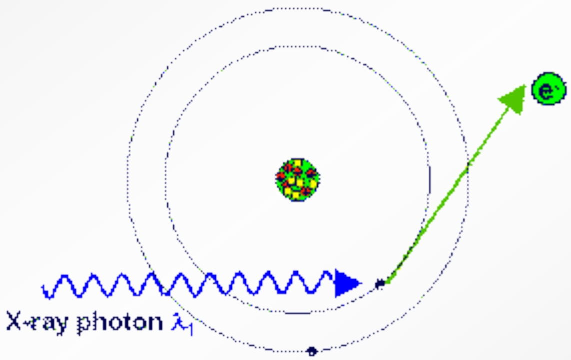
Interaction of Photons with Matter



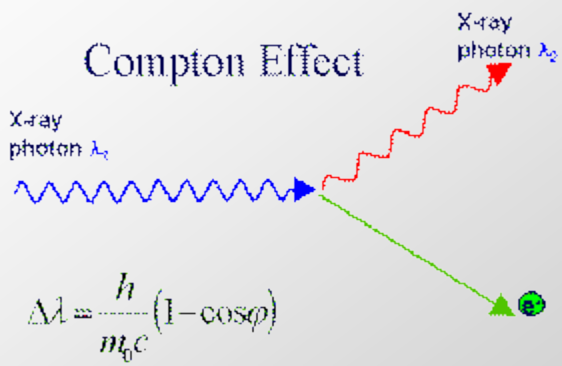
Radiography



Diffraction



Photoelectric Effect

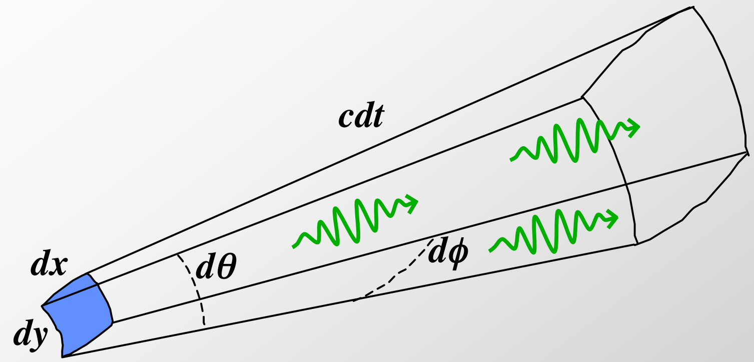


Compton Scattering

Brightness of a Light Source

- Brightness is a principal characteristic of a particle source
  - Density of particle in the 6-D phase space
- Same definition applies to photon beams
  - Photons are bosons & the Pauli exclusion principle does not apply
  - Quantum mechanics does not limit achievable photon brightness

$$\textbf{Brightness} = \frac{\text{\# of photons in given } \Delta\lambda/\lambda}{\text{sec, mrad } \theta\text{, mrad } \varphi\text{, mm}^2}$$

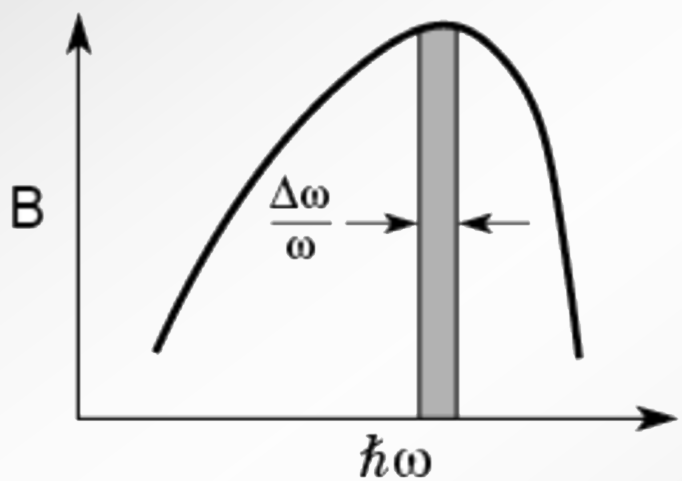


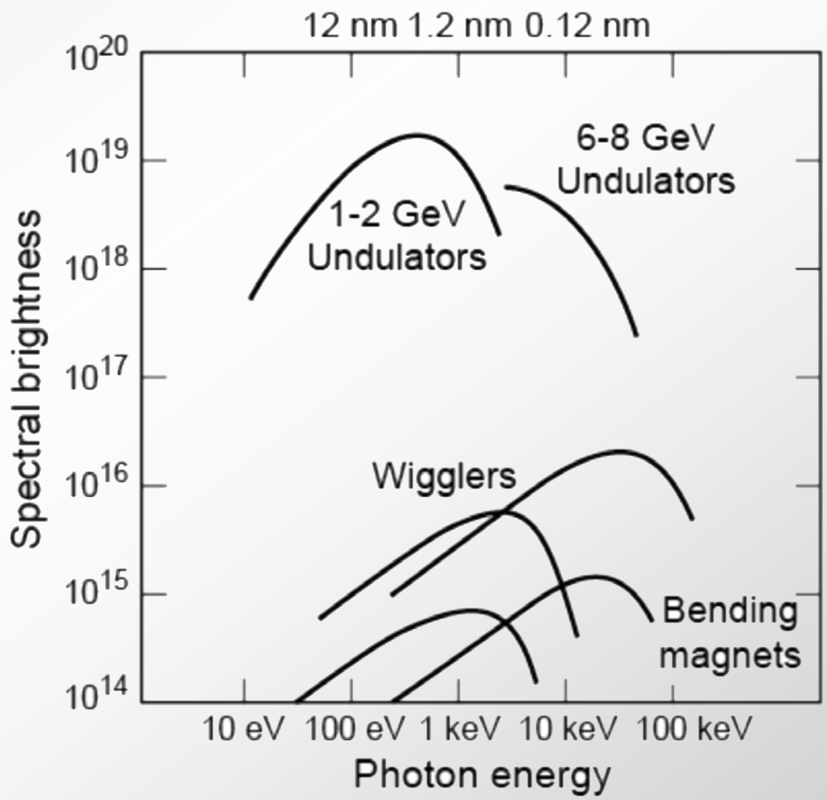
$$\textbf{Flux} = \frac{\text{\# of photons in given } \Delta\lambda/\lambda}{\text{sec}}$$

$$Flux = \frac{d\dot{N}}{d\lambda} = \int Brightness\, dS\, d\Omega$$

Spectral brightness

❖ Spectral brightness is that portion of the brightness lying within a relative spectral bandwidth $\Delta\omega/\omega$:

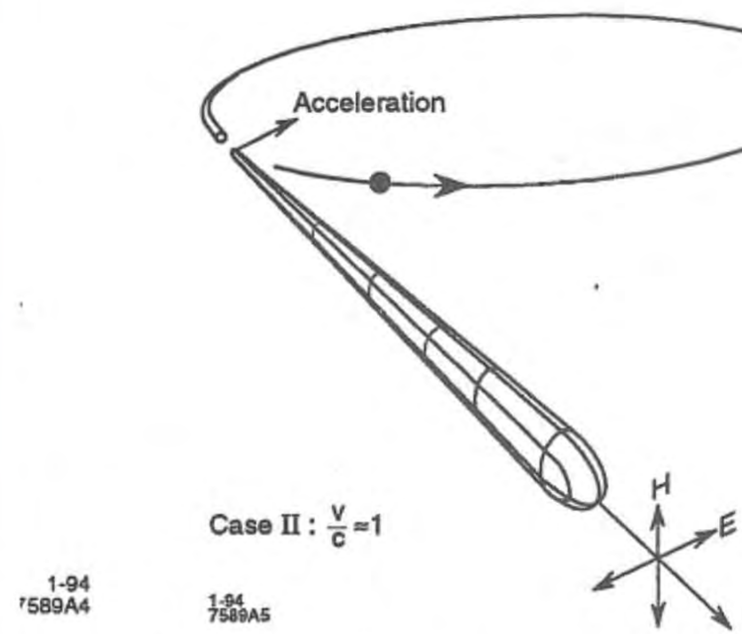




How bright is a synchrotron light source?

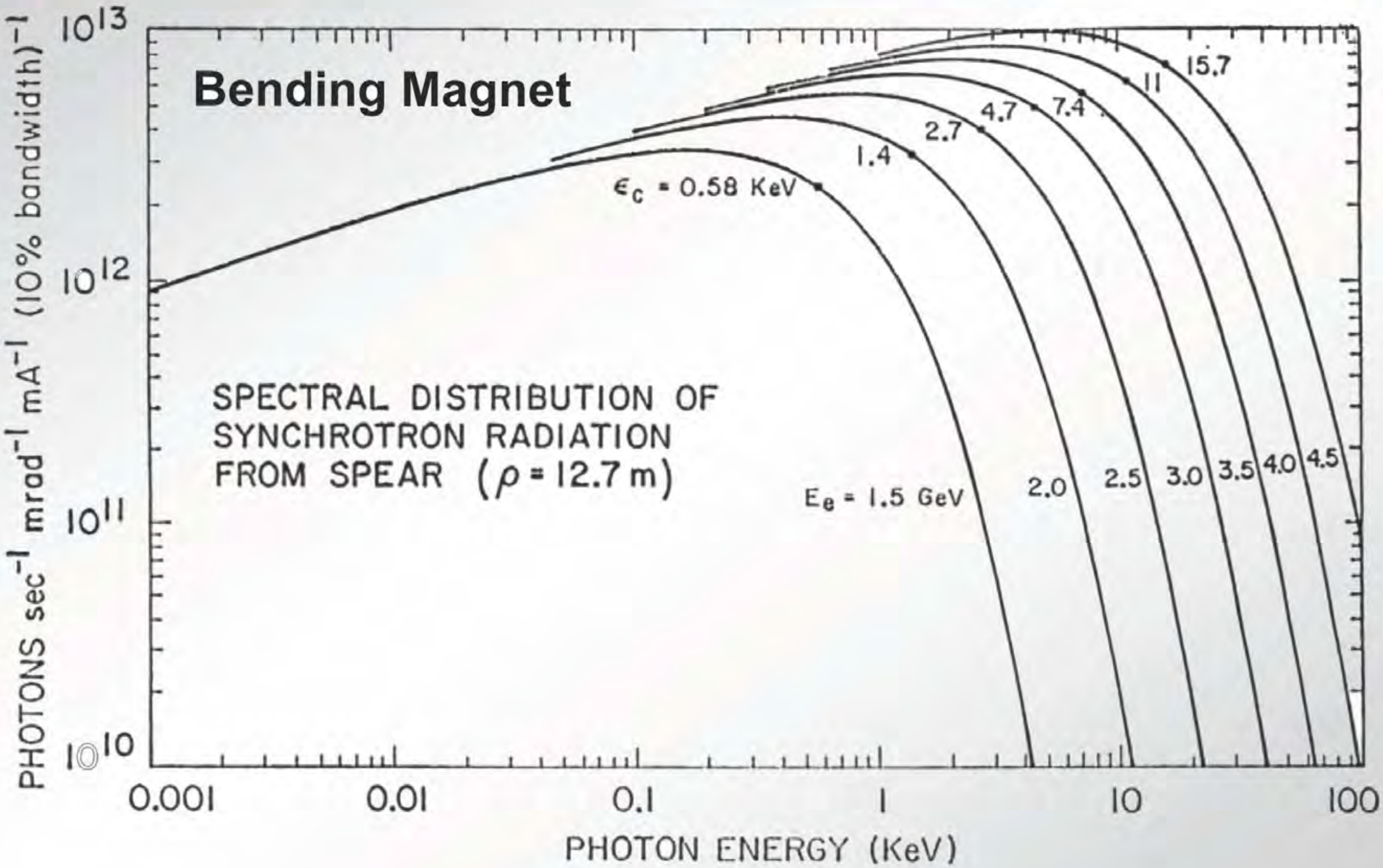
Remark: the sources are compared at different wavelengths!

Angular distribution of SR

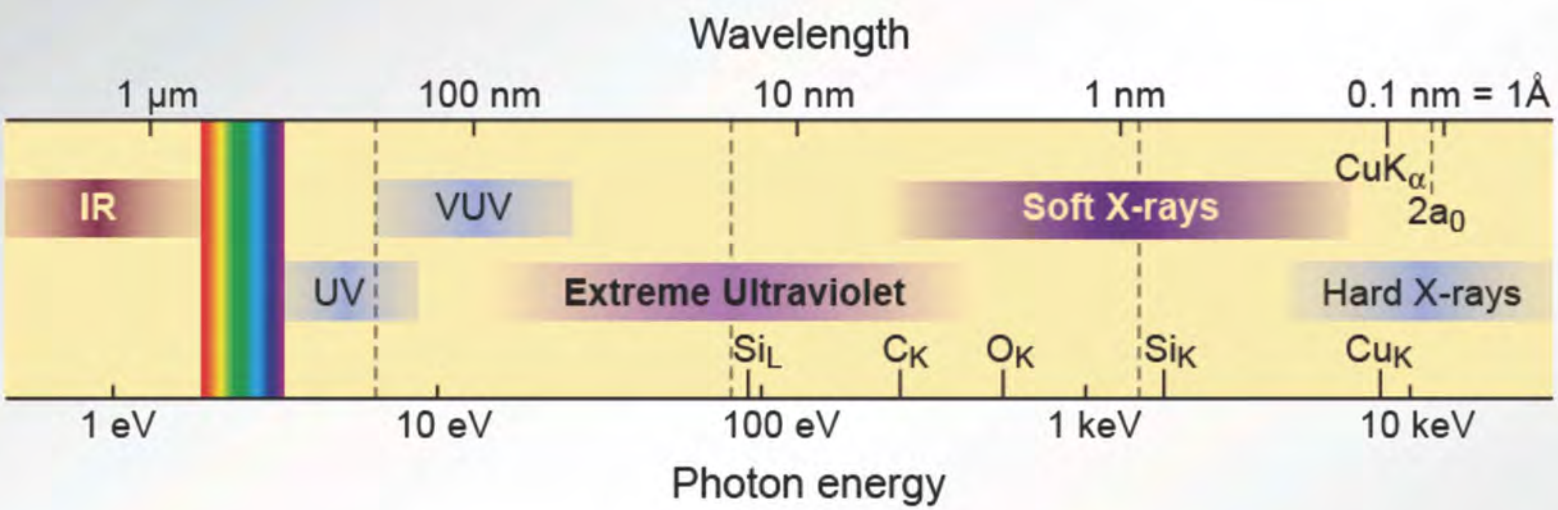


When the electron velocity approaches the velocity of light, the emission pattern is folded sharply forward.

Energy dependence of SR spectrum

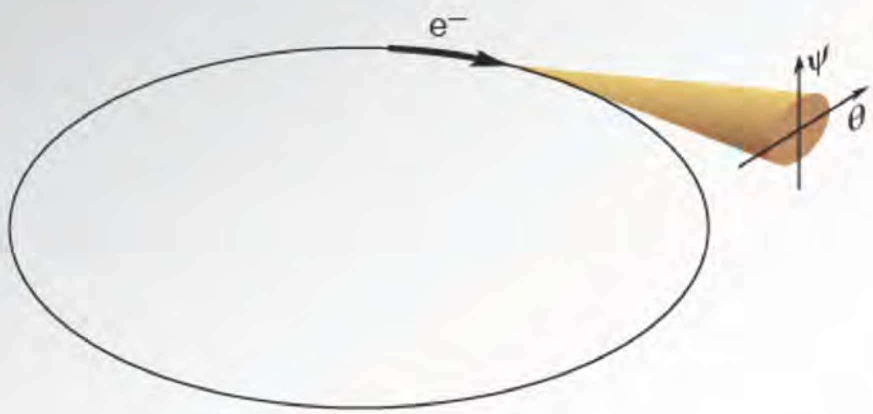


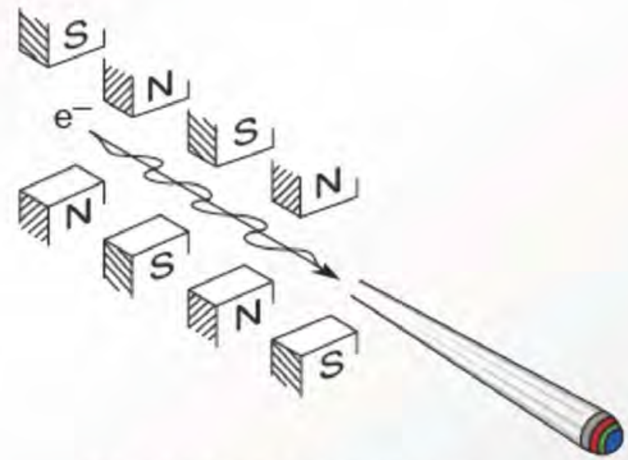
Spectrum available using SR



- **See smaller features**
- **Write smaller patterns**
- **Elemental and chemical sensitivity**

Two ways to produce radiation
from highly relativistic electrons





Synchrotron radiation

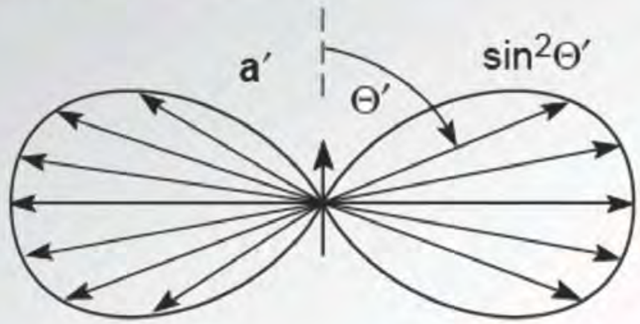
- 10^{10} brighter than the most powerful (compact) laboratory source
- An x-ray "light bulb" in that it radiates all "colors" (wavelengths, photons energies)

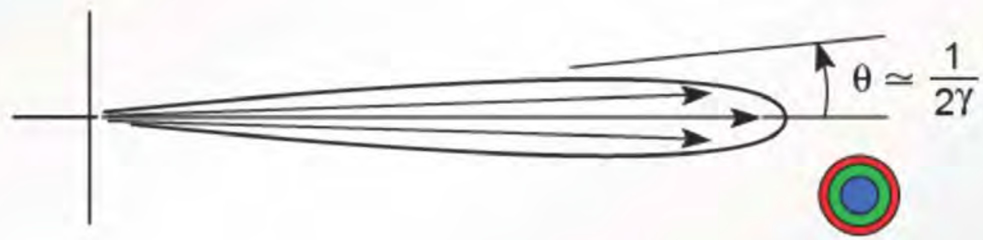
Undulator radiation

- Lasers exist for the IR, visible, UV, VUV, and EUV
- Undulator radiation is quasi-monochromatic and highly directional, approximating many of the desired properties of an x-ray laser

Relativistic electrons radiate
in a narrow cone

Dipole radiation





Frame of reference moving with electrons

Laboratory frame of reference

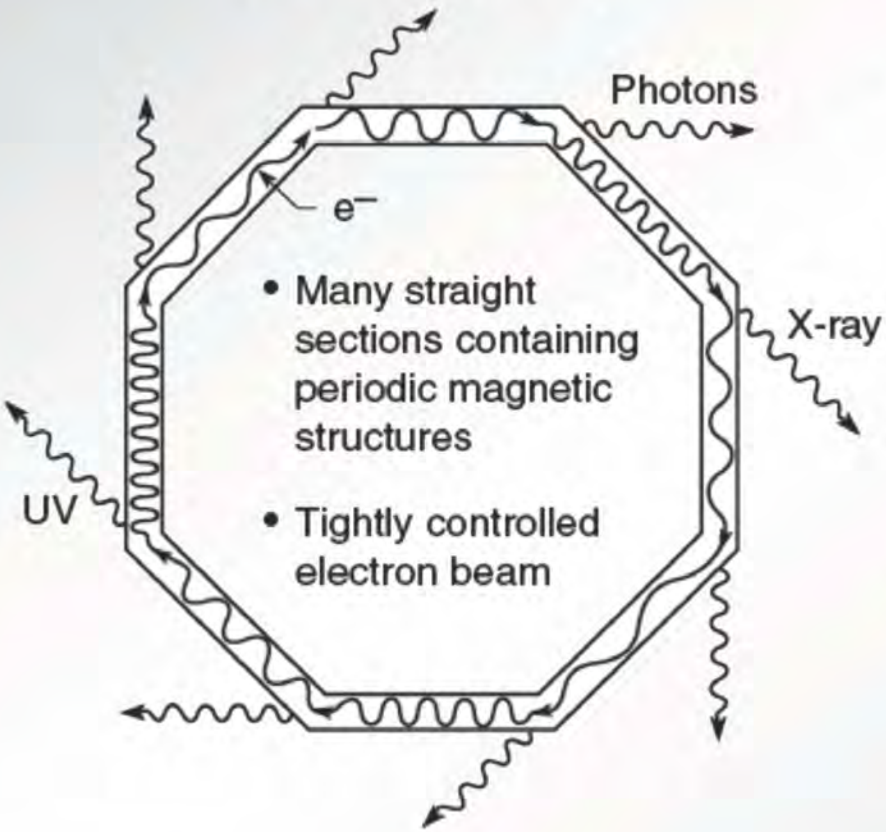
$k' = 2\pi/\lambda'$

$k_z = 2\gamma k'_z$(Relativistic Doppler shift)

$$\theta \simeq \frac{k_x}{k_z} \simeq \frac{k'_x}{2\gamma k'_z} = \frac{\tan\theta'}{2\gamma} \simeq \frac{1}{2\gamma}$$

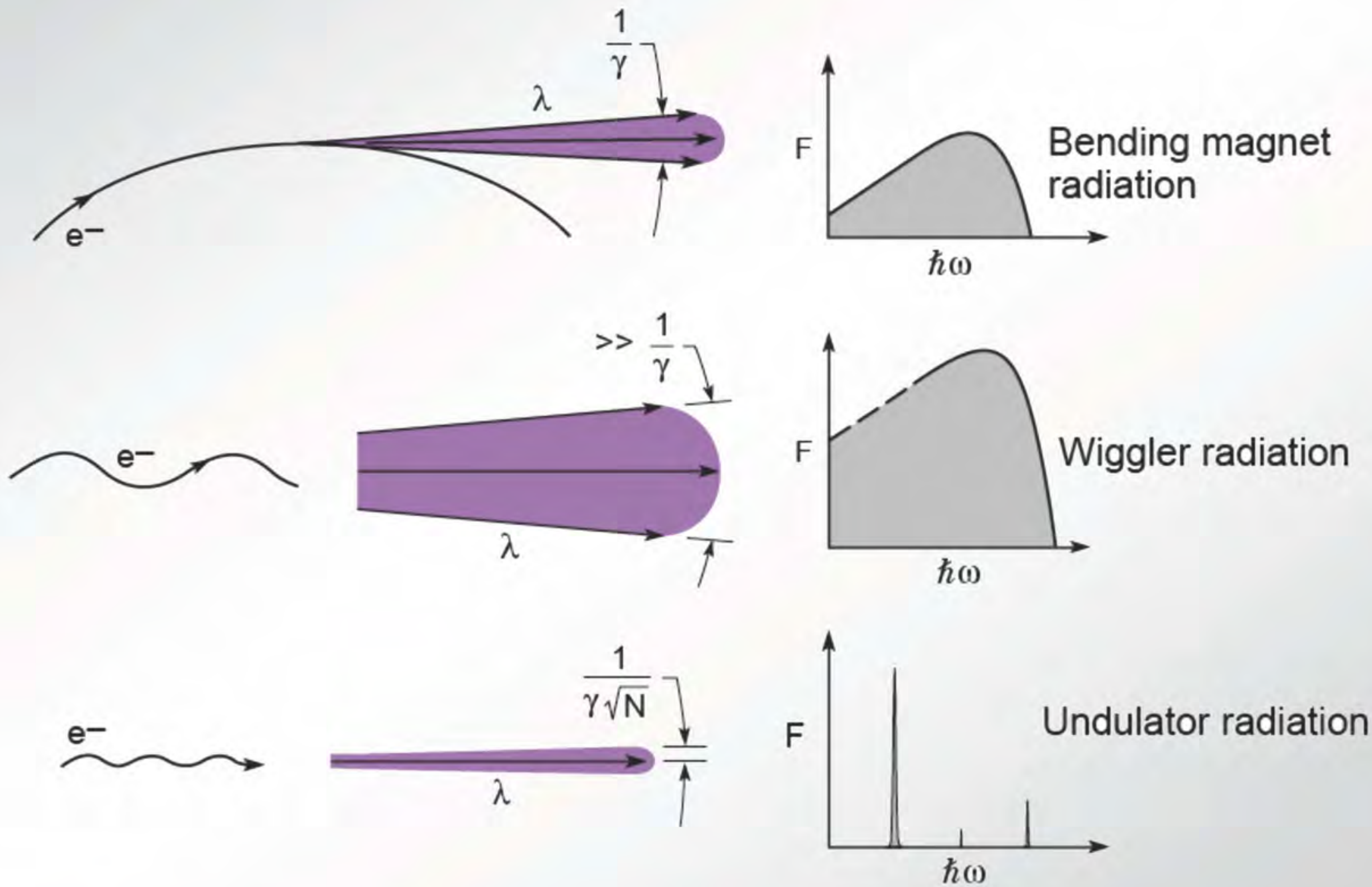
Third generation light sources
have long straight sections & bright e-beams

Modern Synchrotron
Radiation Facility

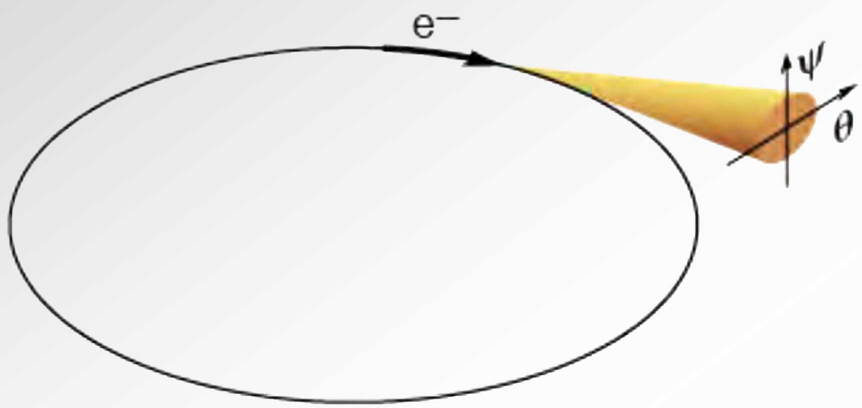


- Many straight sections for undulators and wigglers
- Brighter radiation for spatially resolved studies (smaller beam more suitable for microscopies)
- Interesting coherence properties at very short wavelengths

Light sources provide three types of SR



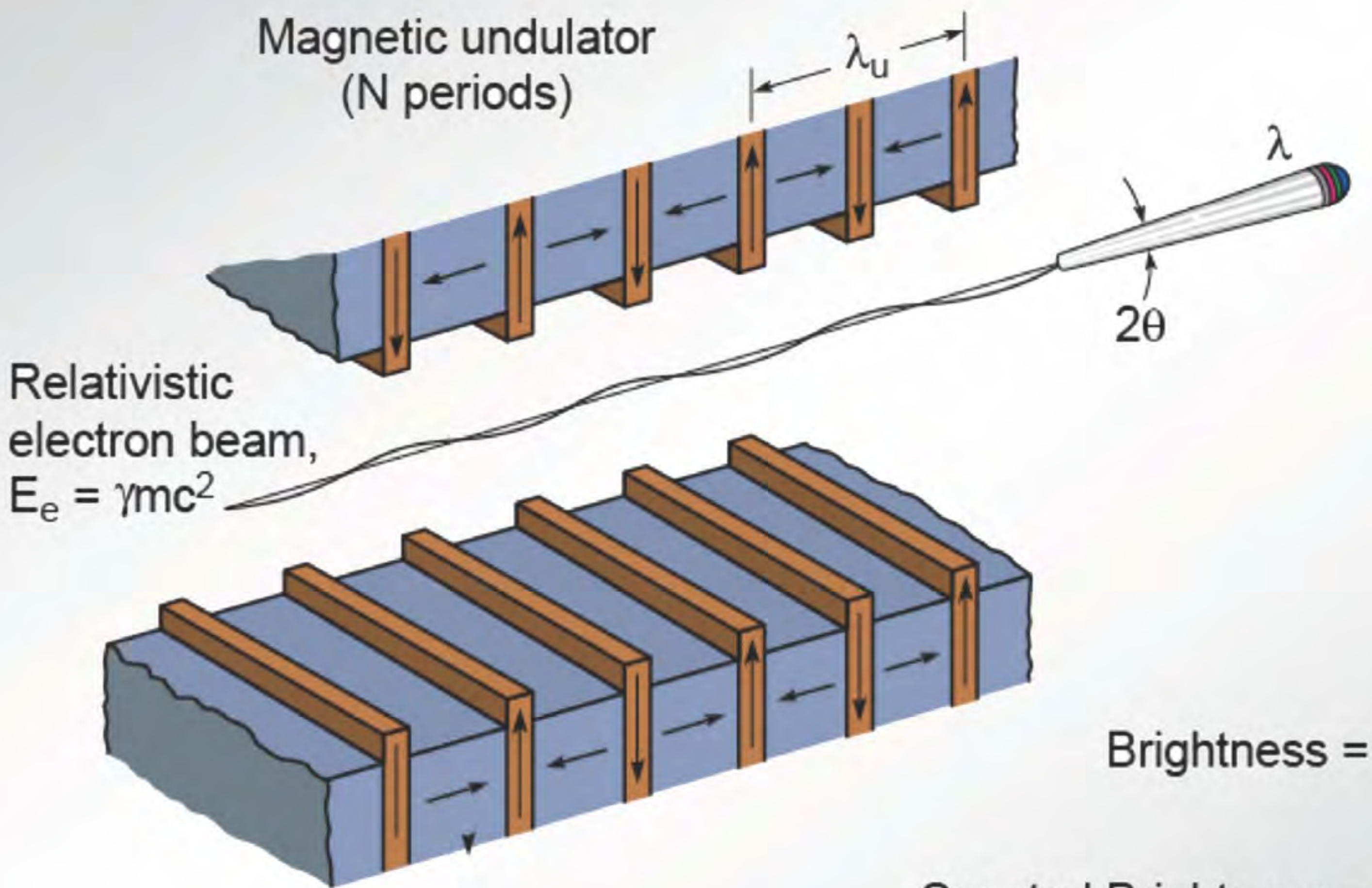
Bend magnet radiation



$$E_c(\text{keV}) = 0.6650 E_e^2(\text{GeV}) B(\text{T})$$

- Advantages:
  - Broad spectral range
  - Least expensive
  - Most accessible
    - Many beamlines
- Disadvantages:
  - Limit coverage of hard X-rays
  - Not as bright at undulator radiation

For brighter X-rays add the radiation
from many small bends



$$\lambda \simeq \frac{\lambda_u}{2\gamma^2}$$

$$\theta_{cen} \simeq \frac{1}{\gamma\sqrt{N}}$$

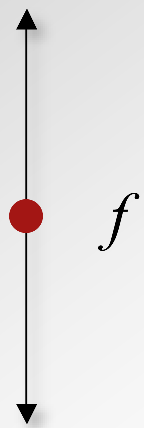
$$\left[\frac{\Delta\lambda}{\lambda}\right]_{cen} = \frac{1}{N}$$

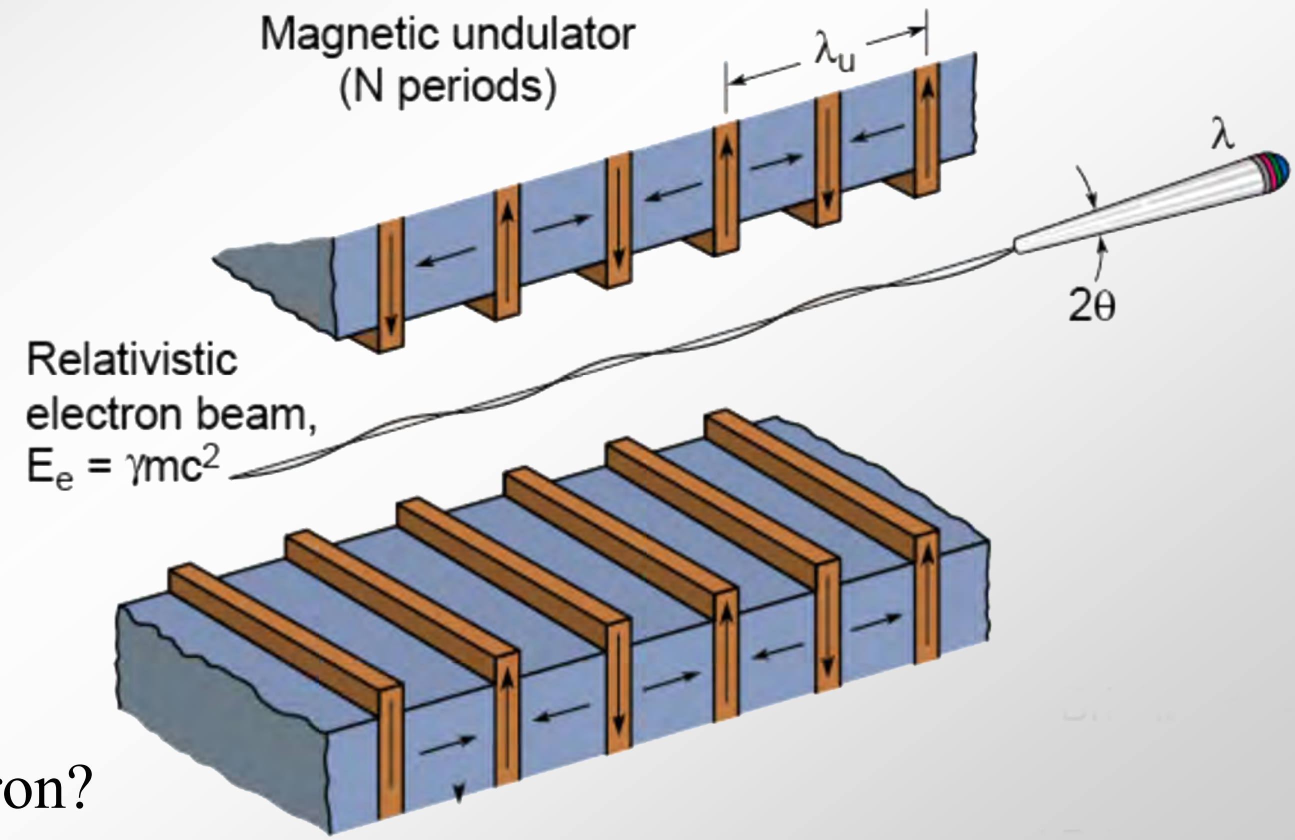
$$\text{Brightness} = \frac{\text{photon flux}}{(\Delta A)\,(\Delta\Omega)}$$

$$\text{Spectral Brightness} = \frac{\text{photon flux}}{(\Delta A)\,(\Delta\Omega)\,(\Delta\lambda/\lambda)}$$

Undulator radiation: What is λ_{rad}?

An electron in the lab oscillating at frequency, f, emits dipole radiation of frequency f





What about the relativistic electron?

Power in the central cone
of undulator radiation

$$\lambda_x = \frac{\lambda_u}{2\gamma^2}\left(1 + \frac{K^2}{2} + \gamma^2\theta^2\right)$$

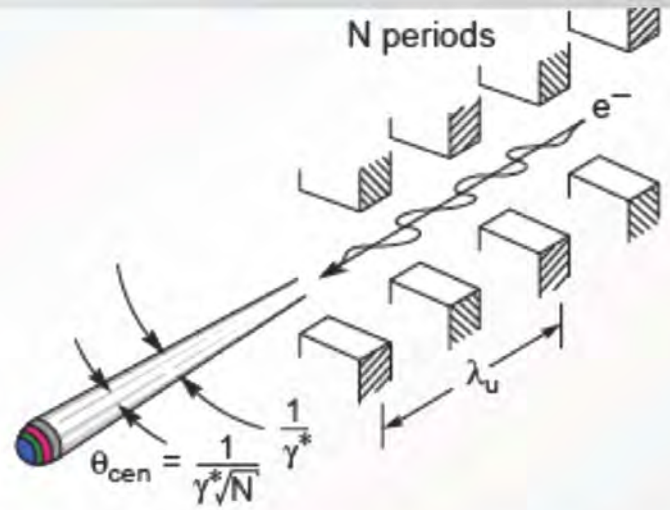
$$\bar{P}_{cen} = \frac{\pi e \gamma^2 I}{\epsilon_0 \lambda_u} \frac{K^2}{\left(1 + \frac{K^2}{2}\right)^2} f(K)$$

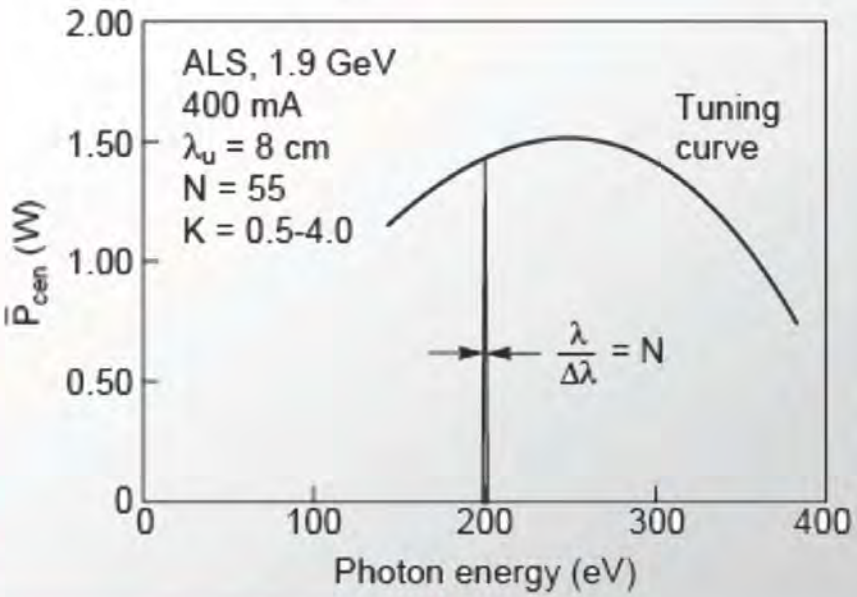
$$\theta_{cen} = \frac{1}{\gamma^* \sqrt{N}}$$

$$\left(\frac{\Delta\lambda}{\lambda}\right)_{cen} = \frac{1}{N}$$

$$K = \frac{e B_0 \lambda_u}{2\pi m_0 c}$$

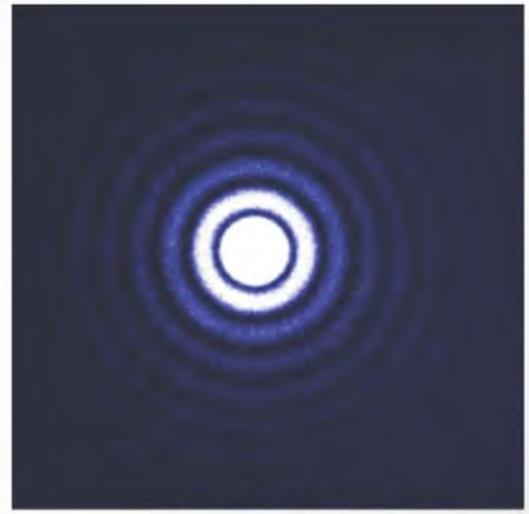
$$\gamma^* = \gamma / \sqrt{1 + \frac{K^2}{2}}$$



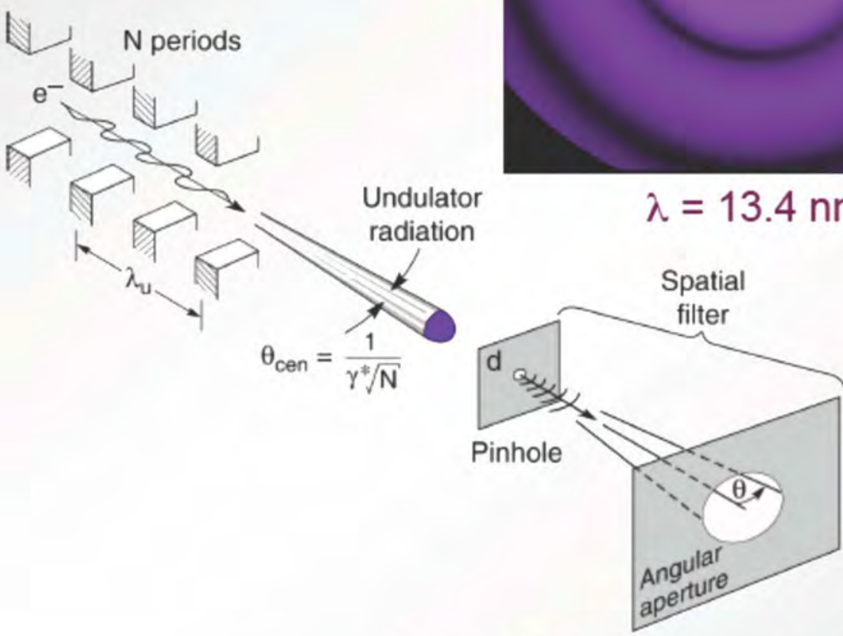


Spatial coherence of undulator radiation

$\lambda = 13.4$ nm



$\lambda = 2.5$ nm



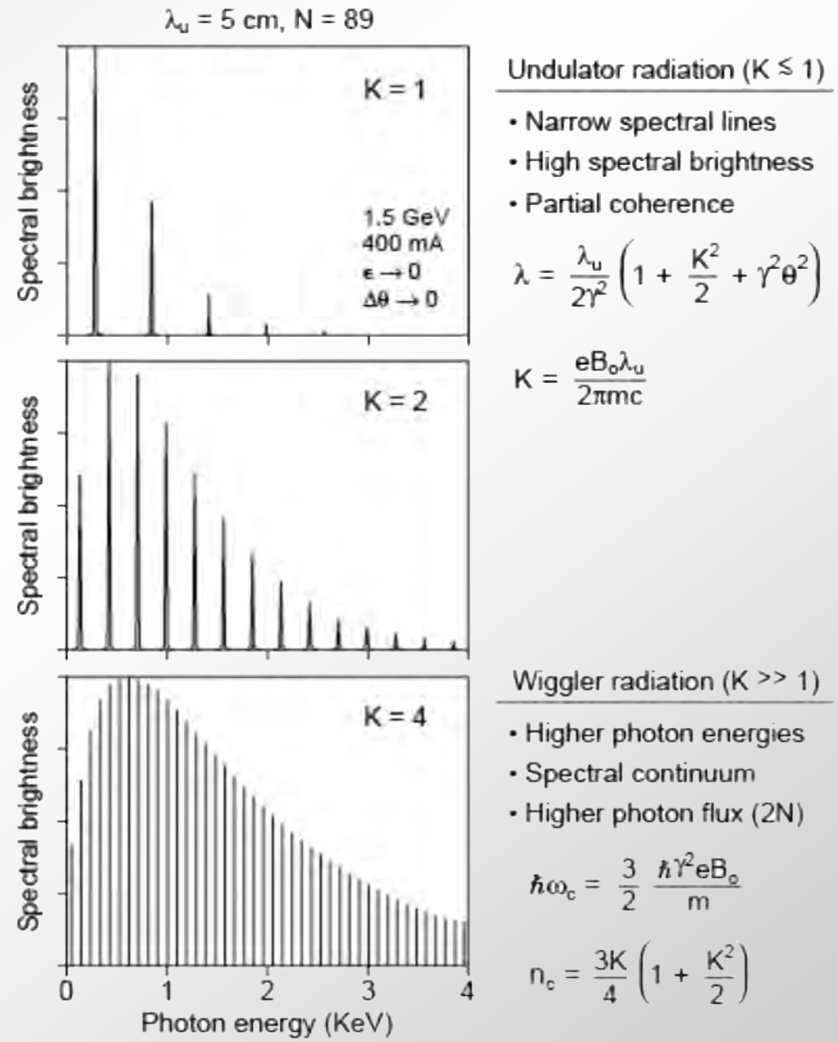
1 µm^D pinhole
25 mm wide CCD at 410 mm

$$\boxed{d \cdot \theta = \frac{\lambda}{2\pi}}$$

Courtesy of Patrick Naulleau, LBNL / Kris Rosfjord, UCB and LBNL

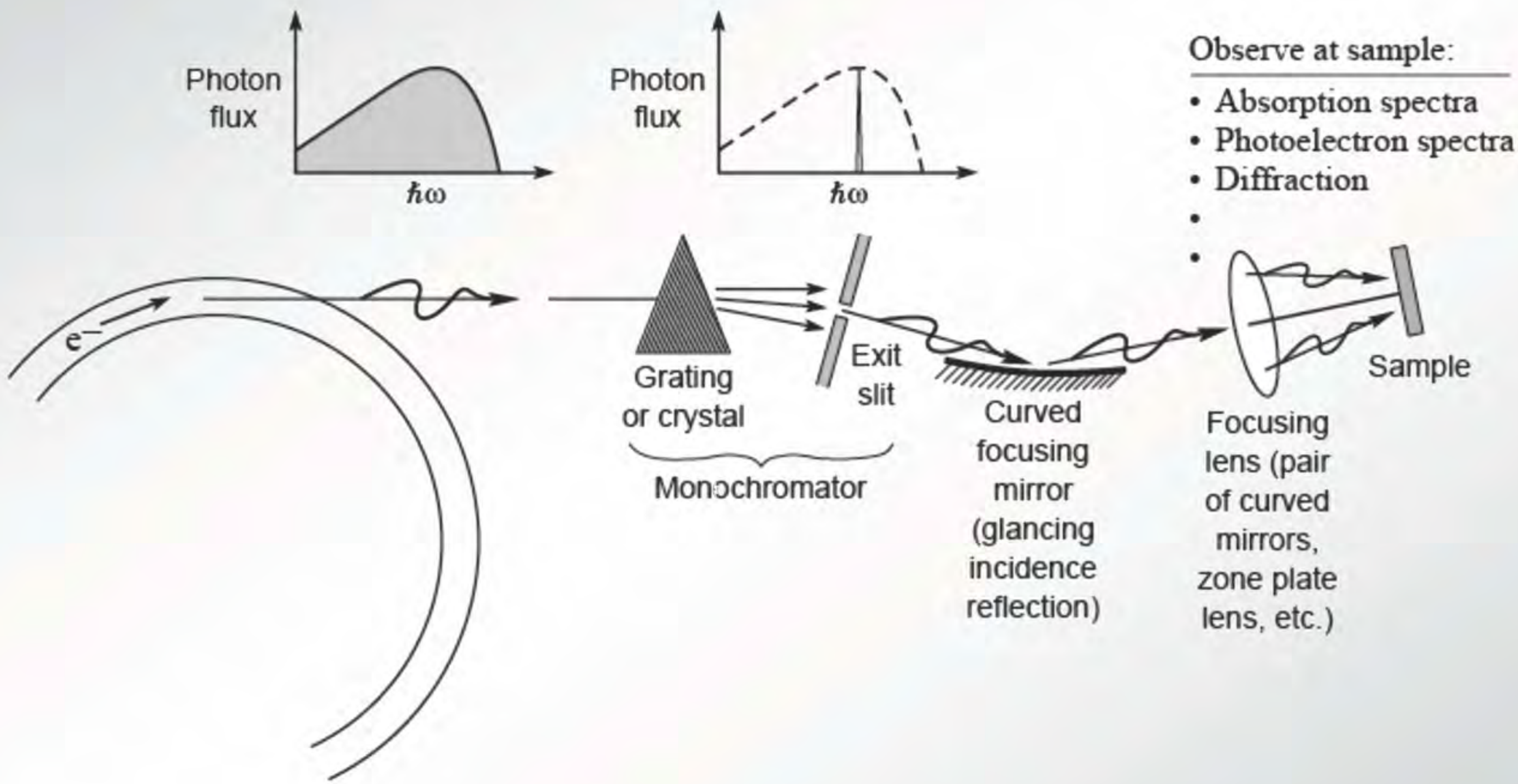
Characteristics of wiggler radiation

- For K >> 1, radiation appears in high harmonics, & at large horizontal angles $\theta = \pm K/\gamma$
  - One tends to use larger collection angles, which tends to spectrally merge nearby harmonics.
  - Continuum at high photon energies, similar bend magnet radiation,
    - Increased by 2N (the number of magnet pole pieces).



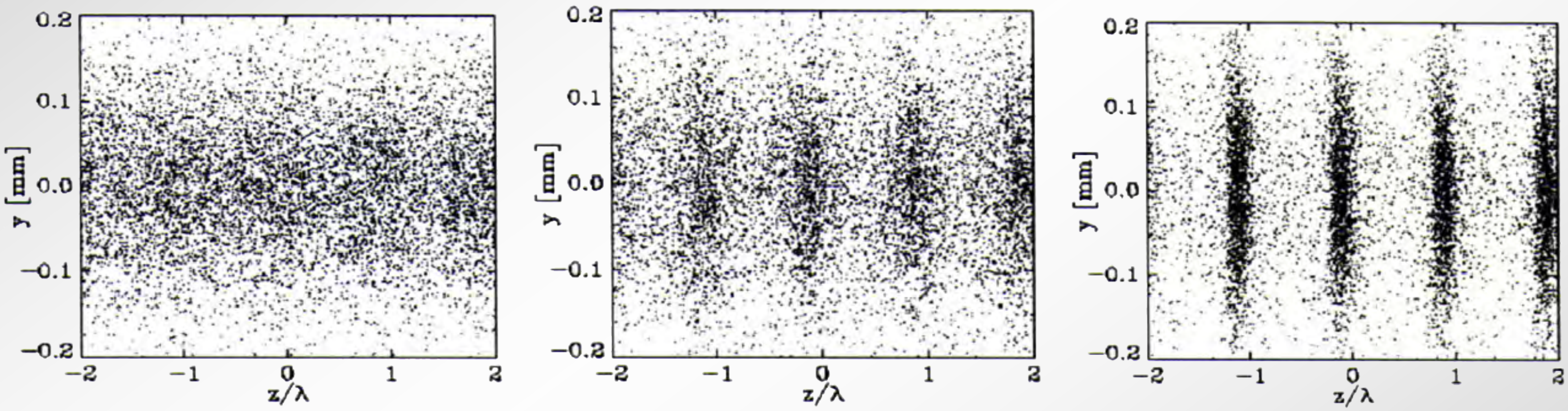
(Courtesy of K.-J. Kim)

X-ray beamlines transport the photons to the sample



To get brighter beams we need another great invention

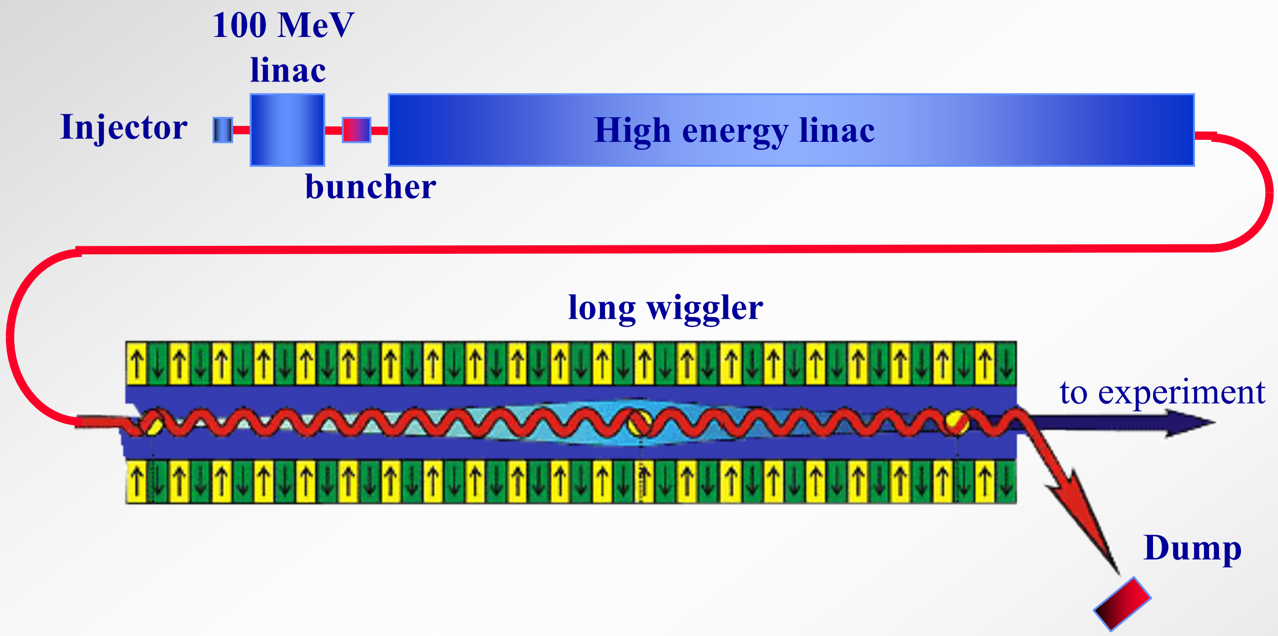
- The Free Electron Laser (John Madey, Stanford, 1976)
- Physics basis: *Bunched electrons radiate coherently*

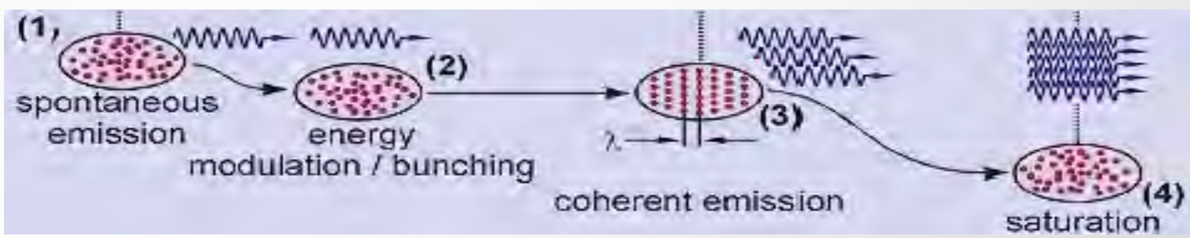


START MIDDLE END

- Madey's discovery: the bunching can be self-induced!

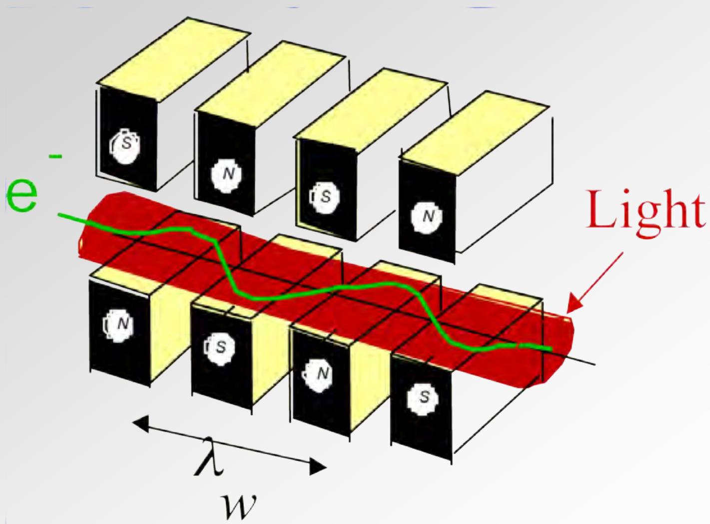
Coherent emission ==> Free Electron Laser



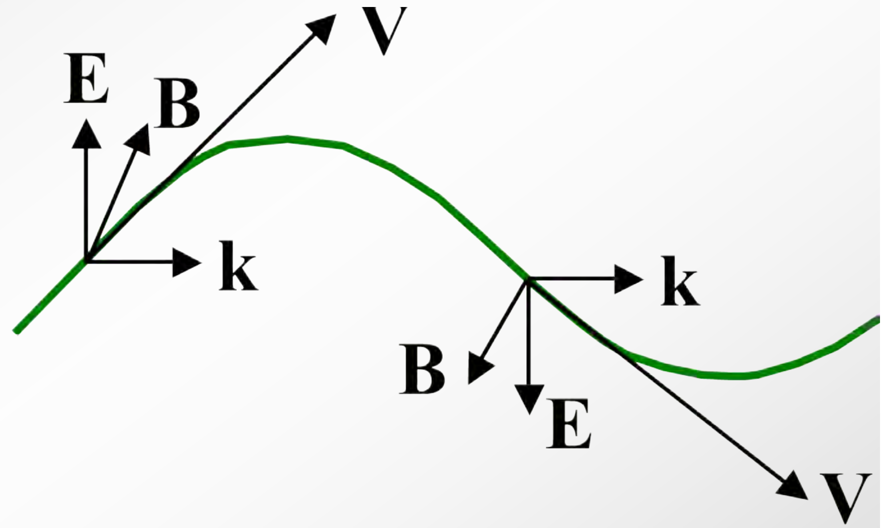


See movie

Laser interaction in the wiggler
manipulates electron beam in longitudinal phase



Electron trajectory through wiggler with two periods



In resonance the electrons always "run uphill" against the E field

Energy lost from the electrons augments the electromagnetic field

Fundamental FEL physics

- Electrons see a potential

$$V(x) \sim |A| \left(1 - \cos\left(x + \varphi\right)\right)$$

where

$$A \propto B_w \lambda_w E_{laser}$$

and φ is the phase between the electrons and the laser field

- Imagine an electron part way up the potential well but falling toward the potential minimum at $\theta = 0$
  - Energy radiated by the electron increases the laser field & consequently lowers the minimum further.
  - Electrons moving up the potential well decrease the laser field

The equations of motion

- The electrons move according to the pendulum equation

$$\frac{d^2x}{dt^2} = |A|\sin(x+\varphi)$$

- The field varies as

$$\frac{dA}{dt} = -J\left\langle e^{-ix}\right\rangle$$

where $x = (k_w - k)\, z - \omega t$

The simulation will show us the bunching and signal growth

DownLoad: FELOde.zip

Basic Free Electron Laser Physics

Resonance condition:

Slip one optical period per wiggler period

FEL bunches beam on an optical wavelength at *ALL* harmonics

Bonifacio et al. NIM A293, Aug. 1990

Gain-bandwidth & efficiency ~ ρ

Gain induces ΔE ~ ρ

$$\rho = \frac{1}{\gamma}\left(\frac{a_w\,\omega_p}{4\,c\,k_w}\right)^{2/3} \propto \frac{I^{1/3}\,B^{2/3}\,\lambda_w^{4/3}}{\gamma}$$

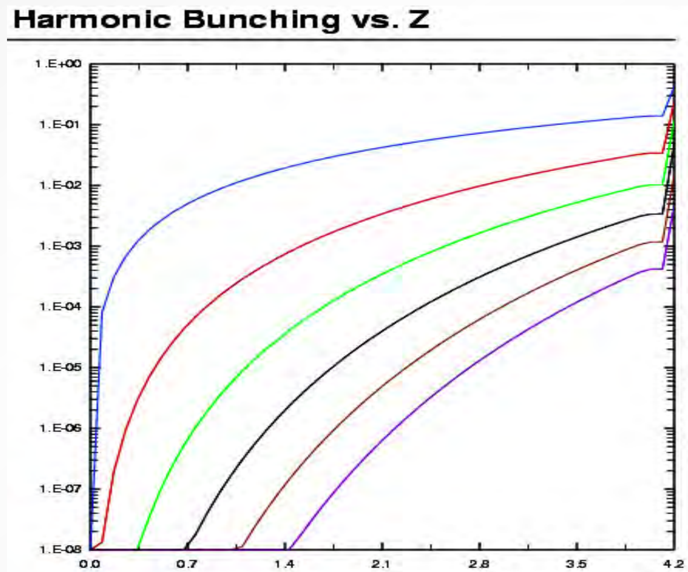
1) Emittance constraint

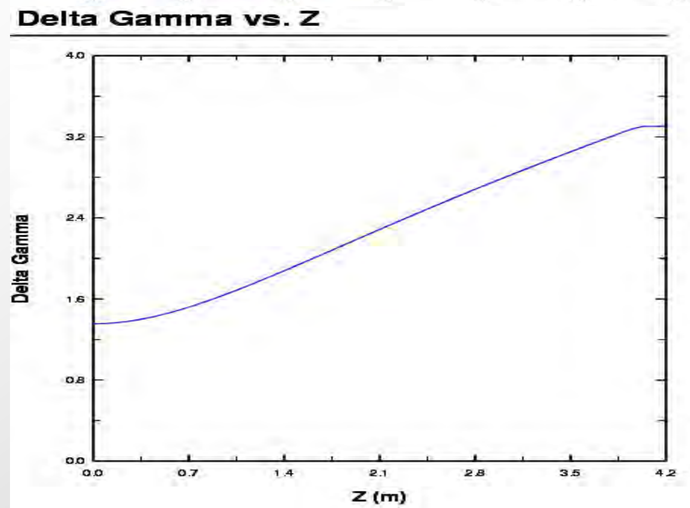
Match beam phase area to diffraction limited optical beam

2) Energy spread condition

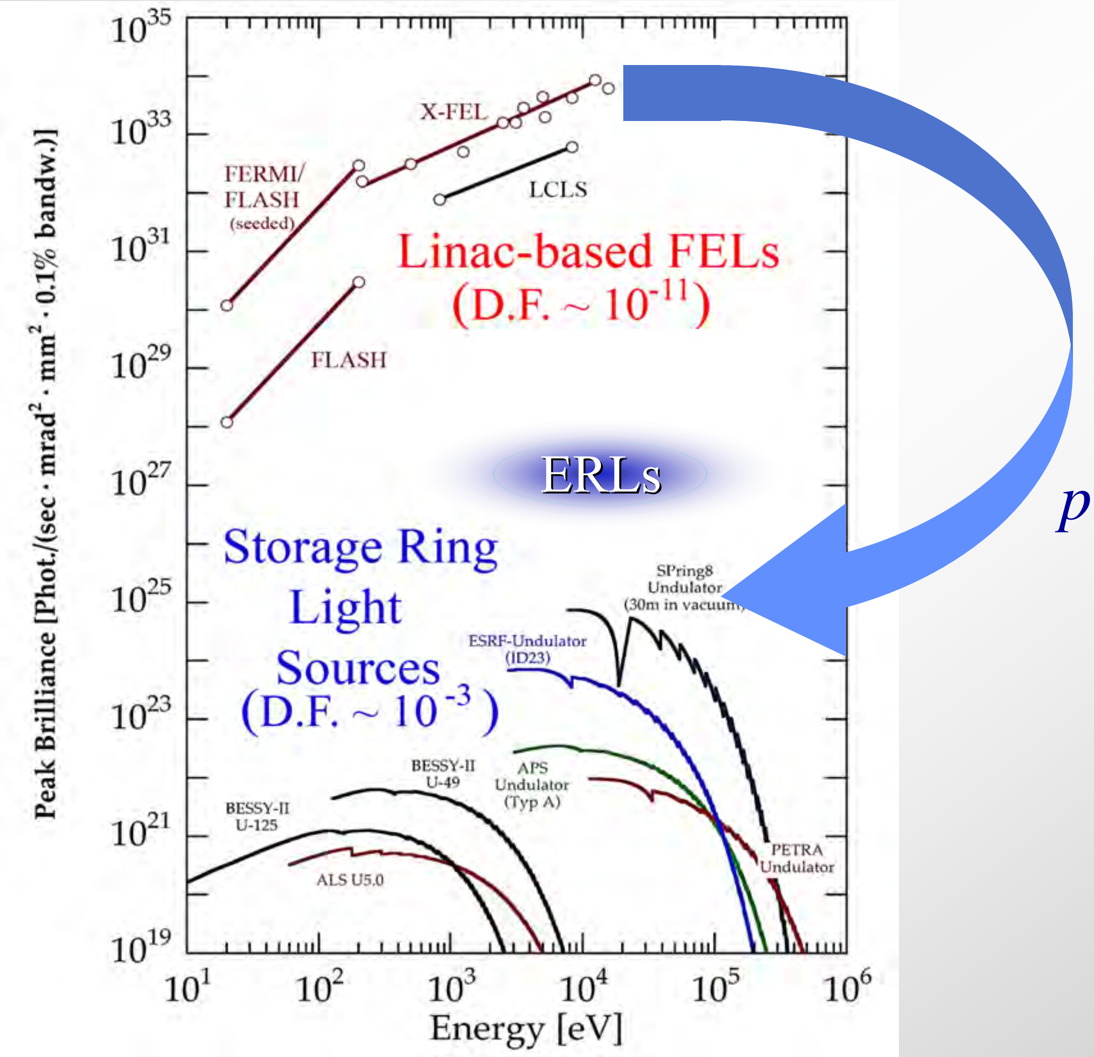
Keep electrons from debunching

3) Gain must be faster than diffraction





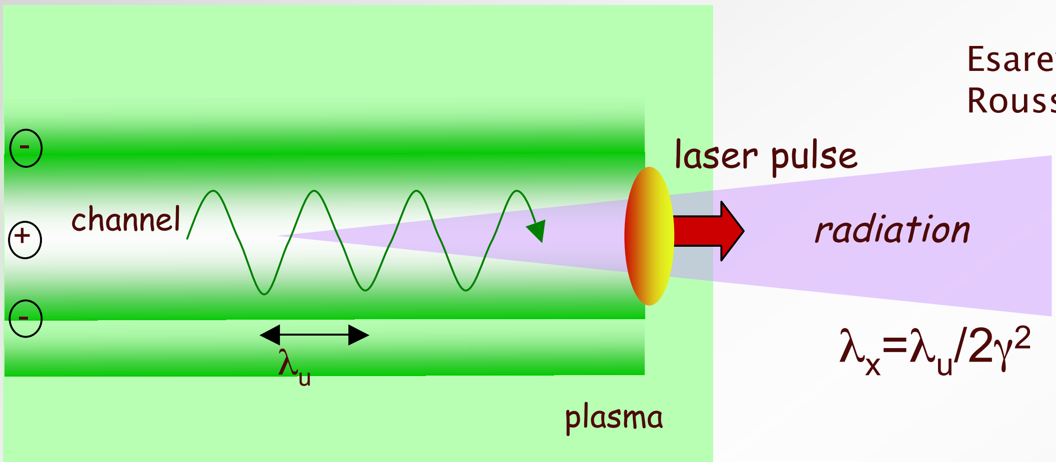
FOM 1 from condensed matter studies: Light source brilliance v. photon energy



Duty factor correction for pulsed linacs

Near term: x-rays from betatron motion and Thomson scattering

Betatron oscillations:



Esarey et al., Phys. Rev E (2002)
Rousse et al., Phys. Rev. Lett. (2004)

$\lambda_x = \lambda_u / 2\gamma^2$

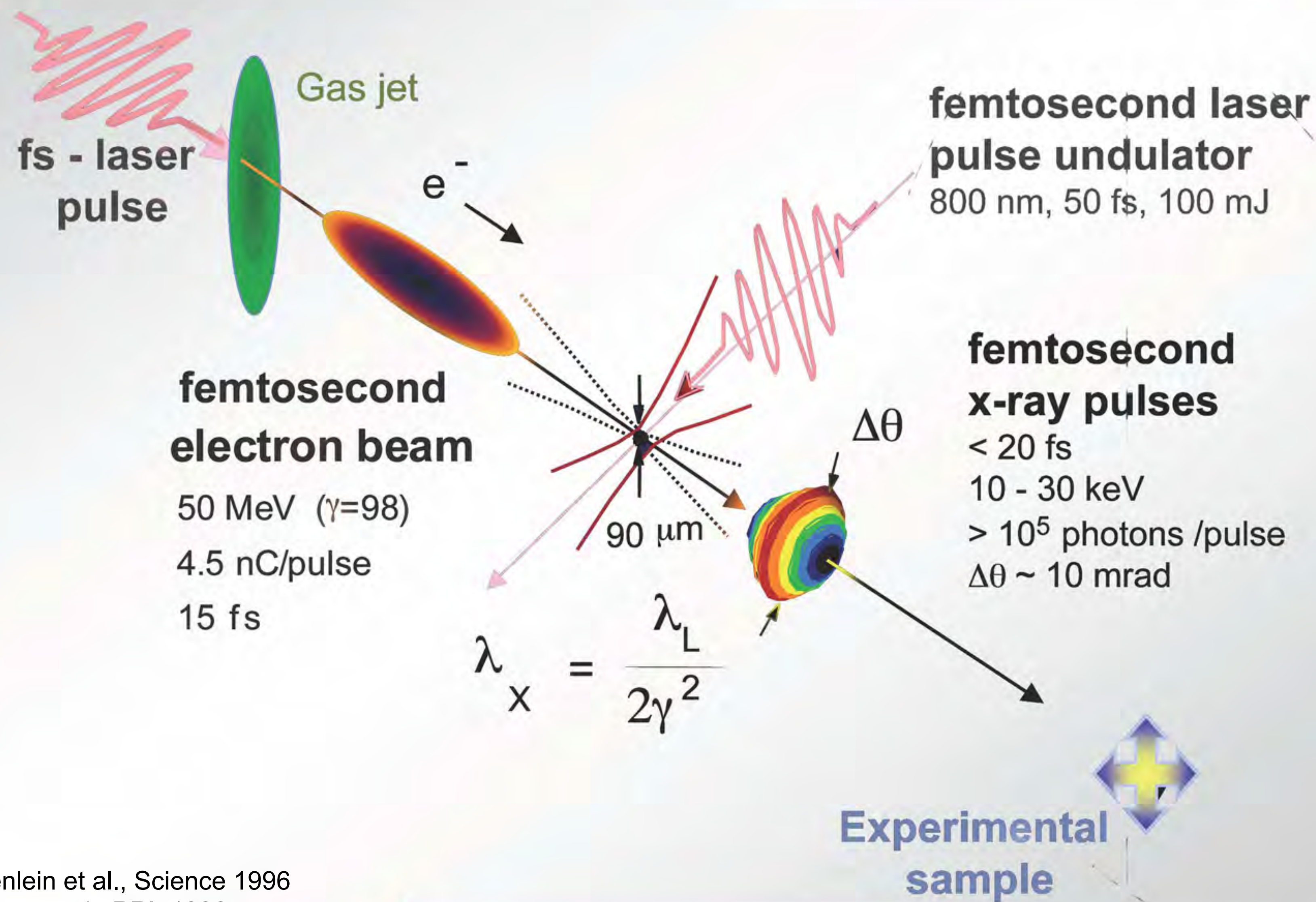
<u>Strength parameter</u>

Betatron: $a_\beta = \pi (2\gamma)^{1/2} r_\beta / \lambda_p$

Thomson scattering: $a_0 = e/mc^2 A$

Radiation pulse duration = bunch duration

Potential Thompson source
from all optical accelerator



Schoenlein et al., Science 1996
Leemans et al., PRL 1996