Transverse geometric wakefields: RF and collimator

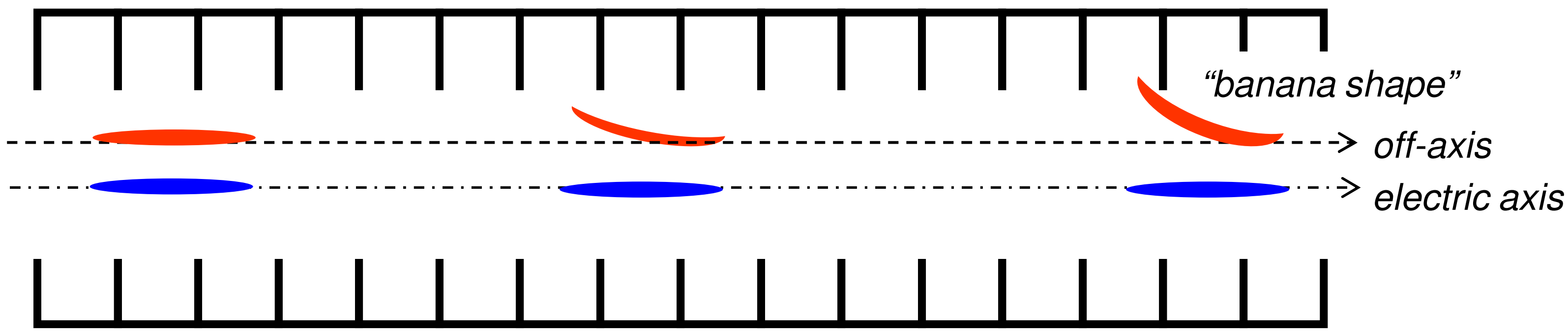
S. Di Mitri (1.5 hr.)

Geometric transverse wakefield in a RF structure

Picture courtesy of S. Milton

Transverse wakefield describes the transverse kick imparted by the image charges to the e-beam as it passes in proximity of a (metallic) surface. The causality principle holds: beam leading particles “hurt” the trailing ones. The **wake kick is correlated with the longitudinal particle position along the bunch**.

Transverse wakefield is generated as the radial symmetry of the e.m. field brought by the beam is broken (“dipole mode”). Namely, it is generated by a **relative misalignment of the beam respect to the cavity electric axis** (*coherent betatron oscillations*).



The induced transverse **projected emittance growth** can be counteracted by “damping” the trailing particles’ oscillation amplitude:

1) either by “modulating” the particle energy along the bunch, so that the bunch tail is focused (back onto the axis) differently from the head (***BNS damping***, *chromatic effect*),
2) or by pushing the beam off-axis on purpose so that multiple wake kicks eventually cancel each other (***emittance bumps***, *geometric effect*).

Short-range wakefield

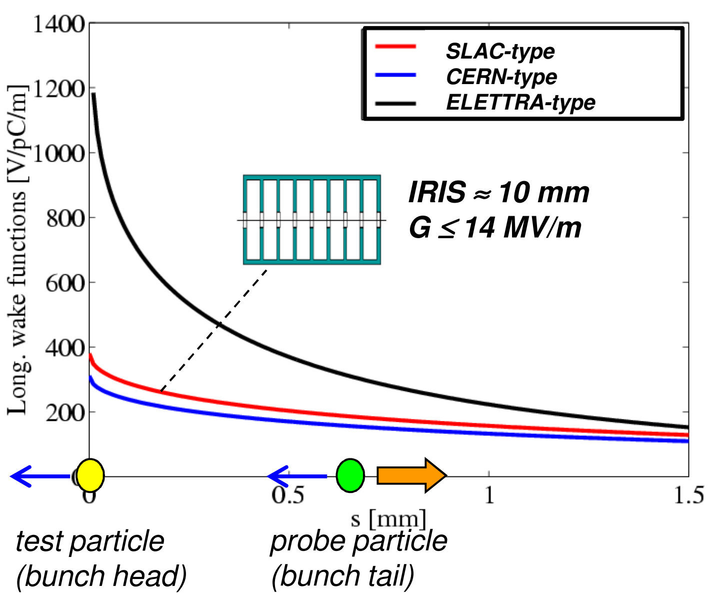
Pictures courtesy of
P. Craievich

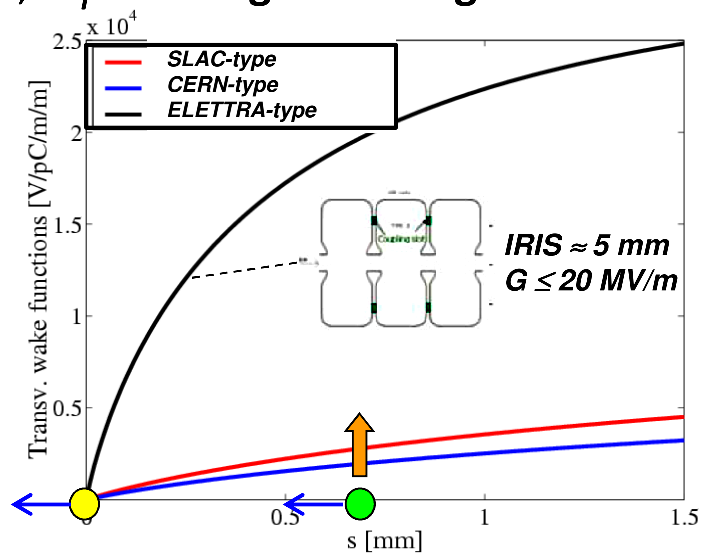
Let us ignore transient regimes of wakefield generation and assume a cyilindrically symmetric, periodic accelerating structure. Then, the following model applies for most of the practical cases (especially for short bunches) if $a^2/2L \ll \sigma_z \ll s_1$:

$$w_T(z) = A\left[1-\left(1+\sqrt{\frac{z}{s_1}}\right)e^{-\sqrt{\frac{z}{s_1}}}\right]\left[\frac{V}{C\cdot m^2}\right], \qquad \text{where } A \approx \frac{Z_0 c s_1}{\pi a^4} \approx 10^3 \div 10^5 \ \frac{V}{pC\cdot m^2}$$

and $s_1 \approx 0.3\div0.8$mm is a cell geometric parameter. **w_T is the wakefield per unit length of the cavity, per unit length of (relative) lateral displacement**.

Notice that, while w_L is stronger for shorter bunches, **w_T is stronger for longer ones**.





Single-bunch beam break-up

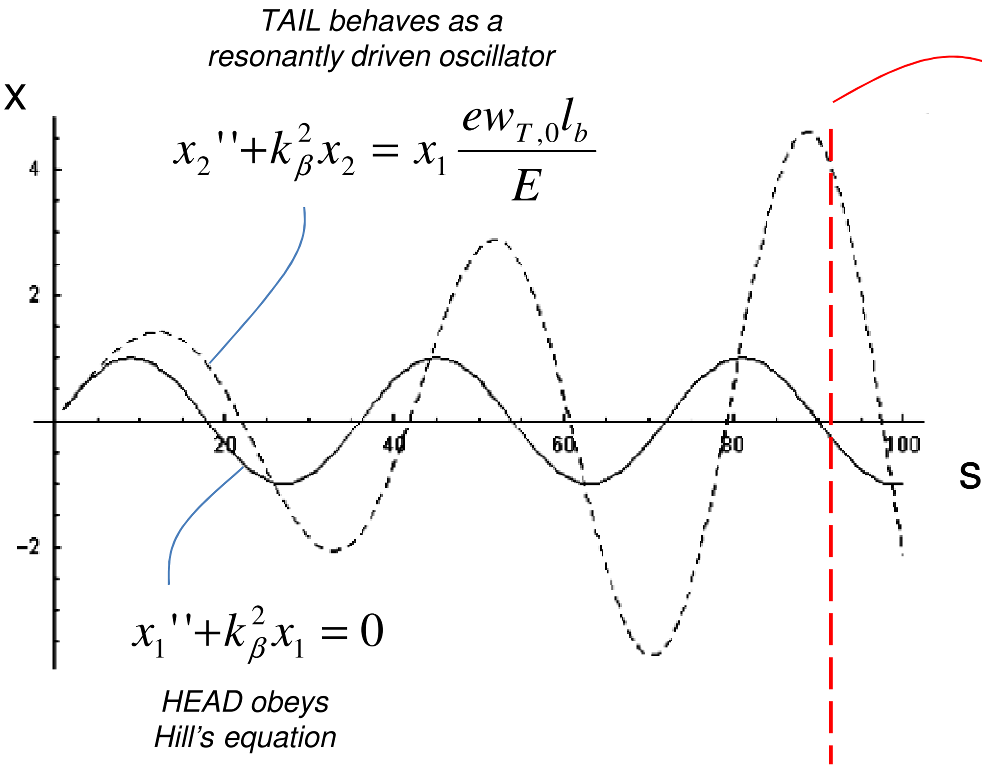
Pictures courtesy of
A. Chao

Equation of motion for x(z,s) in the presence of w_T (exact):

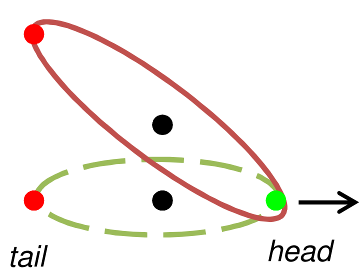
$$\frac{d}{ds}\left[\gamma(s)\frac{d}{ds}x(z,s)\right]+k_\beta^2\gamma(s)x(z,s)=r_e\int_z^\infty dz'\,\rho(z')w_T(z'-z)\left[x(z',s)-d_c(s)\right]$$

acceleration — *β-focusing* — *charge distribution* — *wake function* — *cavity displacement relative to the particle free β-oscillation*

In the two-particle model, at constant energy, the bunch head drives resonantly the tail:



centroid lateral shift and projected emittance growth



Coupling strength

The analytical solution can be found iteratively, by means of perturbative expansion of the wakefield term. At the lowest order, it can be expressed as the product of the unperturbed x_β times the wake driving term. For an **off-axis injection** into a **perfectly aligned linac**, constant accelerating gradient and focusing **k(s)=k**$_\beta$, we have:

$$x^1(z,s)=\sqrt{2J\beta}\left[\underbrace{\frac{1}{q}\cos(k_\beta s)}_{\text{unperturbed }\beta\text{-oscillation}}+\underbrace{\left(\frac{\ln q}{\sqrt{q}(q-1)}\right)s\sin(k_\beta s)}_{\text{additional out-of-phase oscillation, which grows monotonically with } s}\cdot\left(\frac{r_e}{4\gamma_0 k_\beta}\right)\underbrace{\int_z^\infty dz'\rho(z')w_T(z'-z)}_{\text{the integral goes like} \sim Nw_T(l_b)/l_b}\right],\quad q:=\frac{\gamma_f}{\gamma_0}$$

Normalization: ρ/I, z/l_b, s/L and $w_T/w_{T,0}$. From r.h.s. we extract a coefficient that measures the **coupling strength** of the wake to the bunch:

$$\boxed{\varepsilon_r=\frac{4\pi\varepsilon_0}{I_A}\frac{w_{T,0}Il_bL^2}{\gamma_0}}$$

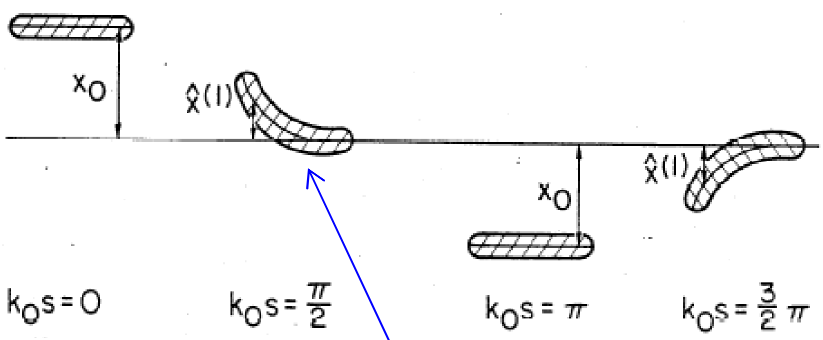
The higher the value of ε_r (>> 1) is, the more important the higher order terms (in *s*) are for the particle motion. In this case, multiple wake integrals and more complicated trigonometric dependencies appear. The additional oscillation terms grow with **powers of s**.

EXERCISE: ε_r is given for a linac 200 m long. What is the bunch current that would imply the same coupling strength for a 50 times longer linac ? (Assume all other parameters constant).

“Banana shape”

Pictures courtesy of
A. Chao

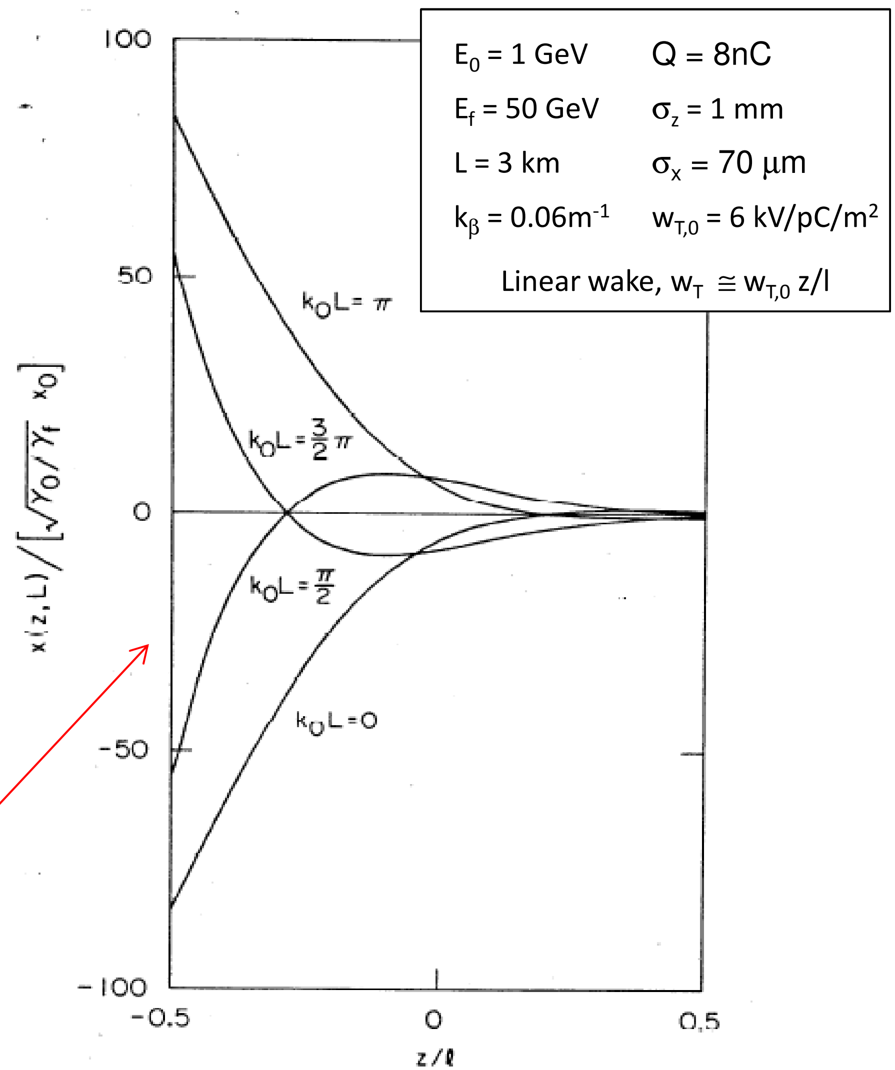
Bunch shape for weak instability ($\varepsilon_r \approx 1$), at four betatron phase advances $\Delta\mu = k_0 s$.



Bunch longitudinal **slices** feel different wake kicks, which **displace** them in the transverse **phase space**, one respect to the other.

As a result, the **projected emittance** grows and “**oscillates**” along the linac according to the wake strength and the betatron phase advance.

“**Banana shape**” in the SLAC linac for strong instability ($\varepsilon_r \gg 1$).



Emittance growth

In real facilities, beam-to-linac misalignment is the result of different and simultanous error sources. Sometimes, some of them dominate over the others.

- **Quadrupoles misalignment** $\Rightarrow$ beam is kicked off-axis. Then assume 1-to-1 trajectory correction at all BPMs; these are located close to focusing and de-focusing quadrupoles.

$$\boxed{\Delta(\gamma\varepsilon) \approx \sigma^2_{y,BPM}\left[\pi\varepsilon_0 r_e N W_\perp(2\sigma_z)\right]^2 \frac{L^2_{cell}}{16\alpha(\Delta\gamma_{str}/L_{str})}\left[\left(\frac{\gamma_f}{\gamma_i}\right)^{2\alpha}-1\right]\frac{\cos(\Delta\mu_{cell}/2)}{\sin^3(\Delta\mu_{cell}/2)}}$$

$$L_{cell}(s) \propto \gamma^\alpha(s)$$

**under auto-phasing, see next slides.*

- **Linac random misalignment** $\Rightarrow$ beam centered in the quads but off-axis in the structures,

$$\boxed{\Delta(\gamma\varepsilon) \approx \sigma^2_{str}\left[\pi\varepsilon_0 r_e N W_\perp(2\sigma_z)\right]^2 \frac{L_{str}\bar{\beta}}{2\alpha(\Delta\gamma_{str}/L_{str})}\left[\left(\frac{\gamma_f}{\gamma_i}\right)^{\alpha}-1\right]}$$

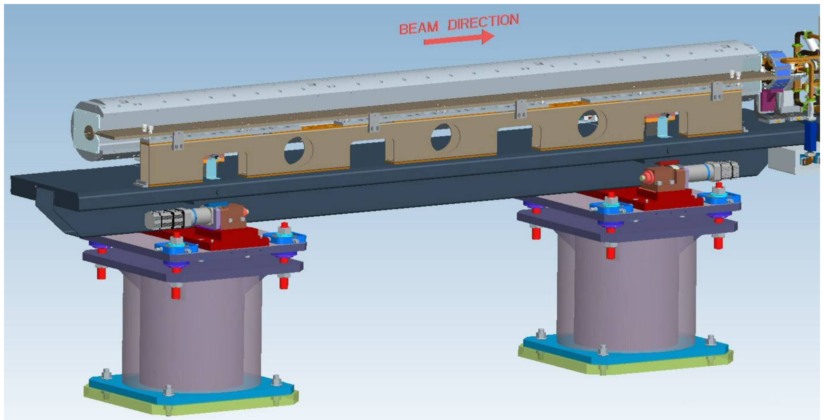
If systematic misalignment of **2-consecutive structures** $\Rightarrow$ slightly stronger effect because more structures are contributing with same sign of the kick,

$$\boxed{\Delta(\gamma\varepsilon) \approx \sigma^2_{str}\left[\pi\varepsilon_0 r_e N W_\perp(2\sigma_z)\right]^2 \frac{L_{cell}\bar{\beta}}{4\alpha(\Delta\gamma_{str}/L_{str})}\left[\left(\frac{\gamma_f}{\gamma_i}\right)^{2\alpha}-1\right]}$$

Linac alignment and layout

Pictures courtesy of
S. Milton et al.

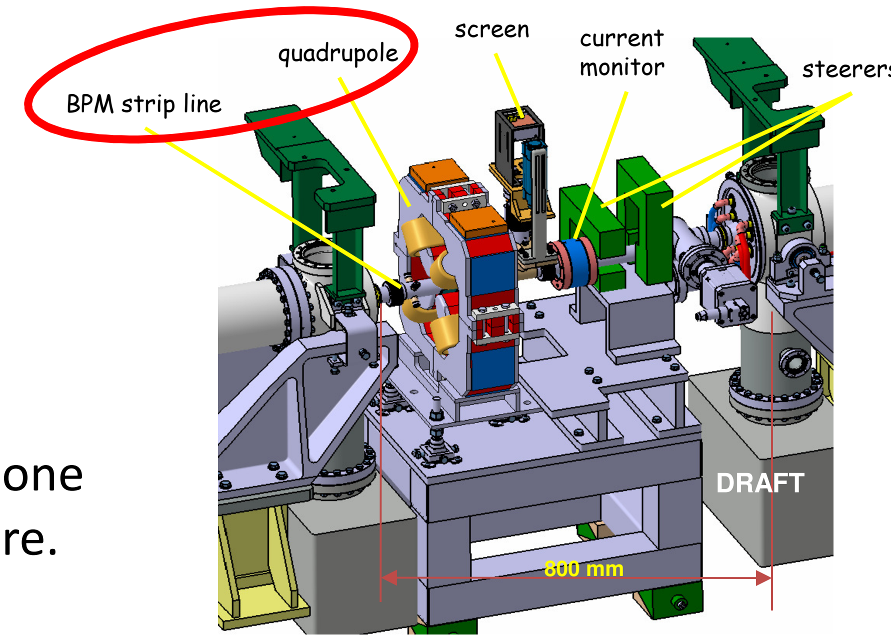
Previous slide points out the importance of the *static* alignment of the main linac components. Some technicial solutions may help for reducing the initial wake effect and allow an accurate trajectory control.



1. Use fixed, stable support (esepcially for RF structures) and girder with 3-D movers on the top of it.

2. *Fiducialize* magnets, RF structures and BPMs (both for piezo and laser tracker)

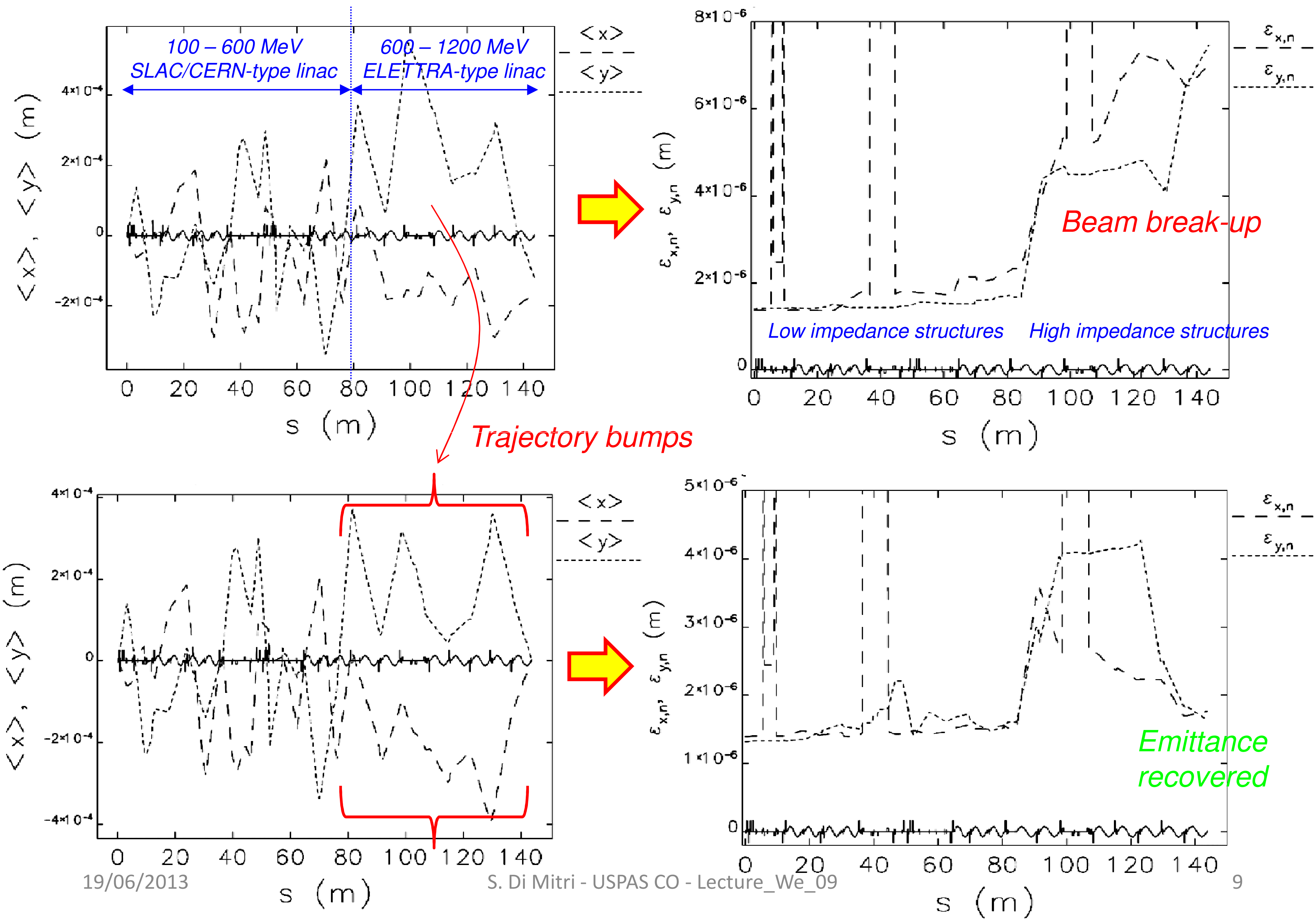
3. Insert BPM inside the Quad, and one Quad (possibly) after every RF structure.



A. Rubino, D. Castronovo, M. Ferianis, L. Rumiz

Emittance bumps

S. Di Mitri, PhD Thesis



Trajectory jitter

S. Di Mitri, PhD Thesis

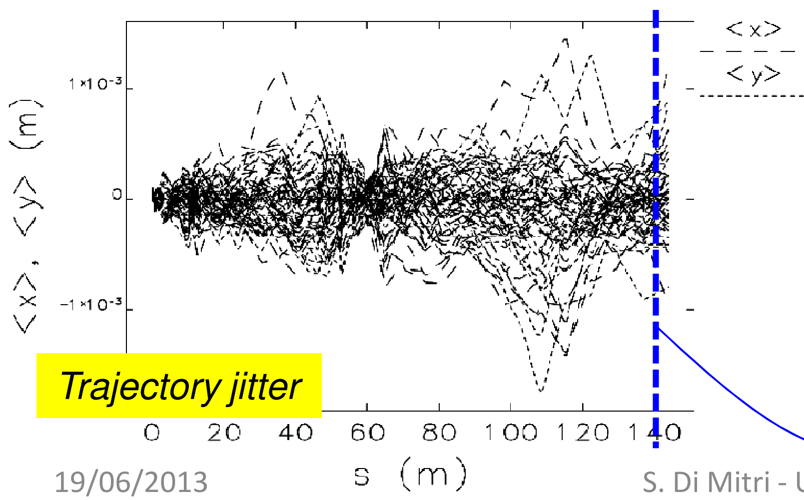
Emittance bumps rely on the trajectory manipulation in a certain linac region. If the beam optics or trajectory changes, the wake suppression is expected to start failing. So, how much is this *scheme sensitive* to ***trajectory jitter***?

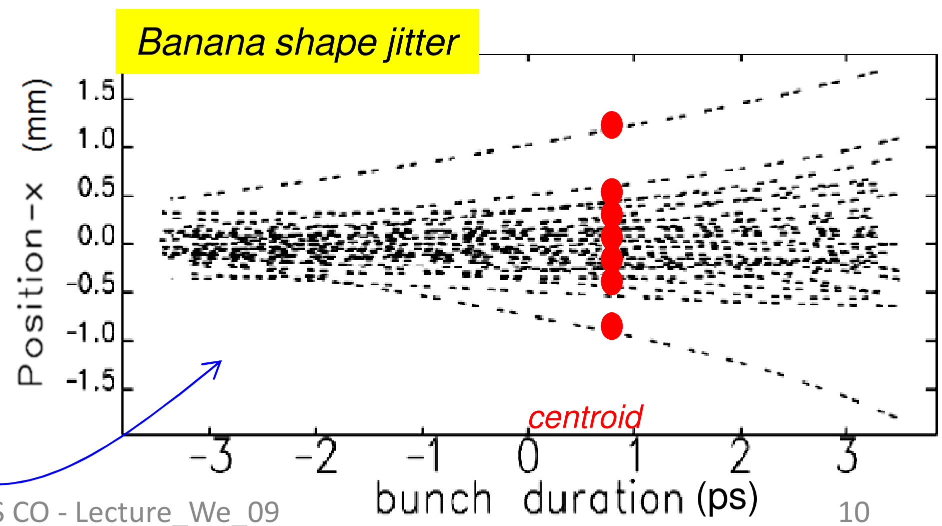
❑ Common short-term sources (say, $f \leq 10$Hz):

- beam launching (injector jitters),
- mean energy (RF jitter),
- magnets' power supply, vibrations (*e.g.,* due to magnet water cooling).

❑ Different trajectories imply (all along the linac and at its end):

- different banana shape,
- different bunch centroid position.





Tolerance jitter budget

- Let us now consider both centroid's position <x> and angular divergence <x'> (any idea on how to measure it?) $\Rightarrow$ built the bunch **centroid Cournat-Snyder invariant**.
- During machine design, we could specify the admitted jitter budget by imposing, *e.g.*, that the **centroid invariant varies less than 10% of the (unperturbed) beam emittance**:

$$A_{T,x} = \sqrt{\frac{x_{CM}^2 + (\alpha_x x_{CM} + \beta_x x'_{CM})^2}{\varepsilon_x \beta_x}} \leq 0.1$$

- The uncorrelated sum of error kicks (*j=1,...M$_n$*, for *n* different jitter sources) must be less than 10%:

$$A_{T,x}^2 \cong \sum_1^M x'^2_{CM,i} \frac{\beta_x}{\varepsilon_x} \equiv 0.1^2 \left[\sum_1^{M_1} \left(\frac{\sigma_{t,1}}{\sigma_{s,j}} \right)^2 + \ldots \sum_1^{M_n} \left(\frac{\sigma_{t,n}}{\sigma_{s,j}} \right)^2 \right] \leq 0.1^2$$

sum of normalized error kicks

ratio of "tolerance" over "sensitivy"

Sensitivity $\sigma_{s,j}$:= trajectory amplitude variation over jitter amplitude variation.

Tolerance $\sigma_{t,n}$:= maximum admitted over all sensitivity amplitudes (per jitter source).

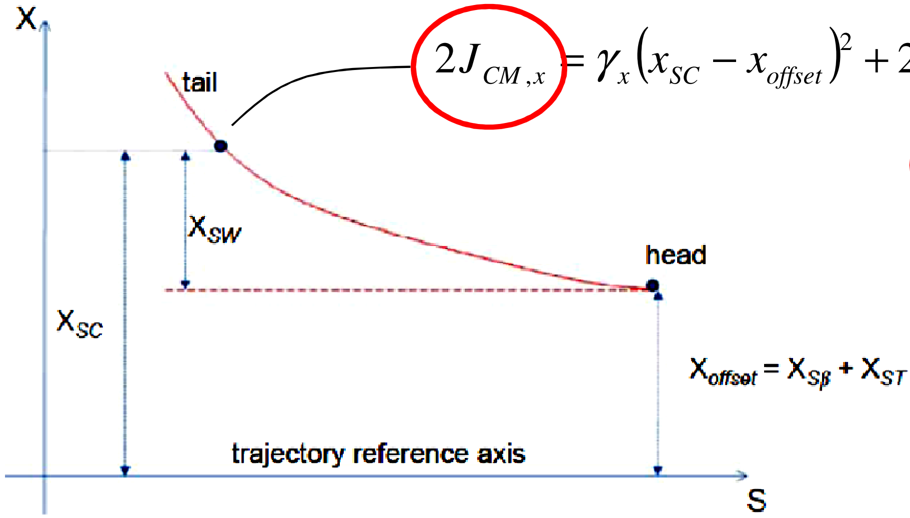
Note 1: sensitivities can be computed (over many tracking runs) including *machine errors.*

Note 2: Tolerances are user-defined: they are arbitrary *weights* for different jitter sources and, to be physical, have to fit technological limits.

Slice centroid Courant-Snyder invariant

P. Craievich, S. Di Mitri, A. Zholents, NIM A 604 (2009) 457

- Since the wake is correlated along the bunch, we additionally require that the position of **each slice centroid varies less than**, say, **one unperturbed RMS beam size**:



$$2J_{CM,x} = \gamma_x\left(x_{SC}-x_{offset}\right)^2 + 2\alpha_x\left(x_{SC}-x_{offset}\right)\left(x'_{SC}-x'_{offset}\right) + \beta_x\left(x'_{SC}-x'_{offset}\right)^2 \equiv \varepsilon_{SW,x}$$

$$\frac{\sigma_{x,SC}}{\sqrt{\varepsilon_x\beta_x}} \le 1,$$ where:

$$\sigma_{x,SC} = \sqrt{\langle x_{SC}^2 - \bar{x}_{SC}^2\rangle} \simeq \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(x_{SW}^i - \bar{x}_{SW}\right)^2} =$$

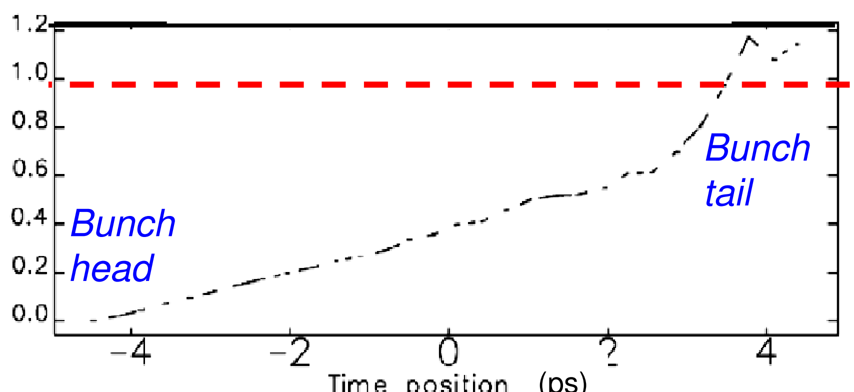
$$= \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left[\sqrt{\beta_x^i\epsilon_{SW,x}^i}\cos\phi_x - \frac{1}{N}\sum_{i=1}^{N}\sqrt{\beta_x^i\epsilon_{SW,x}^i}\cos\phi_x\right]^2} =$$

$$= \sqrt{\beta_x}\cos\phi_x\sqrt{\frac{1}{N}\sum_{i=1}^{N}\left[\sqrt{\epsilon_{SW,x}^i} - \frac{1}{N}\sum_{i=1}^{N}\sqrt{\epsilon_{SW,x}^i}\right]^2}$$

- Assume same optics for all the slices $\Rightarrow$ RMS variation of the *i-th* slice centroid invariant, computed over many shots (trajectories), must be < than the RMS unperturbed emittance, computed over all beam particles.

$$\boxed{\frac{RMS\left(2J_{CM,x}\right)}{\sqrt{\varepsilon_x}} \le 1}$$

for each slice

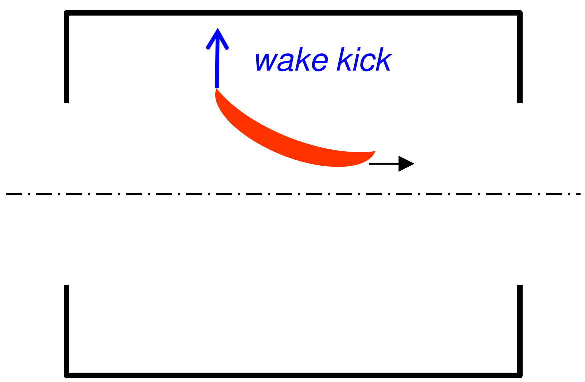


Balakin–Novokhatsky–Smirnov damping

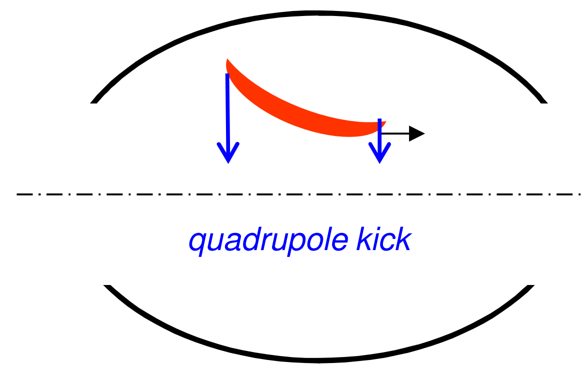
1. The transverse wake deflects the trailing particles of a bunch with positive offset in the positive direction. The idea is to focus back those particles with a negative kick, that is the **bunch *tail*** must be ***over-focused*** **relative to the *head***.

2. In fact, by imposing a lower energy in the bunch tail than in the head, the trailing particles feel a stronger quadrupole focusing that tends to realign the bunch slices in the phase space .

In a RF structure



In a quadrupole



3. Imagine two macroparticles with different β -frequencies (*i.e.*, $k_{\beta,1}$ and $k_{\beta,2}$). The trajectory difference between the two particles is:

$$x_2 - x_1 \cong \hat{x}\left(1 - \frac{e^2 w_T(l_b)}{E}\frac{1}{k_{\beta,2}^2 - k_{\beta,1}^2}\right)\left(\cos k_{\beta,2}s - \cos k_{\beta,1}s\right)$$

Energy spread and "autho-phasing" condition

4. The wake effect can be locally cancelled if (*i.e.*, cancelled at all points in the linac downstream of the location where) the ***"auto-phasing" condition*** holds:

$$\frac{e^2 w_T(l_b)}{E}\frac{1}{k_{\beta,2}^2 - k_{\beta,1}^2} = 1$$

5. As mentioned before, it can be achieved by introducing an **energy difference** between the head and the tail of the bunch. When discrete focusing such as FODO lattice is considered, the **auto-phasing RMS energy spread** is:

$$\boxed{\sigma_{\delta,BNS} \approx \frac{Ne^2 w_T(2\sigma_z)\overline{\beta} L_{cell}}{E\tan\left(\Delta\mu_{cell}/2\right)}}$$

   The BNS energy spread scales as $\sim\gamma^{2\alpha-1}$ along the linac, where $\beta\sim\gamma^{\alpha}$.

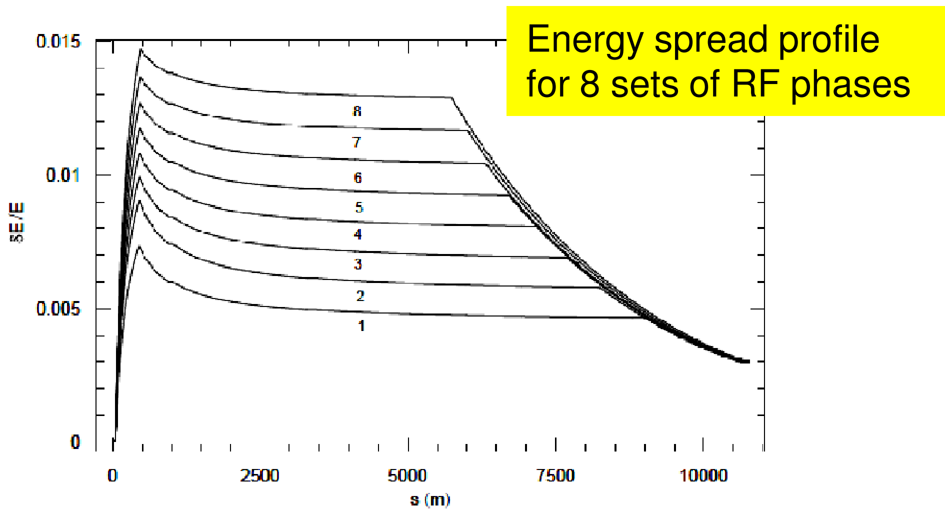
6. As a result of randomly misaligned accelerating structures (perfect FODO focusing along M-cells, with $\beta\sim\gamma^{\alpha}$, and trajectory control is assumed) and *in the absence of any wake suppression scheme*, the final **projected emittance growth** due to transverse wake field instability is:

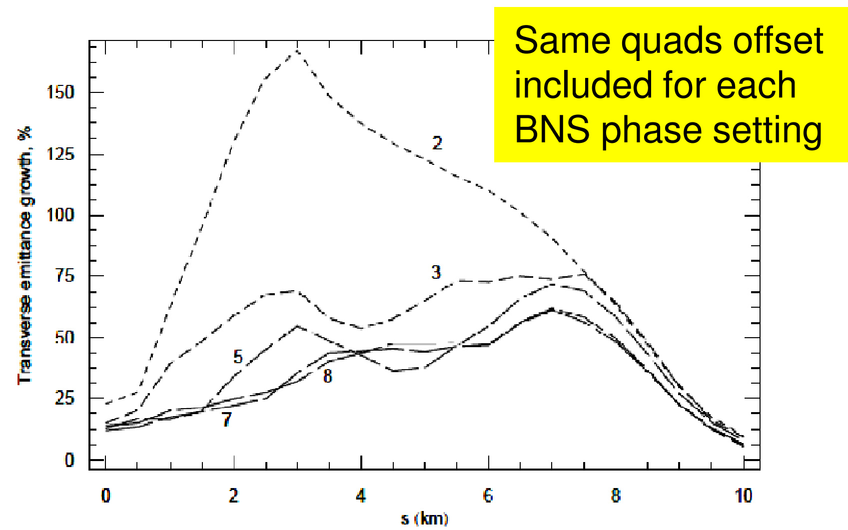
$$\boxed{\Delta\varepsilon \approx \left(\frac{\pi r_e}{Z_0 c e}\right)^2 (Ne)^2 \left(w_T(2\sigma_z)\right)^2 \Delta^2 L_{cell}^2 M\overline{\beta}\,\frac{\left(q^{\alpha}-1\right)}{\alpha}}$$

Linac energy budget

Pictures courtesy of
G. Stupakov

7. The BNS autophasing condition implies an *optimization of the linac RF phasing, for any given quadrupole setting*, in order to: i) *reduce* the ***energy overhead*** that is needed to impose the correlated energy spread, and ii) *minimize* the ***final energy spread*** at the undulator entrance.



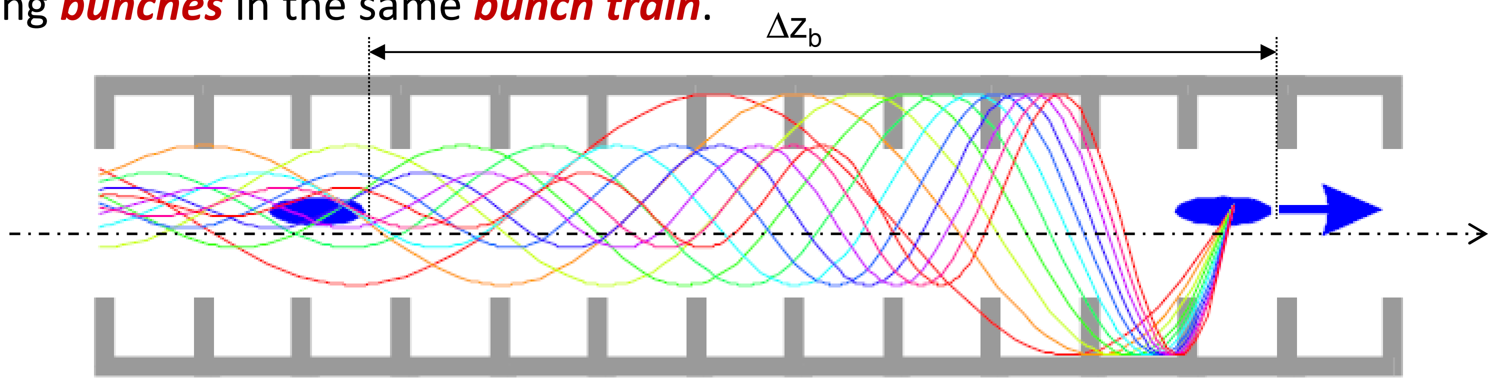


Typically, initial RF structures are run off-crest (+) to generate $\sigma_{\delta,BNS}$, while ending structures are run off-crest (-) to remove the residual energy spread. However, the BNS damping goes in conflict with emittance growth due to ***spurious dispersion***. This is generated by quads traversed off-axis by particles at different energies.

8. Notice that δ_{BNS} ***has opposite sign respect to*** δ ***required for magnetic compression***. This leads to additional complexity for the RF phases managements. In practice, BNS damping has been mostly investigated for long, 10's of GeV linear colliders (*e.g.*, NLC). Emittance bumps are routinely adopted in existing few GeV's linac-driven FELs.

Long-range wakefield

The long-range (transverse) wakefield is the extension of the short-range to multi-bunch patterns. Now, leading and trailing *particles* in the *same bunch* are substitued with leading and trailing ***bunches*** in the same ***bunch train***.



For long-range wakes, tend to consider ***field modes*** rather than wake potential: this is the sum over several high order modes (HOMs) which are excited by the first bunches of a train, which travel off axis in the accelerating structure, and act on the subsequent ones:

$$w_T(z) = \sum_k \frac{r_{s,k}\,\omega_k}{Q_k} e^{-\frac{\omega_k z}{2cQ_k}} \sin\frac{\omega_k z}{c} \left[\frac{V}{Cm^2}\right]$$

The traling bunches are driven even more off axis leading to an even stronger excitation of the modes in the next accelerating section (instability). The transverse kick of the j-th bunch after one structure, for the k-th mode, is:

$$\Delta x'_j = \sum_{i=1}^{j-1} \frac{x_i q_i}{E_i} \frac{r_{s,k}\,\omega_k}{Q_k} e^{-\frac{\omega_k i \Delta z_b}{2cQ_k}} \sin\frac{\omega_k i \Delta z_b}{c}$$

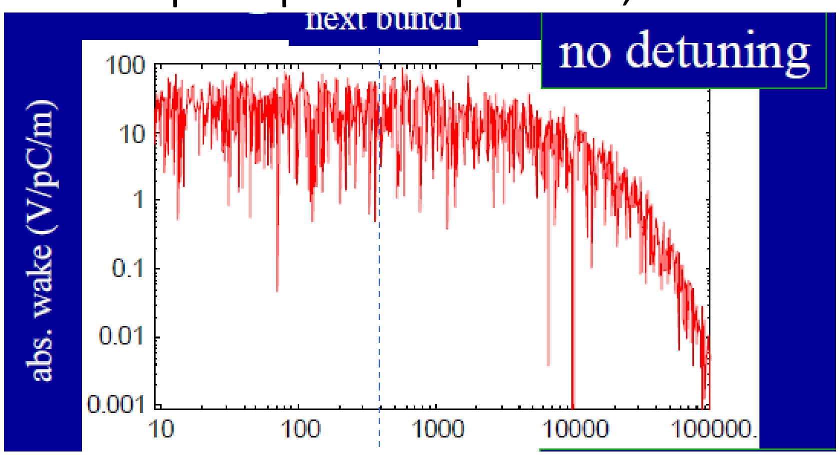
where the **bunch offsets grow exponentially** according to: $\Delta x_f \propto e^{\sqrt{Q w_T}}$

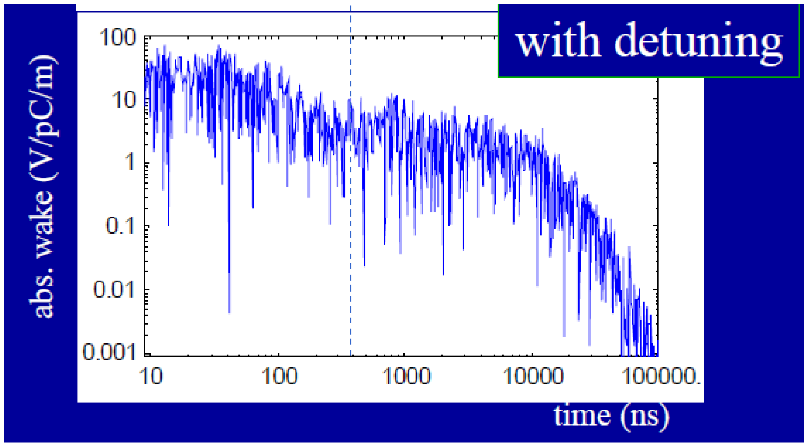
Multi-bunch beam break-up

If the wake is negligible beyond more than one bunch spacing (*daisy chain model*), then the criterion for little or no emittance blow-up is, as in the single-bunch case, $\varepsilon_r < 1$, where the multi-bunch wake function is now evaluated over the single bunch length.

The **multi-bunch instability** can be **suppressed** with a **special design of the structures**.

- ***Detuned structures*** have slightly different cell-to-cell dimensions to introduce a frequency spread of each mode, causing decoherence of the wake function. This is already present, albeit in principle not optimized, in constant-gradient structures.





- For X-band linear colliders, very **low Q (~20) *choke mode structures*** have been designed, which suppress all the deflecting modes.

- In SC linacs, ***HOM loop couplers*** have been designed to couple out lower frequency modes (bewlo a few GHz) and bring them to room temperature loads for absorption.

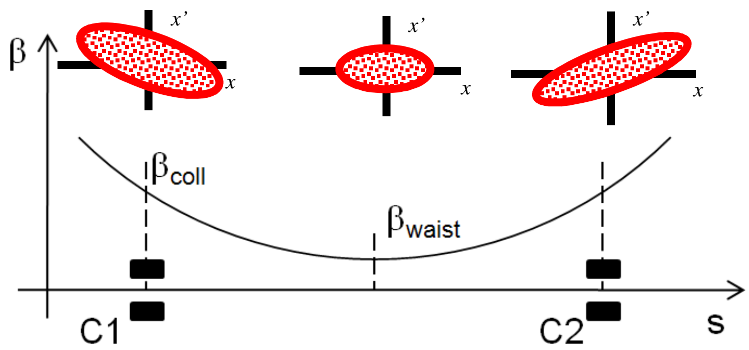
Geometric collimation

S. Di Mitri, PRST-AB 13, 052801 (2010)

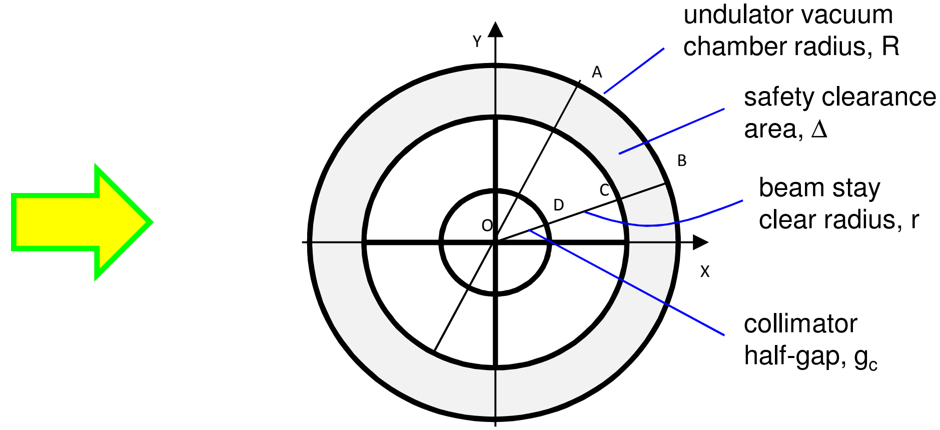
Collimators are high-Z, metallic blocks with **apertures** to intercept, scatter and absorb undesired particles at very large β-amplitudes ("halo", |A| ≥ 20σ) or off-energy (|δ| ≥ 2%). The formers are called ***geometric***, the latters, ***energy* collimators**.

They **protect** the **undulator** from being hitted by e.m. showers generate by primary (halo) or secondary particles (from vacuum chamber). The *beam core should* pass through *untouched*.

To stop halo particles both in position and angular divergence, <u>at least two geometric collimators are needed and ideally separated by Δμ = π/2.</u>



In the linac: *low-β inserttion for 2-stage geometric collimation*



Undulator: *vacuum chamber cross-section*

The optimum **collimator acceptance** and **half-gap** are:

$$a_{\pi/2} = \frac{(R-\Delta)^2}{2\hat{\beta}_{und}} \Rightarrow g_{\pi/2} = \sqrt{a_{\pi/2}\beta_{coll}}$$

Warning! Small iris means strong geometric wakefields (like in small iris structures), but...
...larger gaps may limit the collimation efficiency. Optics tuning is required for a compromise.

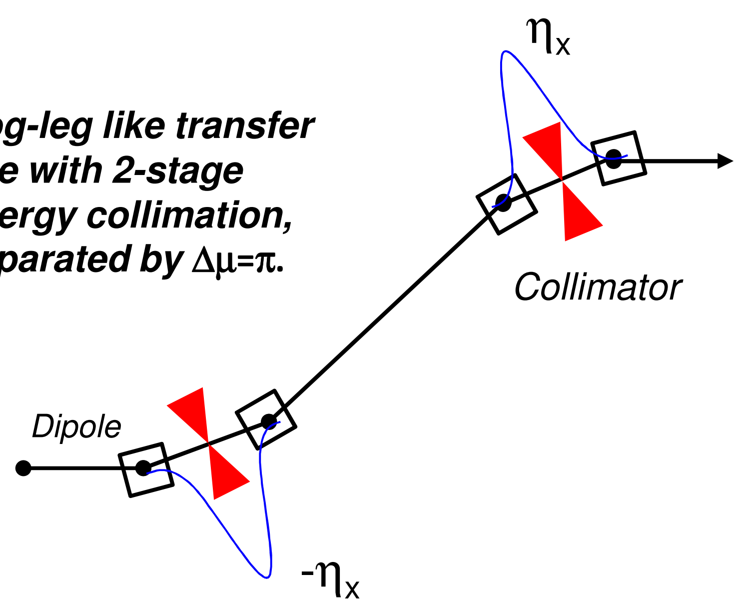
Energy collimation

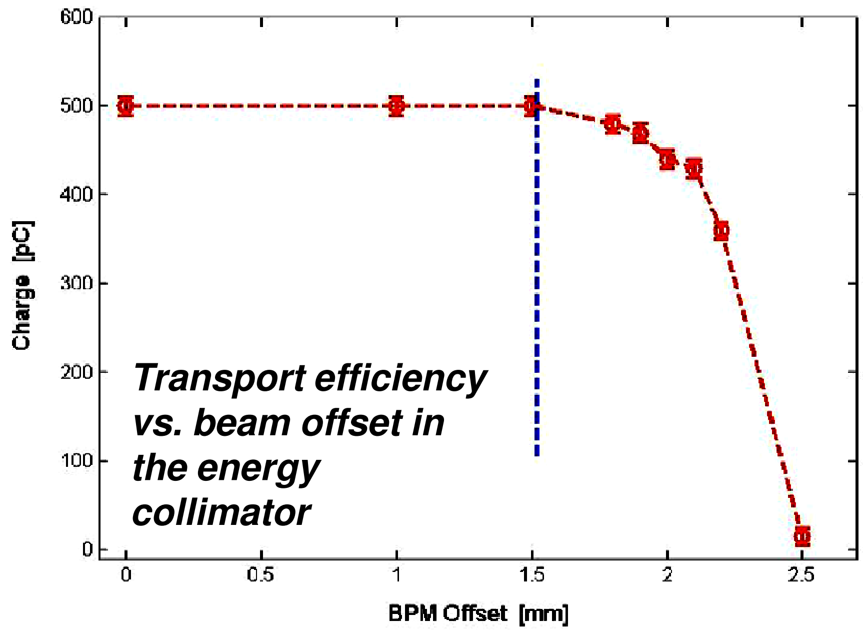
S. Di Mitri, PRST-AB 13, 052801 (2010)

To stop particles with both positive and negative energy deviation respect to the reference energy, at least two collimators placed in a dispersive region are needed, ideally separated by $\Delta\mu = \pi$.

The energy acceptance is $\frac{\Delta E}{E} = \frac{g_c}{\eta_x}$, so that one aims to have small collimator's gap and large momentum dispersion. If the particle motion is dominated by the chromatic contribution respect to the geometric, *i.e.* $\frac{\eta_x \sigma_\delta}{\sqrt{\varepsilon_x \beta_x}} >> 1$, and if the the energy collimators are at $\Delta\mu=\pi$, then all particles with $|\delta| \geq \frac{g_c}{|\eta_x|}$ are expected to be intercepted.

Dog-leg like transfer line with 2-stage energy collimation, separated by $\Delta\mu=\pi$.





Geometric transverse wakefield in a collimator

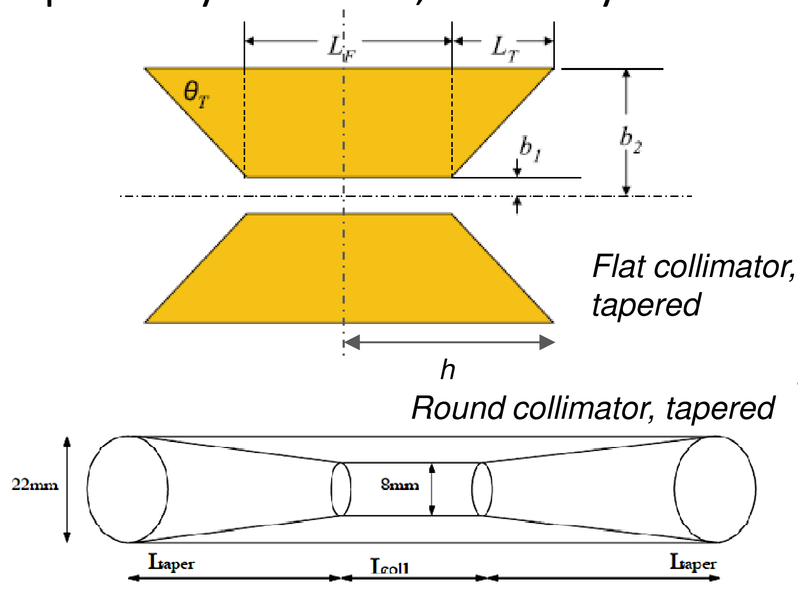
Pictures courtesy of
S. Ferry and E. Karantzoulis

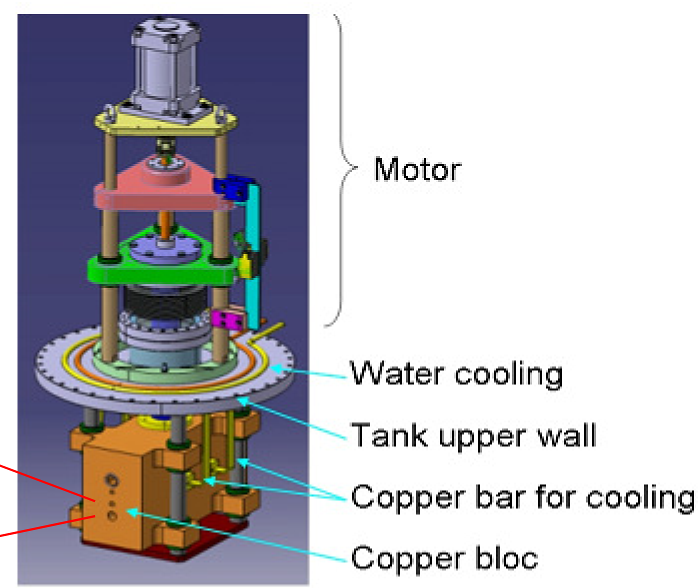
We consider an ultra-relativistic beam passing off-axis by $\Delta y_0 << b_1$ through a collimator with geometric symmetry in the plane of interest (see figure). The beam centroid receives a kick:

$$\boxed{y' = \frac{\Delta y_0 Q}{E}\kappa}$$

where κ is the "**transverse kick factor**" in V/pC/mm, namely the transverse kick averaged over the bunch length.

Analytical formulas for κ can be found in the limits where the parameter $\alpha \equiv \theta_T b_1 / \sigma_z$ is either small or large respect to 1, regimes which we are denoted as ***inductive*** and ***diffractive***, respectively. For $\alpha \approx 1$, the analysis can only provide the order of magnitude of κ.





Transverse kick factor

S. Di Mitri, PRST-AB 13, 052801 (2010)

INDUCTIVE regime (α<<1, θ_T<<1)

$$\kappa = \frac{Z_0 c\alpha}{2\pi^{3/2} b_1^2}\left(1 - \frac{b_1}{b_2}\right)$$

Gaussian bunch in *round*, tapered collimator

$$\kappa \approx \frac{Z_0 c\,\alpha h}{4\pi^{1/2} b_1^3}$$

Gaussian bunch in *flat*, tapered collimator

DIFFRACTIVE regime (α>>1)

$$\kappa \approx \frac{Z_0 c}{2\pi}\left(\frac{1}{b_1^2} - \frac{1}{b_2^2}\right)$$

Long ($L_F \to \infty$) collimator

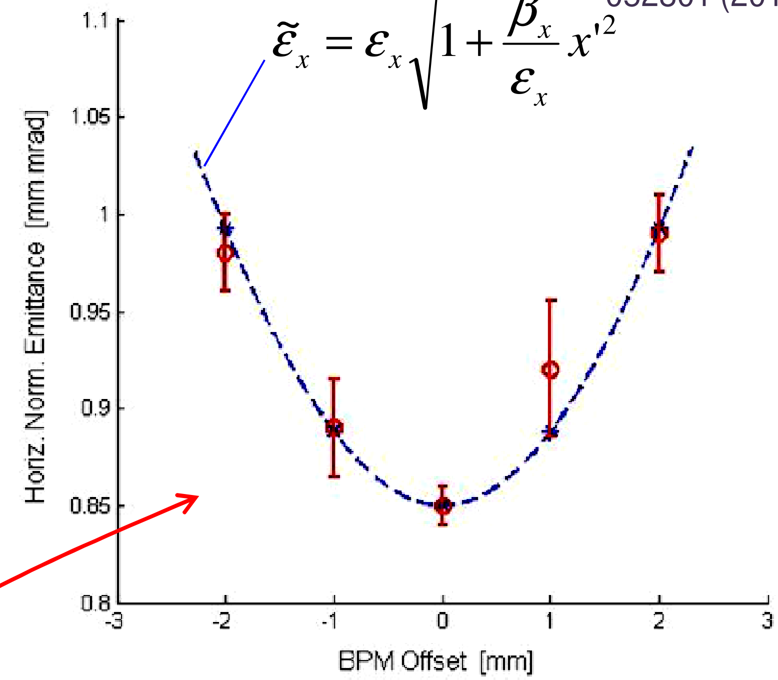
$$\kappa = \frac{Z_0 c}{4\pi}\left(\frac{1}{b_1^2} - \frac{b_1^2}{b_2^4}\right)$$

Short ($L_F \to 0$), round collimator

$$\kappa \approx \frac{Z_0 c}{4\pi b_1^2}$$

Short ($L_F \to 0$), flat collimator

$$\tilde{\varepsilon}_x = \varepsilon_x \sqrt{1 + \frac{\beta_x}{\varepsilon_x} x'^2}$$



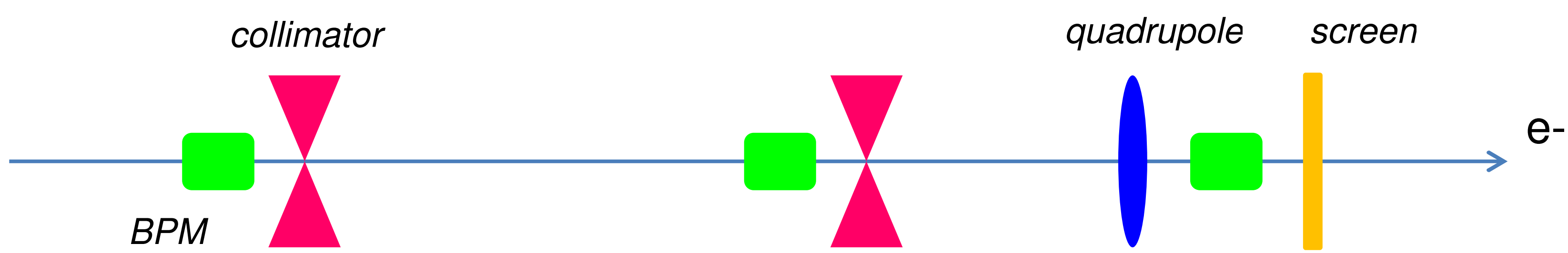
- Beam final normalized emittance vs. horizontal offset in the collimator.
- The geometric collimator is set to 2 mm half-gap hole. The quadratic term of the fitting corresponds to a kick factor of k_{fit} = 2.20 V/pC/mm.
- The dashed curve shows Eq. 2 evaluated for k = k_{fit}.

Overview

- Collimation of very high brightness beams ($I \sim 300$A, $\gamma\varepsilon \sim 1\mu$m rad) with g ~ 1mm, requires **trajectory control** with accuracy at ~10 μm in order to avoid emittance degradation above ~10%. This is normally feasible in modern linacs with now-a-days standard BPMs.

- The **transverse kick factor** can be measured in (at least) two ways:
  1. looking to the emittance growth vs. beam offset in the collimator,
  2. Looking to the downstream beam position vs. the beam offset in the collimator.

  The analytical approximations work well for simple collimator geometries.



- Notice that the **longitudinal kick factor** can usually be neglected because:
  - it is well absorbed by the longitudinal emittance which is usually ~100 times larger tha the transverse one;
  - wakefield induced energy spread is dominated by the stronger wake potential due to the much longer linac structures.