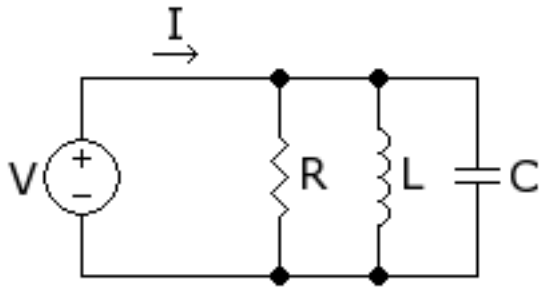
RF Technology

S. Di Mitri (1.5 hr.)

Credits:
S. Milton, JUAS 2011, Erice, I. C. Serpico, M. Dal Forno (E-ST).

RLC circuit ~ Resonant cavity



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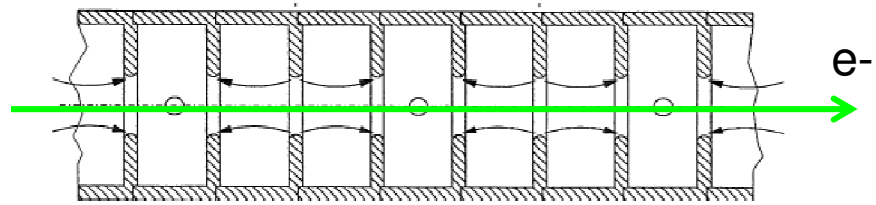


$$U = \frac{1}{2}CV^2 \qquad P_d = \frac{V^2}{R}$$

$$Q = \frac{\omega_{RF} U}{\pi P_d} = \frac{\text{Energy stored in the cavity}}{\text{Energy dissipated in one period}}$$

$$V = \sqrt{P_d R_s} \equiv \sqrt{P_d r_s L}$$ **Accelerating voltage**

$$E_z^2 = \left(-\frac{dV}{dz}\right)^2 \cong \left(\frac{V}{L}\right)^2 = \frac{P_d r_s}{L} \cong r_s \frac{dP_d}{dz}$$ **Accelerating field**

$$\frac{r_s}{\pi Q} = \frac{E_z^2}{\frac{dP_d}{dz}} \frac{P_d}{\omega_{RF} U} = \frac{E_z^2}{\frac{dP_d}{dz}} \frac{\frac{dP_d}{dz}}{\omega_{RF} \frac{dU}{dz}} = \frac{E_z^2}{\omega_{RF} u}$$

This ratio depends on the cavity *geometry only*.

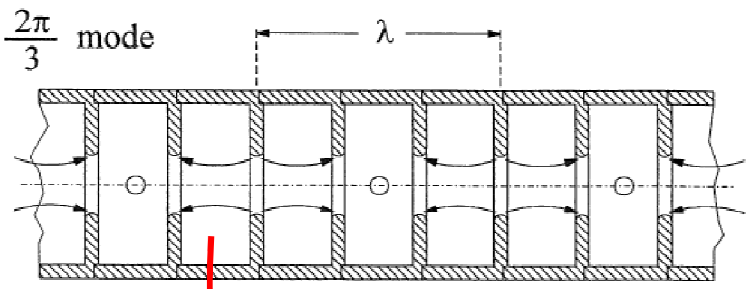
→ High Q, **Quality Factor** ~ high efficiency for energy storing

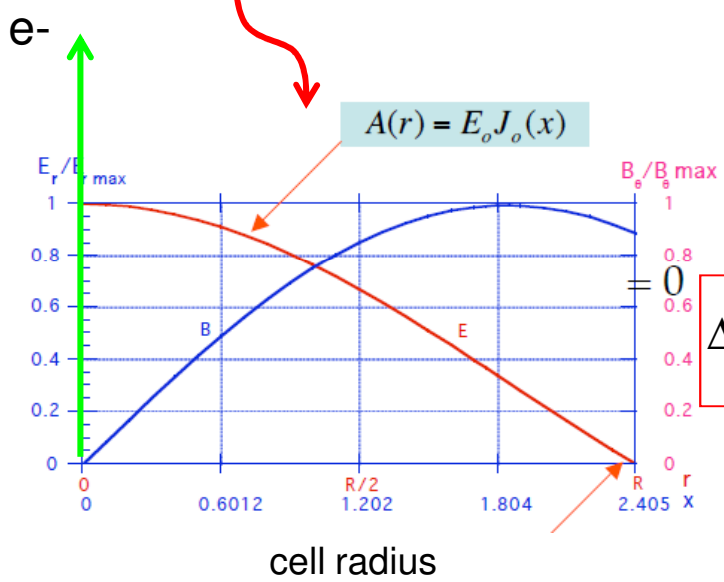
→ High r_s, **Shunt Impedance per unit length** ~ high accelerating field vs. dissipated power

EXERCISE: demonstrate that U(t) decays exponentially with time, with characteristic time constant ~Q/ω_{RF} ("filling time"). *Hint:* P_d is the energy loss per unit time.

Standing wave structure

Pictures courtesy of S. Tazzari



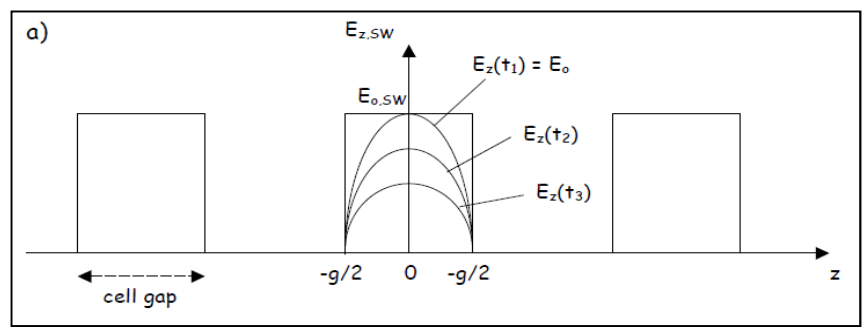


In a standing wave (SW) structure, **E_z is "resonating"** with multiple reflections back and forth.

Loss-free propagation can only occur if the field wavelength is an integer multiple of the iris separation (this makes constructive interference between reflections): $\lambda_z = pd, \quad p = 1,2,3,...; \; k_z d = \frac{2\pi}{p}$

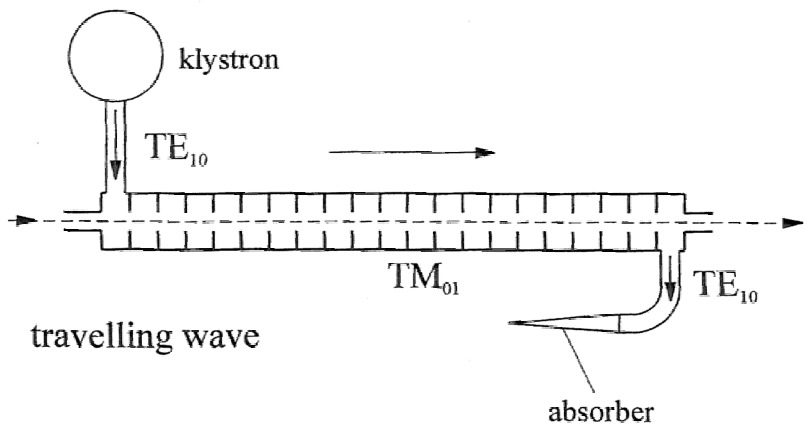
$$E_z \underset{\substack{r=0\\n=0}}{\simeq} \hat{E}_z \cos(\omega_{RF} t + \phi_0) \equiv \hat{E}_z \cos(\phi_{RF})$$

$$\Delta\gamma = -\int_0^{Nd} \frac{e}{m_e c^2} E_z dz = -\frac{e}{m_e c^2} \hat{E}_z N_d d \cos(\phi_{RF}) \equiv \frac{e}{m_e c^2} V_0 \cos(\phi_{RF})$$



EXERCISE: demonstrate that the energy gain per cell is reduced by a factor $T_0 < 1$ ("transient time factor") when a cell length [-g/2, g/2] and finite particle velocity (βc) is considered.

Traveling wave structure



In a traveling wave (TW) structure, the accelerating wave is **co-propagating with the particles**. Still parasitic power reflections have to avoided with "resonant" operating modes as in a SW.

$$E_z \underset{\substack{r=0\\n=0}}{\simeq} \hat{E}_z \cos(\omega_{RF} t - k_z z + \phi_0) \equiv \hat{E}_z(z)\cos(\phi_{RF})$$

- To keep particles and accelerating wavefront synchronous (***phase-matched***) all along the structure, we must have $v_p \cong \beta c$. This is ensured by the irises that "slow down" v_p acting as a capacitive load.
- All previous formulas for SW are now valid with the prescription **P_d (dissipated) → P (propagating)**.
- Since P is now propagting along the structure, the **filling time** becomes by def. $t_f = \frac{L}{v_g} = \frac{uL}{P}$

- If we assume ***$R_s(z)$ = const.*** (all cells are identical), Ez and P is varying exponentially with z, and the TW is said to be ***"constant impedance"***.
- If we impose ***$E_z(z)$ = const.***, P is varying linearly with z and the TW is said to be ***"contant gradient"*** (v_g is varied along the structure to compensate for the power decrease by reducing the iris raiuds.)

Contant Impedance vs. Constant Gradient

$$\begin{cases} P = uv_g = -\dfrac{\pi Q v_g}{\omega_{RF}} \dfrac{dP}{dz} \\ E_z^2 = \dfrac{r_s}{\pi Q} \omega_{RF} u = const. \cdot u \end{cases}$$

EXERCISE: demonstrate that in a TW-CI the acc. field is:

$$E_z(z) = \sqrt{2\tau \frac{P_0 r_s}{L} e^{-\tau \frac{z}{L}}},$$ **Constant impedance**

$$\tau := \frac{\omega_{RF} L}{2\pi Q v_g}$$ **Attenuation factor**

$$\begin{cases} E_z^2 = r_s \dfrac{dP}{dz} \equiv const. \\ P(z) = P_0 + [P(L) - P_0] \dfrac{z}{L} \end{cases}$$

EXERCISE: demonstrate that, once the power attenuation factor is introduced as below, the acc. field in a TW-CG is:

$$P(L) \equiv P_0 e^{-2\tau}$$

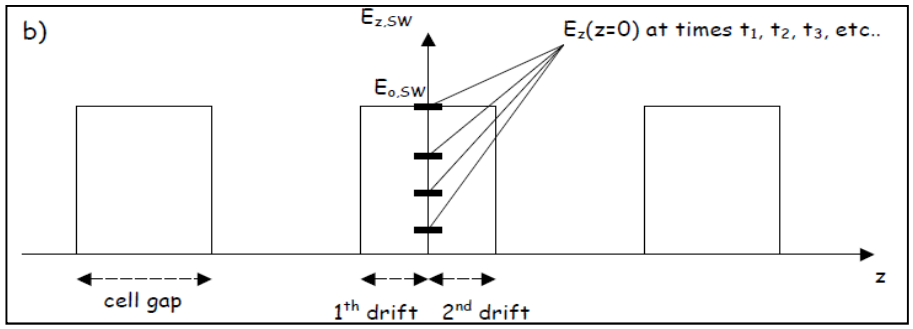
$$E_z(z) = \sqrt{\left(1 - e^{-2\tau}\right) \frac{P_0 r_s}{L}} = const.$$ **Constant gradient**

- SW has typically a higher aceleration efficiency , but longer filling time than TWs.
- TW-CI is quite simple to build (all cells are identical). It usually provides a higher acc. gradient than TW-CG but with a higher peak field. This may induce discharges in the structure.
- TW-CG is less simple to be built (varying iris radius), but it provides same energy gain than TW-CI with a lower peak field. The most used in recent times.

Modeling in tracking codes (β→1)

In many tracking codes for ultra-relativistic particles, the following approximations are routinely adopted both for SW and TW structures:

1) cylindrically symmetric, spatially periodic RF structure;
2) the accelerating field radial dependence is ignored (particles are assumed to be traveling close to the electric axis: $E_z(r)=E_z(0)$);
3) higher harmonics of E_z are neglected (negligible acceleration respect to fund. mode $n=0$);
4) the filling time is short enough respect to the RF pulse to ensure steady-state acceleration;
5) the transient time factor = 1;
6) the TW structure is a "constant-gradient" with attenuation factor = 0;
7) for any cell of finite length, acceleration is provided in the "drift-kick-drift" scheme (single energy kick in the cell's center):

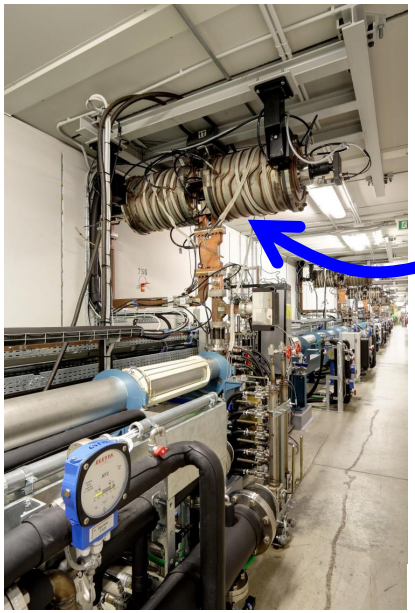


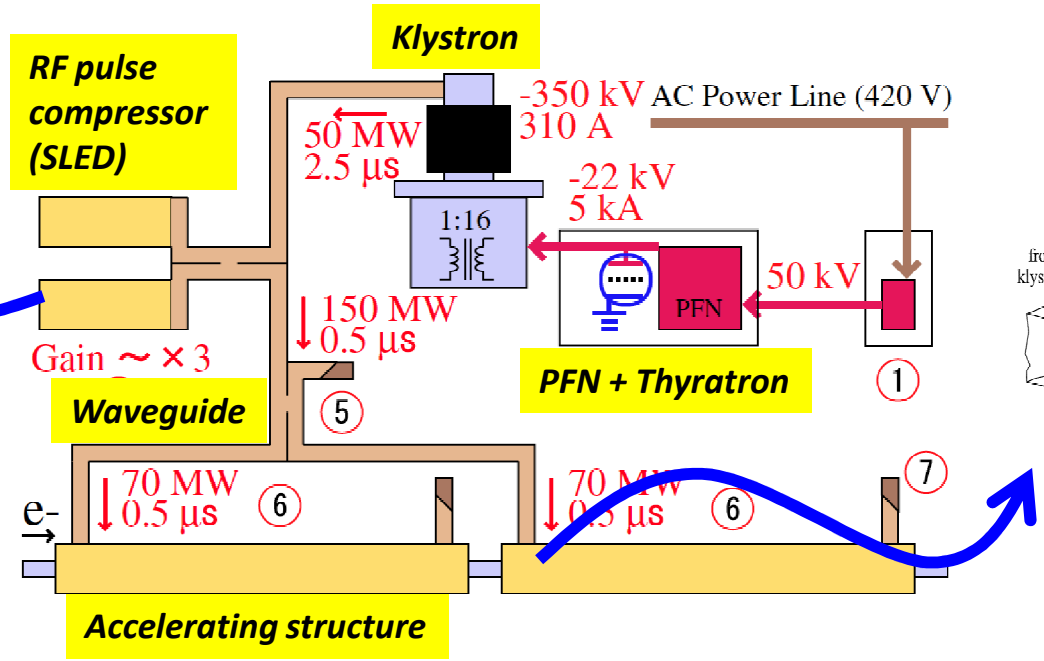
In this very simple picture, the **total energy gain** is just:

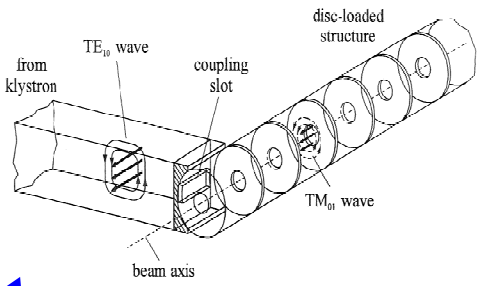
$$\Delta\gamma(L) = \frac{|e|}{m_e c^2} V_0 \cos(\phi_{RF})$$

RF generation (mention)

Pictures courtesy of
S. Milton







- Typical klysrtron power is **30 – 70 MW, over a few µs**.
- **SLED** cavities allow to gain a **factor of 3–4 in peak power**, reducing the RF pulse length to sub- µs duration.
- If the total power is enough to reach the maximum peak field tolerable by the cavity (before inducing discharges in the structure due to surface emission), the waveguide can be split into two lines to **supply multiple structures**.
- The final **RF pulse duration** has to be **longer** than the structure **filling time**.

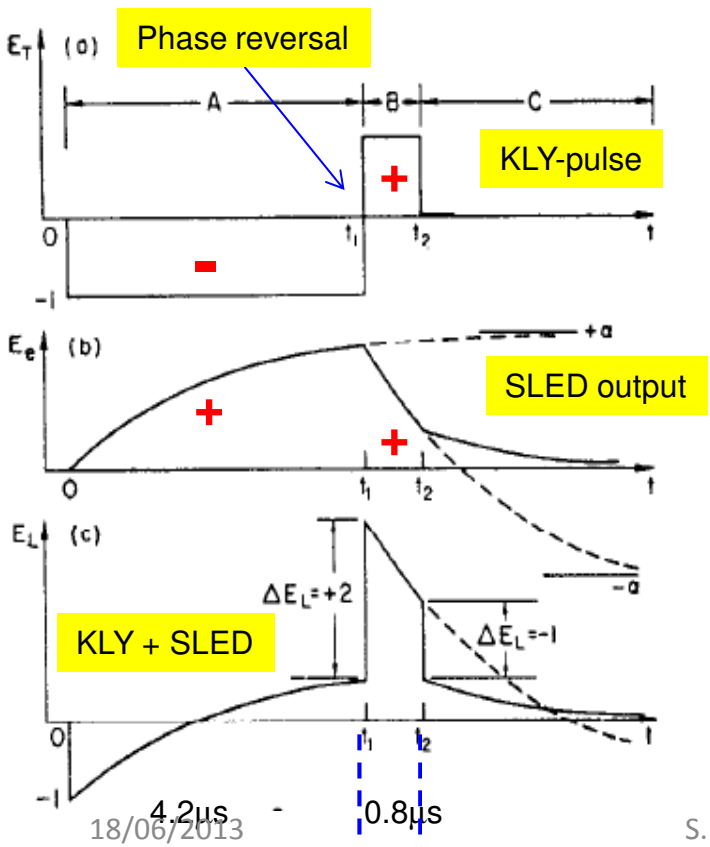
SLED – Slac Energy Doubler through RF pulse compression

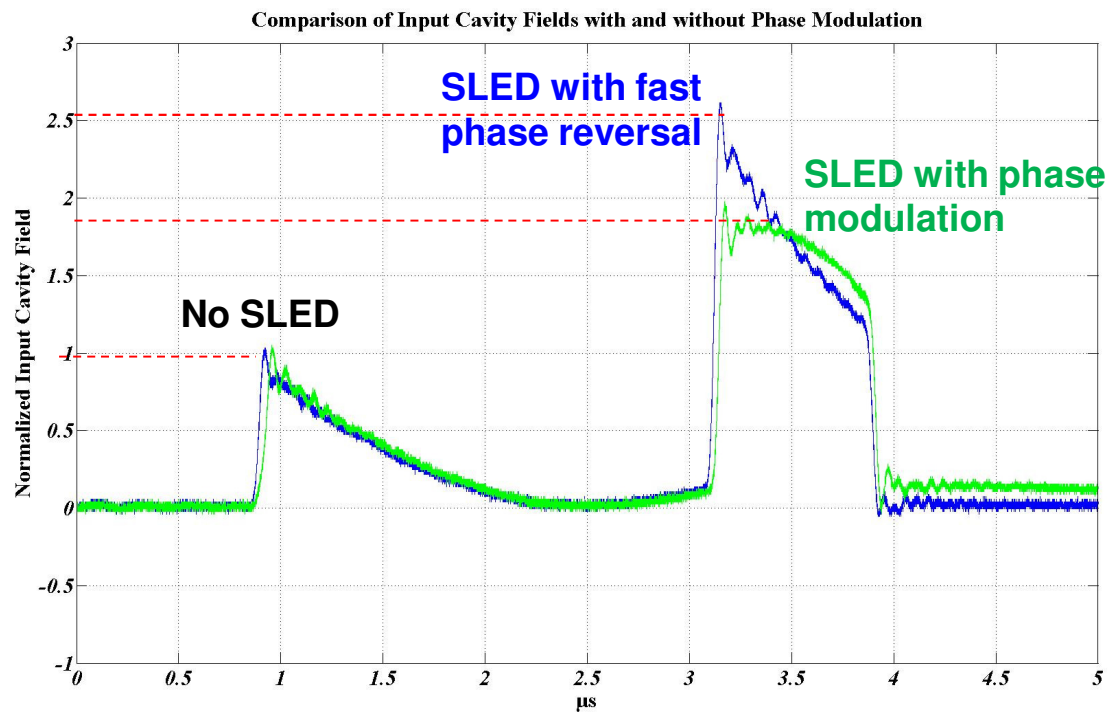
Pictures by C. Serpico

When the *structure breakdown limit is **far** from being achieved*, the *peak accelerating field* (gradient) can be *increased* with a SLED system.

This is a high Q-cavity which stores klystron energy during a large fraction of each RF pulse, and then discharges it rapidly into the accelerator. So, the output RF pulse is shortened, the peak power increased, as well as the available accelerating voltage.

Factors of 3–4 in peak power are typically and readily achieved. These have to be integral part of the RF distribution design.





Choosing the basic RF parameters – 1/2

1. Keep things **compact** → we want a **high acc. gradient** → **high acc. voltage** (for *L* fixed)

2. Have a **large statistics** → we want a **high rep. rate** → keep the **RF power low** (thermal load)

We keep $L = V_0/E_z$ fixed and scale all other dimensions as f^{-1}. This will allow us to identify the workable frequency range depending on the linac specific application:

NC stays for "**normal-conducting**": warm linacs whose thermal load threshold usually limits the max. rep. rate.

SC stays for "**super-conducting**": He-liquid frozen structures that allow higher rep. rates than NC linacs.

Surface resistance: $$R_{surf} \propto \begin{cases} f^{1/2} & for\ NC \\ f^{2} & for\ SC \end{cases}$$

Dissipated RF power: $$P_D = \frac{R_{surf}}{2}\int \left|\frac{B}{\mu_o}\right|^2 dA \propto \begin{cases} f^{-1/2} & for\ NC \\ f & for\ SC \end{cases}$$

Quality factor: $$Q = \frac{\omega_{RF} U}{\pi P_D} \propto \begin{cases} f^{-1/2} & for\ NC \\ f^{-2} & for\ SC \end{cases}$$

Shunt impedance per unit length: $$r_s = \frac{\hat{E}_z^{\,2} L}{P_D} \propto \begin{cases} f^{1/2} & for\ NC \\ f^{-1} & for\ SC \end{cases}$$

Available f (sort of): L-band (1.3GHz), S-band (2.9GHz), C-band (5.7GHz) and X-band (11.4GHz)

Choosing the basic RF parameters – 2/2

- ***r_s/Q*** $\propto$ acc. gradient for a given stored energy (only geometry dependent) ***$\propto f$ in all cases***

- ***NC linacs*** gain from **high *f*** (*e.g.*, X-band now under study for accelaration). Doing so, they maximize the energy gain, while being **weakly dependent from dissipative processes** (see previous slide).

  - However, generally ***$f \propto 1/a$***, with a = cell's iris radius.
  - Image charges on the walls kick the bunch ("***wakefield***") with a strength ~ $1/a^2$ (longitudinal kick) and ~ $1/a^4$ (tansverse kick).
  - ***High frequencies start exciting wakefield instability*** *(mainly transverse).* Beam manipulation techniques are typically adopted to overcome this issue.

- ***SC linacs*** gain from **high *f*** but they are more **stressed by a high thermal load** than NC because of their (additional and expensive) cryogenic system.

  - They are well suited for ***low / intermediate f*** (*e.g.*, L-band are well tested since many years).
  - Single-bunch wakefields are suppressed by the rather ***large a*** at low frequency (see above).
  - However, trains of bunches start exciting ***multi-bunch wakefields*** resonant in the structures. High order mode dampers are typically adopted in the structure design to suppress them.

Examples

Parameter	S-band, NC	X-band, NC
Working frequency, f [GHz]	2.998	11.424
Type	TW, Constant impedance	TW, Constant gradient
Phase Advance per Cell	$2\pi/3$	$2\pi/3$
Length, L [m]	3	1
Average Iris Radius, *a* [mm]	10.0	3.5
Group Velocity, v_g [c]	0.01	0.01 (average)
Filling Time [μs]	?	?
Quality Factor, πQ	12000	8600
Attenuation Factor, τ [neper]	?	?
Shunt Impedance, r_s [MΩ/m]	55	80
Input Peak Power, P_{RF} [MW]	45	80
RF Pulse Duration [μs]	**2.0**	**1.5**
Acc. Gradient, eV_0/L [MV/m]	?	?
Repetition Rate [Hz]	50	50

Design and parameters optimization really depends on what you take care more of....

RF distribution

1. For any **design energy**, I want at the same time to:
   - minimize the overall linac length,
   - minimize the RF power comsumption,
   - be able to operate at high repetition rate.

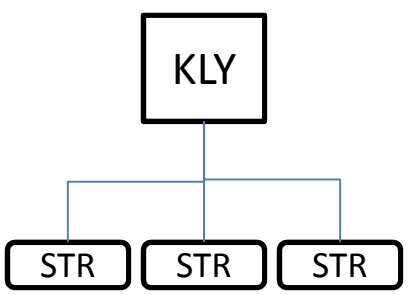
2. I need **high shunt impedance**, **high gradient** structures requiring **low imput power**.
   - The trade off is in RF power (cost of RF suppliers and electricity) and space (cost of accelerating structures and linac building).

3. The problem can thus be faced as follows:
   - as initial step, fix the structure's geometry (frequency, length, shunt impedance);
   - balance the RF power per structure with the **number of klystrons** needed to achieve the goal final energy, E_f.

acc. gradient

$$E_f \cong NGL = eNV_0 = eN\sqrt{P_{RF} r_s L}$$

# of structures structure's length



M ≡ *multiplicity* = number of structures per klystron

Klystron multiplicity

- **Small *M*** (<2:1) implies high P_{RF} that is:
  - high V_0, small N
  - a *compact* linac but *many klystrons*

This choice is limited by the **structure breakdown limit** at high accelerating gradients.

Many klystrons and modulators may imply a tighter packing of RF components thus more space needed in the "klystron gallery".

Well suited for **compact, NC** linacs at **low repetition rate**.

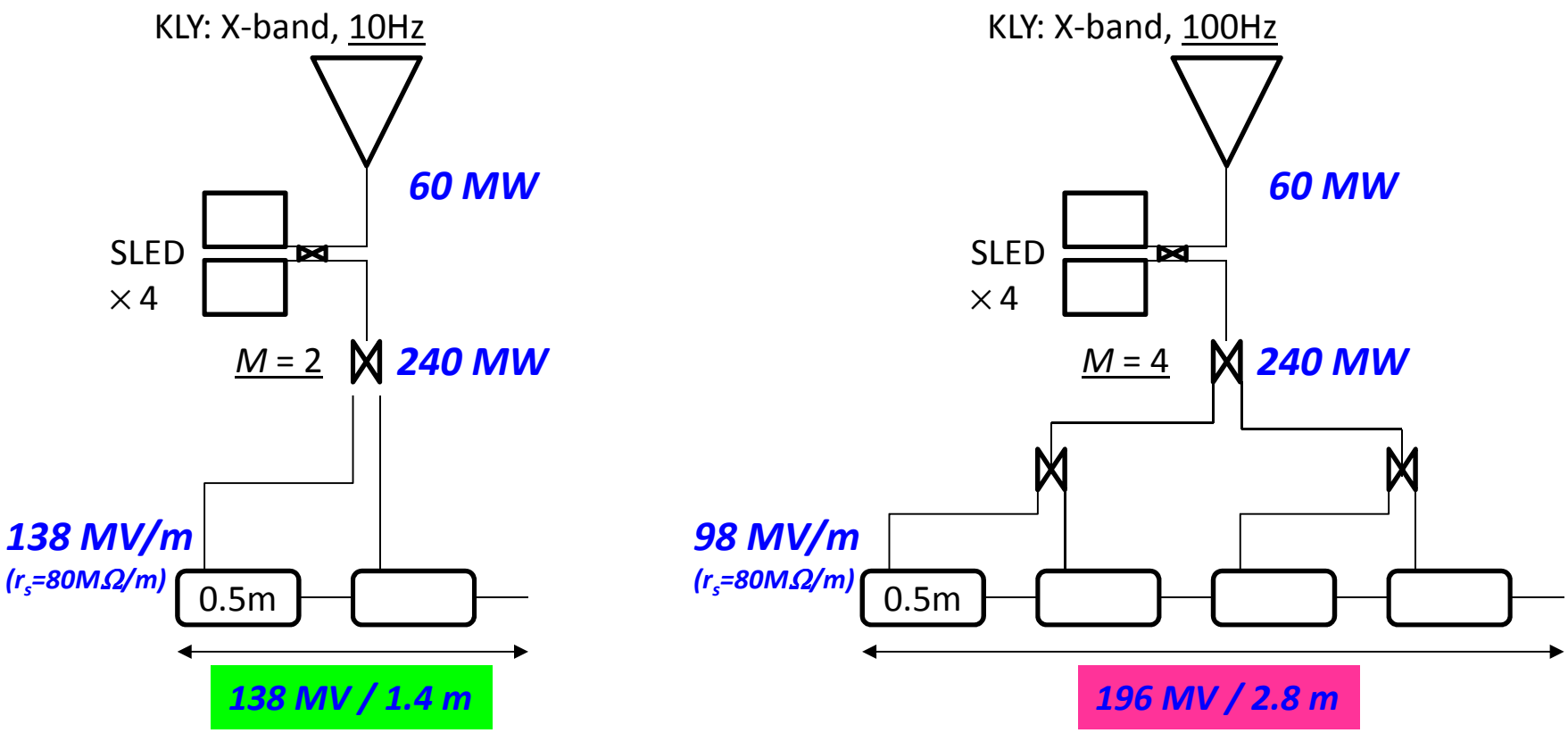
- **Large *M*** (>4:1) implies low P_{RF} that is:
  - low V_0, large N
  - *few klystrons* but a *long linac*

This choice is limited by the **available space** for the linac.

Many structures may imply a more complex cooling system.

Well suited for **SC** linacs or **NC** linacs at **high repetition rate**.

Examples



Assume E_0 = 100MeV, E_f = 3GeV, and a periodic linac layout.

Case *M = 2* gives (3000–100)MeV / 138MeV = *21 KLYs, 21 girders* and a *29 m* long linac *@10Hz*

Case *M = 4* gives (3000–100)MeV / 196MeV = *15 KLYs, 30 girders and a 42 m* long linac *@100Hz*

Power consumption

The *average* power sent to the structure is $\left\langle P_{RF,0} \right\rangle = P_{RF,0} \cdot \Delta t_{RF} \cdot rep.rate$

Case M = 2: $\langle P_{RF,0} \rangle$ = 120MW · 0.7μs · 10Hz = <u>0.84 kW / structure</u>
Case M = 4: $\langle P_{RF,0} \rangle$ = 60MW · 0.7μs · 100Hz = <u>4.20 kW / structure</u>
We do not expect strong cooling requirements.

For such a structure, the attenuation factor is $\tau \approx 1.5$ neper. Being TW-CG, we evaluate that $(1 - e^{-3})$ = 5% of input power goes to the load, while <u>95% of that is used for acceleration</u>. Thus, we have designed an efficient RF acceleration system.

i. Klystron power = 60MW · 1.4μs · 100Hz / 0.8 (efficiency) = 10.5 kW
ii. Modulator power = 10.5kW / 0.9 (efficiency) = 11.7 KW
iii. Using 1 klystron per 4 structures (M = 4), we require 11.7KW · 15 RF stations = <u>0.18 MW</u>

iv. We should add about 0.1 MW for the injector, 0.1 MW for the water cooling, 0.1 MW for the magnets and 0.1 MW for basic building power. In total we require <u>0.6 MW for a system this size</u>.