Velocity bunching & more on magnetic-chicane bunch compression

MV

last revised 15-June

Brief summary of what we learned yesterday

- **Beam energy change through rf structure**
  - Ultrarelativistic approx.
- **Setting of RF phase to control the beam energy chirp**
- **Concept of magnetic-chicane bunch compressor**
  - Orbit path-length dependence on particle energy
  - Correlation between particle energy and its position along the bunch (energy chirp)
- **Momentum compaction R_{56} for 4-bend C-shaped chicane**
- **Compression factor**
- **Linearization of the longitudinal phase-space using harmonic rf cavities**

Outline

1. **Compressing low-energy bunches**
   - Ballistic compression
   - Velocity bunching

2. **More on magnetic-chicane compression**
   - Sensitivity of beam jitters to RF parameters
   - One-stage vs. multiple-stage compression.

Compression at low energy: velocity bunching

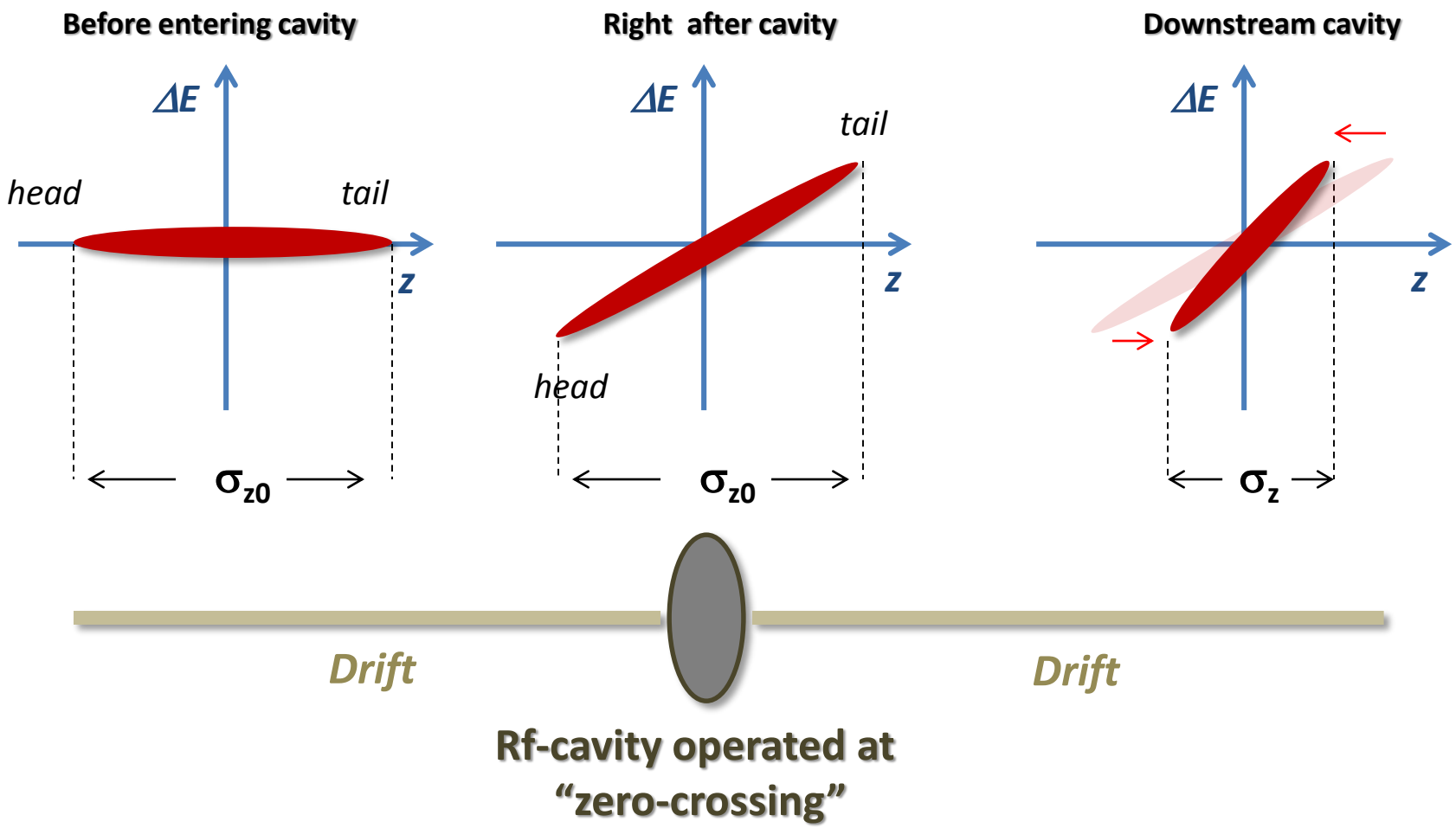
- High beam energy **(v~c)**: different energies -> no difference in velocity (c).
  - Bends provide the dispersion needed to establish a dependence of the path-length on beam energy that can be exploited to do compression

- Low beam energy **(v<c)**: different energies -> difference in velcity can be significant
  - Exploit different times of arrival by particles with different velocities to do compression in straight non-dispersive channels.

- **In some cases low-energy compression is desirable**
  - Typically when the e-source generates beams with too low a current.
  - Desirable feature compared to magnetic compression (no CSR effects). But it is more vulnerable to space-charge effects

- **Note on use of terms:**
  - Velocity-bunching compression can be done during acceleration
  - The term "**Ballistic compression**" is often used when compression is done *without* accelerating
  - "Velocity bunching" is the more generic term
  - "RF compression" also used

Conceptual picture of ballistic compression



- For compression, particles in the tail should have larger energy (velocity)
  - Same sign of energy chirp as for magnetic compression in 4-bend C-Shape chicane

Example of Ballistic Compression in action: Studies for the NGLS injector

186 MHz Normal Conducting RF-Gun
Accelerating electrons to 1.25 MeV energy (~750keV kinetic energy)
Accelerating gap: ~4cm (19MV/m field at cathode)
(APEX- prototype presently under commissioning)

Solenoids *(emittance compensation)*

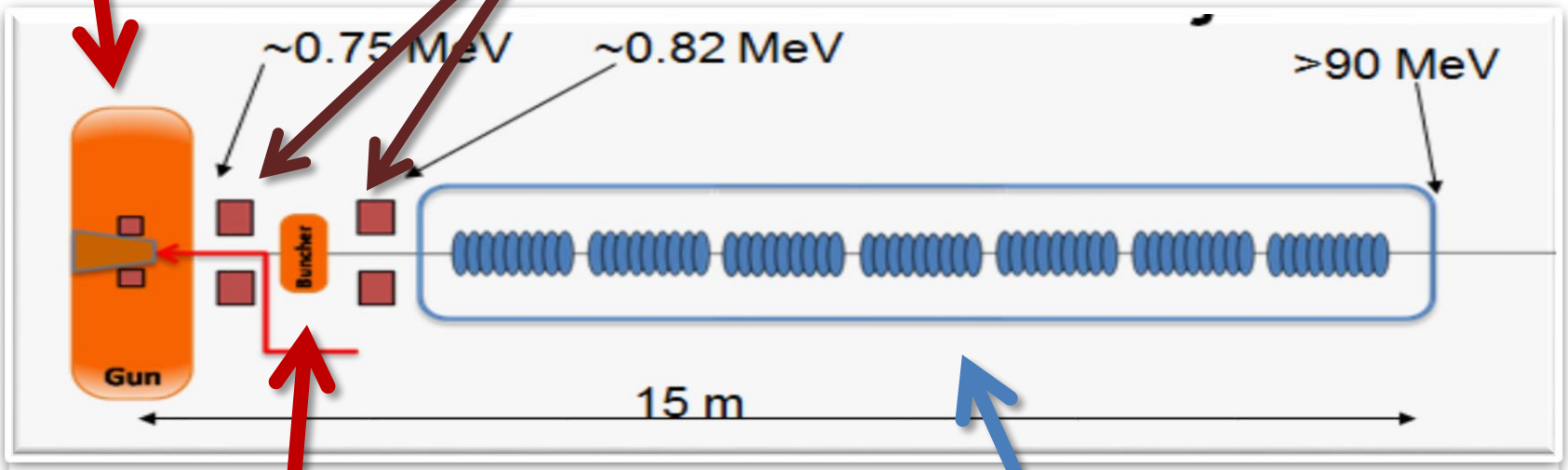
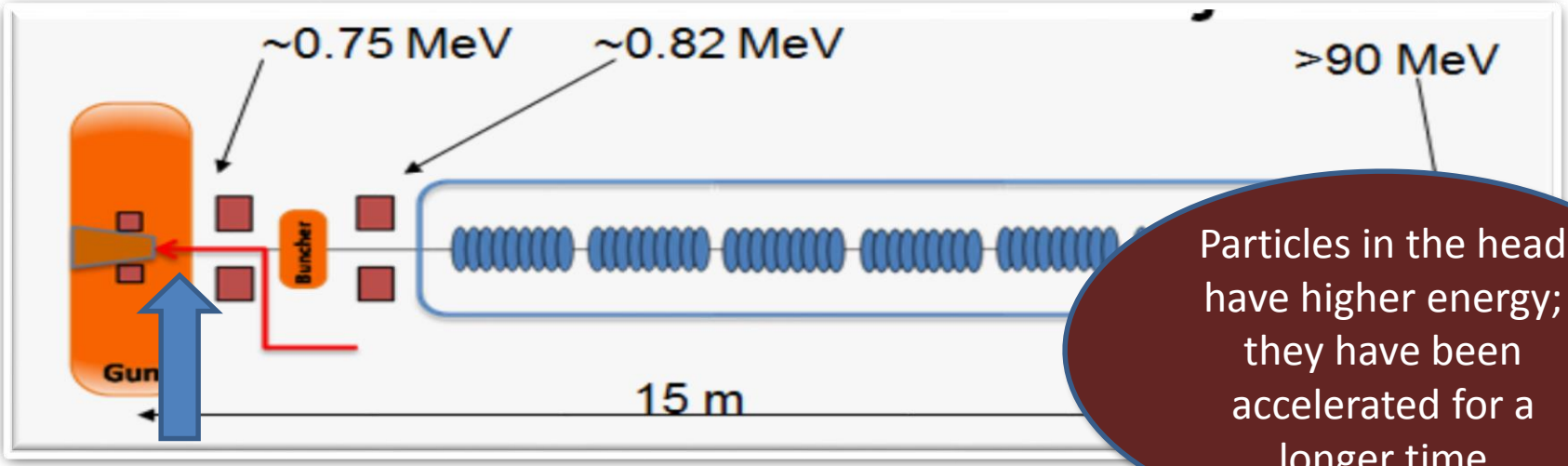


Fig courtesy of C. Papadopoulos

"Buncher"
Single 1.3 GHz RF cavity
(Normal Conducting)

"Booster"
1.3 GHz Cavities (Tesla style)
(Super Conducting)

Beam half-way through gun accelerating gap

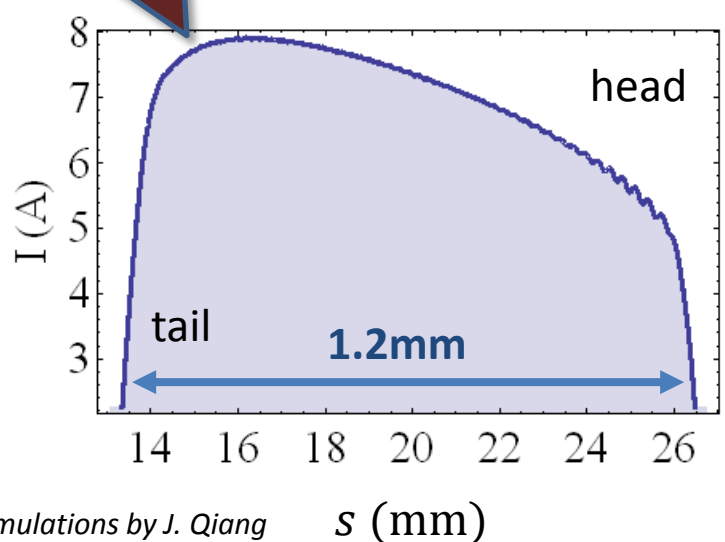


Particles in the head have higher energy; they have been accelerated for a longer time

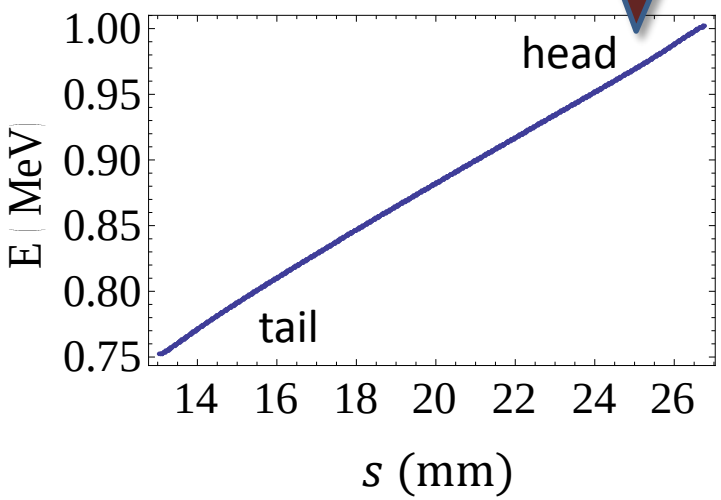
Relatively low-current

Beam snap-shot in time*

Current profile

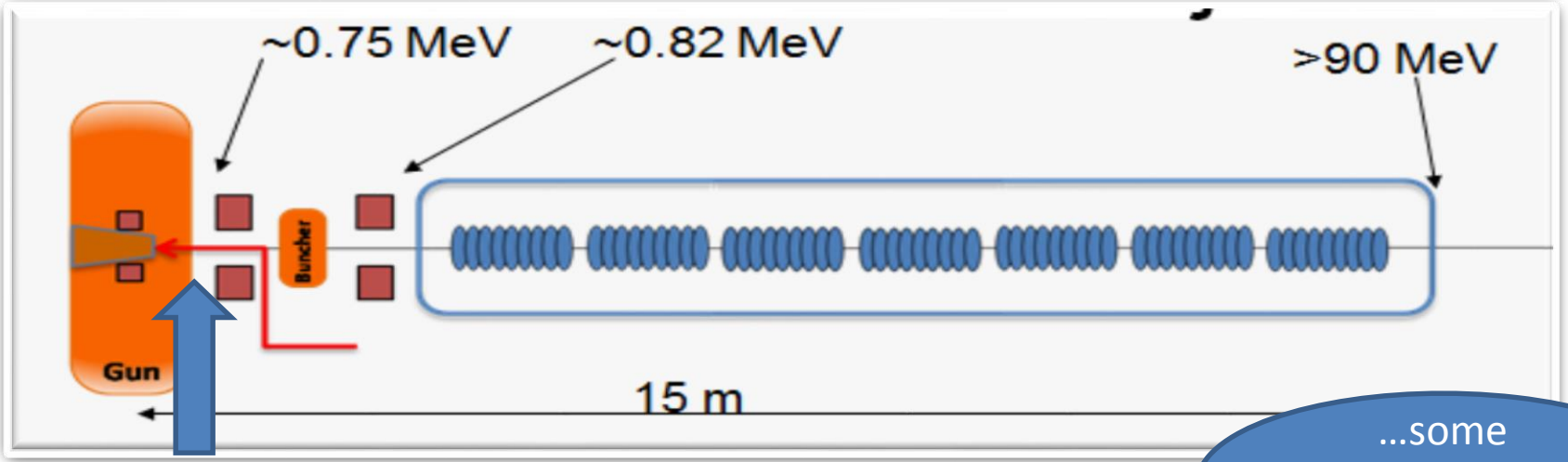


Energy profile



**IMPACT simulations by J. Qiang*

Beam at exit of gun accelerating gap

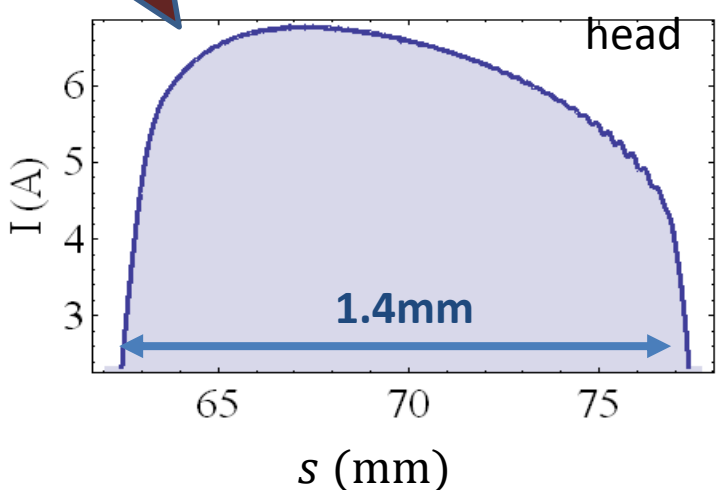


Current drops slightly

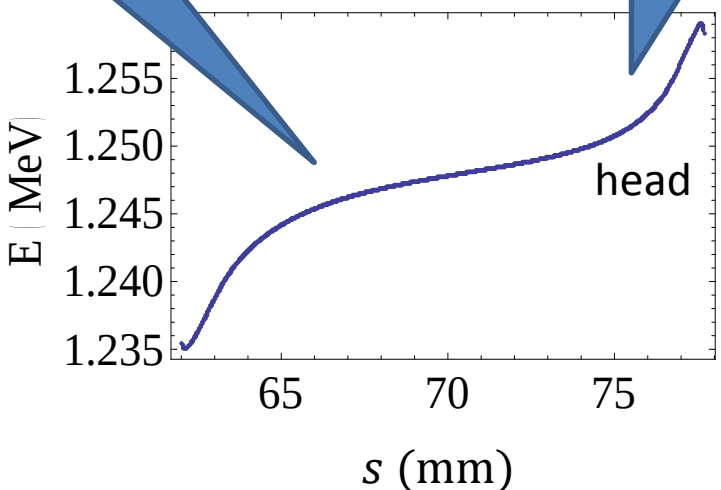
Largely flattened…

…some space-charge induced chirp

Current profile



Energy profile



Beam at entrance of buncher

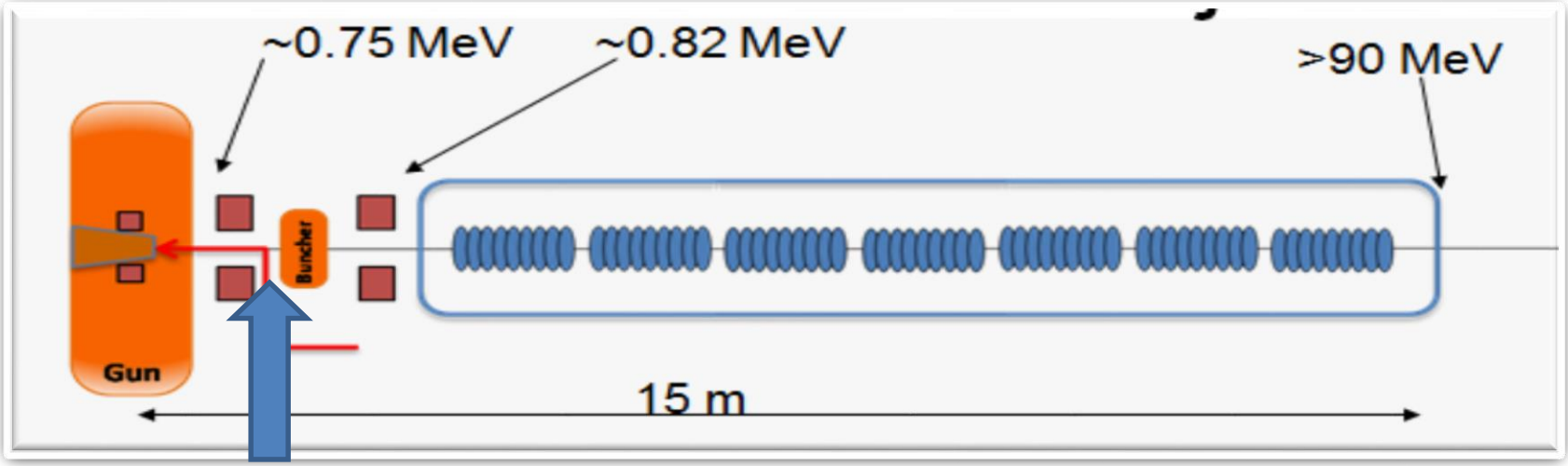
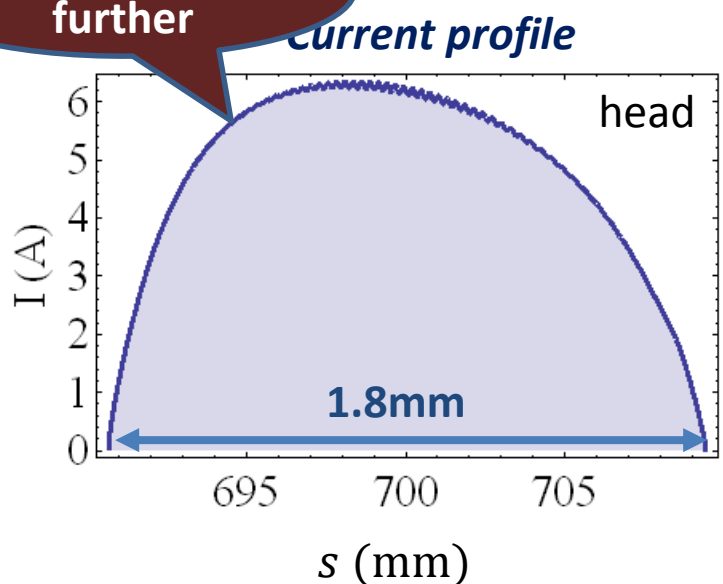
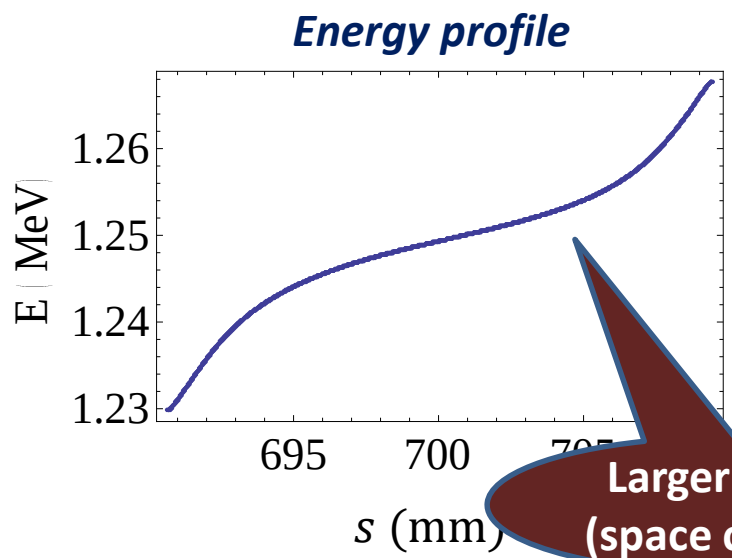


Fig. from C. Papadopoulos

Current drops further





Larger Chirp (space charge)

Beam at exit of buncher

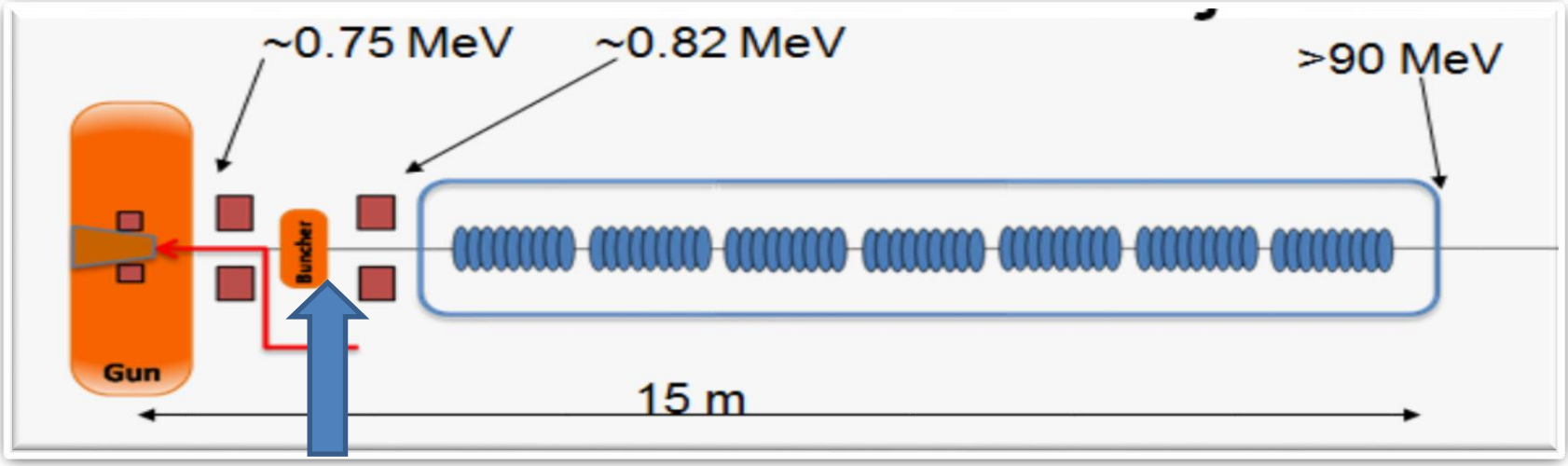
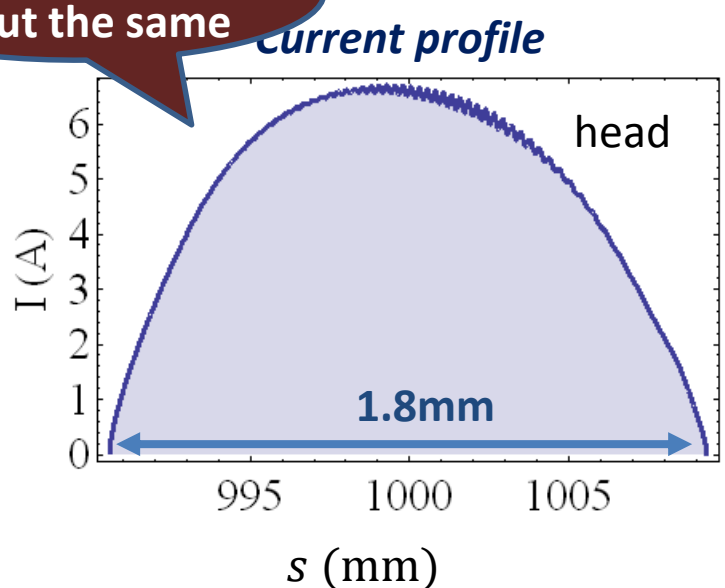
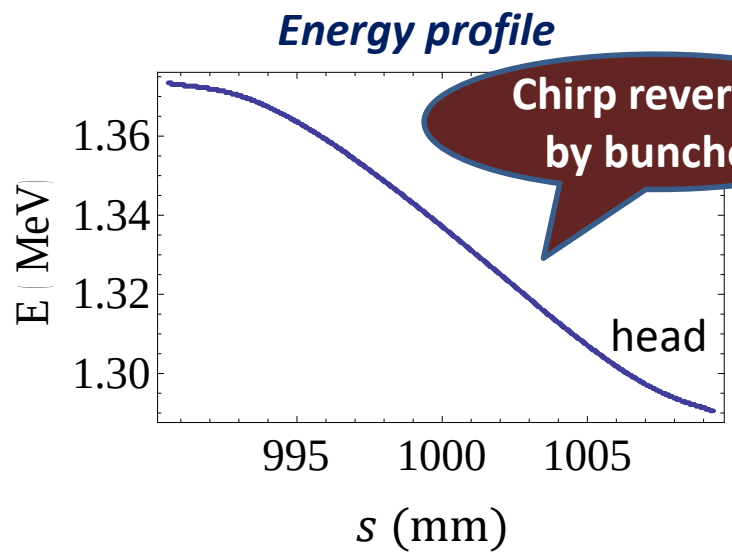


Fig. from C. Papadopoulos

Current is about the same





Beam just downstream the buncher

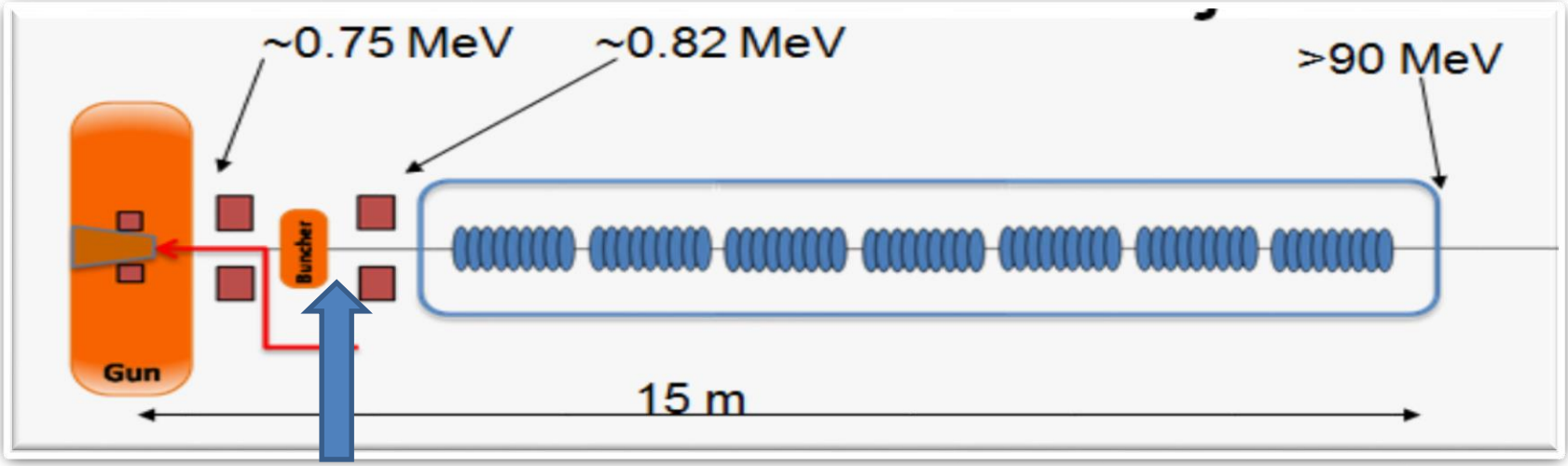
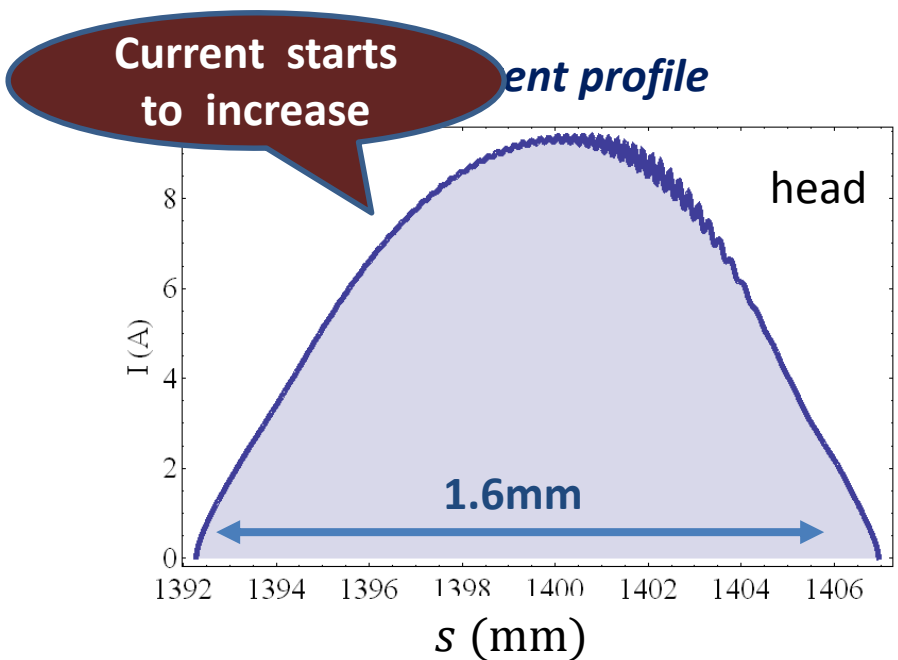
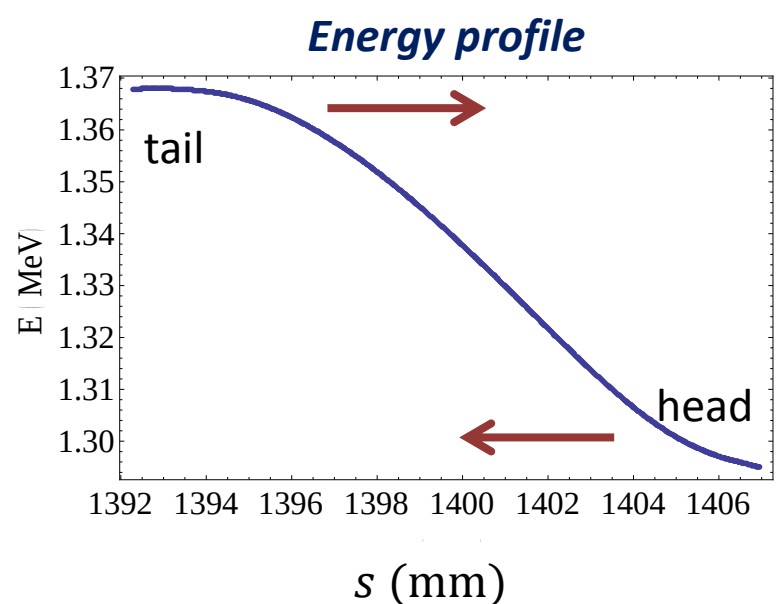


Fig. from C. Papadopoulos





Beam 1.2m downstream the buncher

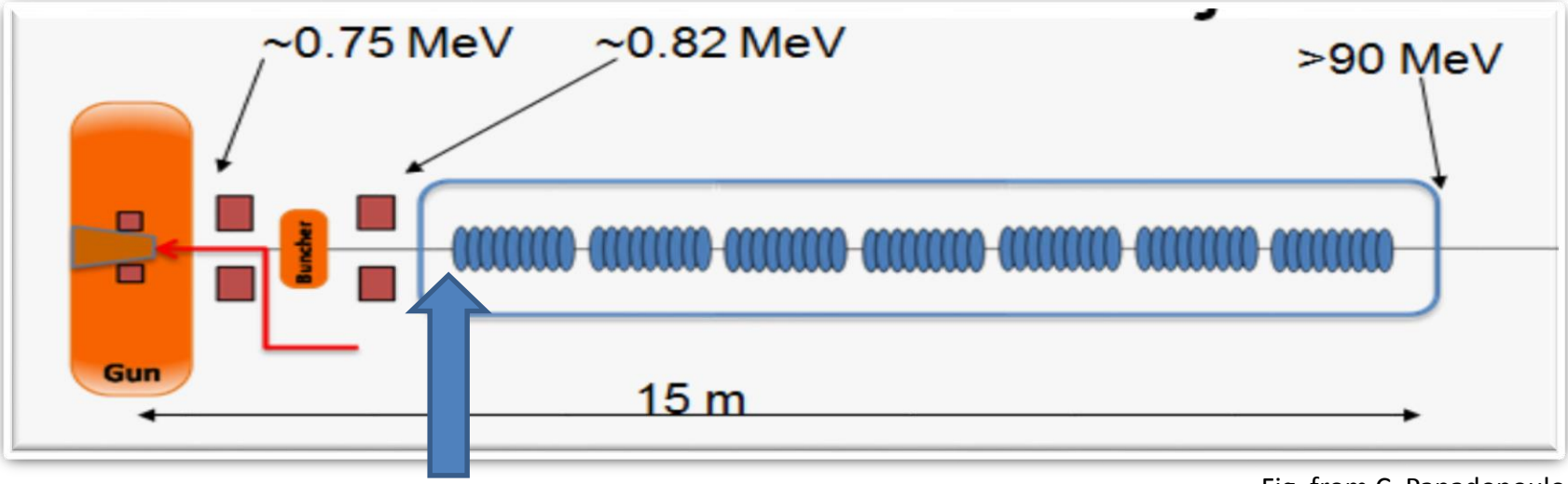
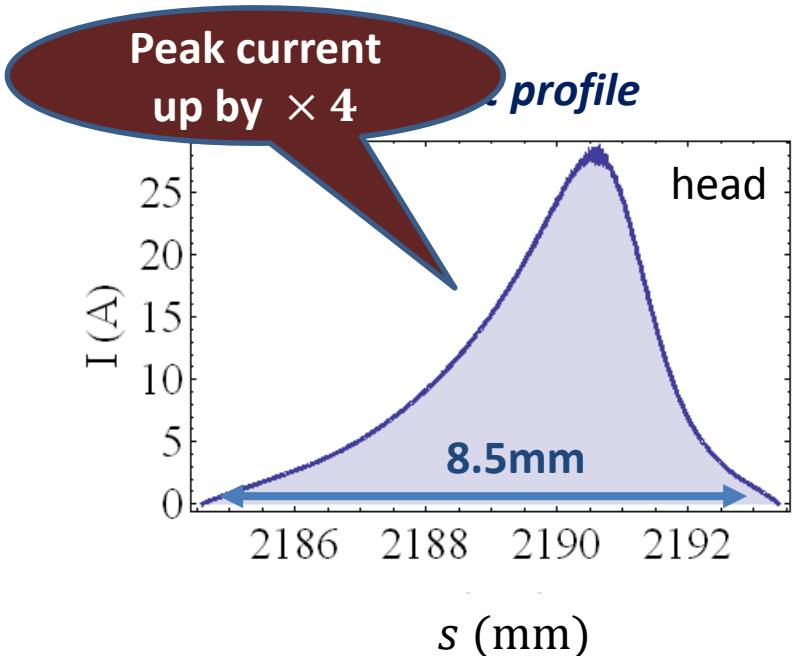
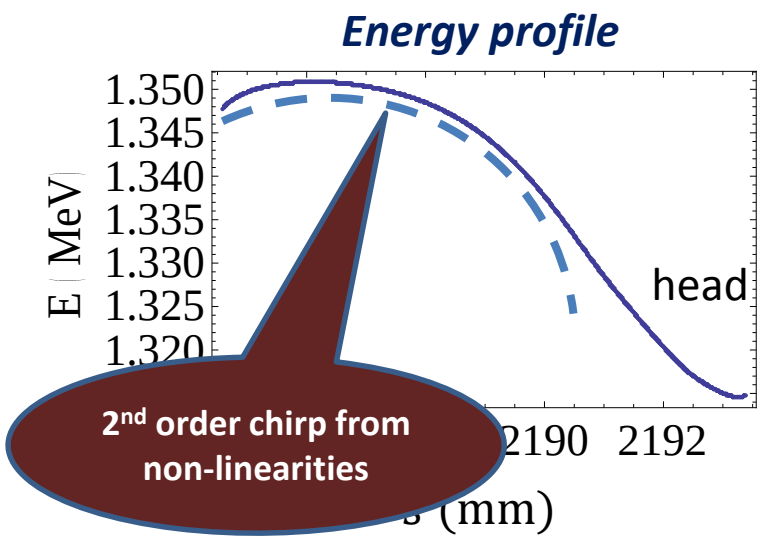


Fig. from C. Papadopoulos



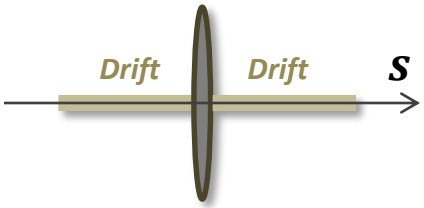


Ballistic compression: expression for compression factor (linear approximation)

Relativistic equation For longitudinal motion (with s as the independent variable)

$$\begin{cases} \dfrac{dt}{ds} = \dfrac{\gamma}{c\sqrt{\gamma^2-1}} \\ \dfrac{d\gamma}{ds} = -\dfrac{eE_{s0}(s)}{mc^2}\cos(\omega_{\mathrm{rf}}t+\varphi_{\mathrm{rf}}) \end{cases}$$

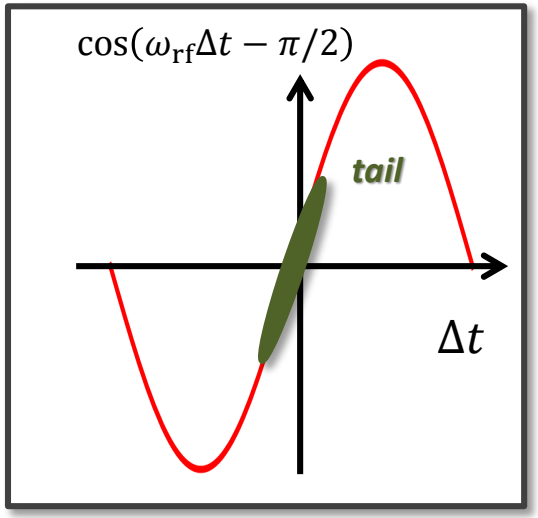
Standing wave RF cavity



Rf-cavity operated at "zero-crossing"

- Assume kick approximation for cavity (0-length cavity length at $s=0$).

$$-\frac{eE_{s0}(s)}{mc^2} = \frac{eV_0}{mc^2}\delta(s)$$ ← *Dirac δ function*



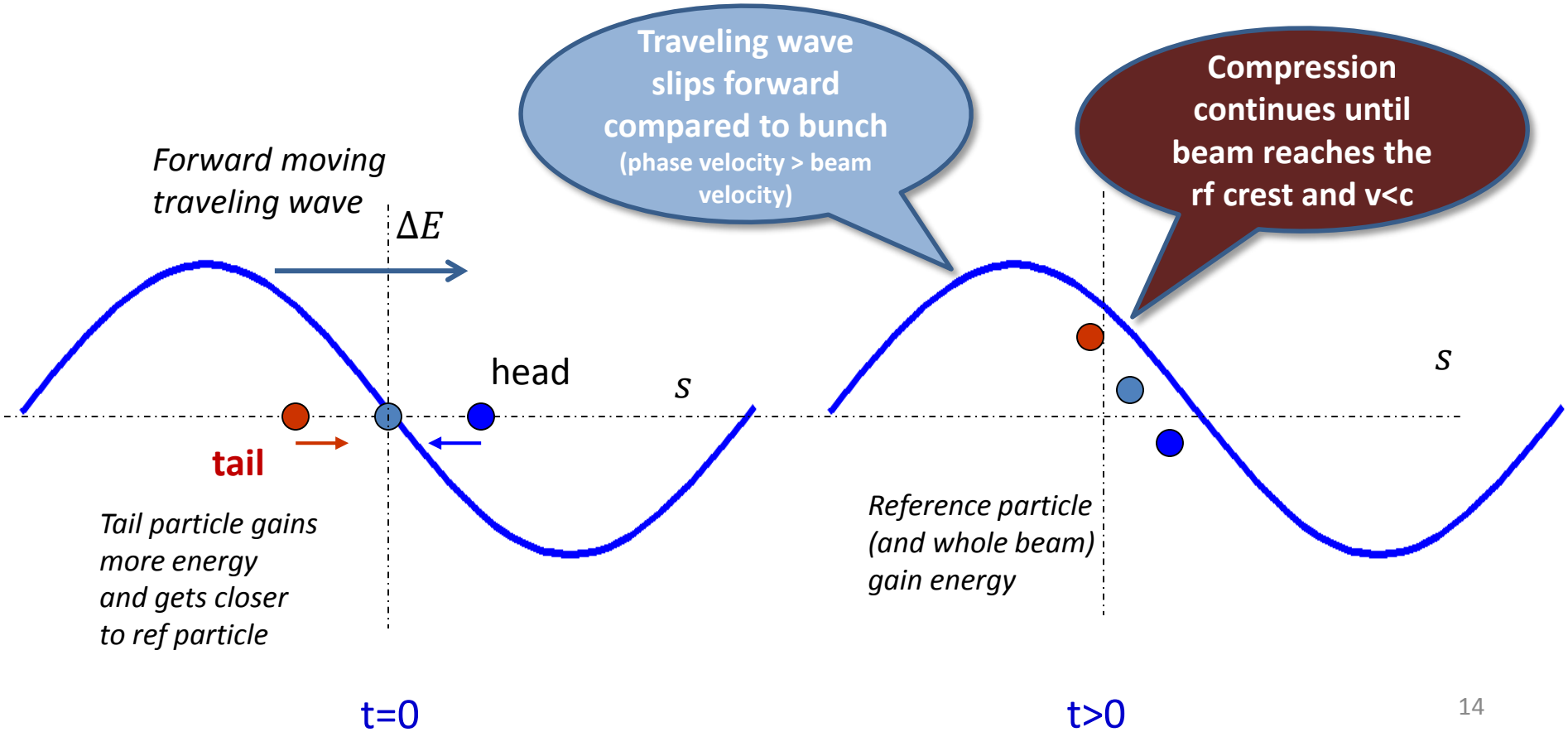
- Expand equations about reference orbit $t = t_r + \Delta t$, $\gamma = \gamma_r + \Delta\gamma$
- Find the equations for Δt and $\Delta\gamma$ (first order)
- Define time so that $t=0$ when the ref. particle crosses the cavity
- Set rf phase so that the reference particle goes through the cavity at "zero crossing" : $\cos(\varphi_{\mathrm{rf}}) = 0$ (i.e. energy of reference particle doesn't change) -> $\varphi_{\mathrm{rf}} = \pm\pi/2$
- Choose $\varphi_{\mathrm{rf}} = -\pi/2$. (To have the right energy chirp - higher energy particles in the tail.)
- Compression factor:

$$C \equiv \frac{\Delta t(s)}{\Delta t(s=0)} = \left[1 - \frac{eV_0}{mc^2}\frac{sk_{\mathrm{rf}}}{(\gamma^2-1)^{\frac{3}{2}}}\right]^{-1}$$

Work-out details as an exercise

Velocity bunching: compress while accelerating

- If current out-of gun is already high further compression is best done while also accelerating (to reduce effect of space-charge forces; ease emittance compensation)
- Compression takes place over longer, multi-cavity structure
- Dynamics of compression is best illustrated in travelling-wave structures



Single-stage or multi-stage compression?

- Because of collective effects (space charge) max. amount of velocity bunching compression at low energy is limited by desire to preserve beam quality
  - Magnetic compression is still needed (usually done at sufficiently high energy to limit adverse impact of collective effects)

- Magnetic compression: is it better to do all the compression at once, or break it up through two or more chicanes?

- Favoring **multi-stage** magnetic compression:
  - CSR effects on transverse emittance (2nd and further stage compression done at higher energy to reduce CSR effects)
  - Reduced sensitivity to rf jitters

- Favoring **single-stage** magnetic compression:
  - Control of microbunching
  - Reduced system complexity (not critical)

Examples of velocity-bunching compression experiments

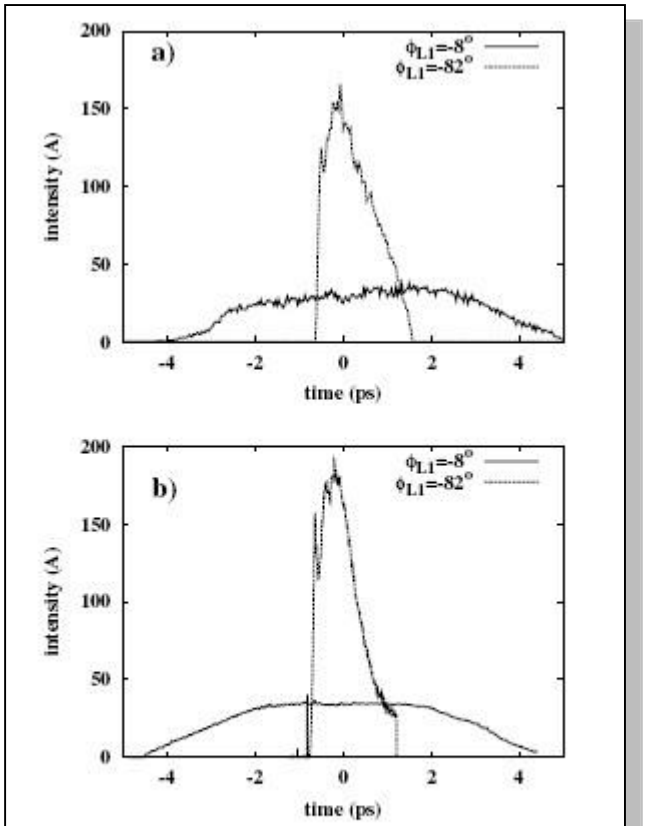
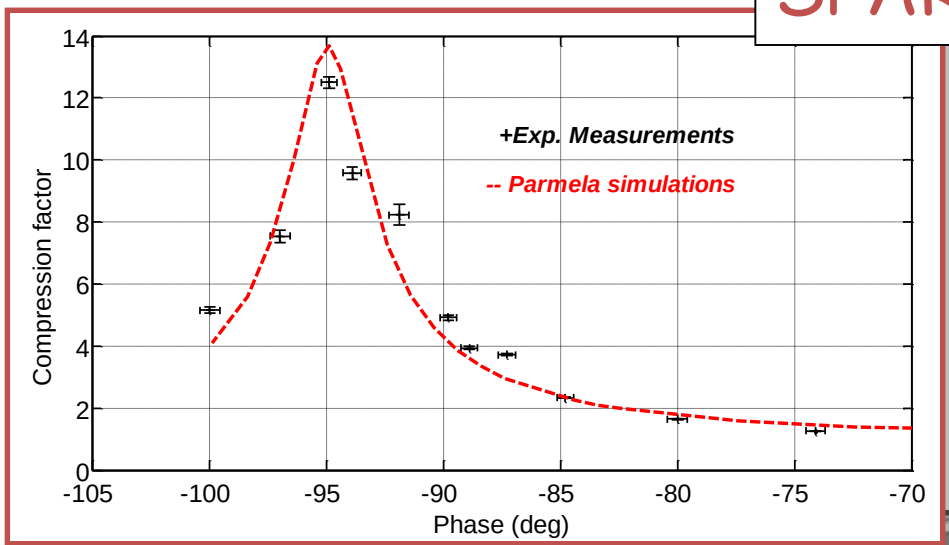
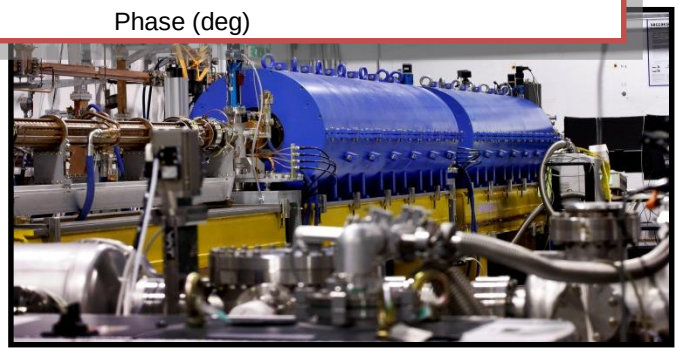


FIG. 6. Comparison of the bunch time profile for L1 −8° and −82° off crest. (a) was generated by tracking simulation; (b) is a direct measurement using the zero-phasing method. The time > 0 corresponds to the bunch tail

Piot et al. Phys. Rev. ST Accel. Beams 6, 033503 (2003)

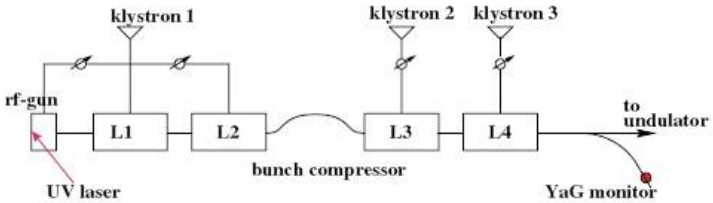
SPARC





Ferrario et al.
Phys. Rev. Lett. 104, 054801 (2010)

DUV-FEL
BNL



More on Magnetic Compression

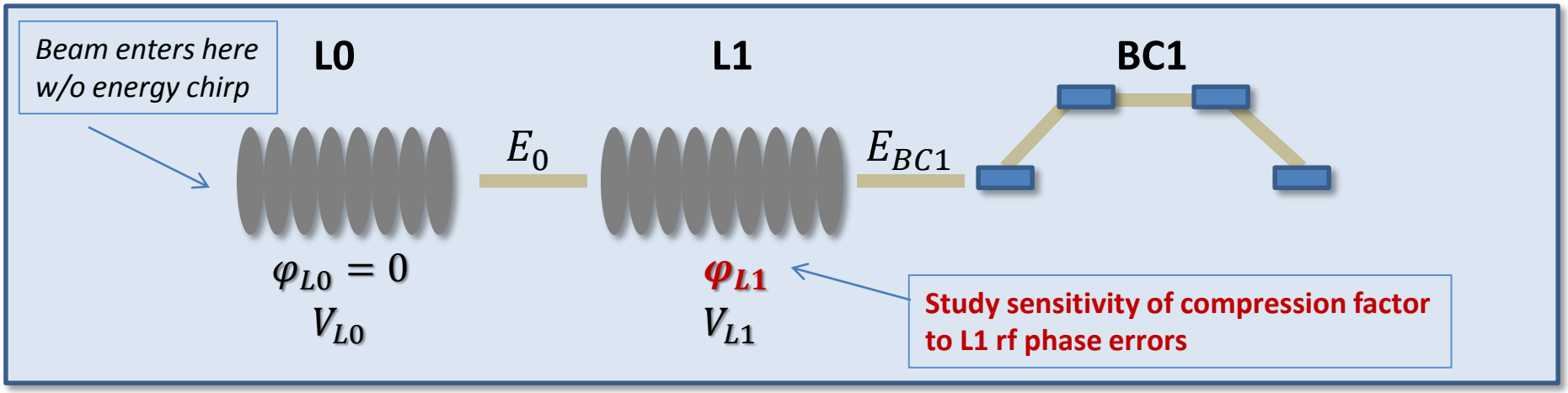
Multi-stage magnetic compression is more robust against various sources of jitters

- **Fluctuations of rf structure parameters (voltage, phase) around set values are unavoidable.**

- **They cause undesirable "jitters" in**
  - Final electron beam energy
  - Beam arrival time
  - Beam peak current

- **E.g. for Superconducting Structures (typically easier to stabilize) aggressive but not unreasonable targets for max. rf fluctuations are**
  - 0.01deg (rf phases)
  - 0.01% (rf voltages)

Example 1: Sensitivity of compression factor to RF phase. Single-stage magnetic compression



Compression factor: $C = \dfrac{1}{1 + hR_{56}}$

Linear chirp after L1: $h = -\dfrac{eV_{L1}\mathrm{k_{rf}}\sin\varphi_{L1}}{E_{BC1}}$

$$\frac{\Delta C}{C} = \frac{1}{C}\frac{\partial C}{\partial h}\Delta h = -\frac{1}{C}C^2R_{56}\Delta h = -CR_{56}h\frac{\Delta h}{h} = -C(C^{-1}-1)\frac{\Delta h}{h} = (C-1)\frac{\Delta h}{h}$$

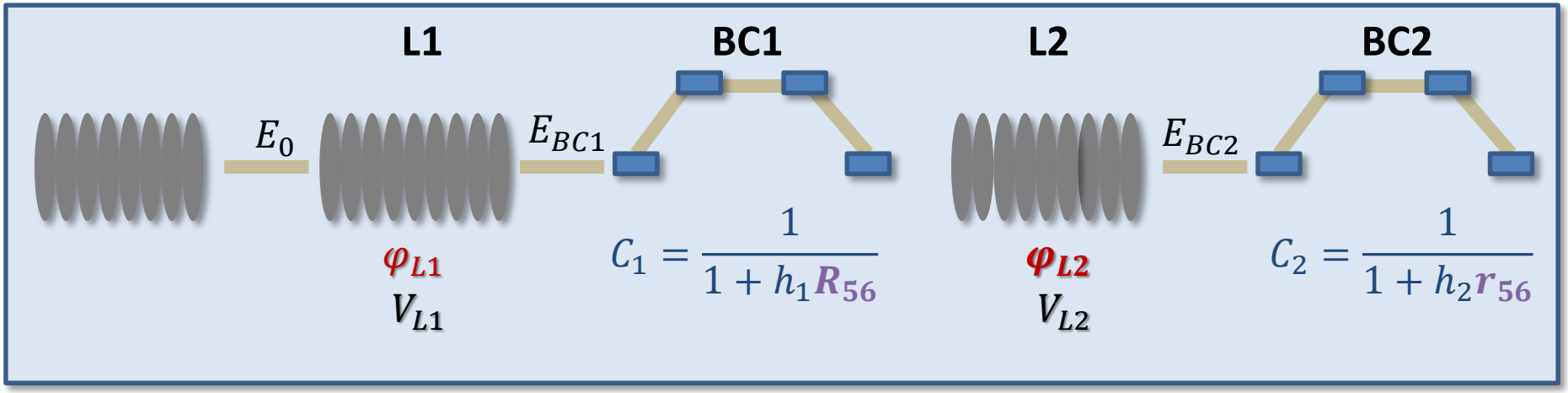
$$\frac{\Delta h}{h} = \frac{1}{h}\frac{\partial h}{\partial \varphi_{L1}}\Delta\varphi_{L1} = -\frac{E_{BC1}}{eV_{L1}\mathrm{k_{rf}}\sin\varphi_{L1}}\left(-\frac{eV_{L1}\mathrm{k_{rf}}\cos\varphi_{L1}}{E_{BC1}}\right)\Delta\varphi_{L1} = \frac{\Delta\varphi_{L1}}{\tan\varphi_{L1}}$$

$$\boldsymbol{\frac{\Delta C}{C} = (C-1)\frac{\Delta\varphi_{L1}}{\tan\varphi_{L1}}}$$

singular at 0-phase (for fixed compression C)

Proportional to $C - 1 \simeq C$

Example 2: Sensitivity of compression factor to RF phase. Double-stage magnetic compression



chirp contributed by L1

chirp contributed by L2

$$a_1 \equiv -\frac{eV_{L1}\mathrm{k_{rf}}\sin\varphi_{L1}}{E_{BC1}}$$

$$a_2 \equiv -\frac{eV_{L2}\mathrm{k_{rf}}\sin\varphi_{L2}}{E_{BC2}}$$

$$h_1 = a_1$$

$$h_2 = C_1 a_1 \frac{E_{BC1}}{E_{BC2}} + a_2$$

Suppose $C_1 a_1 \frac{E_{BC1}}{E_{BC2}} \ll a_2$ therefore $h_2 \simeq a_2$, and $C_2 \simeq \frac{1}{1+a_2 r_{56}}$

Total compression

$$C = C_1 C_2 \simeq \frac{1}{1 + a_1 R_{56}} \times \frac{1}{1 + a_2 r_{56}}$$

$$\frac{\Delta C}{C} = -\frac{1}{C}\left(C_1^2 R_{56} \Delta a_1 C_2 + C_1 C_2^2 r_{56} \Delta a_2\right)$$

Example 2: Sensitivity of compression factor to RF phase. Double-stage magnetic compression (cont'd)

$$C_1 = \frac{1}{1 + a_1 R_{56}} \qquad C_2 \simeq \frac{1}{1 + a_2 r_{56}}$$

$$\frac{\Delta C}{C} = -\frac{1}{C}\left(C_1^2 R_{56} \Delta a_1 C_2 + C_1 C_2^2 r_{56} \Delta a_2\right) = -C_1 R_{56} \Delta a_1 - C_2 r_{56} \Delta a_2$$

$$= -C_1 R_{56} a_1 \frac{\Delta a_1}{a_1} - C_2\, r_{56} a_2 \frac{\Delta a_2}{a_2}$$

$$= -C_1 \left(C_1^{-1} - 1\right) \frac{\Delta a_1}{a_1} - C_2 \left(C_2^{-1} - 1\right) \frac{\Delta a_2}{a_2}$$

$$\boldsymbol{\frac{\Delta C}{C} = \left(C_1 - 1\right)\frac{\Delta a_1}{a_1} + \left(C_2 - 1\right)\frac{\Delta a_2}{a_2} \sim 2\sqrt{C}\,\frac{\Delta \varphi}{\tan \varphi}}$$

Assuming $C_1 \sim C_2 \gg 1$,
$C = C_1 C_2$
$\Delta\varphi_{L1} \sim \Delta\varphi_{L2} \sim \Delta\varphi$
$\varphi_{L1} \sim \varphi_{L2} \sim \varphi$

For comparison, for single-stage we found $\frac{\Delta C}{C} = (C - 1)\frac{\Delta h}{h} \sim \boldsymbol{C\frac{\Delta \varphi}{\tan \varphi}}$

- Conclusion: in two-stage compression there is the potential to make sensitivity to rf phase jitter smaller
  - The condition for the most benefit is $C_1 a_1 \frac{E_{BC1}}{E_{BC2}} \ll a_2$. (Although it may be difficult to enforce)
  - In practice, compression sensitivity to phase jitters will be higher than predicted by the formula above but still reduced compared to one-stage compression

Summary

- RF compression is one more tool in the tool-box for beam manipulations
  - It has to be done at low energy (++ and --)
- Multi-stage magnetic compression is usually preferred as a way to reduce certain collective effects (CSR impact on transverse emittance)
  - It can make the beam more sensitive to other collective effects (microbunching instability).
- Sensitivity to rf parameter errors is smaller in multi-stage compression
  - E.g. compression (peak current) jitter dependence on rf phase errors

$$\frac{\Delta C}{C} = (C - 1)\frac{\Delta\varphi_{L1}}{\tan\varphi_{L1}}$$

Bonus material

Velocity bunching: analytical model

- **Travelling wave structure**
  - Straightforward extension of formalism to standing-wave structures (Decompose travelling wave into forward + backward travelling waves; effect of backward wave will average out)

$$\begin{cases} \dfrac{dt}{ds} = \dfrac{\gamma}{c\sqrt{\gamma^2 - 1}} \\ \dfrac{d\gamma}{ds} = -\dfrac{eE_{s0}(s)}{mc^2}\cos(k_{\mathrm{rf}}s - \omega_{\mathrm{rf}}t + \varphi_0) \end{cases}$$

Chang dynamical Coordinate from t to φ

$$\varphi = k_{\mathrm{rf}}s - \omega_{\mathrm{rf}}t(s) + \varphi_0$$

$$\frac{d\varphi}{ds} = k_{\mathrm{rf}} - \omega_{\mathrm{rf}}\frac{dt}{ds} =$$

$$= k_{\mathrm{rf}}\left(1 - \frac{1}{\beta}\right) = k_{\mathrm{rf}}\left(1 - \frac{\gamma}{\sqrt{\gamma^2 - 1}}\right)$$

New equations

$$\begin{cases} \dfrac{d\phi}{ds} = k_{\mathrm{rf}}\left(1 - \dfrac{\gamma}{\sqrt{\gamma^2 - 1}}\right) \\ \dfrac{d\gamma}{ds} = -\dfrac{eE_{s0}(s)}{mc^2}\cos(\phi) \end{cases}$$

Derivation of invariant

$$\begin{cases} \dfrac{d\phi}{ds} = k_{\mathrm{rf}}\left(1 - \dfrac{\gamma}{\sqrt{\gamma^2 - 1}}\right) \equiv \dfrac{\partial H}{\partial \gamma} \\ \dfrac{d\gamma}{ds} = \alpha\cos(\phi) \equiv -\dfrac{\partial H}{\partial \varphi} \end{cases}$$

System of equations can be thought of as a canonical system with effective Hamiltonian H:

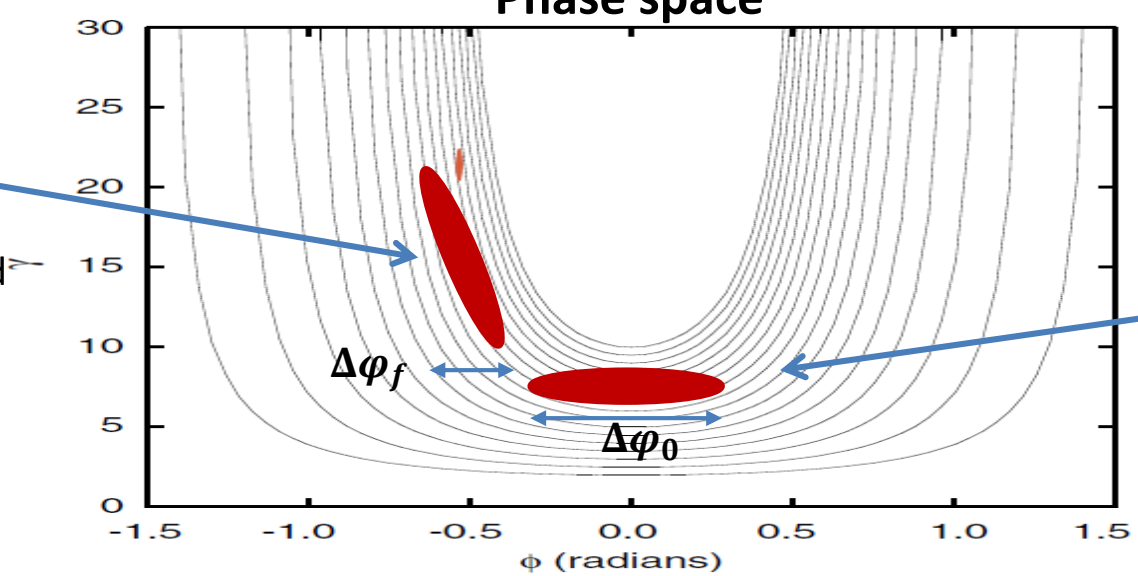
$$\alpha = -\frac{eE_{s0}(s)}{mc^2}$$

$$\boldsymbol{H = k_{\mathrm{rf}}\left(\gamma - \sqrt{\gamma^2 - 1}\right) - \alpha k_{\mathrm{rf}} \sin\varphi}$$

- H is independent of s → H is a constant of motion

Phase space

Beams moves to higher energy, is compressed



Beam injected at zero crossing

Compression factor in the linear approximation

- Use invariance of Hamiltonian:

$$H_{0r} = k_{\mathrm{rf}}\left(\gamma_{0r} - \sqrt{\gamma_{0r}^2 - 1}\right) - \alpha k_{\mathrm{rf}} \sin\varphi_{0r}$$

Invariant @injection
For reference partice

- Assume for simplicity that $\left(\gamma - \sqrt{\gamma^2 - 1}\right) \sim 0$ at exit of Compressing structure

$$k_{\mathrm{rf}}\left(\gamma_0 - \sqrt{\gamma_0^2 - 1}\right) - \alpha k_{\mathrm{rf}} \sin\varphi_0 = \sin\varphi_1 \equiv$$

- Compression factor

$$C = \frac{|\Delta t_0|}{|\Delta t_1|} = \frac{|\Delta\varphi_0|}{|\Delta\phi_1|} \simeq \frac{\cos\varphi_{r0}}{\cos\varphi_{r1}}$$

Max. compression
Is when phase
$\varphi_{r1} = \frac{\pi}{2}$