Chapter 2:

SRF Accelerating Structure

2.1 Understanding of cavity parameters

2.2 Elliptical cavity design

2.3 Acceleration in multi-cell cavity

2.4 Higher order mode

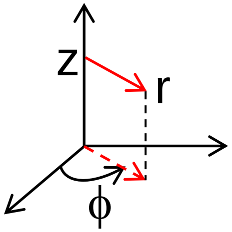
2.1 Understanding of cavity parameters

Understandings of cavity parameters are extremely important.
In order to get real practical numbers, self-consistent models and notations will be introduced with examples.

We will approach basic concepts from simple examples.
Most of complex problems can be (easily) attacked with understandings of definition, physical meaning, dimension, their relations, etc.
Most of SRF cavities are using standing wave and TM010 (or TM010-like pi-mode) structures.

Let's look back the cavity parameters with simple case first.

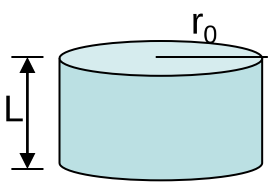
Pillbox: simplest but basis of most structures



Reminder) wave equation in cylindrical coordinates

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\varphi}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2\varphi}{\partial\theta^2}+\frac{\partial^2\varphi}{\partial z^2}=\frac{1}{c^2}\frac{\partial^2\varphi}{\partial t^2}$$

$\varphi : E \ or \ H$



Wave is bouncing back and forth between walls→
Degeneration→ modes
Set z: wave propagation direction
TE mode: transverse electric→ Ez=0
TM mode: transverse magnetic→ Hz=0

$$\text{TE mode}: H_z = H_{nmp} J_n\left(\frac{q_{nm}}{r_0} r\right)\cos(n\theta)\sin\left(\frac{p\pi z}{L}\right)e^{j\omega t}$$

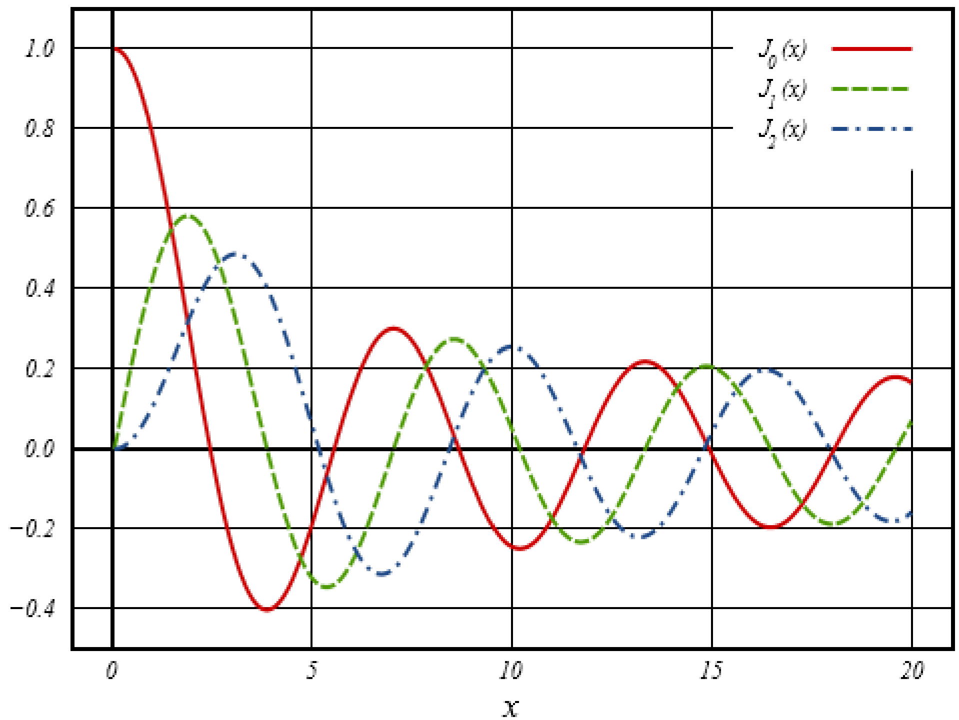
$$\text{TM mode}: E_z = E_{nmp} J_n\left(\frac{P_{nm}}{r_0} r\right)\cos(n\theta)\sin\left(\frac{p\pi z}{L}\right)e^{j\omega t}$$

J_n : n-th order Bessel function

$n = 0, 1, 2, 3, \ldots$ is the number of complete cycles of variation for $0 \le \theta \le 2\pi$

$p = 0, 1, 2, 3, \ldots$ is the number of half cycles of variation in the z-direction

Bessel function



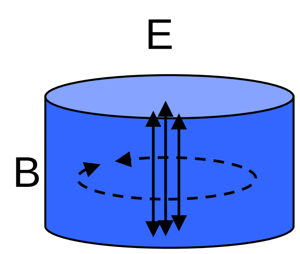
TE mode			
n	qn1	qn2	qn3
0	3.832	7.016	10.174
1	1.841	5.331	8.536
2	3.054	6.706	9.970
3	4.201	8.015	11.346

TM mode			
	pn1	pn2	pn3
0	2.405	5.520	8.654
1	3.832	7.016	10.174
2	5.135	8.417	11.620
3	6.380	9.761	13.015

$$TE_{nmp} : f_r = \frac{c}{2}\left[\frac{q_{nm}^2}{\pi r_0^2} + \frac{p^2}{l^2}\right]^{1/2}$$

$$TM_{nmp} : f_r = \frac{c}{2}\left[\frac{p_{nm}^2}{\pi r_0^2} + \frac{p^2}{l^2}\right]^{1/2}$$

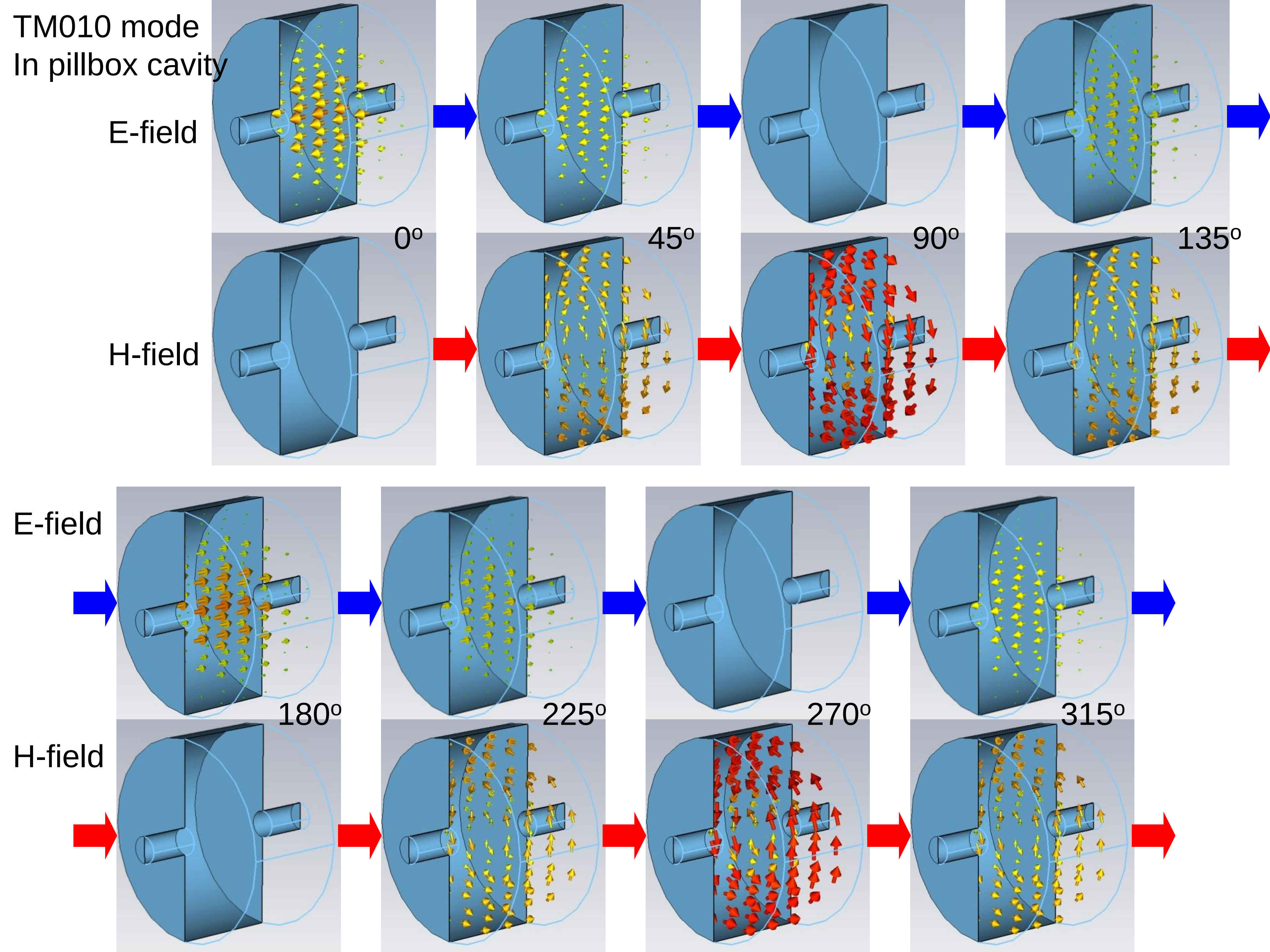
TM010 mode in the pillbox cavity



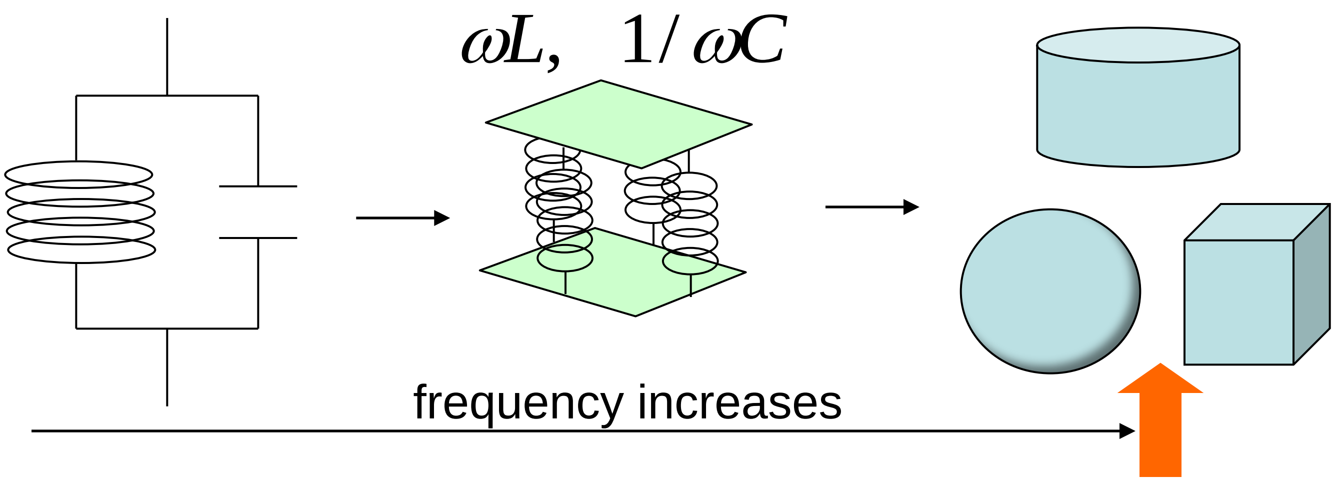
$$f_0 = \frac{2.405 \cdot c}{2\pi r_0}$$

$$E_z = E\, J_0\left(\frac{2.405\, r}{r_0}\right)e^{i\omega t}$$

$$H_\theta = -i\frac{\omega\, \varepsilon\, r_0}{2.405} E\, J_0'\left(\frac{2.405\, r}{r_0}\right)e^{i\omega t}$$

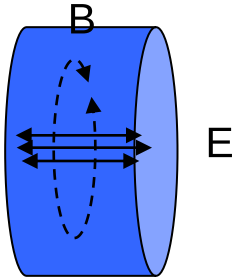

TM010 mode
In pillbox cavity
E-field
H-field
0o
45o
90o
135o
E-field
H-field
180o
225o
270o
315o

RF Cavity is a device that can store electromagnetic energy

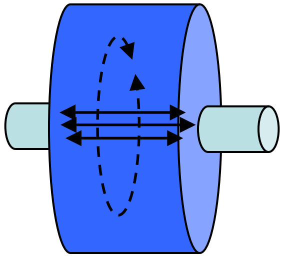


Since any surfaces or a part of surfaces can act as either capacitor or inductor in RF, there are infinite numbers of modes that can be excited in a cavity.

Among them
TM010 mode is good
for charged particle acceleration, since



And then make holes for beam



High frequency field (above cut-off) will pass through pipes→ Number of modes that can resonate are limited.

Stored energy, $U=U_E+U_H$

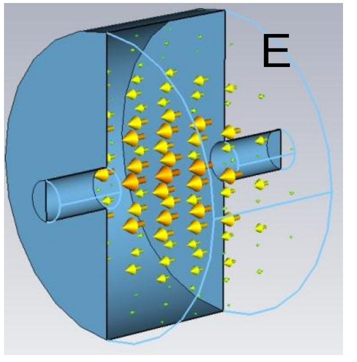
U_E; time averaged stored energy on account of electric field

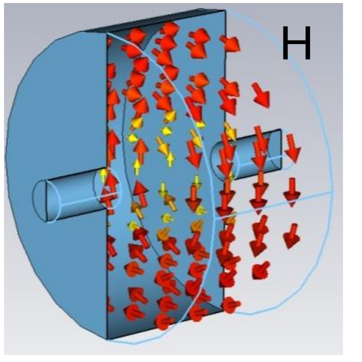
U_H; time averaged stored energy on account of magnetic field

$U_E=U_H$ in a cavity

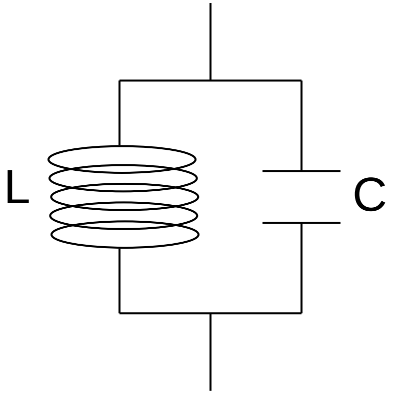
$$U_E = \frac{1}{4}\varepsilon\int_{\text{cavity volume}} \vec{E}\cdot\vec{E}^{*}dv$$

$$U_H = \frac{1}{4}\mu\int_{\text{cavity volume}} \vec{H}\cdot\vec{H}^{*}dv$$





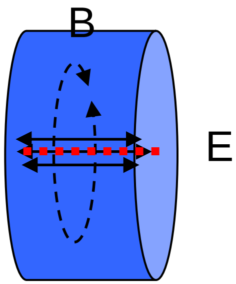
In many cases, we use a lump circuit to describe a resonator or RF/MW circuit. Let's go back to a lossless LC tank circuit.



$$\omega_0^{\,2} = \frac{1}{LC}\ ,\ \omega_0\text{; resonance frequency}$$

$$U_E = \frac{1}{4}CVV^{*} = \frac{1}{4}\varepsilon\int_{\text{cavity volume}} \vec{E}\cdot\vec{E}^{*}dv$$

We need to define the voltage across the resonator, V.



We need to define the two end points for the integration of E-field.
In most cases, we define integration path between the two end points along the line of maximum E-field.
For TM010 mode, line integration along the axis;

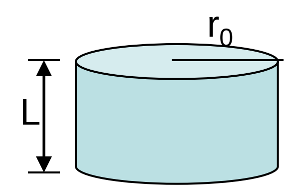
$$V_0 = -\int \vec{E} \cdot d\vec{l}$$

Now we can calculate equivalent C & L.

V is not a potential !! It is only a line integral of E and has voltage unit.
It is the reference value of an energy gain for the charged particle.

When you want to have a confidence in your calculation, do a benchmarking.

Example) cylindrical cavity with r_0=10 cm, L=5 cm for TM010 mode
1.Resonance frequency?
2. Stored energy?



1. First, analytic solution

$$f_0 = \frac{2.405 \cdot c}{2\pi r_0} = 1.147425 \text{ GHz}, c; \text{speed of light}$$

Only depends on r_0 for TM010 of a pillbox cavity

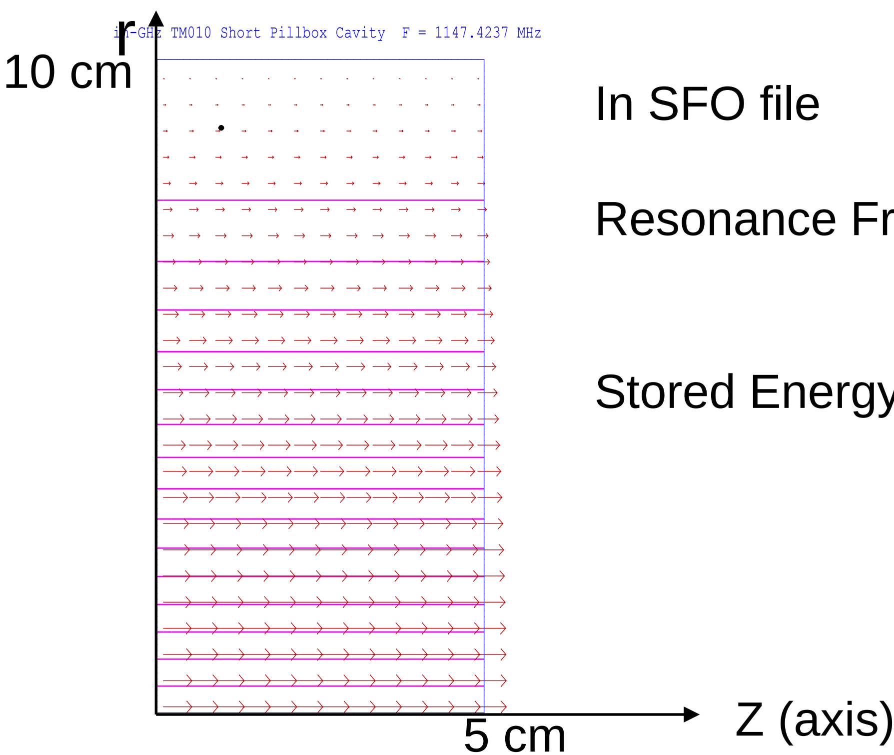
2. Stored energy

$$E_z = E\ J_0\left(\frac{2.405\ r}{r_0}\right)e^{i\omega t}$$

$$U = 2U_E = \frac{1}{2}\varepsilon\int_0^{r_0} E_z E_z^*(L2\pi r dr) = \pi L\varepsilon E^2 \frac{r_0^2}{2} J_1^2(2.405) = 1.8731\,\text{mJ at } E = 1\,\text{MV/m}$$

Ex) Numerical Calculation (SUPERFISH input file: TEST1_1.af)

```
$reg kprob=1,            ; Superfish problem
dx=.1,                   ; X mesh spacing
freq=1000.,              ; Starting frequency in MHz
xdri=1.,ydri=9.0 $       ; Drive point location

$po x=0.0,y=0.0 $        ; Start of the boundary points
$po x=0.0,y=10. $
$po x=5.,y=10. $
$po x=5.,y=0.0 $
$po x=0.0,y=0.0 $
```

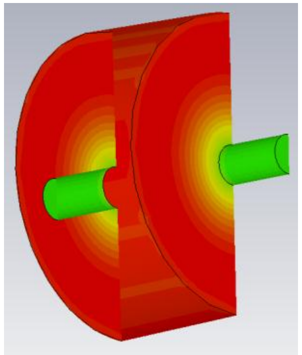


In SFO file

Resonance Frequency; 1147.424 MHz
cf. 1147.425 MHz from analytic solution

Stored Energy; 1.8731 mJ at field normalization; 1MV/m
cf. 1.8731 mJ from analytic solution

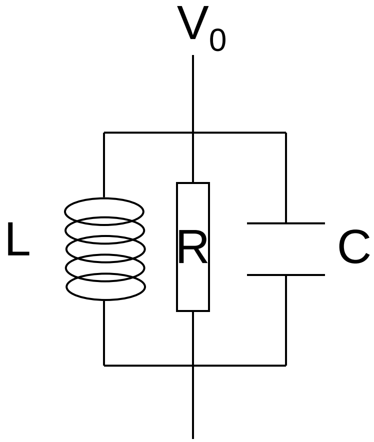
Cavity wall dissipation; P_c



H_t (tangential component of magnetic field) is continuous on cavity surfaces.

Metal surfaces have surface resistances.

(superconducting materials have surface resistances too, but very small.)



Time averaged power dissipation

$$P_c = \frac{VV^*}{2R} = \frac{V_0^2}{2R} = \frac{R_S}{2}\int_{\substack{\text{cavity}\\\text{surface}}} H_t \cdot H_t^* ds$$

$$\text{in normal conducting cavity} \rightarrow \text{surface resistance } R_S = \frac{1}{\sigma\delta}$$

$$\sigma : \text{conductivity}, \delta : \text{skin depth}$$

$$\text{in superconducting cavity} \rightarrow \text{surface resistance } R_S = R_{BCS} + R_{res} + R_{others}$$

R_S, R,… (more will come later on)
; confusing, but remember/understand physical meanings

Unloaded quality factor Q_0

$$U_E = \frac{1}{4}\varepsilon\int_{\substack{\text{cavity}\\\text{volume}}} \vec{E}\cdot\vec{E}^* dv$$

$$U_H = \frac{1}{4}\mu\int_{\substack{\text{cavity}\\\text{volume}}} \vec{H}\cdot\vec{H}^* dv$$

For any resonant circuit, the bandwidth Δf between frequencies of 3 dB down response or energy storage is given by f_0/Q, where the quality factor Q is a measure of the frequency selectivity of a given resonant circuit.
Also measure of ratio between stored energy and power dissipation per cycle.

$$P_c = \frac{VV^*}{2R} = \frac{V_0^2}{2R} = \frac{R_S}{2}\int_{\substack{\text{cavity}\\\text{surface}}} H_t \cdot H_t^* ds, \ (V = V_0 \exp(i\omega t))$$

$$Q_0 = \frac{\omega_0 U}{P_c} = \frac{2\omega_0 U_E}{P_c} = \frac{2\omega_0 U_H}{P_c} = \frac{\frac{1}{2}\omega_0\mu\int_{\substack{\text{cavity}\\\text{volume}}} \vec{H}\cdot\vec{H}^* dv}{\frac{R_S}{2}\int_{\substack{\text{cavity}\\\text{surface}}} H_t \cdot H_t^* ds}$$

$$= \omega_0 \frac{\frac{1}{2}CVV^*}{\frac{1}{2}\frac{VV^*}{R}} = \omega_0 CR = \frac{R}{\omega_0 L}$$

Q_0 is a measure of circuit characteristics without any external coupling (unloaded), only by RF power dissipation inside of resonant circuit (surface power dissipation in RF cavity).

For pure copper;

Conductivity $\sigma = 5.8\times10^{7}$ S/m (or mhos/m or ℧/m)

Skin depth $\delta = 66.1/\sqrt{f}$ μm (f in MHz)=1.95 μm (at f=1147.424 MHz)

Surface resistance R_S=8.836 mΩ

TM010 pillbox $$Q_0 = \frac{\omega_0 U}{P_c} = \frac{377}{R_S}\frac{2.405}{2(\frac{r_0}{L}+1)} = 17100$$

SUPERFISH calculation

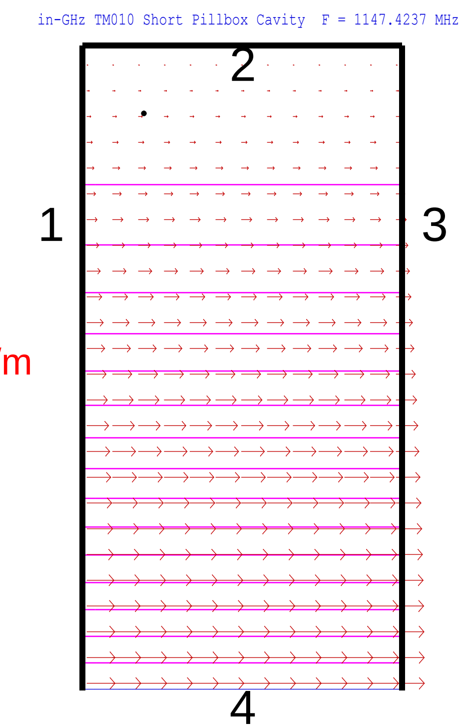
The metal surface should be defined for power loss calculation

TEST1_1.seg

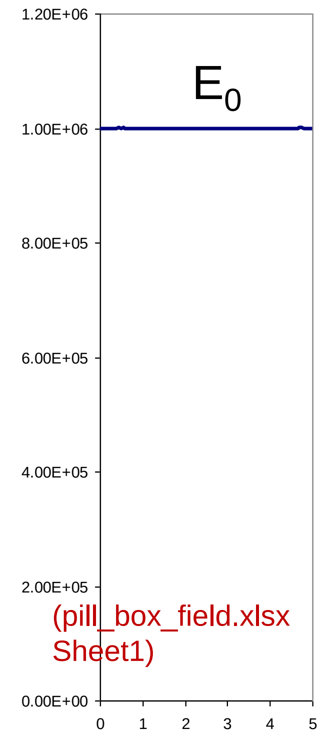
```
FieldSegments
1 2 3                    ; segment numbers
EndData
End
```

In SFO file

Field normalization (NORM = 0): EZERO = 1.00000 MV/m
Frequency = 1147.42365 MHz
Normalization factor for E0 = 1.000 MV/m = 5619.656
Stored energy = 0.0018731 Joules
Using standard room-temperature copper.
Surface resistance = 8.83737 milliOhm
Power dissipation = 790.3723 W
Q = 17085.3

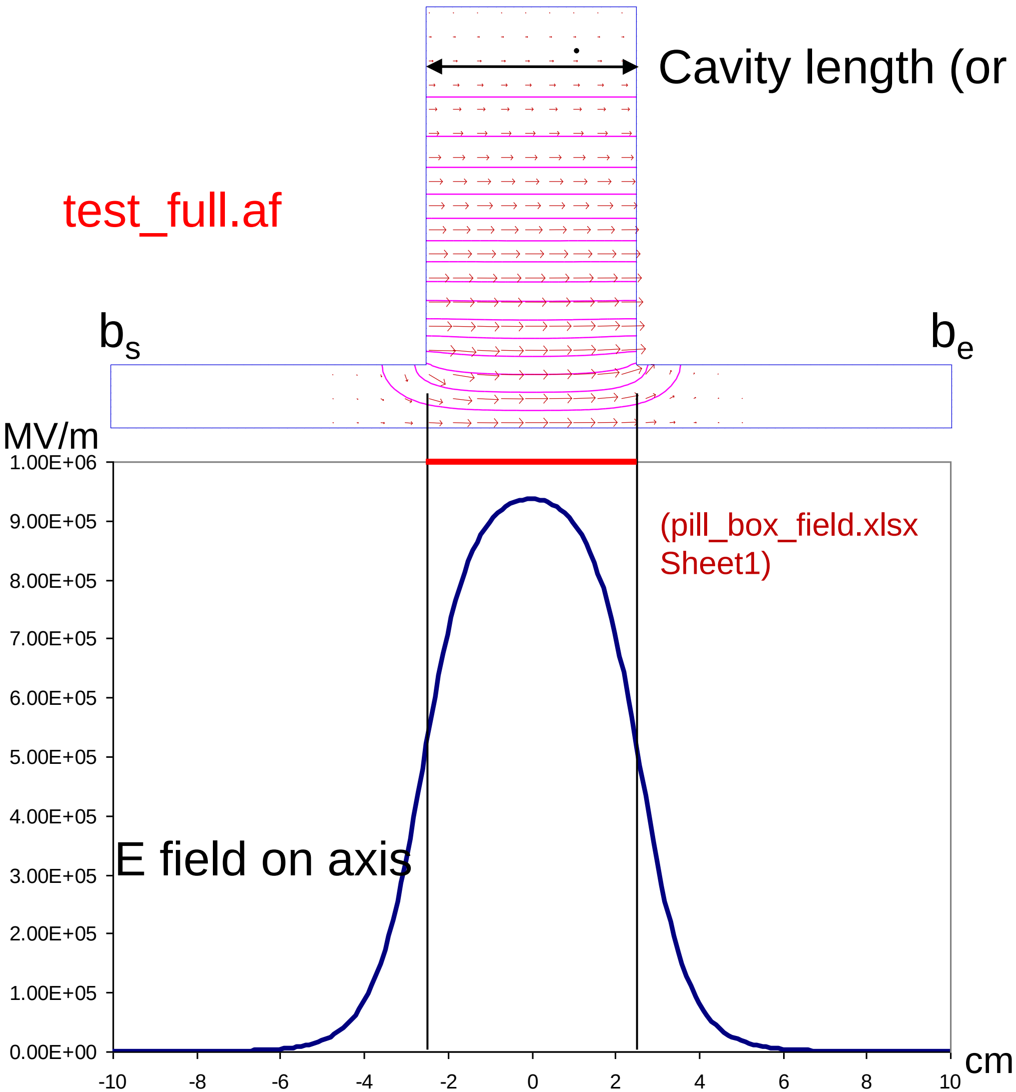


E field on axis



E, E_0, Cavity length (or cell length)

Let's make a hole (bore radius=1.5 cm) to allow a beam to pass through.
Then, analytic calculation gives only a rough estimation.



$$E_0 = \frac{1}{L}\int_{-\infty}^{\infty} |E(z)| dz = \frac{V_0}{L}$$

In simulation

$$E_0 = \frac{1}{L}\int_{bs}^{be} |E(z)| dz = \frac{V_0}{L}$$

E_0 should be defined with a corresponding length L

Boundary should be set in a way that the field should not be affected by the boundary. In some geometry or higher-order-mode analysis, it can result in a large error.

Shunt impedance R_{sh}

One of ‘figure of merits’.
Integral of axial electric field (axial voltage) per unit power dissipation.
Independent of cavity field.

$$P_c = \frac{VV^*}{2R} = \frac{VV^*}{R_{sh}} = \frac{V_0^2}{R_{sh}} = \frac{R_S}{2}\int_{\substack{\text{cavity}\\ \text{surface}}} H_t \cdot H_t^* ds, \ \ (V = V_0 \exp(i\omega t)) \ [W]$$

$$\text{in normal conducting cavity} \rightarrow \text{surface resistance } R_S = \frac{1}{\sigma\delta}$$

$$\text{in superconducting cavity} \rightarrow \text{surface resistance } R_S = R_{BCS} + R_{res} + R_{others}$$

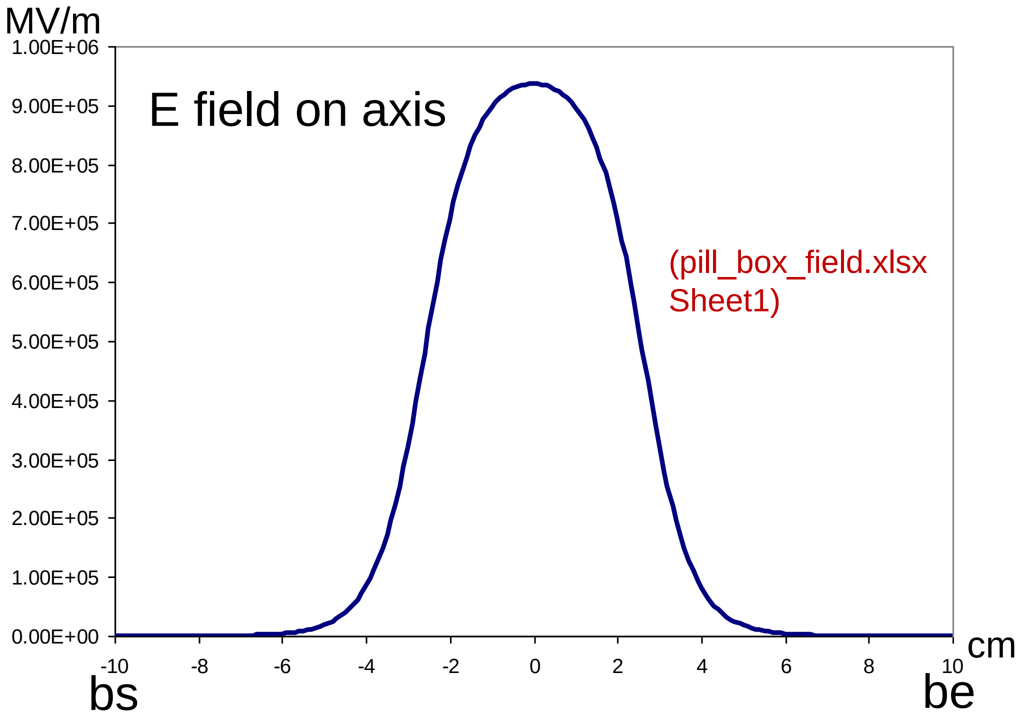
$$R_{sh} = \frac{(E_0 L)^2}{P_c} = \frac{V_0^2}{P_c} \ [\Omega]$$

In linacs, R_{sh} (shunt impedance) refers a time averaged power dissipation.

Shunt impedance per unit length Z (superfish notation)

$$Z = \frac{R_{sh}}{L} = \frac{E_0^2 L}{P_c} [\Omega]$$

Energy gain and transit time factor



Let's calculate energy gain only using axial electric field.
Direct integration

$$\text{energy gain} \quad \Delta W = q\int_{-\infty}^{\infty} E(z)\cos(\omega t + \Phi)dz$$

Example) 200 MeV proton is entering into this cavity. We will calculate energy gain by changing Φ.

$$\Delta W = q\int_{-10cm}^{10cm} E(z)\cos(\omega t + \Phi)dz$$

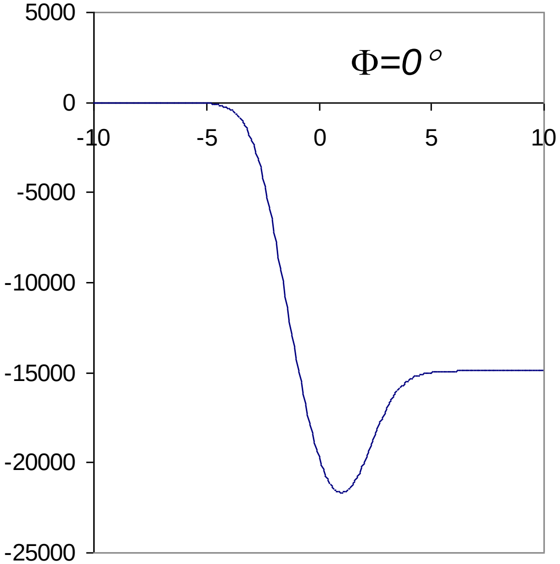
Φ; arbitrary → for convenience, set t=0 (can be arbitrary too) when proton enters into field boundary (-10 cm), t; time to take for proton to reach at z

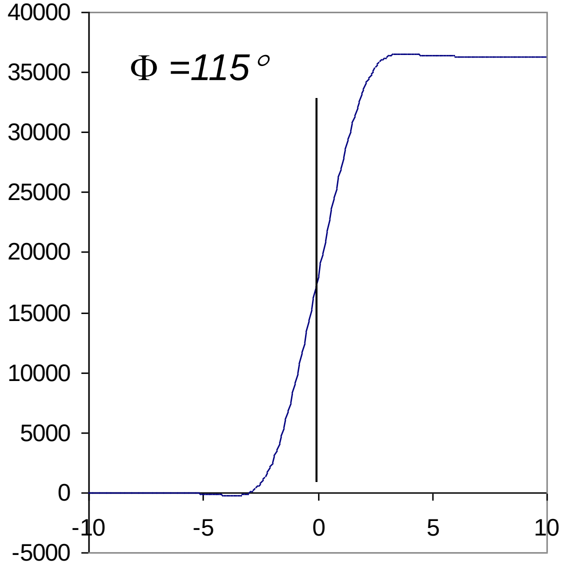
$$t = \int_{bs}^{z} v(z)dz$$

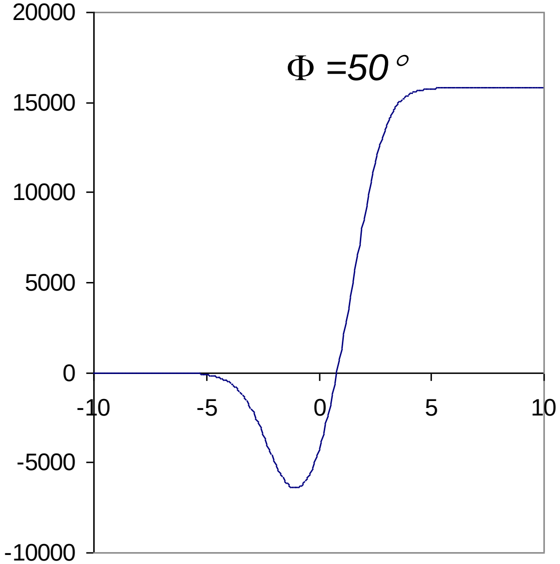
$$v = \beta c = c\sqrt{1-\left(\frac{938.272}{938.272+KE}\right)^2}$$

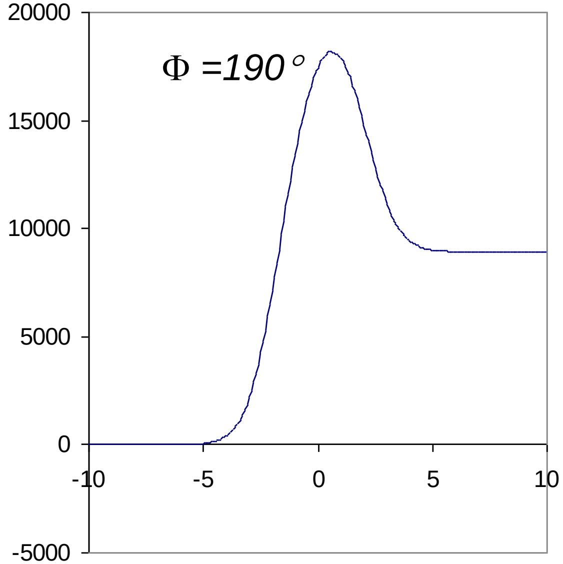
KE; kinetic energy of proton in MeV

Now we have all information. On axis field in SFO file will be used directly.
(pill_box_field.xls)

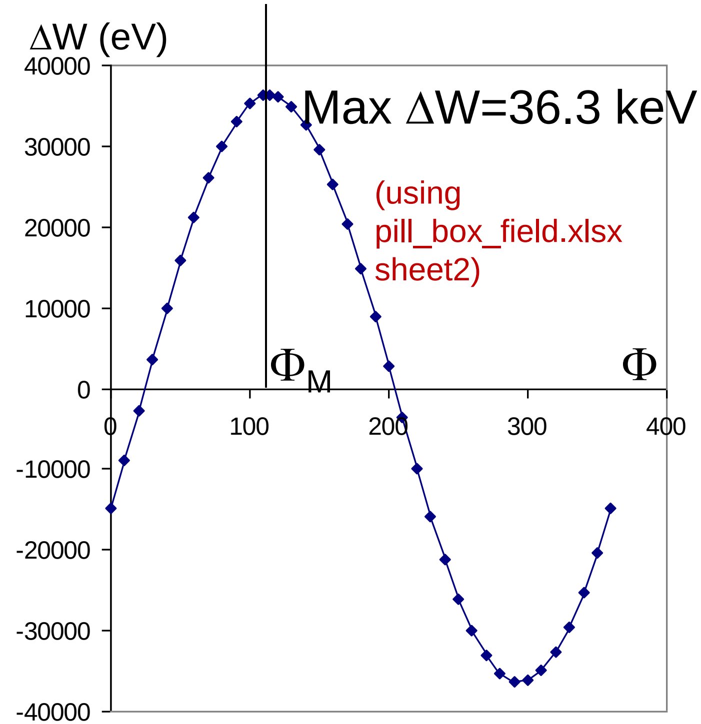








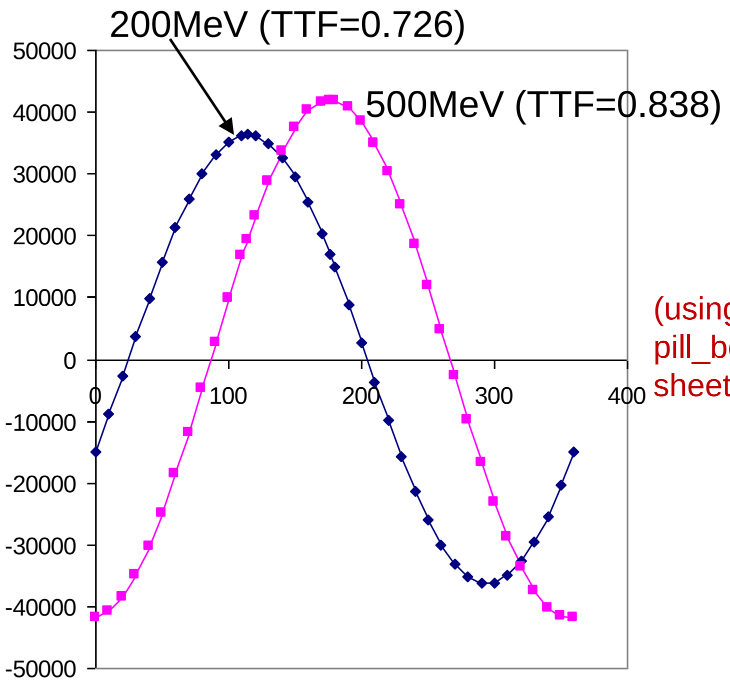
(pill_box_field.xlsx
sheet2)



So, 36.3/50=0.726
Maximum energy gain for
200 MeV proton is 72.6 %
of $V_0=E_0L=1MV/m \times 0.05m$

0.726 is the **transit time factor** of this structure for 200 MeV proton

As expected, TTF is a function of particle velocity. TTF increases as particle velocity increases in this example (single cell structure)



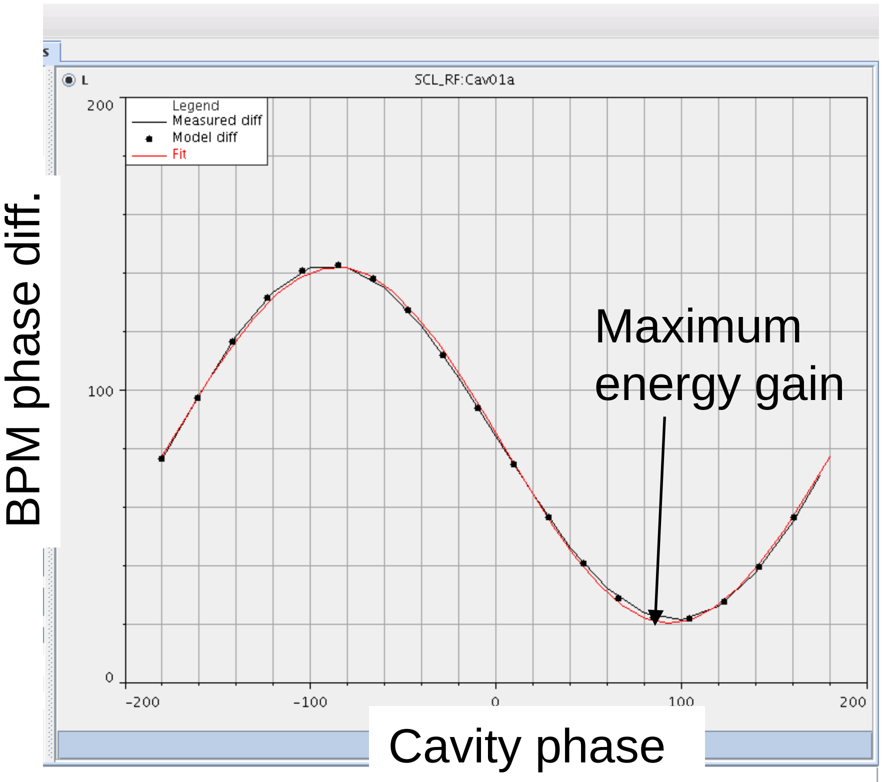
(using pill_box_field.xlsx sheet2)

In this example the time to get z=0 is about 245° for (200 MeV) or 176.5° for (500 MeV)

→The maximum energy gain happens;

proton arrives at the gap center when the field is maximum **in this example** (since symmetric field distribution)

Actually this is a very similar way how you find the beam phase relative to the RF phase in SRF cavities and set a cavity phase (by tradition we sometimes use term ‘synchronous phase’ in SCL,)

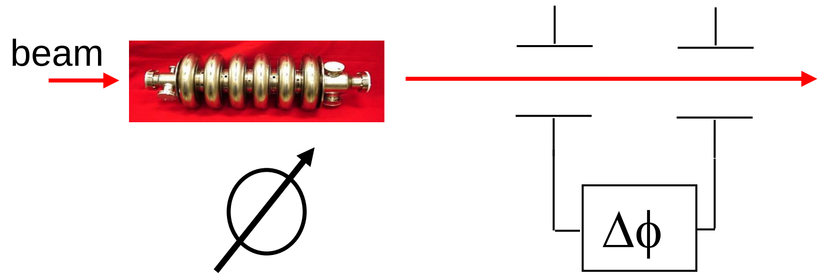


SCL phase scan for first cavity

Solid = measured BPM phase diff

Dot = simulated BPM phase diff

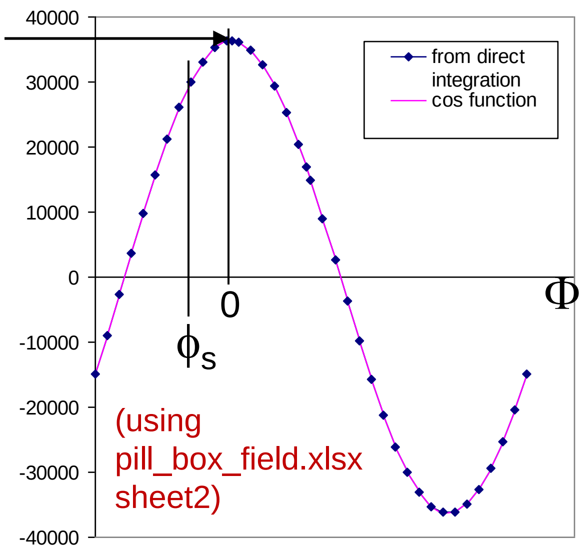
Red = cosine fit



Time of flight between two BPM at a certain RF phase in cavity

- In real world, all phases are only defined with reference RF phase→ relative
- Fitting involves varying input energy, cavity voltage and phase offset in the simulation to match measured BPM phase differences
- Relies on absolute BPM calibration
- With a short, low intensity beam, results are insensitive to detuning cavities intermediate to measurement BPMs

E_oT at a given β



$$\Delta W = qE_0TL\cos\phi_s = qV_0T\cos\phi_s = qV_a\cos\phi_s$$

E_0T; accelerating gradient, E_a or E_{acc}

usually for non-relativistic beam

ϕ_s (synchronous phase) <0

for longitudinal focusing

If we set t=0 when proton arrives at the electrical center for symmetric field distribution (in this case also the electric center), the energy gain can be expressed with cosine function

$$\Delta W = q\int_{bs}^{be} E(z)\cos(\omega t + \phi_s)dz$$

$$= q\int_{bs}^{be} E(z)\left[\cos\omega t\cos\phi_s - \sin\omega t\sin\phi_s\right]dz = qV_0T\cos\phi_s$$

$$T = \frac{\int_{bs}^{be} E(z)\cos\omega t\, dz - \tan\phi_s \int_{bs}^{be} E(z)\sin\omega t\, dz}{E_0L}$$

As we defined, the particle arrive at the center of the gap in this example when the field is at maximum→ ϕ_s=0

In this example we ignored the particle velocity variation for the calculation of t, assuming dW<<Win.

→

$t=(z-z_c)/\beta c$ (z_c; gap center), $\omega=2\pi f= 2\pi c/\lambda$

$\omega t\sim 2\pi(z-z_c)/\beta\lambda=k(z-z_c)$, k; wave number

Superfish notation

$$\mathbf{T} = \frac{\int_{bs}^{be} E(z)\cos k(z-z_c)\,dz - \tan\phi_s \int_{bs}^{be} E(z)\sin k(z-z_c)\,dz}{V_0}$$

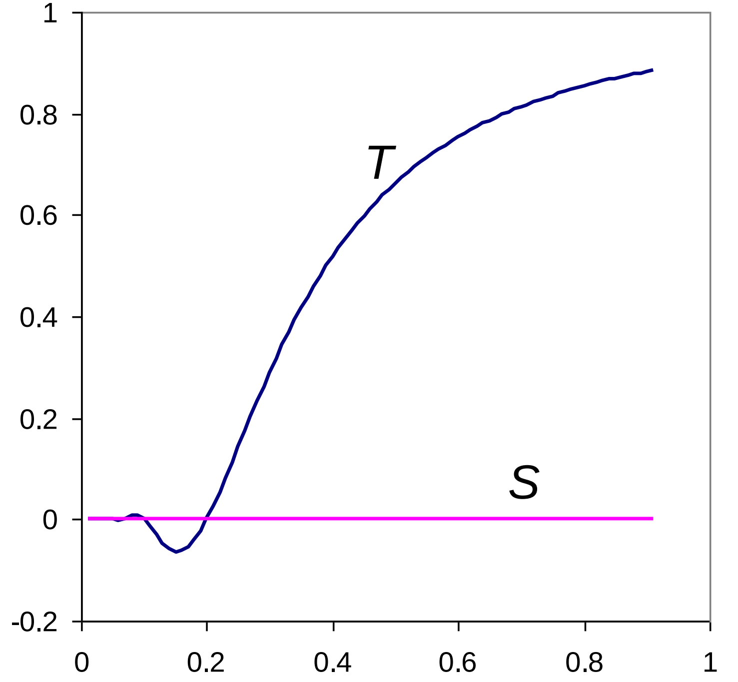
$$= T - \tan\phi_s\, S$$

If z_c is chosen to coincide with cell's electrical center→ integration of sine function=0 → TTF; Independent of synchronous phase

Add TTF table in SUPERFISH file

```
IBETA=2            ; Make T vs beta table, use electrical center
BETA1=0.01         ; Starting velocity for table
BETA2=1.0          ; Ending velocity for table
DBETA=0.1D-001     ; Velocity increment for table
```

(pill_box_field.xlsx sheet3)



Transit time factor can be defined in a different way:

$$E = E(z)\exp(i\omega t) \rightarrow T = \left| \frac{\int_{bs}^{be} E(z)\cos\omega t\, dz + i\int_{bs}^{be} E(z)\sin\omega t\, dz}{E_0 L} \right| = \frac{1}{E_0 L}\left|C + iS\right|$$

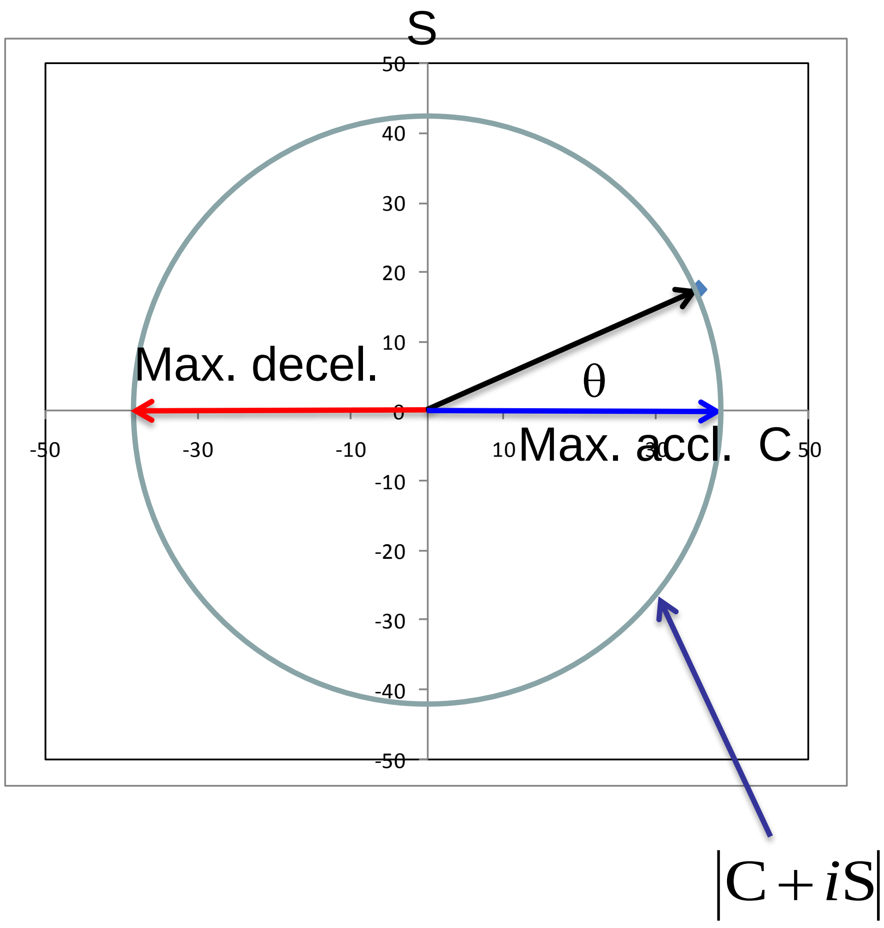
If the particle enters at

$$E = E(z)\exp(i\omega t + \Phi)$$

Where Φ: arbitrary phase.
Makes θ in the plot, then the entrance phase at Φ = -θ results in
-maximum energy gain,

We defined this corresponds particle phase=0

The location between boundaries that makes the sine integral zero is called electric center.



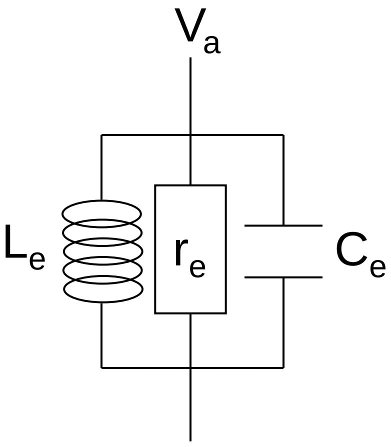
HOMEWORK 2-1: (for extra credit)
We only know the cavity information, L=5 cm, f=1.159GHz, and axial field profile as in the fields.xls

1. Calculate Eo for L=5 cm

2. Calculate electric centers and TTF For 100, 200, 300, 400 MeV proton.

Effective quantities; seen by a particle

→ include transit time factor (don't be confused with notation for electron machines, where transit time factor is a constant for β=1)



In accelerating cavity, energy gain of a particle is a more interesting quantity.
V, R, L, C in an equivalent circuit are lumped quantities. We can construct an equivalent circuit for 'accelerating voltage V_a.

$$P_c = \frac{V_a^2}{2r_e} = \frac{V_a^2}{r_{sh}} = \frac{(V_0 T)^2}{r_{sh}} = \frac{V_0{}^2}{R_{sh}} = \frac{V_0^2}{2R} \ [W]$$

$$r_{sh} = \frac{(E_0 TL)^2}{P_c} = \frac{(V_0 T)^2}{P_c} = \frac{V_a^2}{P_c} = R_{sh} T^2 \ [\Omega]$$

Effective shunt impedance $\mathbf{r_{sh}}$;
Square of accelerating voltage per unit power dissipation.
Effectiveness of delivering energy to a particle per unit power dissipation.
One of major concern for normal conducting cavity; maximize r_{sh}

In electron machines, T is mainly for β=1→ can be treated as a constant.
But in proton/ion machines, T is a function of particle velocity.

r over Q;

Effectiveness of energy gain to a particle per stored energy per a cycle.

$$\left(\frac{r}{Q}\right) \equiv \frac{(E_o T L)^2}{\omega U} = \frac{V_a^2}{\omega U} \; [\Omega]$$

r over Q is a figure of merit.

Only depends on cavity geometry at a given β (not related with surface properties).

A very useful parameter for cavity analysis

$$\frac{r}{Q} = \frac{V_a^2}{\omega U} = \frac{V_a^2}{P_c} \cdot \frac{P_c}{\omega U} = \frac{r_{sh}}{Q_0} = \frac{2r_e}{Q_0}$$

We are only concerning a cavity side now.

We will expand relations for external loads later.

cf) we can define a similar quantity for V_0:

$$\left(\frac{R}{Q}\right) \equiv \frac{R_{sh}}{Q_0} = \frac{2R}{Q_0} = \frac{(E_o L)^2}{\omega U} = \frac{V_0^2}{\omega U} \; [\Omega]$$

in electron machines, T is a constant and, sometimes it is already in R_{sh}

Geometrical factor $Q_0 \cdot R_S$

Since a surface resistance R_S is a function of material, quality, and many other practical parameters, R_S can be taken out from the measured Q_0.

$$P_c = \frac{R_S}{2} \int_{\substack{\text{cavity}\\\text{surface}}} H_t^2 ds$$

$$Q_0 \cdot R_S = \frac{\omega U}{P_c} \cdot R_S = \frac{\frac{1}{2}\mu \int_{\substack{\text{cavity}\\\text{volume}}} H^2 dv}{\frac{1}{2} \int_{\substack{\text{cavity}\\\text{surface}}} H_t^2 ds}$$

can be calculated numerically or analytically

Peak Surface Field

E_p: Peak surface electric field
B_p: Peak surface magnetic field
And
E_p/E_a (or E_p/E_0)
B_p/E_a (or B_p/E_0)
E_p/B_p:

SFO file for simple pillbox cavity with a hole in the previous example. Let's understand what they mean and what they correspond to..

```
Field normalization (NORM = 0):     EZERO      =     1.00000 MV/m
Length used for E0 normalization               =     5.00000 cm
Frequency                                      =  1152.74636 MHz
Particle rest mass energy                      =  938.272029 MeV
Beta =  0.3845148         Kinetic energy       =      78.143 MeV
Normalization factor for E0 =  1.000 MV/m      =    5641.263
Transit-time factor                            =   0.4896499
Stored energy                                  =   0.0018721 Joules
Using standard room-temperature copper.
Surface resistance                             =     8.85784 milliOhm
Normal-conductor resistivity                   =     1.72410 microOhm-cm
Operating temperature                          =     20.0000 C
Power dissipation                              =    793.1324 W
Q            =   17096.0       Shunt impedance =      63.041 MOhm/m
Rs*Q         =   151.434 Ohm             Z*T*T =      15.115 MOhm/m
r/Q  =    44.205 Ohm  Wake loss parameter      =     0.08004 V/pC
Average magnetic field on the outer wall       =     1383.56 A/m, 0.847796 W/cm^2
Maximum H (at Z,R = 2.5,7.65816)               =     1548.33 A/m, 1.06175 W/cm^2
Maximum E (at Z,R = 2.5,1.5)                   =      1.2515 MV/m, 0.041094 Kilp.
Ratio of peak fields Bmax/Emax                 =      1.5547 mT/(MV/m)
Peak-to-average ratio Emax/E0                  =      1.2515
```

$\beta\lambda/2 = L_c$ (→ Length used for E0 normalization, Frequency)

HOMEWORK 2-2:

1. Convert effective quantities here for 200 MeV proton
2. Convert values for E_0=5 MV/m

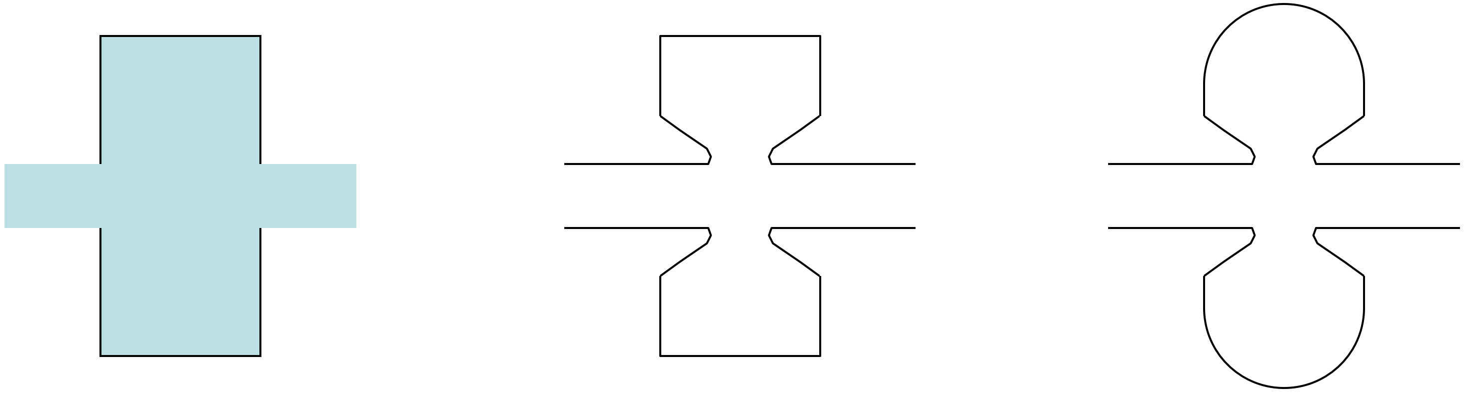
Basis of Design Consideration

Now we can talk about cavities using cavity parameters.

cf) first let's take a short look about
design concerns for **normal conducting cavities**;

Maximize shunt impedance or r/Q
;maximize acceleration voltage seen by the particles at a given stored energy
As a gap length is increase, Transit time factor is decreased.
Too small gap→ Va becomes smaller at a certain peak surface field.
→nose cone shape + gap length (increase acceleration efficiency)

Further increase of r/Q by increasing Qo → sphere has the minimum of S/V
;spherical shape (decrease power dissipation at the same stored energy)



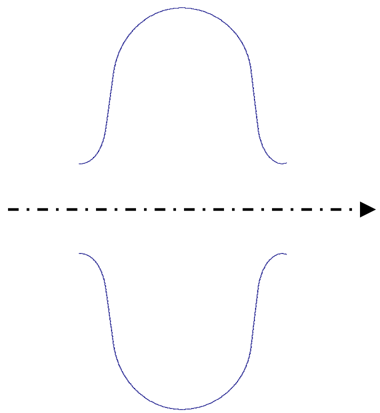
Let estimate the differences with the same example just by changing input for material.

(stest_full.af)

```
IRTYPE=1               ; Rs method: Superconductor formula
TEMPK=2                ; Superconductor temperature, degrees K
TC=9.2                 ; Critical temperature, degrees K
RESIDR=0.1D-007        ; Residual resistance in Ohm
```

		Cu
Field normalization (NORM = 0): EZERO	= 1.00000 MV/m	
Length used for E0 normalization	= 5.00000 cm	
Frequency	= 1152.74636 MHz	
Particle rest mass energy	= 938.272029 MeV	
Beta = 0.3845148 Kinetic energy	= 78.143 MeV	
Normalization factor for E0 = 1.000 MV/m	= 5641.263	
Transit-time factor	= 0.4896499	
Stored energy	= 0.0018721 Joules	
Superconductor surface resistance	= 23.2065 nanoOhm	= 8.85784 milliOhm
Operating temperature	= 2.0000 K	= 20.0000 C
Power dissipation	= 2077.9165 uW	= 793.1324 W
Q	= 6.5255E+09	= 17096.0
Shunt impedance	= 2.4063E+07 MOhm/m	= 63.041 MOhm/m
Z*T*T	= 5.7692E+06 MOhm/m	= 15.115 MOhm/m
Rs*Q = 151.434 Ohm r/Q = 44.205 Ohm		
Wake loss parameter = 0.08004 V/pC		
Average magnetic field on the outer wall	= 1383.56 A/m, 2.22113 uW/cm^2	
Maximum H (at Z,R = 2.5,7.65816)	= 1548.33 A/m, 2.78167 uW/cm^2	
Maximum E (at Z,R = 2.5,1.5)	= 1.2515 MV/m, 0.041094 Kilp.	
Ratio of peak fields Bmax/Emax	= 1.5547 mT/(MV/m)	
Peak-to-average ratio Emax/E0	= 1.2515	

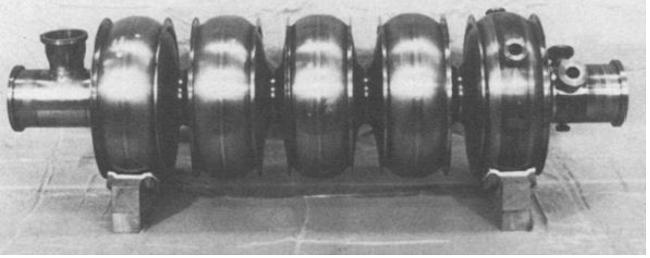
SRF cavity design concerns are mainly to
Minimize peak surface fields + other concerns
Shunt impedance is a minor issue due to much lower surface resistance (intrinsically very high)
→Larger bore radius, round shape everywhere, optimization for other concerns
→Elliptical cavity shape (one of most popular shapes)



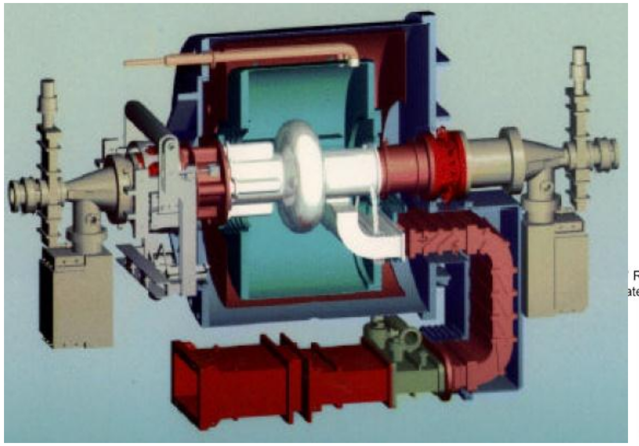
Reduced beta for proton beam in 2000 for SNS
β=0.61, 0.81
(pulsed, the first operational SCL for proton beam)

Examples of elliptical cavities

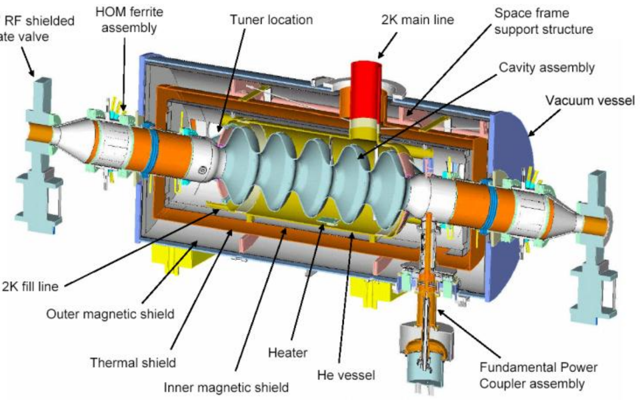
Ring for electrons



KEK TRISTAN 500 MHz



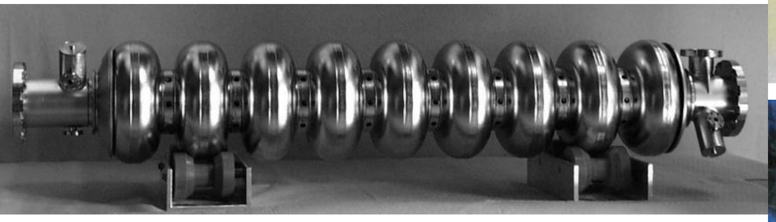
CORNELL 500 MHZ



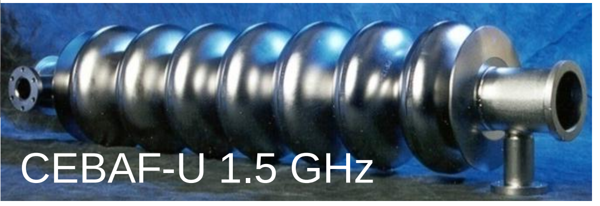
BNL ERL 700 MHz

Linac for electrons

FERMI 3.9 GHz



XFEL/TESLA/ILC 1300 MHz

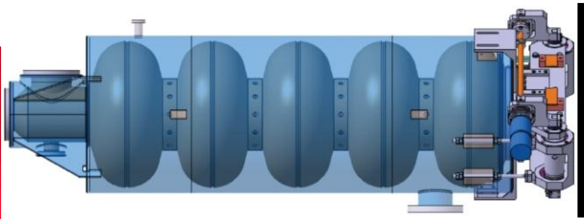


CEBAF-U 1.5 GHz

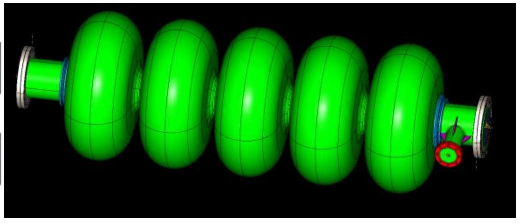
LANL APT 700 MHz

Linac for protons

SNS 805 MHz



CERN SPL, ESS 704 MHz



FERMI Project-X 650 MHz

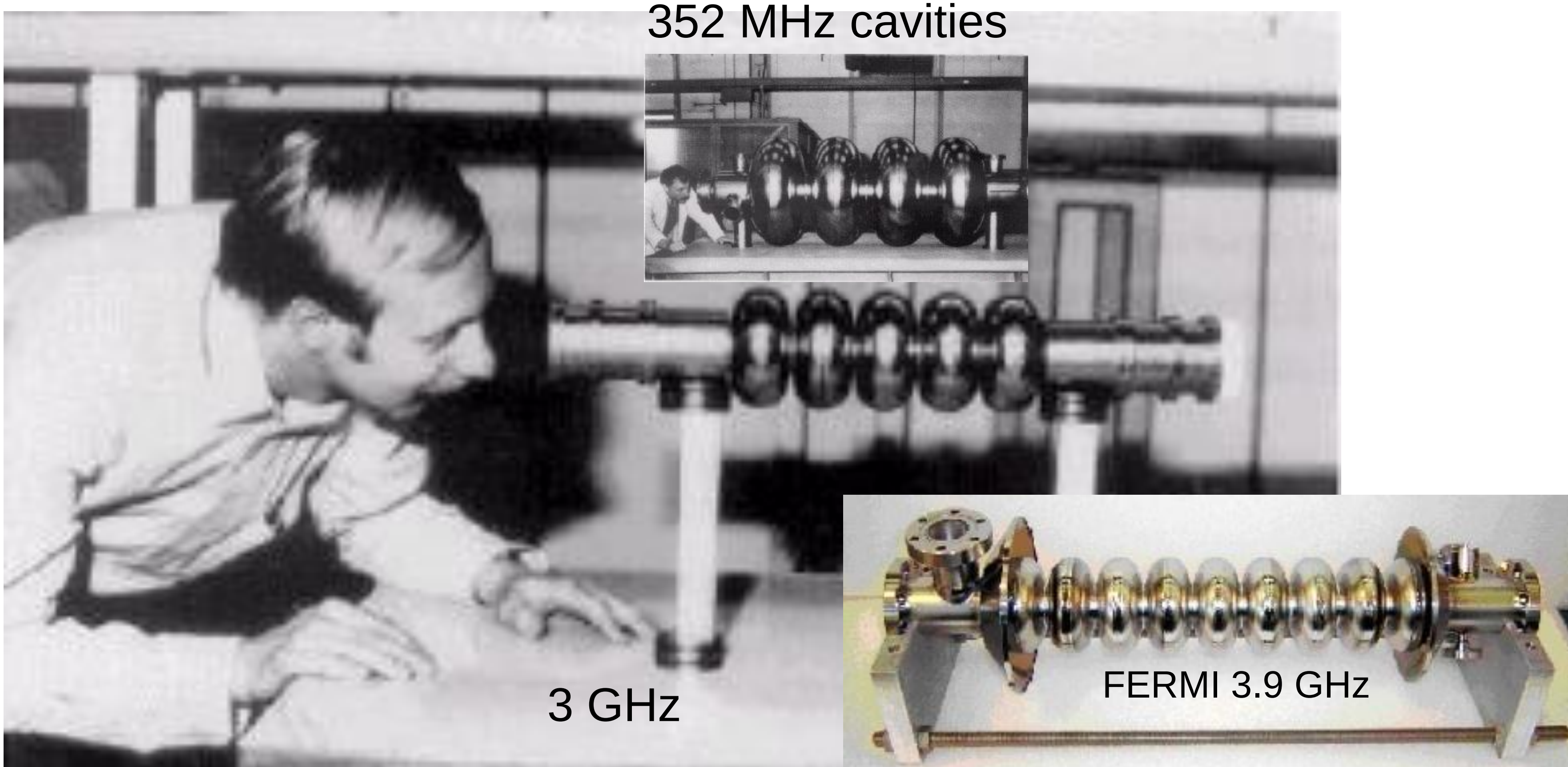
Frequency ranges for elliptical cavities

mostly 350 MHz-4GHz

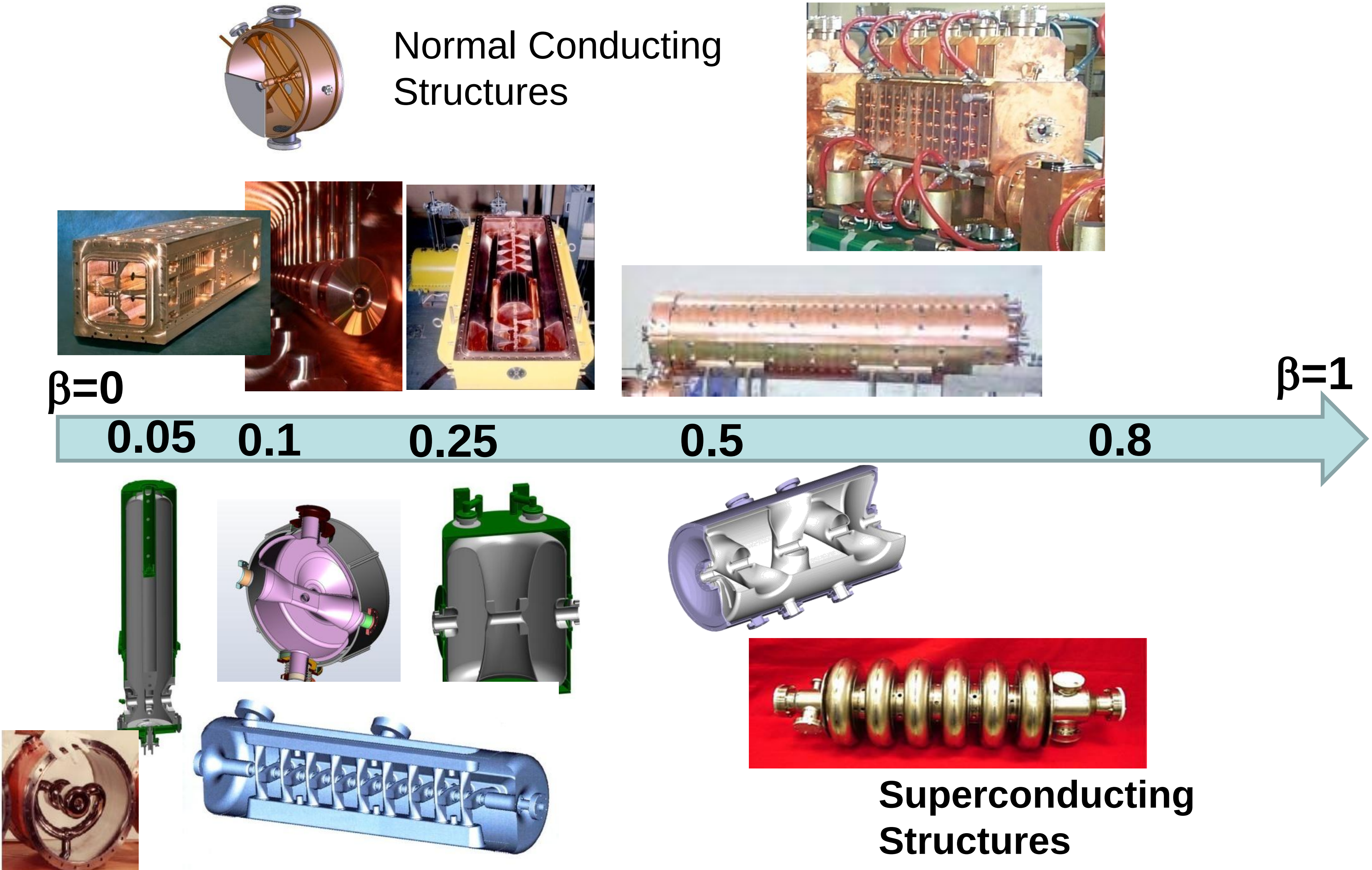
low frequency; sizes become big

other shapes are better like HWR, QWR, Spoke…

high frequency; BCS loss becomes high (high duty, CW)



RF Structures for β < 1 Acceleration



SRF applications are expanding for lower beta region using different cavity shape (Quarter wave, half wave, spoke-type, etc)

Elliptical Cavity

Design and optimization are always iterative works like those for any others.

We will visit a higher level consideration for global architecture design in chapter 6/7.

We will here learn cavity parameters of elliptical cavities and their correlations with design parameters.

And examples of optimization procedure will be introduced in relations with design criteria.

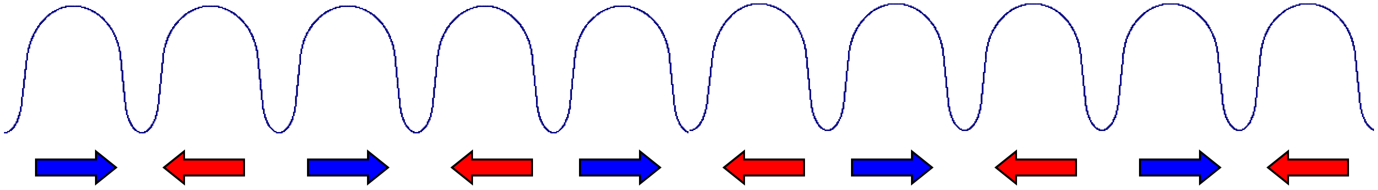
Multi-cell structure;

is composed of an array of single-gap resonators. Each resonator is called 'cell'.

For SRF cavities, pi-mode standing wave structures are mostly used.

Pi-mode means 180-degree phase shift between cells.

→ one cell length=$\beta_g\lambda/2$, β_g: geometrical beta



Pi-mode

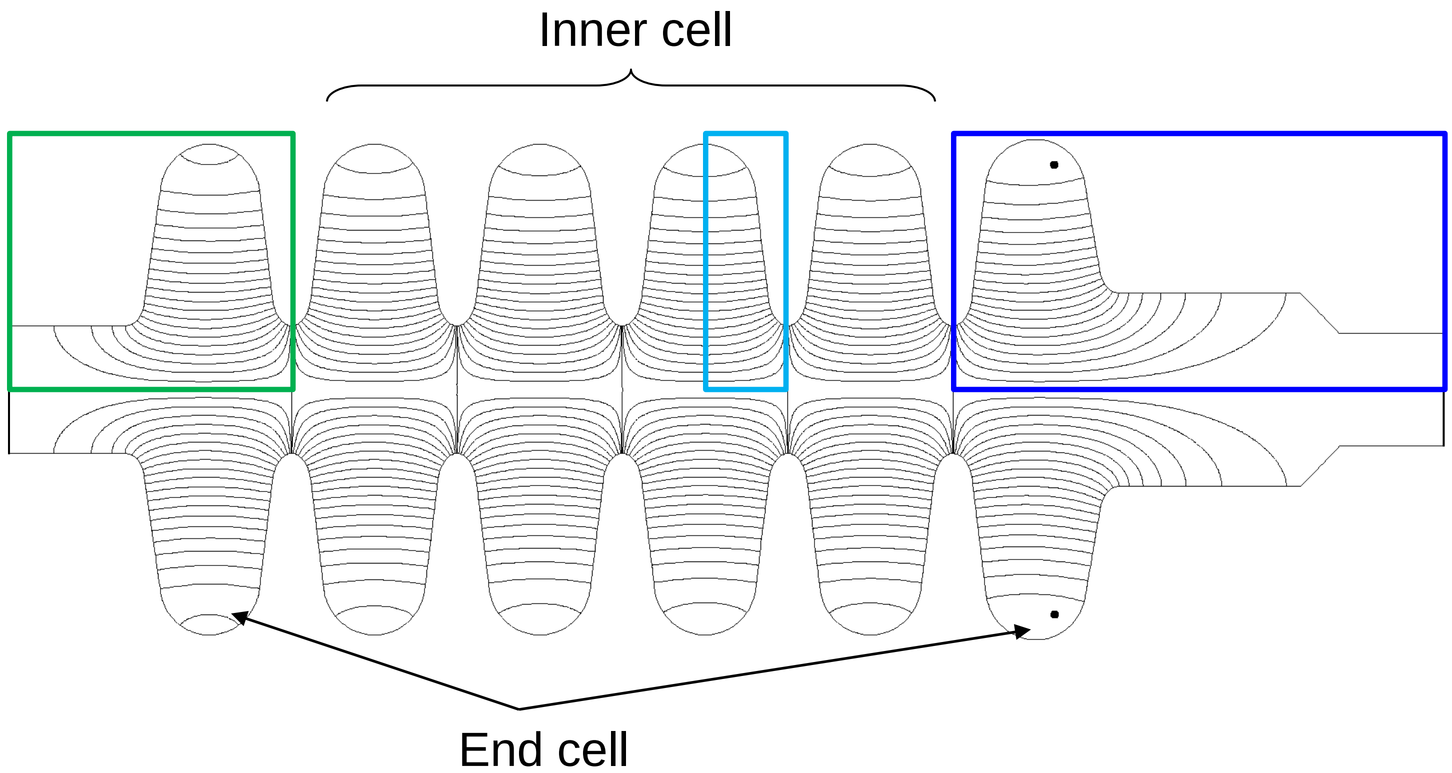
Multi-cell cavity vs. single cell cavity;

what should one take into account?

- Cost
  - (actual acceleration)/(total accelerator length); filling factor, real estate Eacc
  - number of sub systems/equipment; tuner, coupler, helium circuits, controls
- Trapped mode (HOM)
- Field flatness: inter-cell coupling
- Input power coupler power rating (gradient, beam current)
- Cavity processing quality
  - statistically more chance in multi-cell cavity to have bad actors
  - one bad actor can affect whole system
- Beam dynamics especially in lower beta region
  - longitudinal phase slip
  - acceleration efficiency (transit time factor); particle beta covering range

Careful iterations are needed in a connection with a global architecture design.

2.2 SRF Cavity Design



Inner cell

Mid (equator)-plane symmetry
Electric boundary condition
Iris Plane & Axis:
magnetic boundary condition
Cylindrical symmetry (2-D)
Modeling for a half cell is enough.

End cell

No mid (equator)-plane symmetry
Iris Plane & Axis
magnetic boundary condition
Beam pipes for other equipments
Cylindrical symmetry (2-D)
Full cell modeling needed.

We will follow a design procedure one can use for an actual machine design.

Review the general consideration

Half cell design

Static Lorentz force detuning

Multi-cell concern

End cell design

Elliptical Cavity Design considerations

- Minimize the peak surface electric field (Ep/Ea)
  - field emission is strongly related to the surface condition
- Set the peak magnetic field (Bp) with sufficient margin
  - thermal breakdown is related with peak magnetic field
- Have reasonable mechanical stiffness
  - stiff against Lorentz force detuning and microphonics ✓
  - reasonable tuning force
- Slope angle (for rinsing process)
- (Increase r/Q)
- Adequate Inter-cell coupling constant ✓
- Efficient use of RF energy (end-cell design) ✓
  - Have good field flatness
  - Have equal or lower peak surface fields in end cells
- Satisfy required external Q, Q_{ex} (end-cell design) ✓

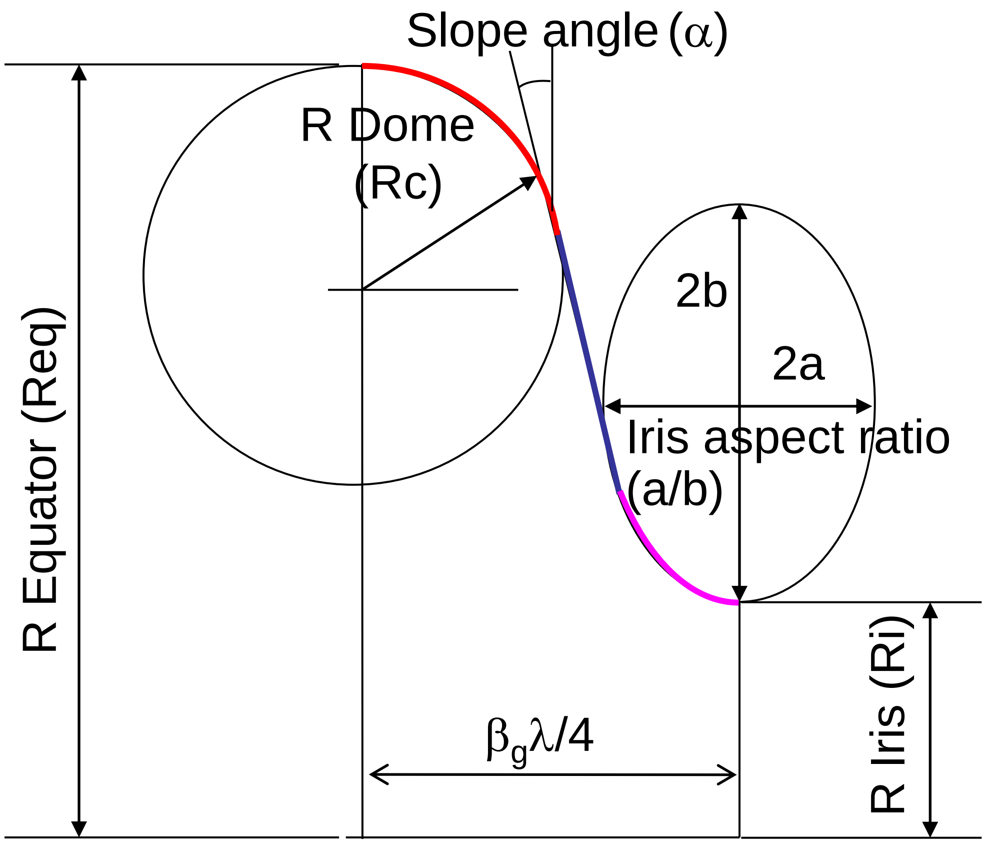
Design criteria are machine-specific.

Need OPTIMIZATION/ITERATION in the parameters space with design criteria.

Most of these design concerns are strongly related with geometry.

✓ Not yet introduced

Geometrical parameters for elliptical cavity with circular dome

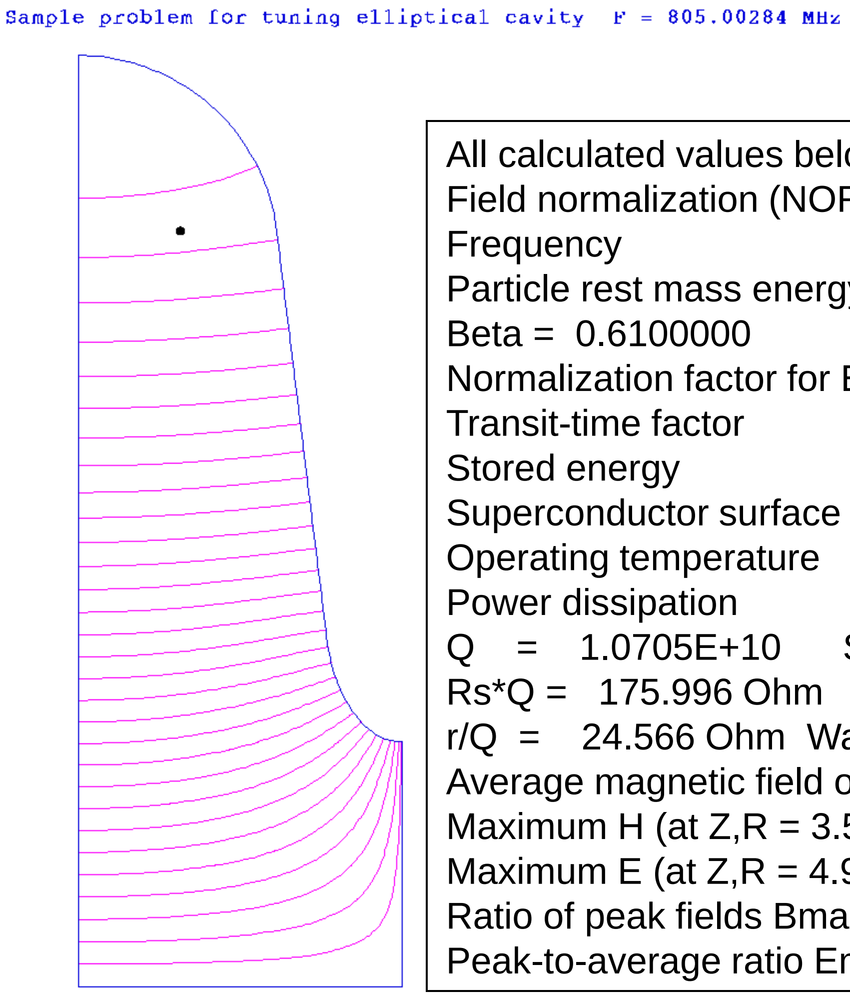


For circular dome

Rc, Ri, α, one of (a/b, a, b); 4 controllable parameters
Req (for tuning)

ELLFISH for elliptical cavity tuning (*61B*.ell)
: pre-defined tuning program

```
Title
Sample problem for tuning elliptical cavity
Design beta = 0.61
Resonant frequency = 805 MHz, Bore radius = 4.3 cm
ENDTitle

PLOTting OFF
PARTICLE              H+
SUPERConductor        2   9.2  2.00000E-08

NumberOfCells         6    ; used by the ELLCAV code
HALF_cavity

FILEname_prefix       61B
SEQuence_number       1
FREQuency             805
BETA                  0.61
DIAMeter              32.75
E0T_Normalization     1
DOME_B                3.5
DOME_A/B              1

WALL_Angle                  7
EQUATOR_flat                0
IRIS_flat                   0
RIGHT_BEAM_tube             0
IRIS_A/B                    0.59
BETASTART                   0.1
BETASTOP                    1.0
BETASTEP                    0.05
BETATABLE                   2
BORE_radius                 4.3
SECOND_Beam_tube            0
SECOND_TUBE_Radius          0
DELTA_frequency             0.01
MESH_size                   0.1
INCrement                   2
START                       2

ENDFile
```



All calculated values below refer to the mesh geometry only.
Field normalization (NORM = 1): EZEROT = 1.00000 MV/m
Frequency = 805.00284 MHz
Particle rest mass energy = 938.272029 MeV
Beta = 0.6100000 Kinetic energy = 245.815 MeV
Normalization factor for E0 = 1.292 MV/m = 16528.062
Transit-time factor = 0.7740364
Stored energy = 0.0259588 Joules
Superconductor surface resistance = 16.4405 nanoOhm
Operating temperature = 2.0000 K
Power dissipation = 12.2652 mW
Q = 1.0705E+10 Shunt impedance = 7.7286E+06 MOhm/m
Rs*Q = 175.996 Ohm Z*T*T = 4.6304E+06 MOhm/m
r/Q = 24.566 Ohm Wake loss parameter = 0.03106 V/pC
Average magnetic field on the outer wall = 3963.96 A/m, 12.9164 uW/cm^2
Maximum H (at Z,R = 3.53007,12.8456) = 4329.41 A/m, 15.4078 uW/cm^2
Maximum E (at Z,R = 4.99373,4.61716) = 2.62773 MV/m, 0.100847 Kilp.
Ratio of peak fields Bmax/Emax = 2.0704 mT/(MV/m)
Peak-to-average ratio Emax/E0 = 2.0340

Inter-cell coupling

-Each cell is weakly coupled to the neighboring cells in a multi-cell cavity.

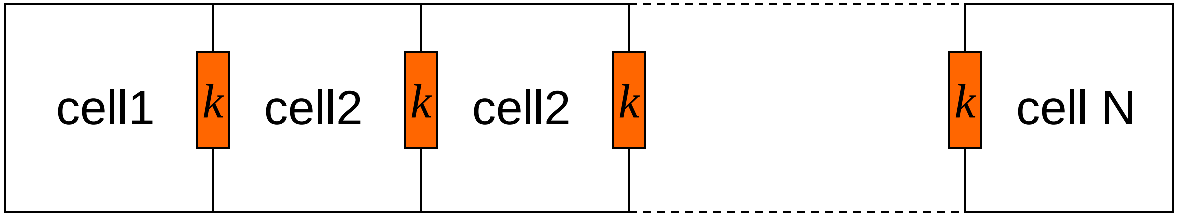
-The RF coupling between cells are through iris or other coupling mechanism.

-One mode of a single cell cavity is split into N (number of cells) modes.

-These N modes have slightly different frequencies and form a ‘passband’.

-Modes in a passband have different phase shift in each cell.

-Fundamental passband refers to a passband associated with the lowest mode, usually accelerating mode TM010.



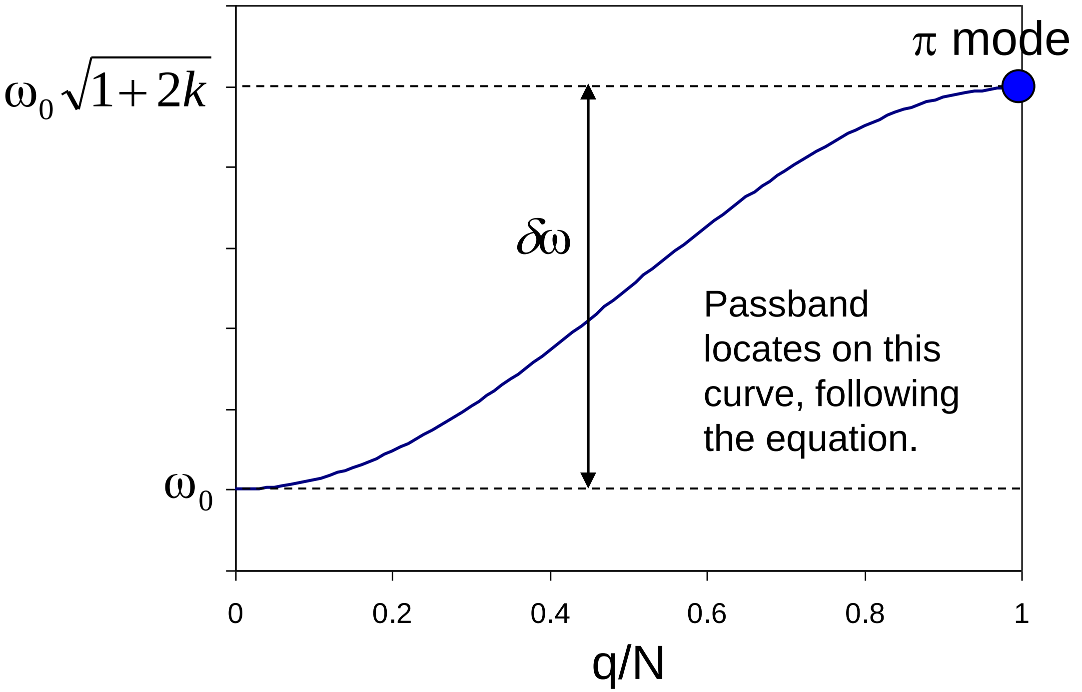
Let’s assume each cell is identical and resonate at f_0 (before having coupling k).

$$\frac{\omega_q^{\ 2}}{\omega_0^{\ 2}} = 1 + k\left(1 - \cos\frac{q\pi}{N}\right),\ q = 1,2,\ldots N$$

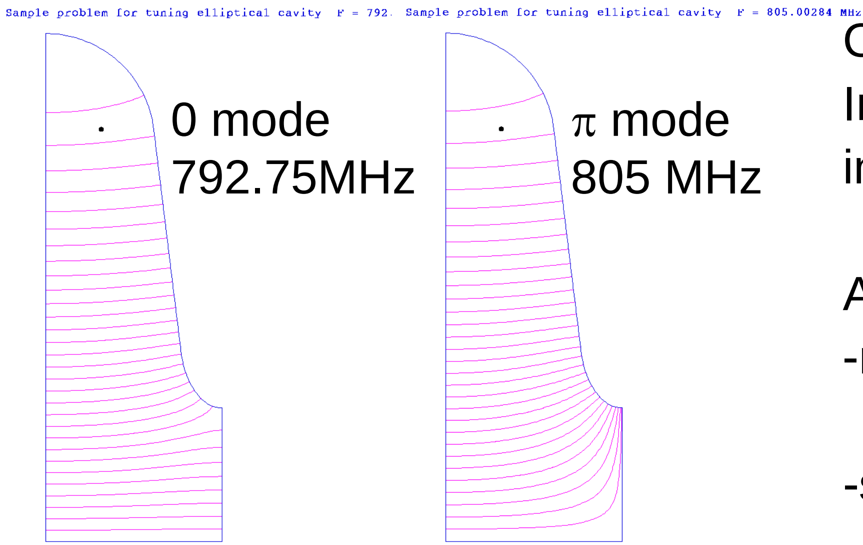
$$\frac{\omega_q^{\ 2}}{\omega_0^{\ 2}} = 1 + k(1 - \cos\frac{q\pi}{N}), \ q = 1,2,...N$$

If q=N→ π mode.

If N→∞, 0 mode exists.



0 mode can be found with the electric boundary condition at the bore (iris).



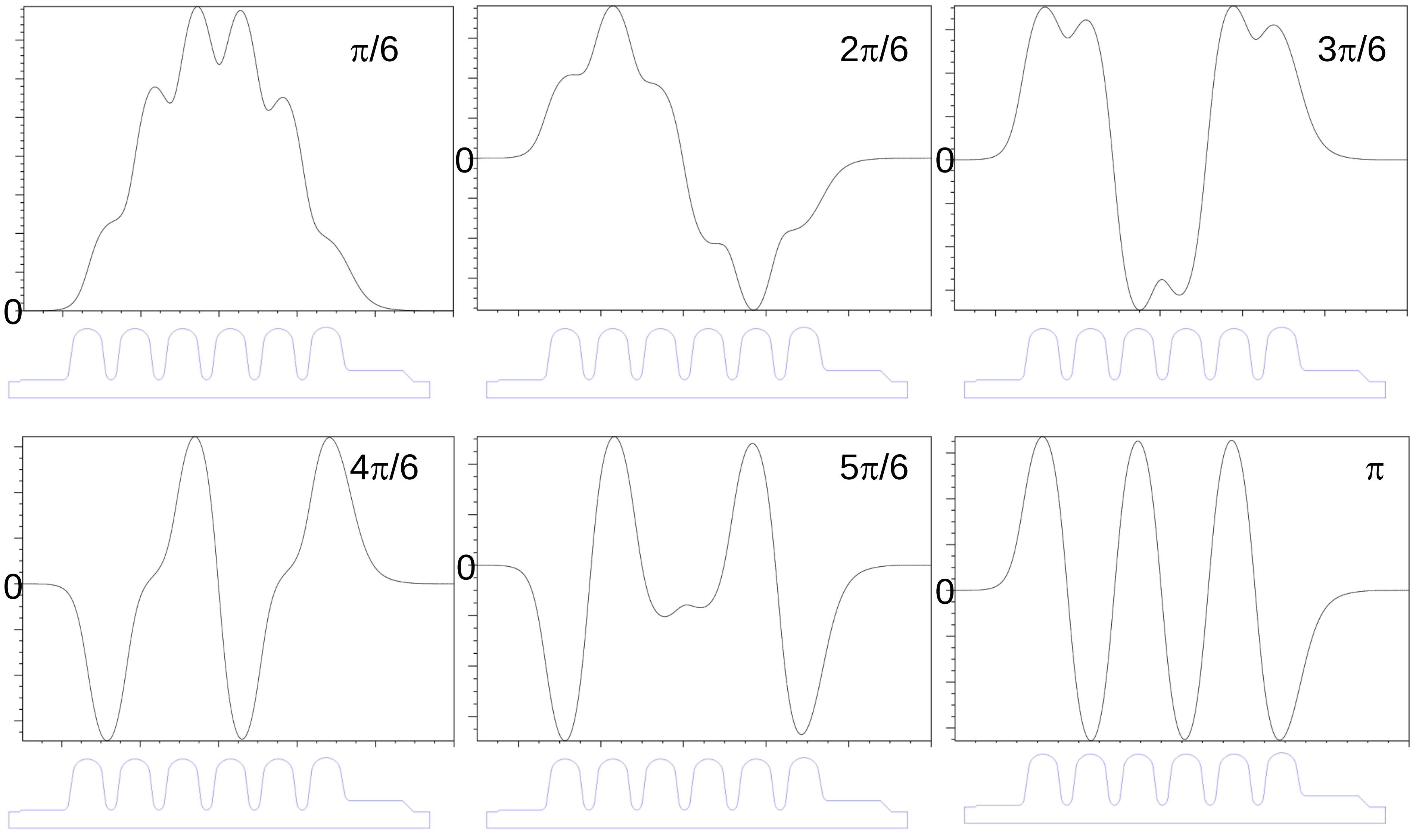
k=0.0156 (1.56 %)

Once a cell geometry is fixed,
Inter-cell coupling coefficient *k* is determined, independent of N.

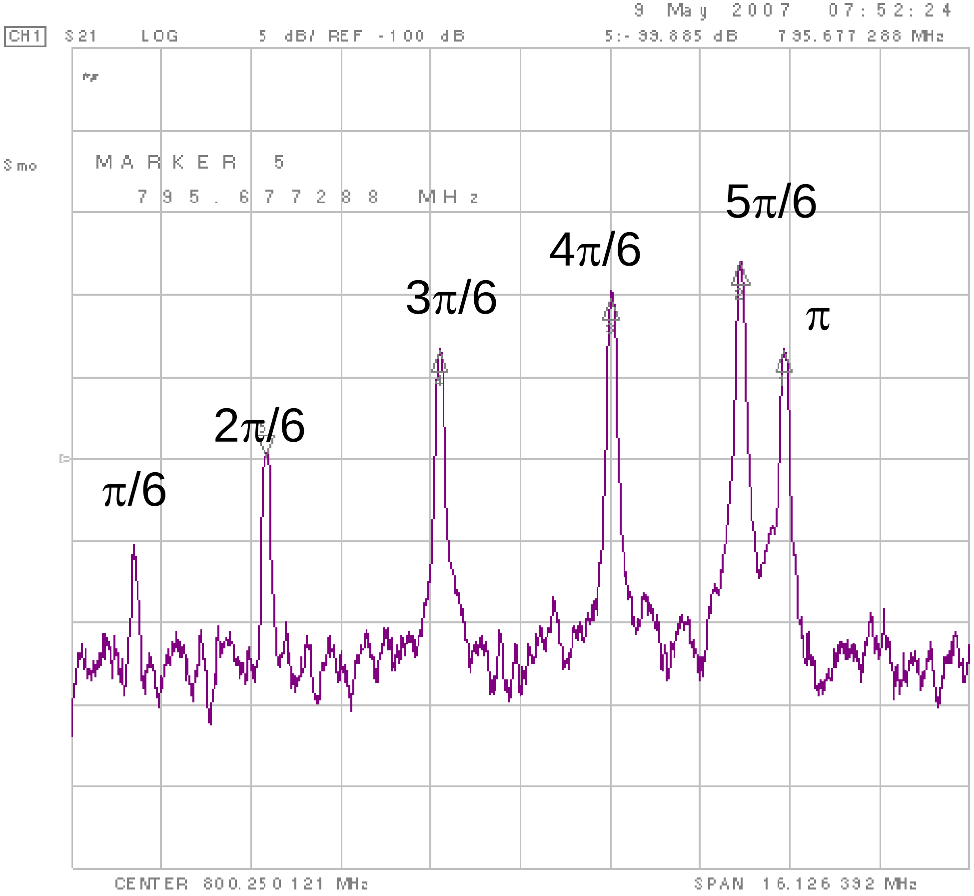
As N increases,
- -mode separation becomes narrower
  - →generator can excite neighboring mode
- -slope at π-mode becomes smaller
  - →lower energy flow; sensitive to perturbation.
- -field flatness sensitivity $\propto N^2/(k\beta)$

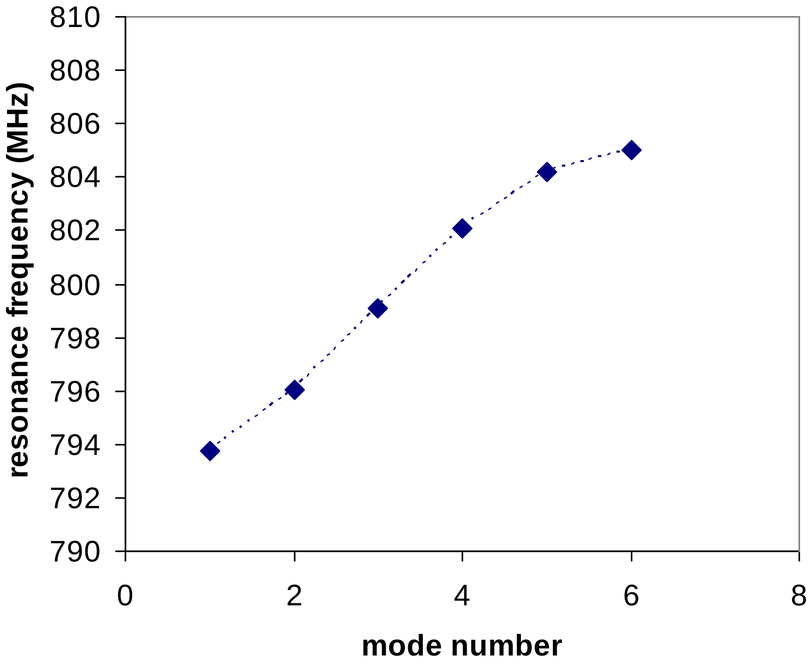
$$\text{power flow scaling} \rightarrow P_{\text{flow}} = v_g \frac{U}{l},\ v_g;\text{group velocity},\ U;\text{stored energy},\ l;\text{length}$$

Phase advance per cell of fundamental pass-bands in 6-cell cavity: fields on axis



Fundamental pass-band in 6-cell cavity (ex. SNS 6cell cavity)





Cell shape optimization

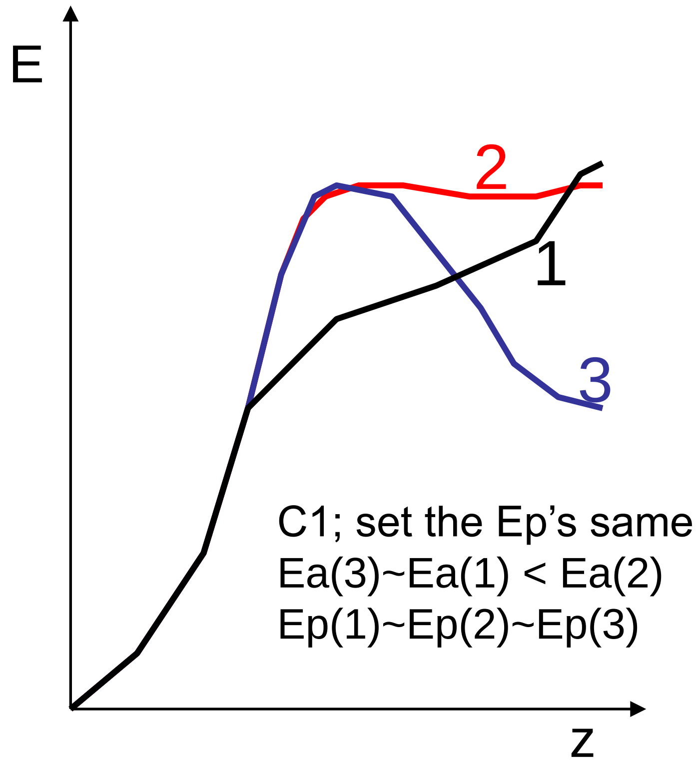
-As mentioned, most of design considerations should be carefully taken into account during shape design.

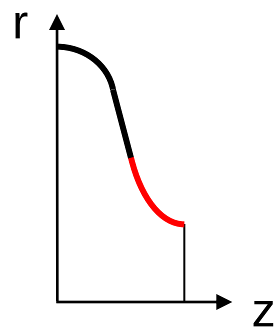
-There are 4 geometrical parameters that determines some of cavity properties.

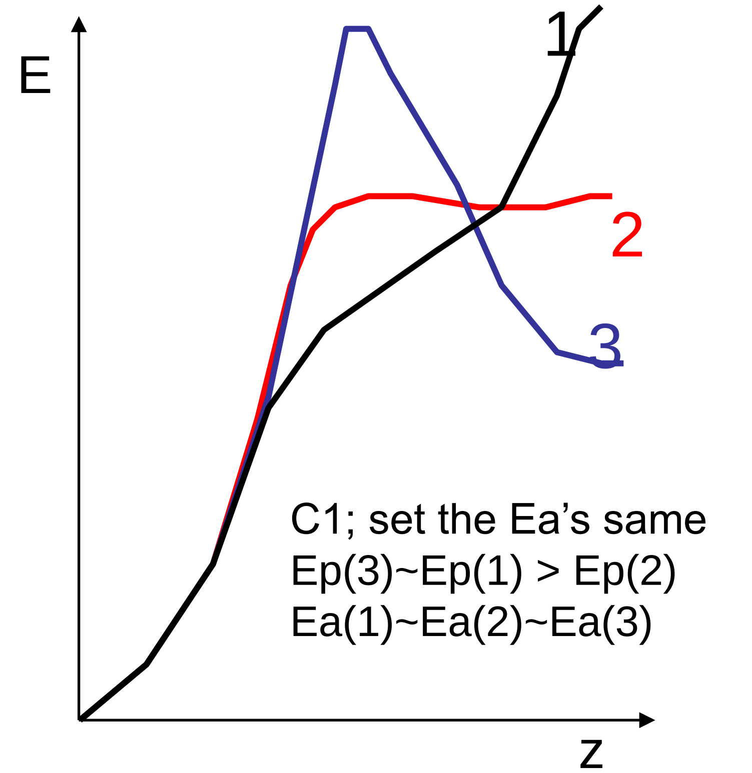
-Best way for the optimization is scanning all 4 variable parameters systematically. Here one optimization procedure will be introduced based on this approach.

-By doing this one can understand better about cavity parameters and their interplays for the real case.

Magnetic and electric regions are well separated out in TM010 cavity
We can control the E profiles by adjusting only iris ellipses
: only touching capacitive region where surface electric fields are high





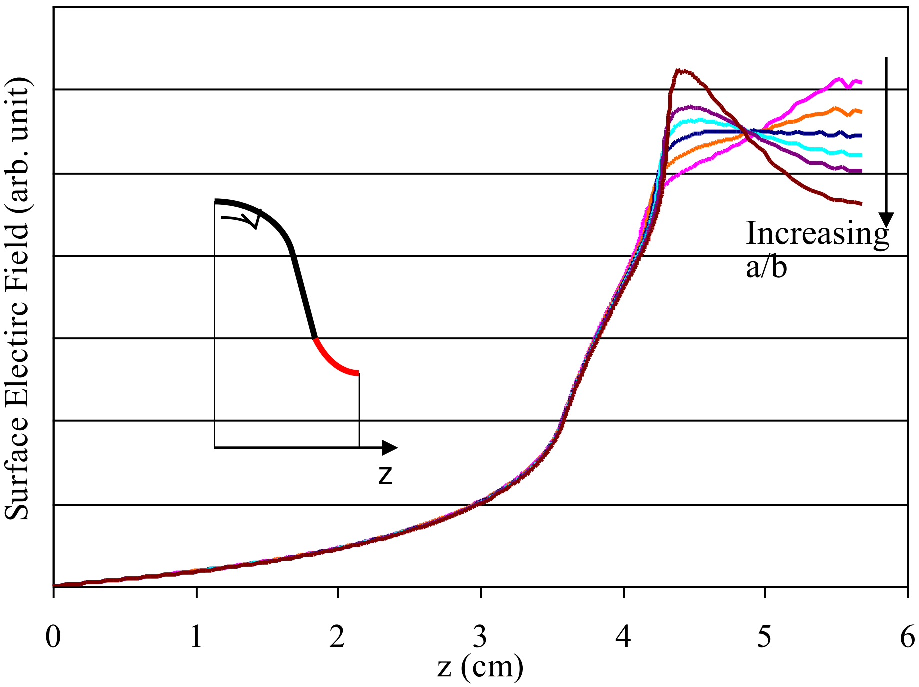


Linac with 1or 3 cavity has lower Eacc

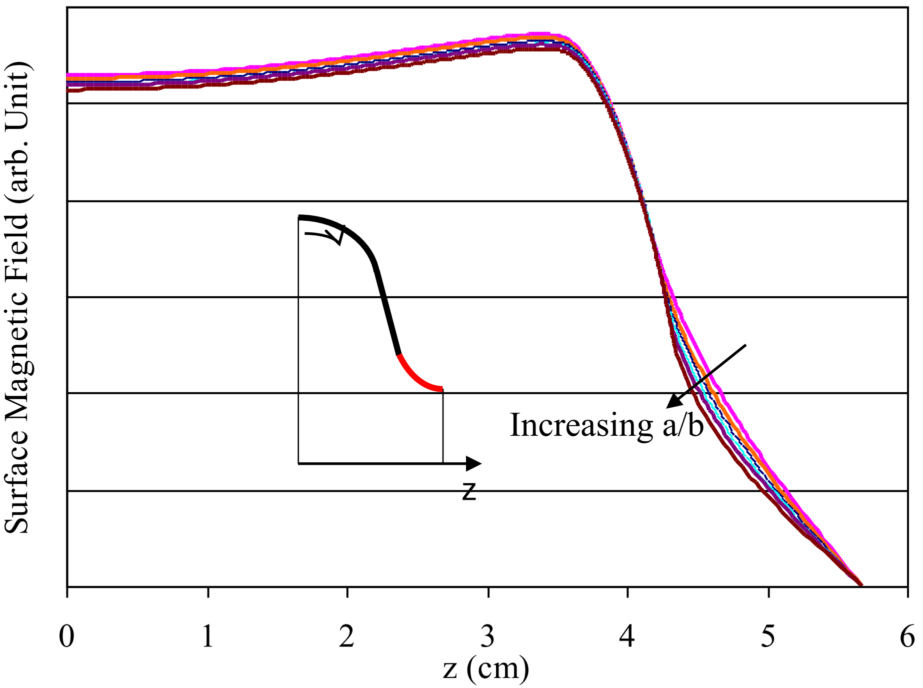
Linac with 1or 3 cavity need higher Ep criterion

Surface fields distribution at the same Accelerating gradient

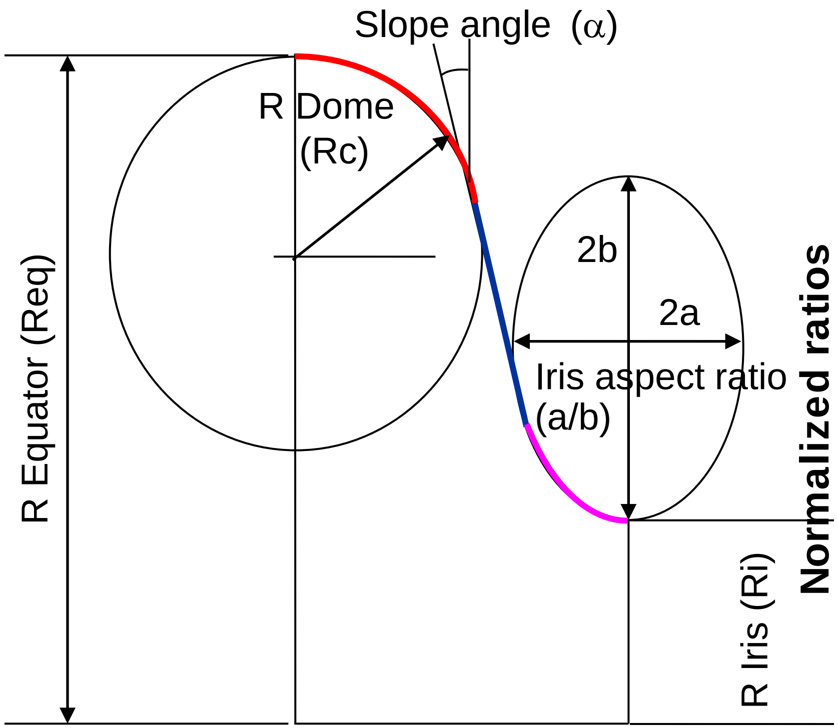
Surface electric field profile



Surface magnetic field profile

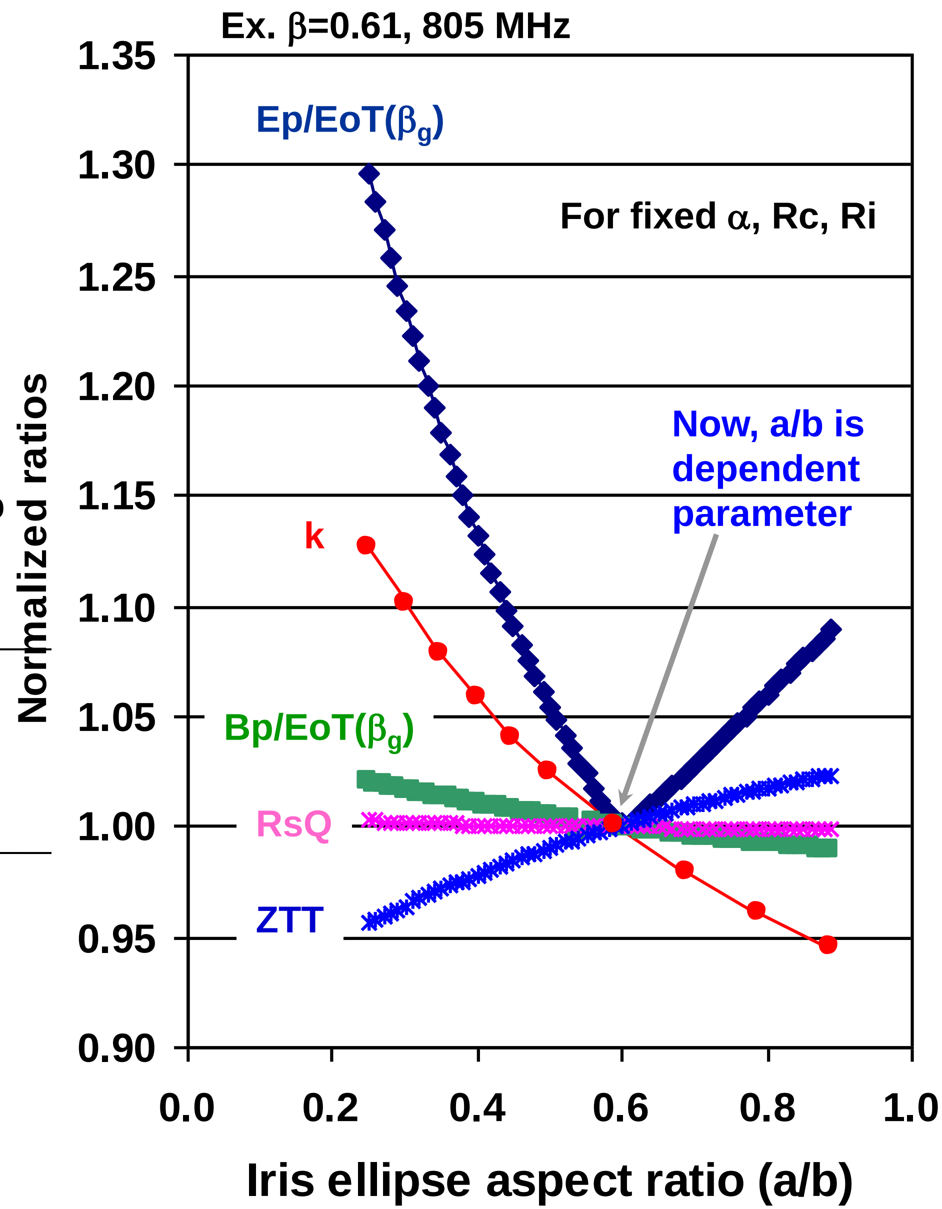


At a certain a/b (blue line) gives minimum peak surface electric field.
Peak surface magnetic field distributions are about same within a few %.
How about other cavity parameters?



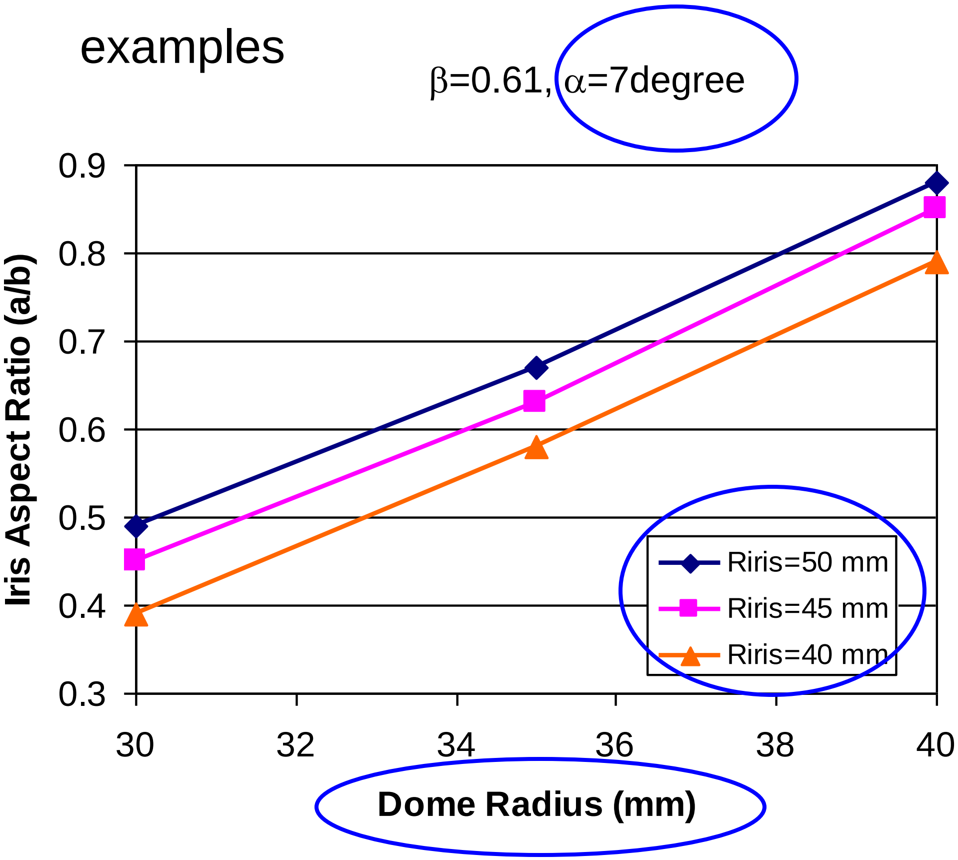
For circular dome
(Elliptical dome cases are same)

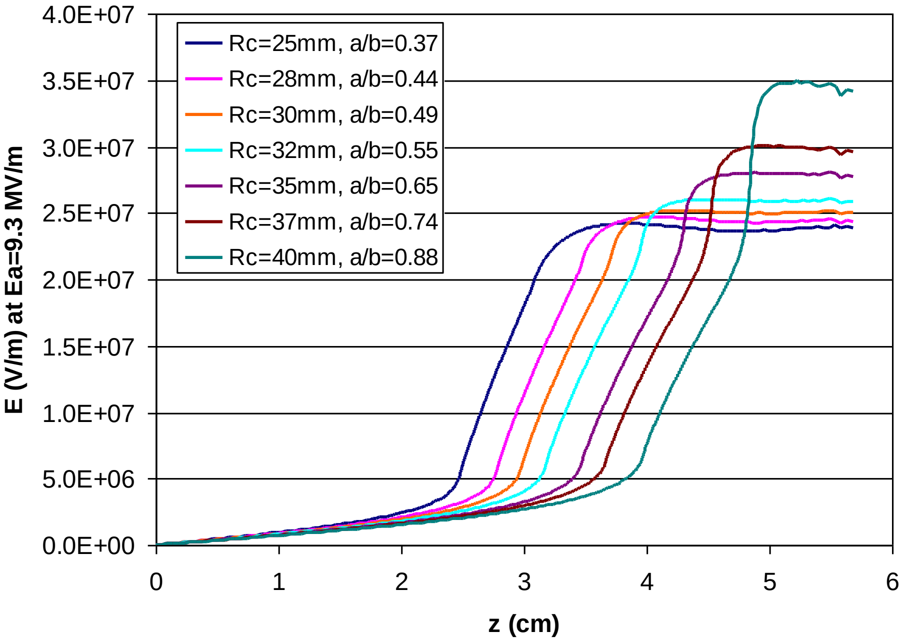
Rc, Ri, α, one of (**a/b**, a, b)
; 4 controllable parameters
Req (for tuning)



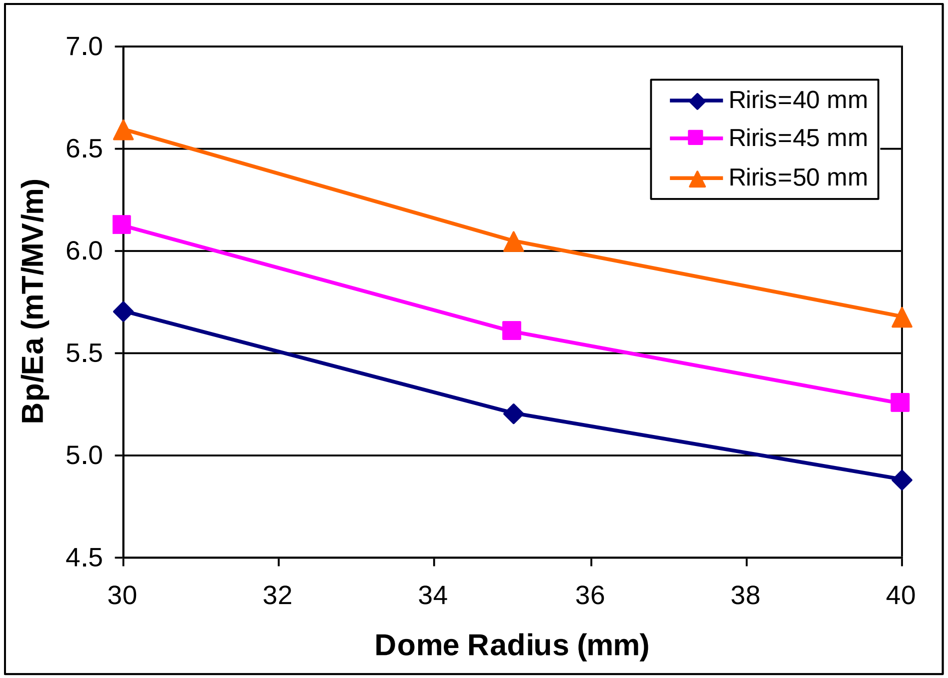
Since the a/b's are automatically determined at given Ri, Rc, and α simper (4 parameter-space→3 parameter-space) while taking the most efficient-set.

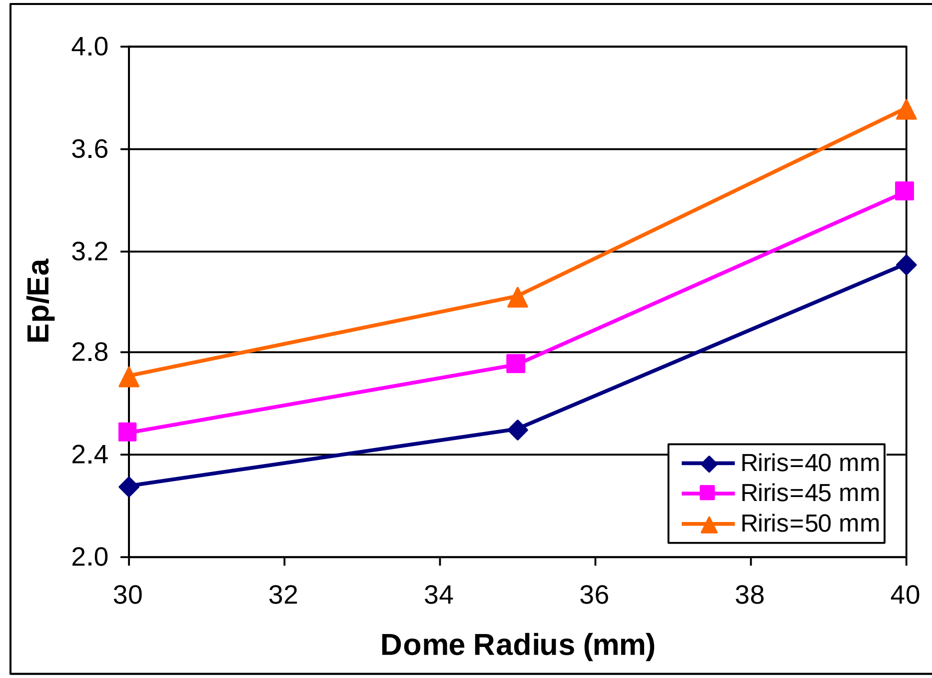
examples

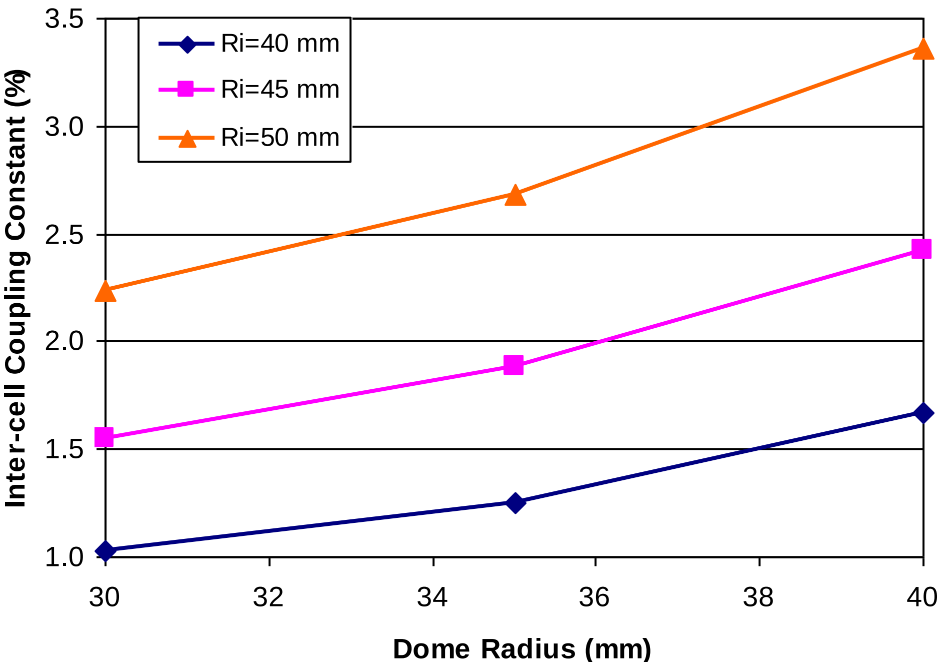


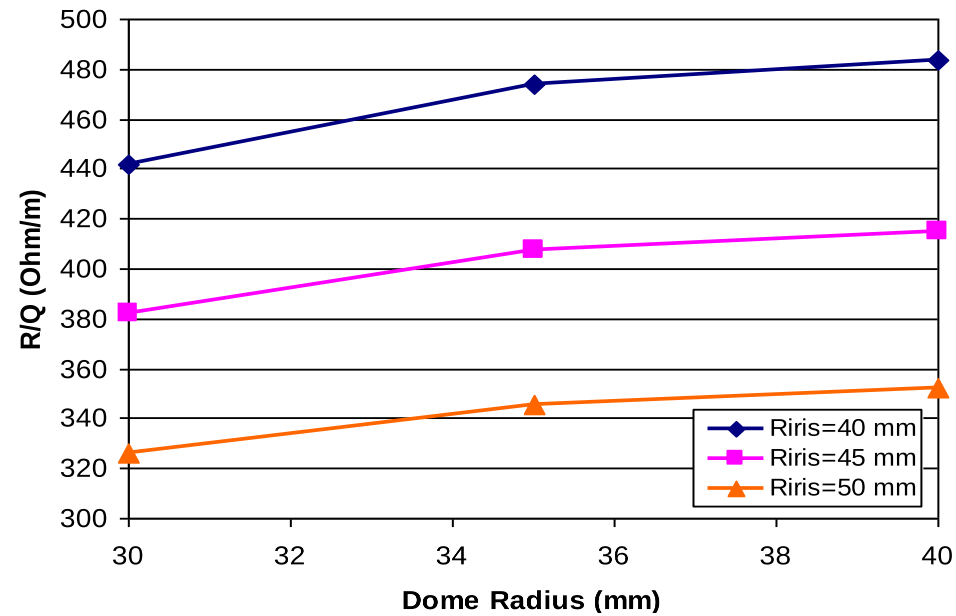


All the points on these lines satisfy the 'efficient set' condition.

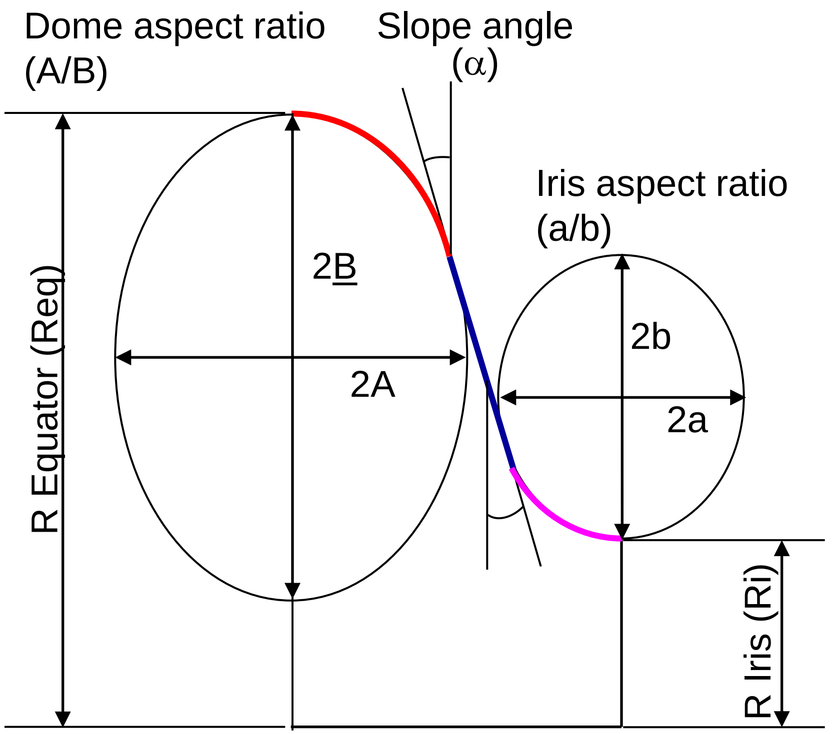

Bp/Ea (mT/MV/m)
Dome Radius (mm)
Riris=40 mm
Riris=45 mm
Riris=50 mm
7.0
6.5
6.0
5.5
5.0
4.5
30
32
34
36
38
40


Ep/Ea
Dome Radius (mm)
Riris=40 mm
Riris=45 mm
Riris=50 mm
4.0
3.6
3.2
2.8
2.4
2.0
30
32
34
36
38
40


Inter-cell Coupling Constant (%)
Dome Radius (mm)
Ri=40 mm
Ri=45 mm
Ri=50 mm
3.5
3.0
2.5
2.0
1.5
1.0
30
32
34
36
38
40


R/Q (Ohm/m)
Dome Radius (mm)
Riris=40 mm
Riris=45 mm
Riris=50 mm
500
480
460
440
420
400
380
360
340
320
300
30
32
34
36
38
40

More general: elliptical dome



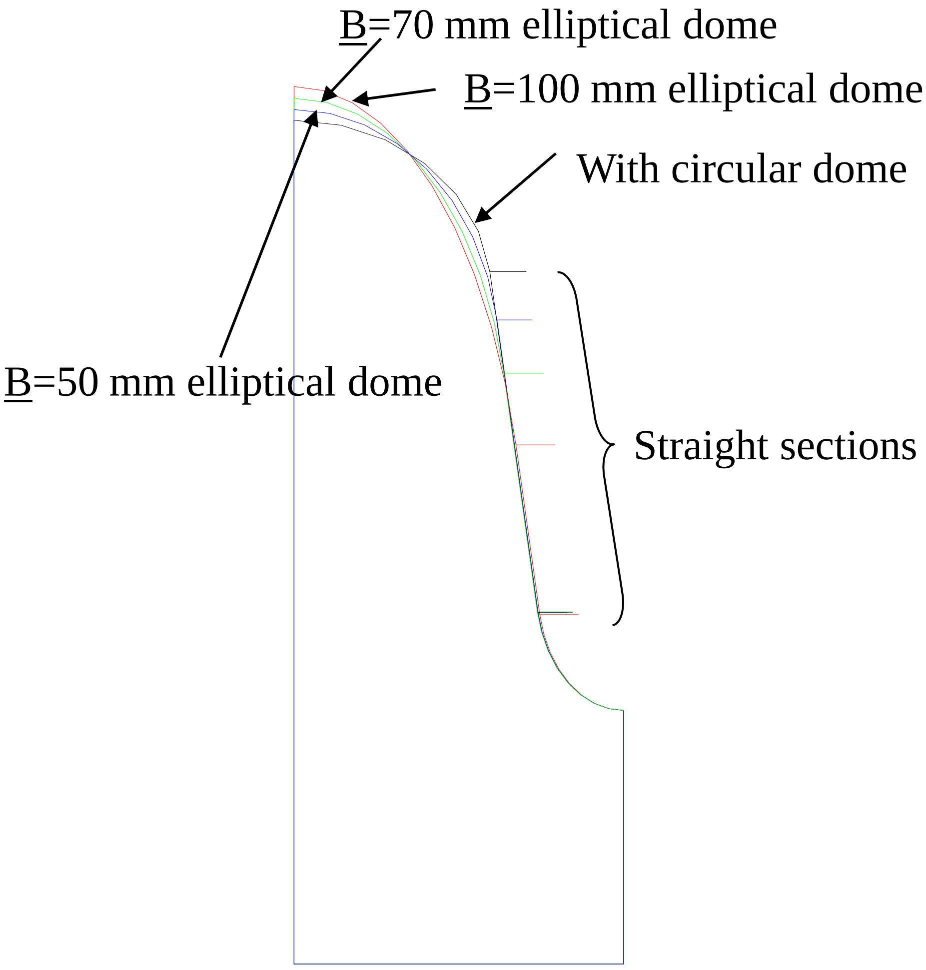
For elliptical dome

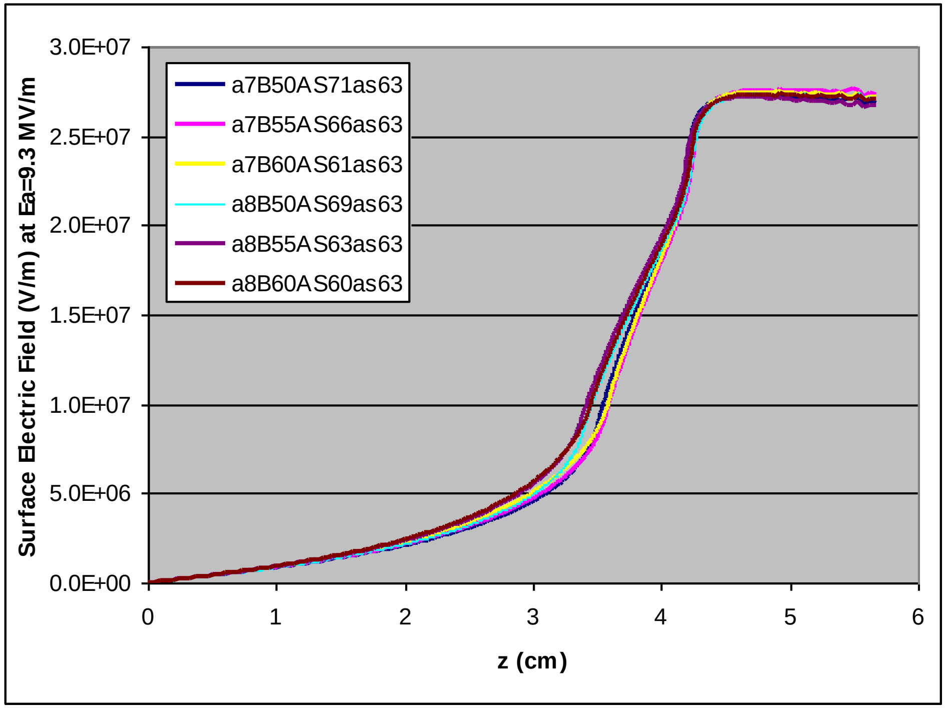
Ri, α, two of (A, B, A/B), one of (a/b, a, b)

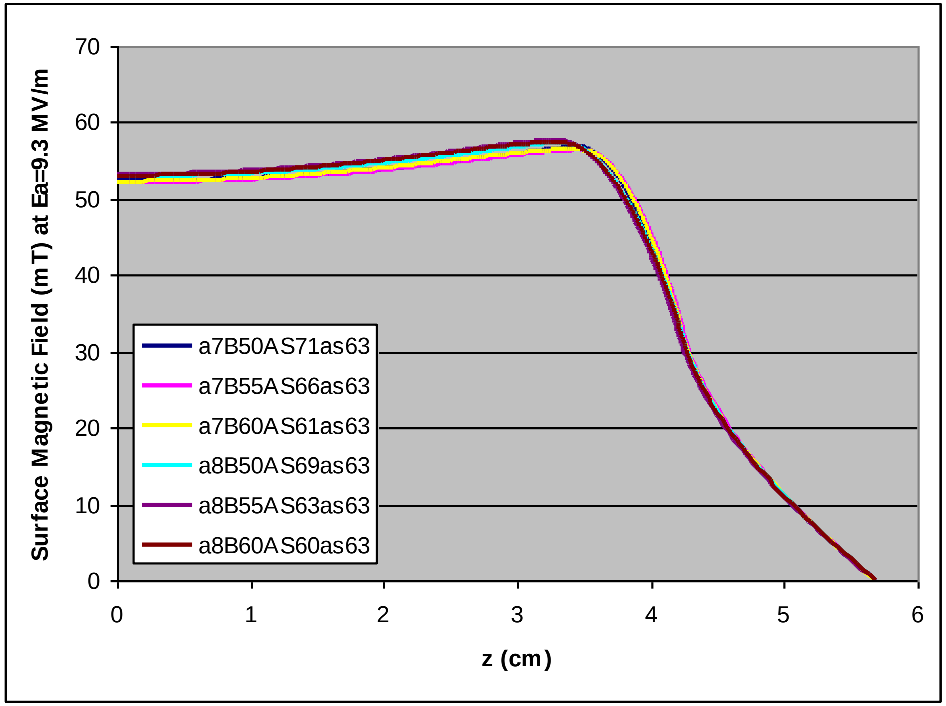
or Ri, α, one of (A, B, A/B), two of (a/b, a, b)

; 5 controllable parameters

Req (for tuning)







While keeping capacitive region same,
Their can be many different shapes.
RF properties are exactly same.
Mechanical properties; slightly different

→ Circular dome is enough

Radiation Pressure on the RF surface, P_{LF}

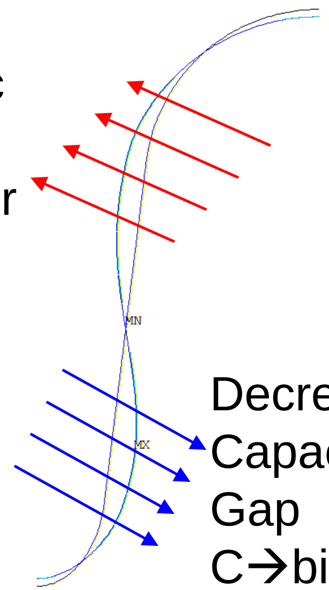
Radiation pressure; electromagnetic field interaction on the surface.

$$P_{LF} = \frac{1}{4}\left(\mu_0 \vec{H}\cdot\vec{H}^* - \varepsilon_0 \vec{E}\cdot\vec{E}^*\right) = \frac{1}{4}\left(\mu_0 H^2 - \varepsilon_0 E^2\right) \qquad \vec{H} = He^{i\omega t}, \quad \vec{E} = Ee^{i\omega t}$$

Outward pressure

Inward pressure

Increase
Magnetic
Volume
L→bigger



Decrease
Capacitive
Gap
C→bigger

Cavity wall slightly deforms under the radiation pressure.
(This example is for the SNS β_g=0.61 cavity. No stiffening ring at fixed boundary condition.)
Maximum displacement~ 1μm

$$\omega_0^{\ 2} = \frac{1}{LC} \quad , \quad \omega_0 \text{; resonance frequency}$$

Resonance frequency decreases

We will only look at the static behaviors first.
In Section 4, some dynamic natures will be introduced.

Slator's perturbation theory

-As learned previously, the stored electric and magnetic energies in a cavity are same at its resonance.

-Small perturbations in a cavity wall will change one type of energy more than the other.

-Resonance frequency will shift by an amount necessary to again equalize the energies between electric and magnetic.

-Slator (J. Slator, *Microwave Electronics*, D. Van Nostrand, Princeton, New jersey, 1950, p.81) gave an expression for the change in frequency when the volume of the cavity is reduced slightly by ΔV

$$\frac{\Delta f}{f_0} = \frac{\int_{\Delta V} (\mu \vec{H} \cdot \vec{H}^* - \varepsilon \vec{E} \cdot \vec{E}^*) dv}{\int_{\substack{\text{cavity} \\ \text{volume}}} (\mu \vec{H} \cdot \vec{H}^* + \varepsilon \vec{E} \cdot \vec{E}^*) dv} = \frac{\int_{\Delta V} (\mu H^2 - \varepsilon E^2) dv}{4(U_H + U_E)}$$

where $\vec{H} = He^{i\omega t}$, $\vec{E} = Ee^{i\omega t}$

-Due to the High Q_L of SRF cavities, the Lorentz force can detune a cavity large enough to affect significantly the coupling.

-It affects RF power needed and/or RF control.

-We will drive equations for this and deal with practical examples in Section 3.

-For CW machines, the Lorentz force detuning is static.

Slow corrections by mechanical tuner while ramp-up.

But cavity stiffness is important for microphonics issue in high Q machine.

-For pulsed machines, the Lorentz force detuning is dynamic.

Enhancing mechanical rigidity of a cavity is an essential part.

corrections are needed during a pulse:

fast tuner and/or additional RF power

-Using a stiffening ring is the most popular way to increase the stiffness.

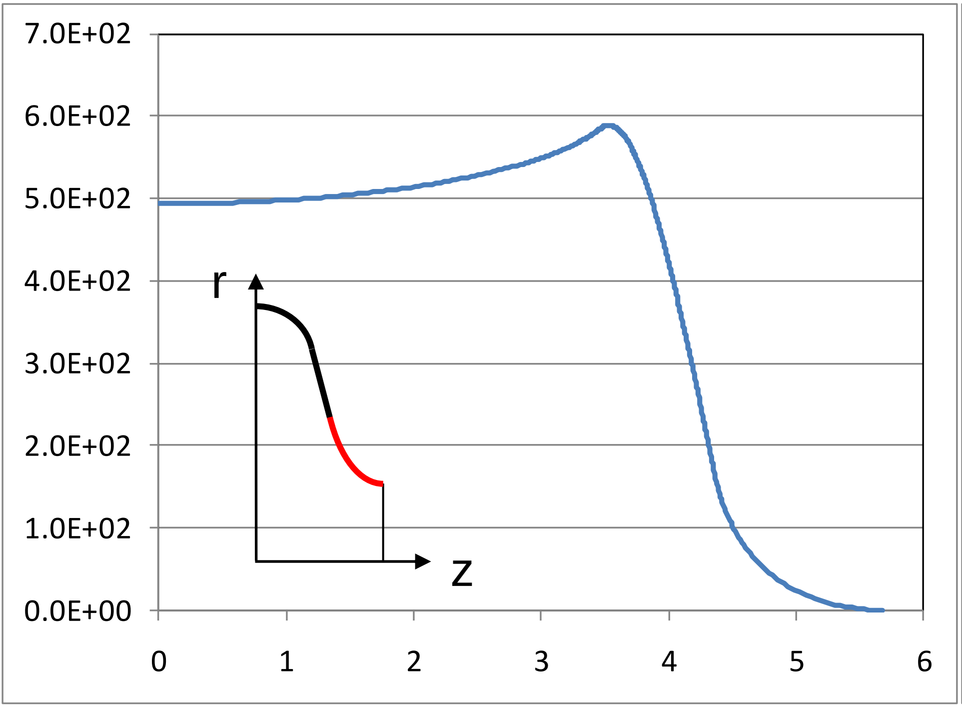
mainly for longitudinal direction.

If a cavity is too stiff, required force for a slow tuner can be unrealistic.

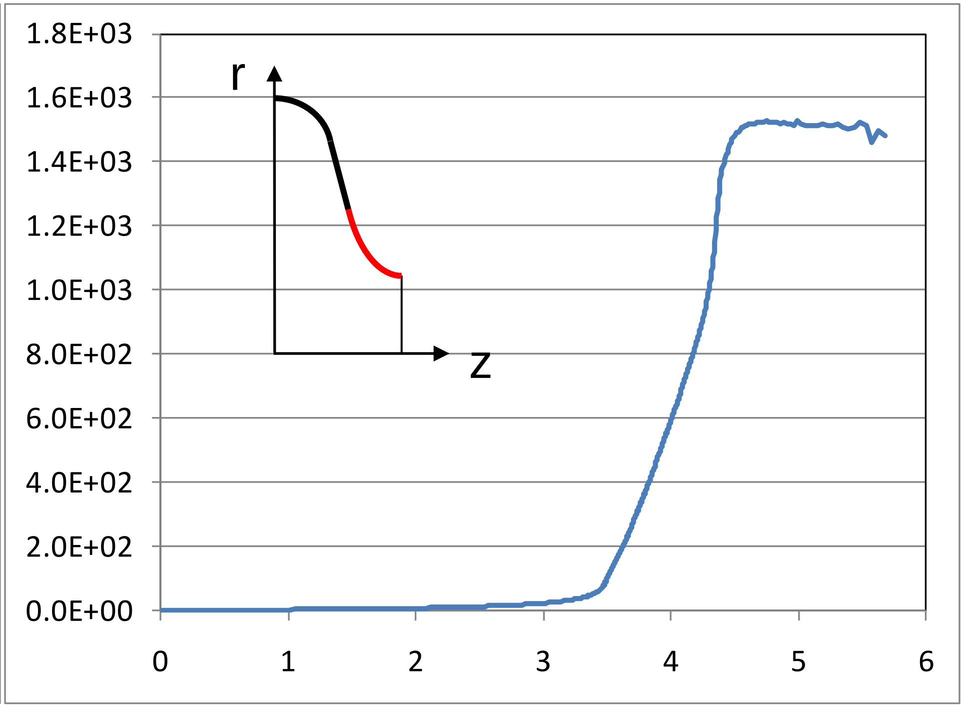
-So again some optimization is needed.

Surface pressure example
Using the same example (half cell only) at Ea=10 MV/m

From magnetic field (in Pa)

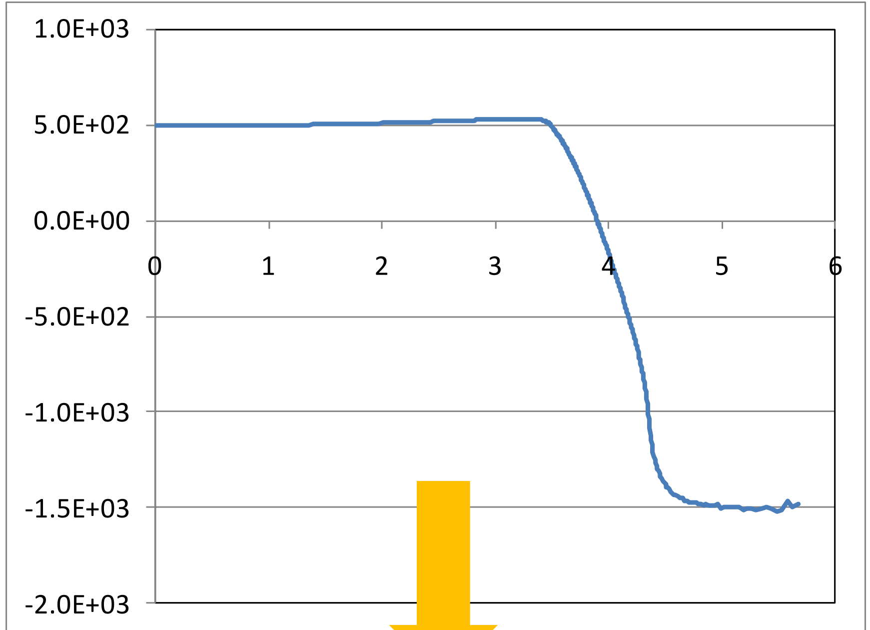


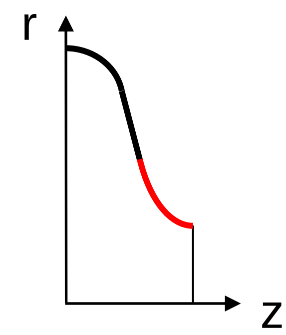
From electric field (in Pa)

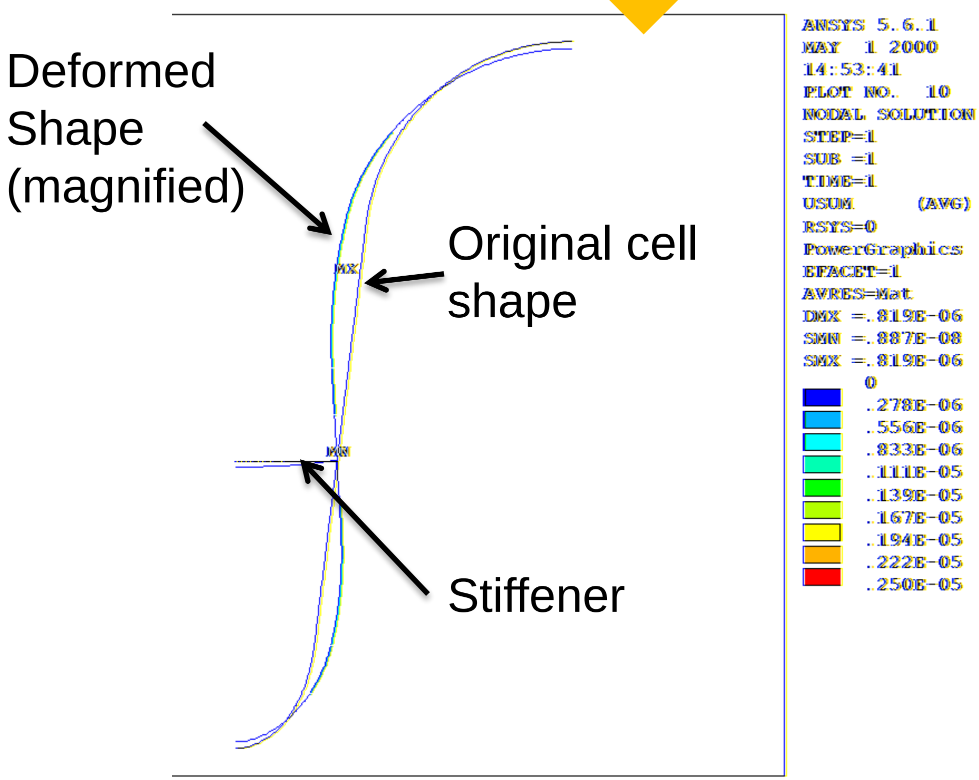


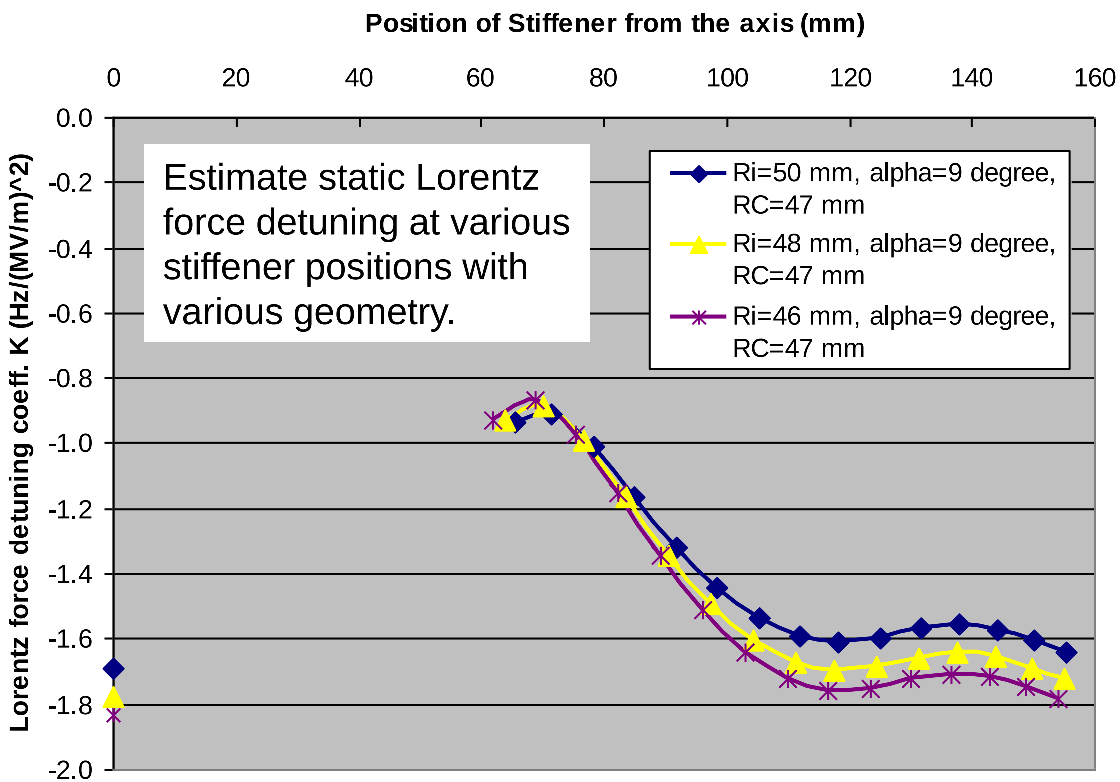
z (cm)

Total Radiation pressure in Pa

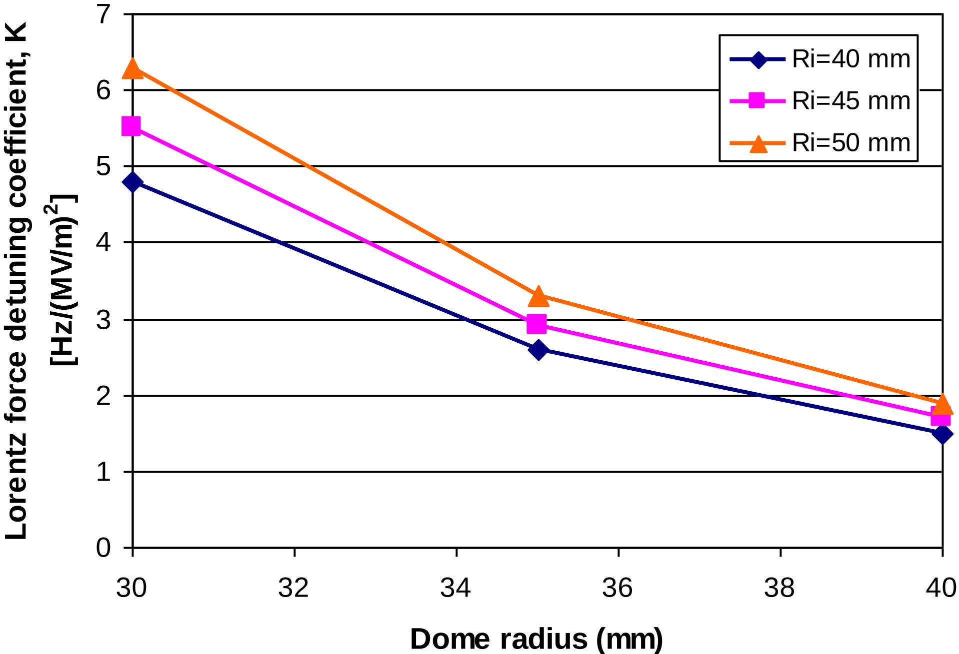








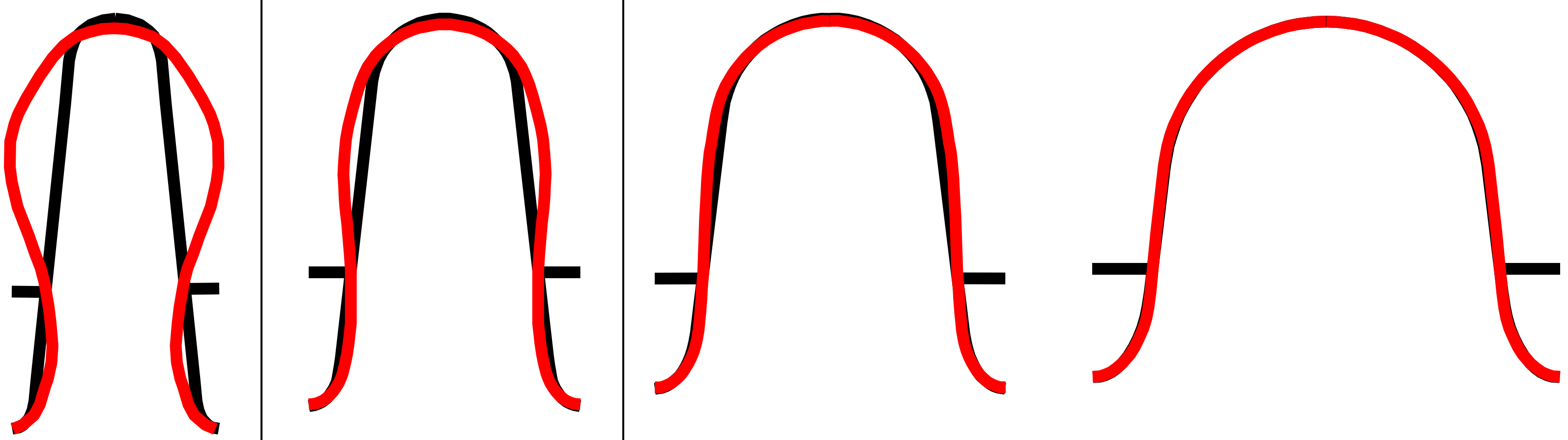
Position of Stiffener from the axis (mm)
0
20
40
60
80
100
120
140
160
0.0
-0.2
-0.4
-0.6
-0.8
-1.0
-1.2
-1.4
-1.6
-1.8
-2.0
Lorentz force detuning coeff. K (Hz/(MV/m)^2)
Estimate static Lorentz force detuning at various stiffener positions with various geometry.
Ri=50 mm, alpha=9 degree, RC=47 mm
Ri=48 mm, alpha=9 degree, RC=47 mm
Ri=46 mm, alpha=9 degree, RC=47 mm



7
6
5
4
3
2
1
0
Lorentz force detuning coefficient, K [Hz/(MV/m)^2]
Ri=40 mm
Ri=45 mm
Ri=50 mm
30
32
34
36
38
40
Dome radius (mm)

How low can we go with β_g in elliptical cavities ?

- **Static Lorentz force detuning (LFD) at EoT(β_g)=10 MV/m, 805 MHz** (Magnification; 50,000)
- **In CW application LFD is not an issue,**
  but static LFD coeff. provides some indication of mechanical stability of structure

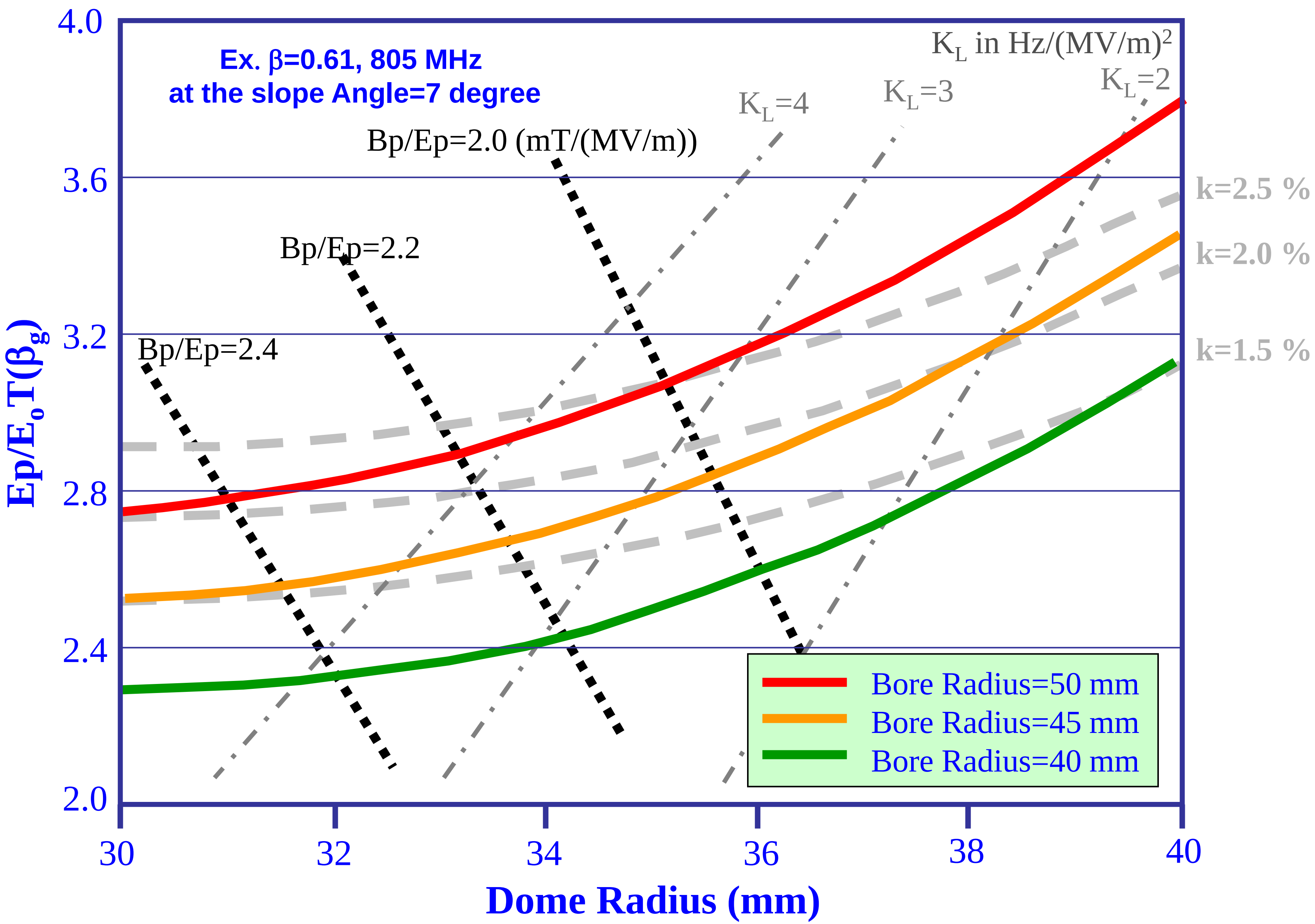


β_g=0.35 | **β_g=0.48** | **β_g=0.61** | **β_g=0.81**

β_g=0.35	β_g=0.48	β_g=0.61, β_g=0.81
RF efficiency; x	**Will work in CW**	**Suitable for all CW & pulsed applications**
Mechanical Stability; x	**Pessimistic in Pulsed application**	**Recent test results of SNS prototype cryomodule, β_g=0.61 ; quite positive; piezo compensation will work**
Multipacting; Strong possibility		

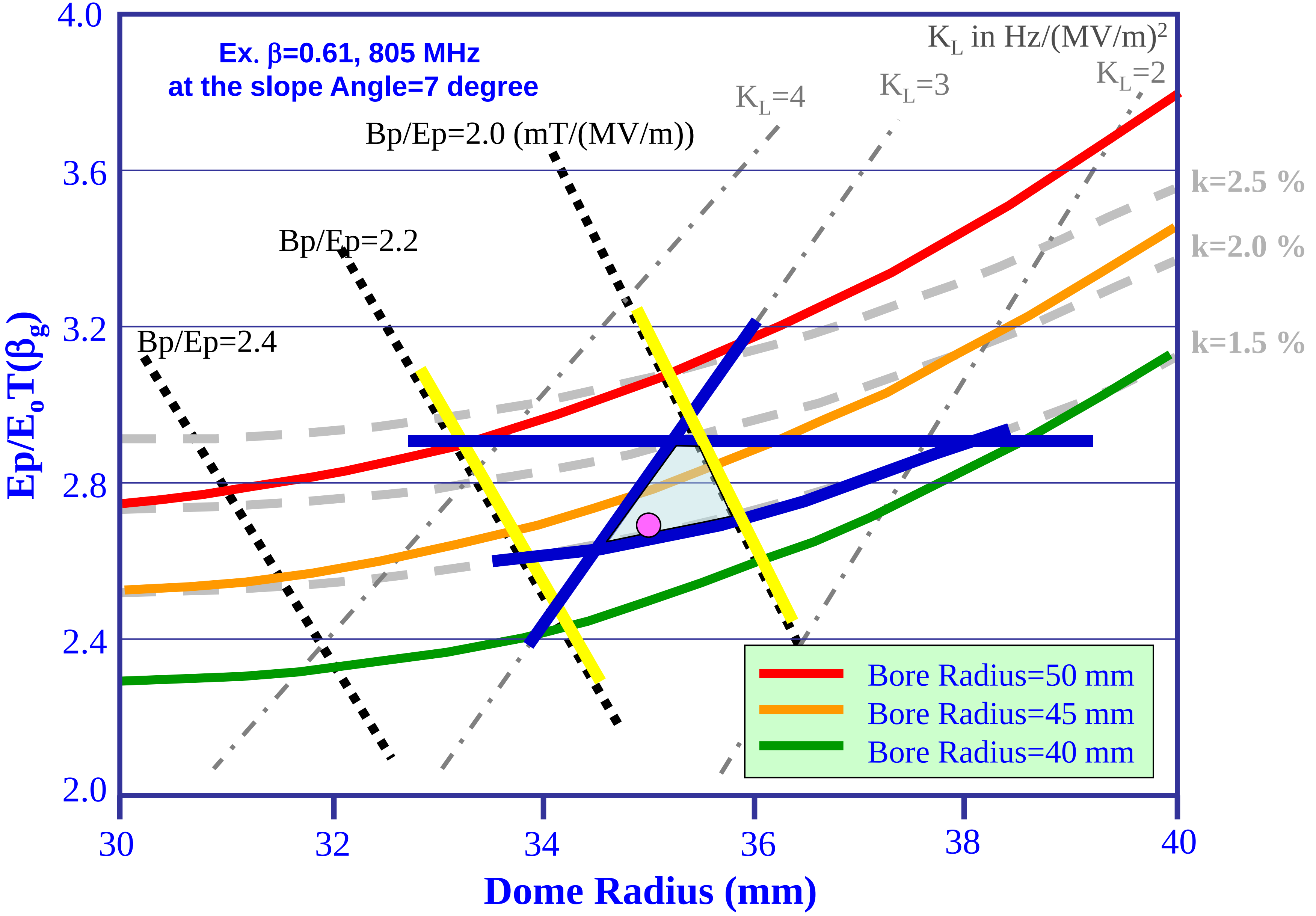
Would be a competing Region with spoke cavity

SNS β=0.61 ; parameter space of cavity



Design criteria (machine specific & technology dependent)

SNS β=0.61 ; parameter space of cavity



Multi cell vs. transit time factor

Let’s add up number of cells with magnetic boundary conditions at both ends. This boundary condition is not realistic, but we can quickly build up model and compare the transit time factors. It will give us good pictures about geometric beta, number of cells, transit time factor, possible acceleration band in beta, etc.

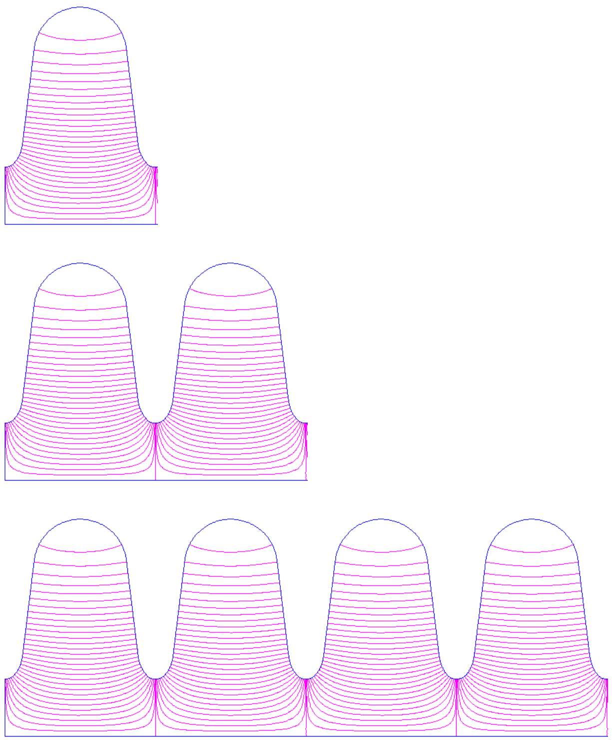
When we finish the full cavity design with end-cells, we will get a real one.

Other concerns on ‘number of cells’

RF power needed (coupler, rf source) with beam loading

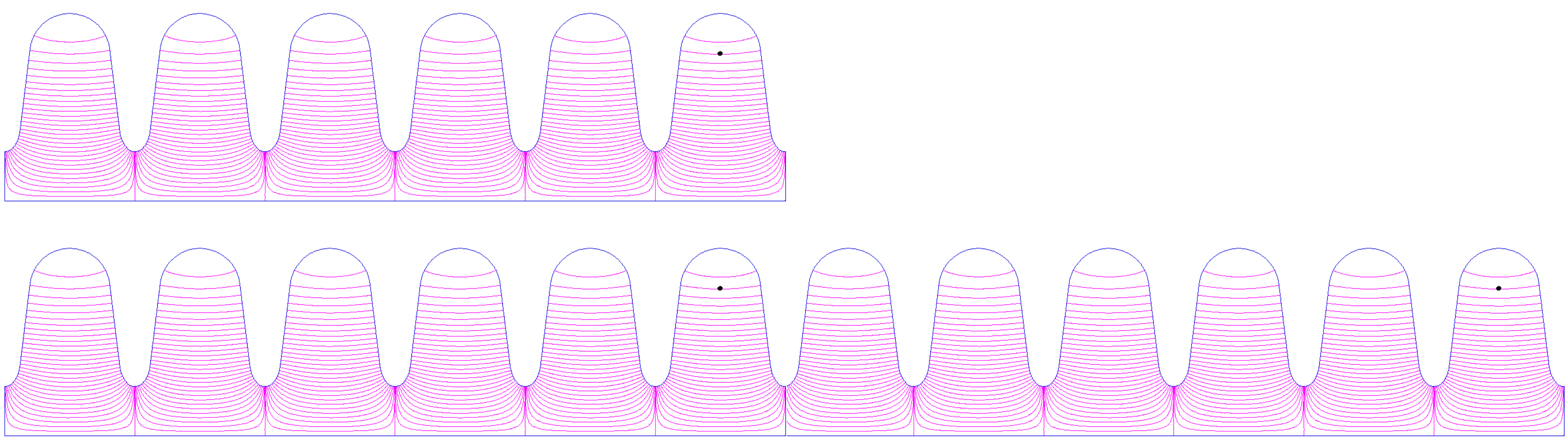
longitudinal phase slips

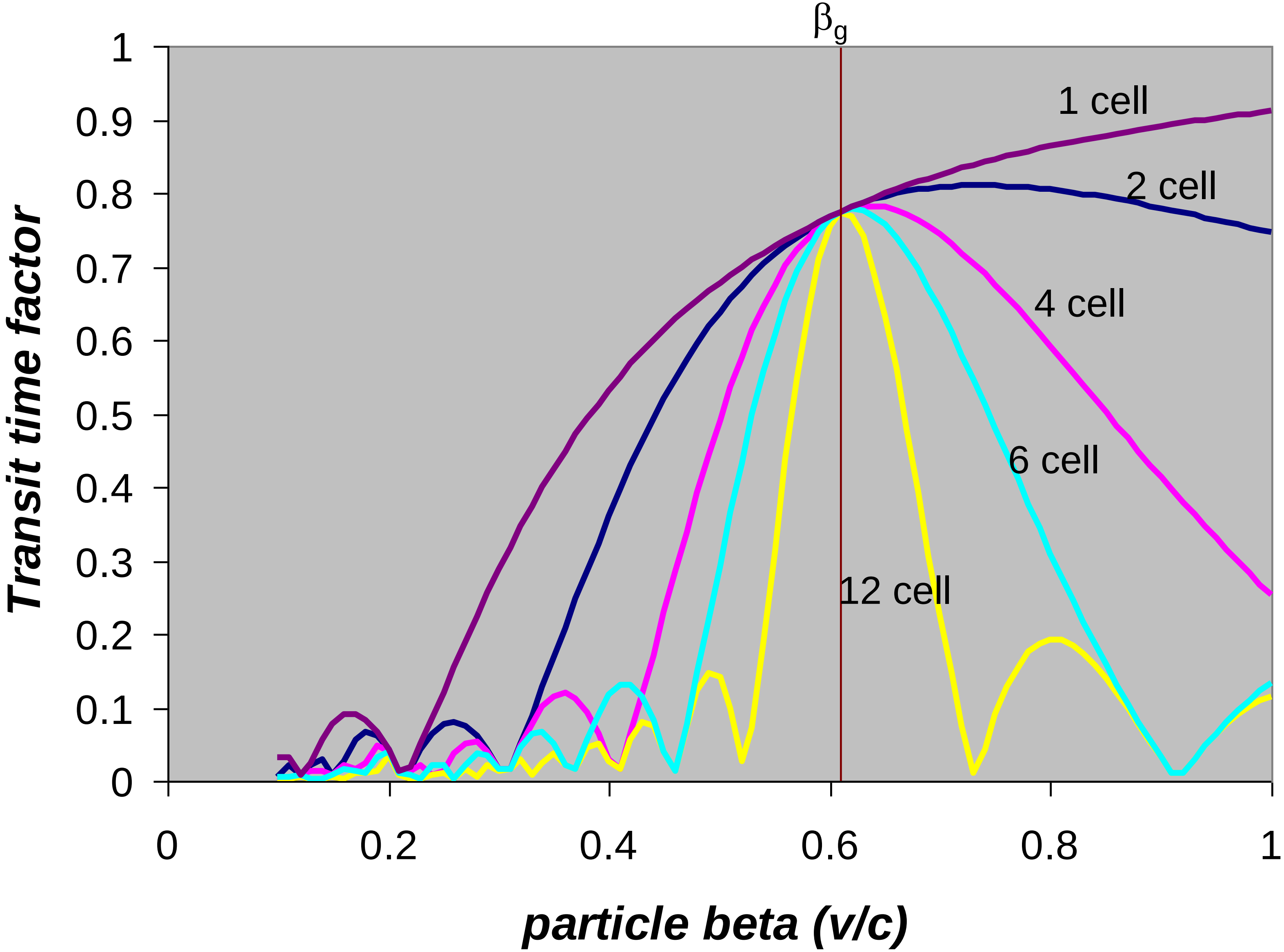
Will be covered in the following sections.



Let’s generate superfish input files for multi-cell structures (1, 2, 4, 6, 12, …)
and compare the transit time factors as a function of particle velocity (b=0.1 to 1.0) and other cavity parameters in SFO files.

Since each cell is identical, peak field distributions are same.
But effective quantities (function of particle velocity, transit time factor) will be different, as one can expect.





An efficient acceleration range is getting narrower as number of cells increases.

End-cell design and RF coupling

Different tuning algorithm because..

Beam pipe connection → naturally field penetrates to the beam pipes

Equipments/parts around beam pipe

field probe: measure cavity field

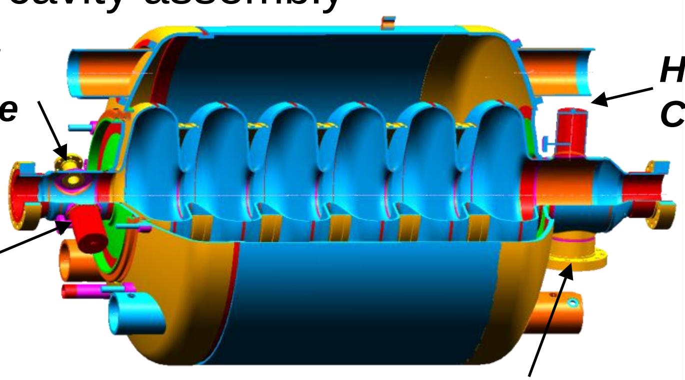
(HOW coupler: damp HOM and extract HOM power)

fundamental power coupler: feed RF power

higher beam loading structure needs higher coupling

→ large beam pipe

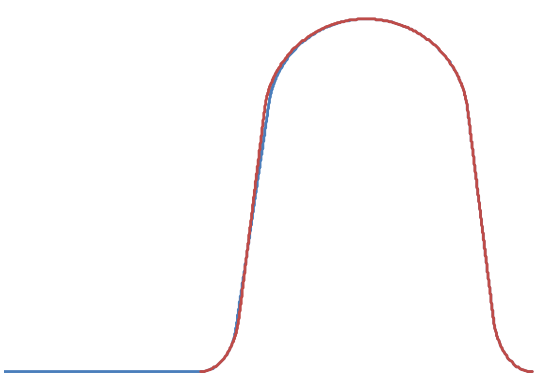
Ex.) SNS cavity assembly



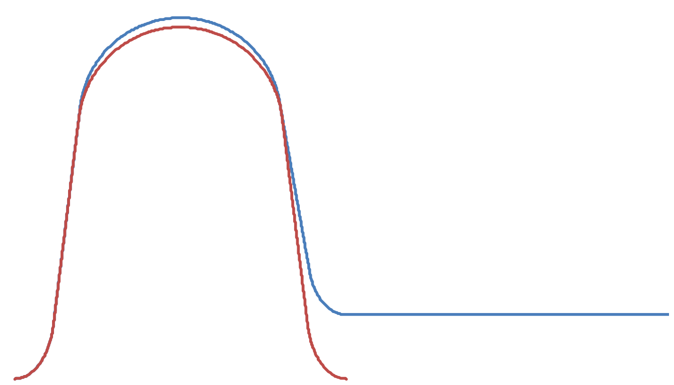
While satisfying,

- have equal or lower peak surface fields than inner cells
- achieve a required Qex
- obtain a good field flatness

Due to stray fields to the beam pipe, peak electric field at the end cell is usually lower than that for the inner cell.
Tuning with magnetic volume (for the large beam pipe side) and/or with slope angle (for the small beam pipe side) are the typical way.
Many combinations can satisfy the requirements.



End-cell at the small beam pipe side



End-cell at the large beam pipe side

External Q, Q_{ex}

$$Q_{ex} = \omega_0 U / P_{ex}$$

ω_0 : resonance frequency
U : stored energy
P_{ex} : power flowing out from the cavity through the coupler
when the RF generator is turned off

We can define an equivalent quality factor for beam loading like Q_b.

$$Q_b = \omega_0 U / P_b$$

P_b : RF power goes to beam

$$P_b = I_0 E_0 T L \cos\phi_s = I_0 E_a L \cos\phi_s = I_0 V_a \cos\phi_s$$

Ex) Q_b for L=70cm, E_a=10MV/m, U=35J, 805MHz, ϕ=-20 degree, I_0=40mA?
→Q_b~6.7×10^5
How about for I_0=1mA →Q_b~2.7×10^7

When $Q_{ex}=Q_b$ (matched condition), RF efficiency is highest. Ideally >99 % of RF power goes to the beam.
(more details will be dealt in Sec. 3)

What can affect Qex ?

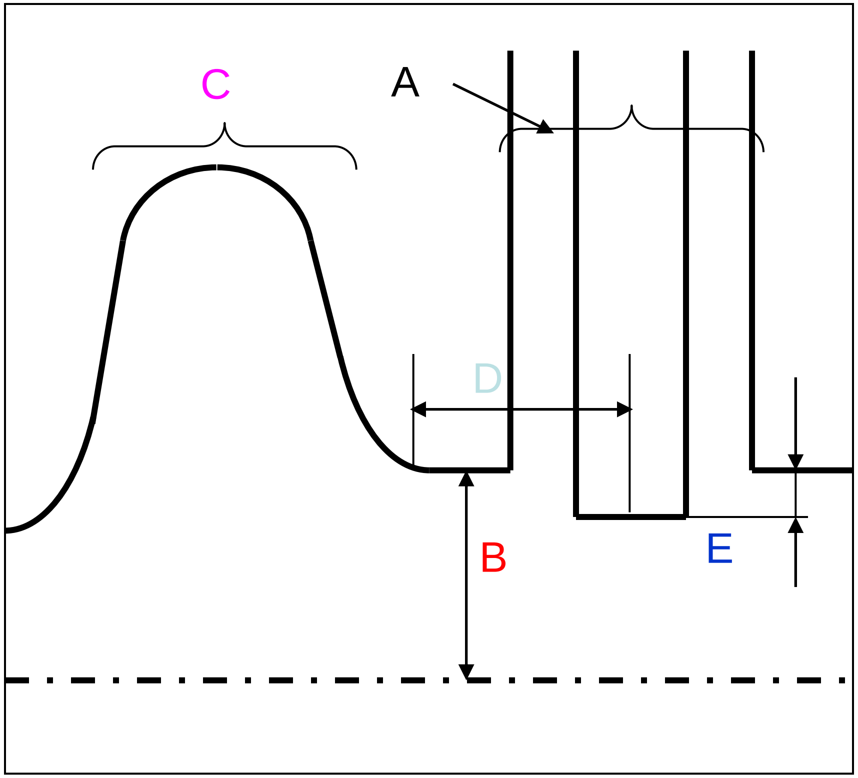
1) A (Geometry of Coupler); typically 50 Ω coaxial

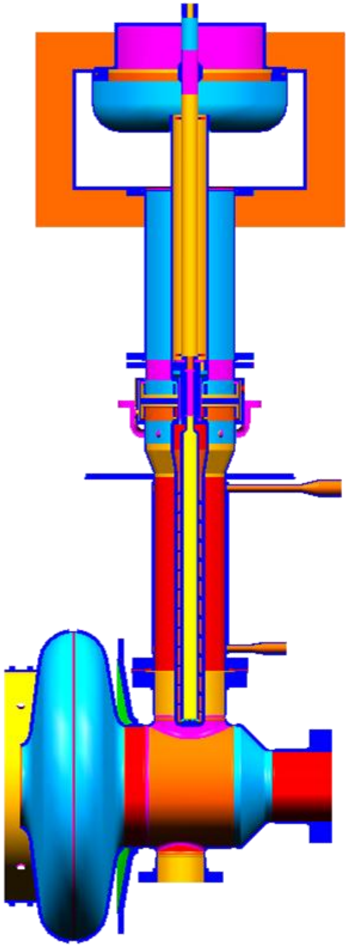
2) B (Beam Pipe Radius)

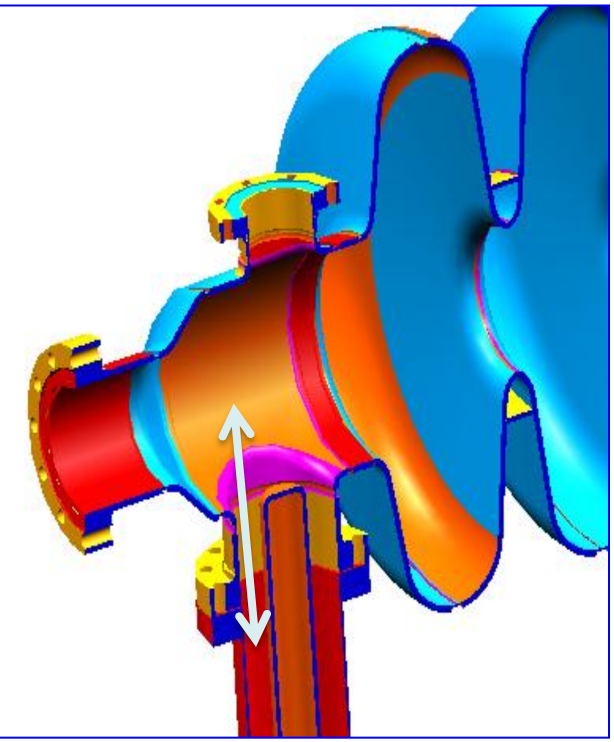
3) C (Right End-cell Geometry)

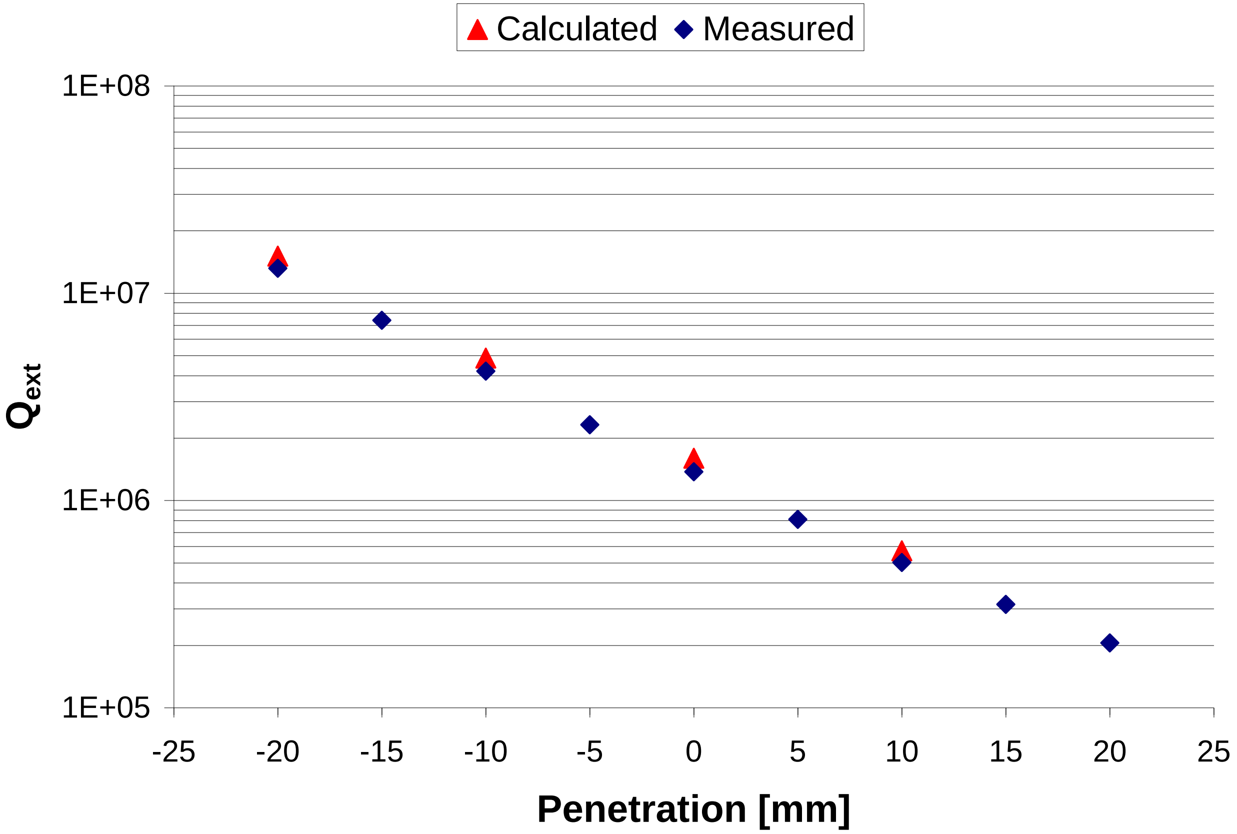
4) D (Distance between Cavity and Coupler)

5) E (Antenna Penetration): strongest



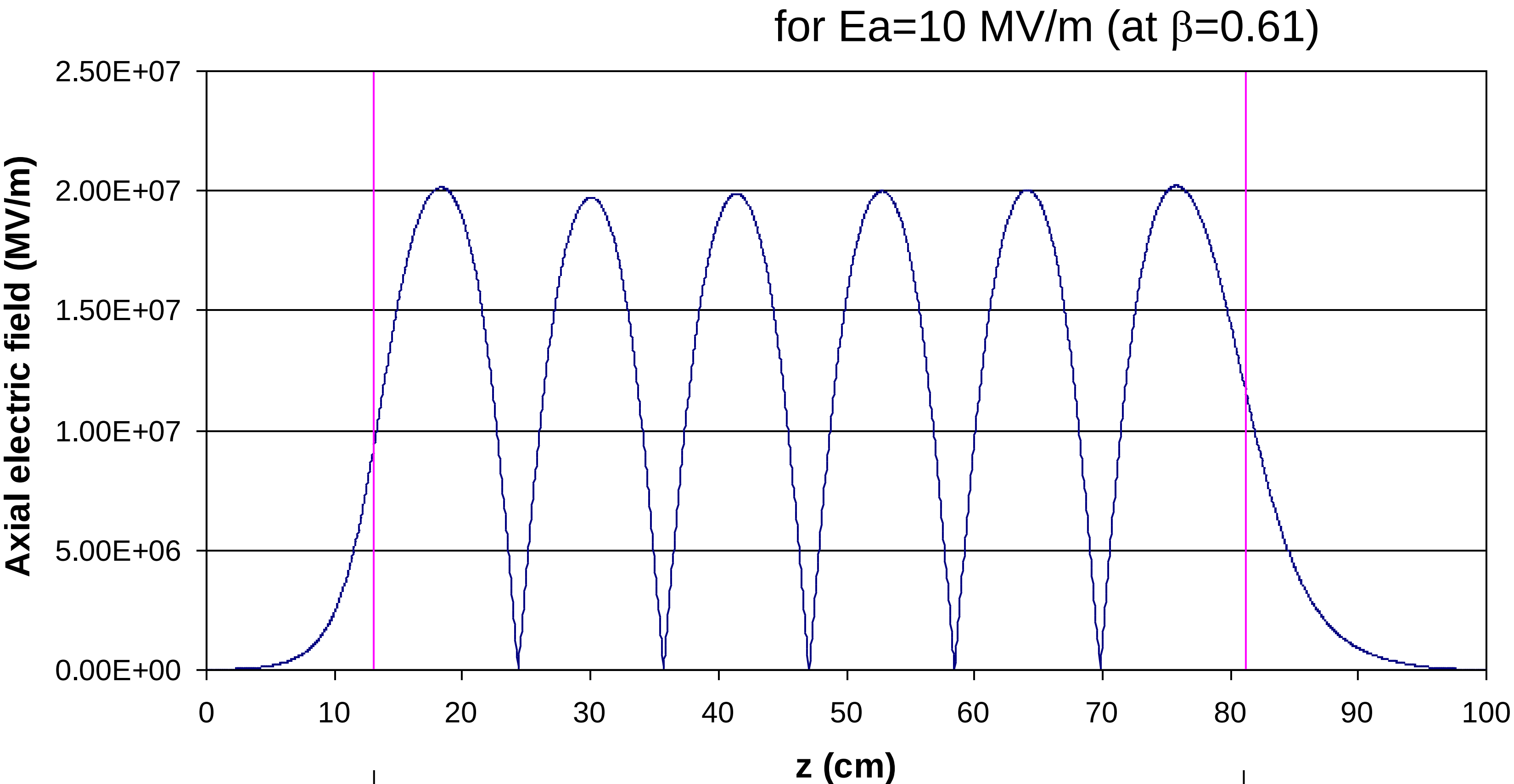


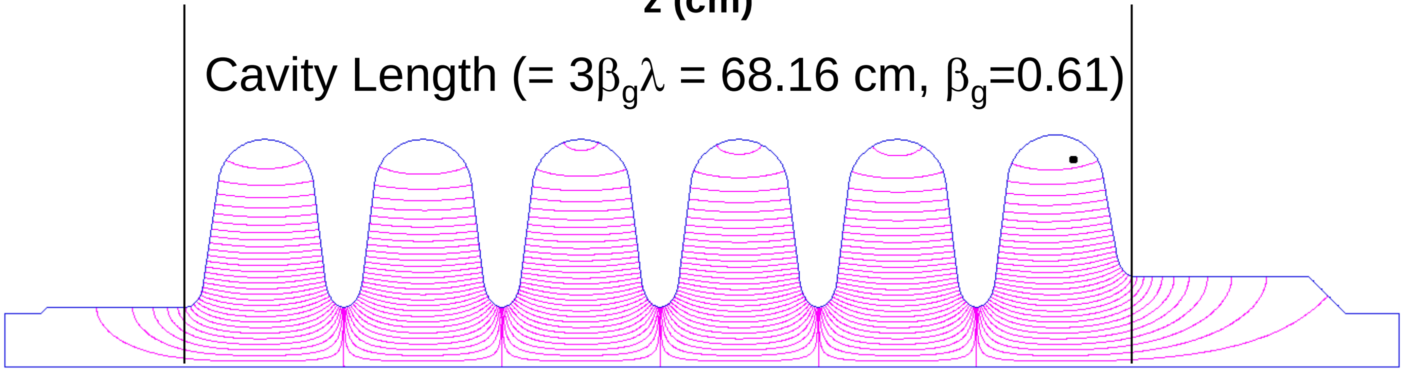




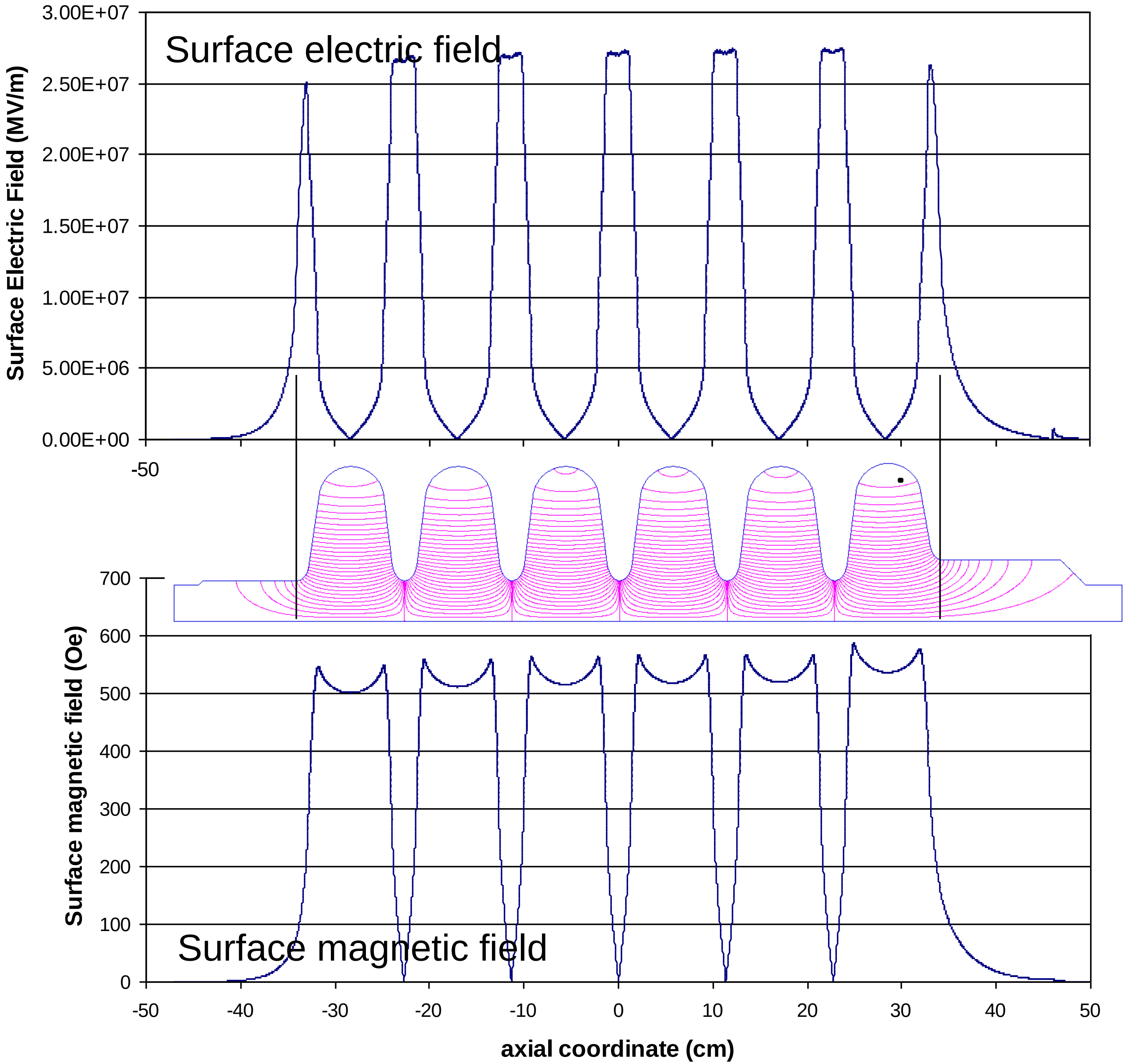
Calculated
Measured
1E+08
1E+07
1E+06
1E+05
Q_{ext}
-25
-20
-15
-10
-5
0
5
10
15
20
25
Penetration [mm]

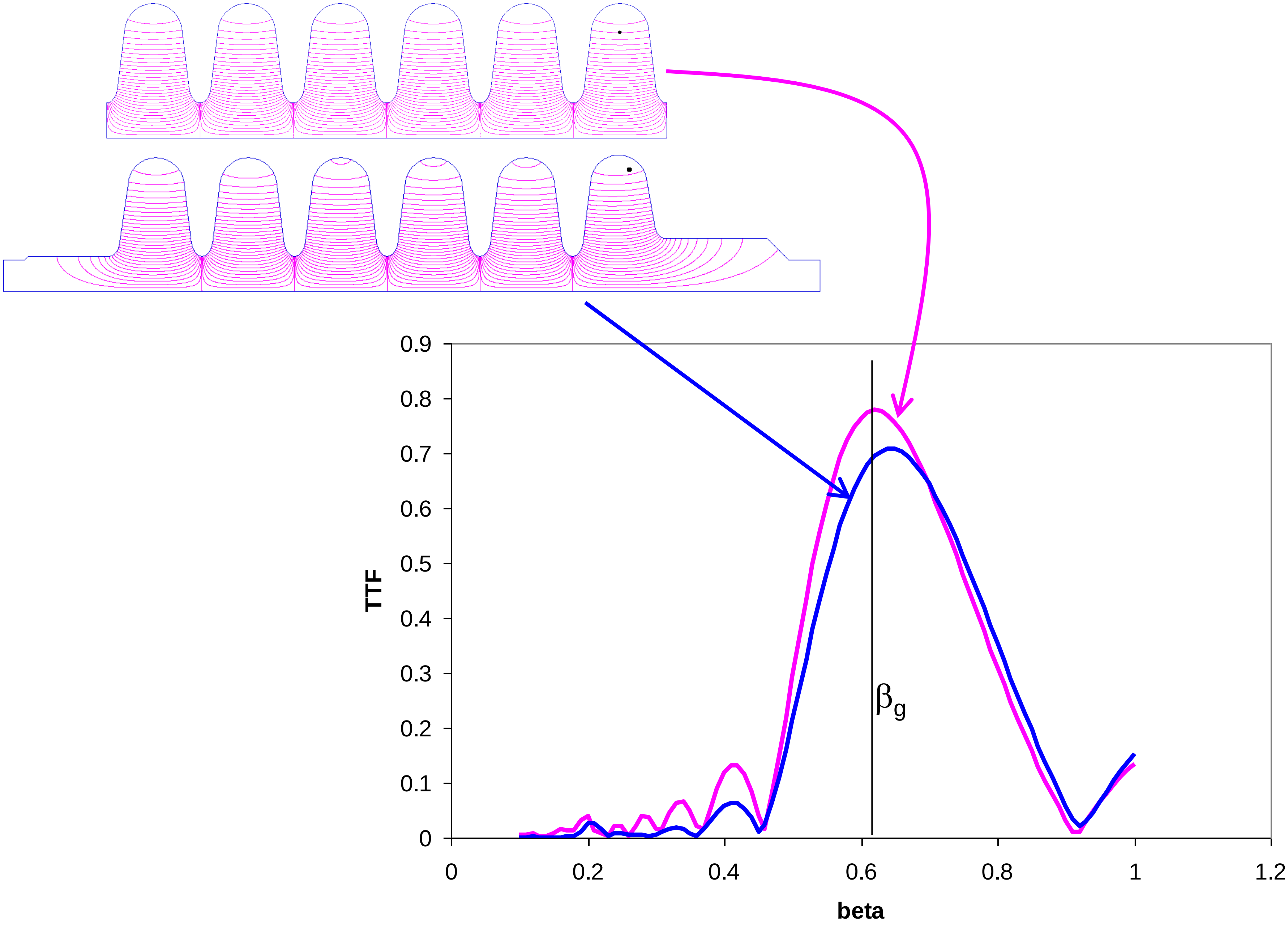
Ex. 805 MHz, β_g=0.61 6-cell cavity (med1.af)





at Ea=10 MV/m (at β=0.61)





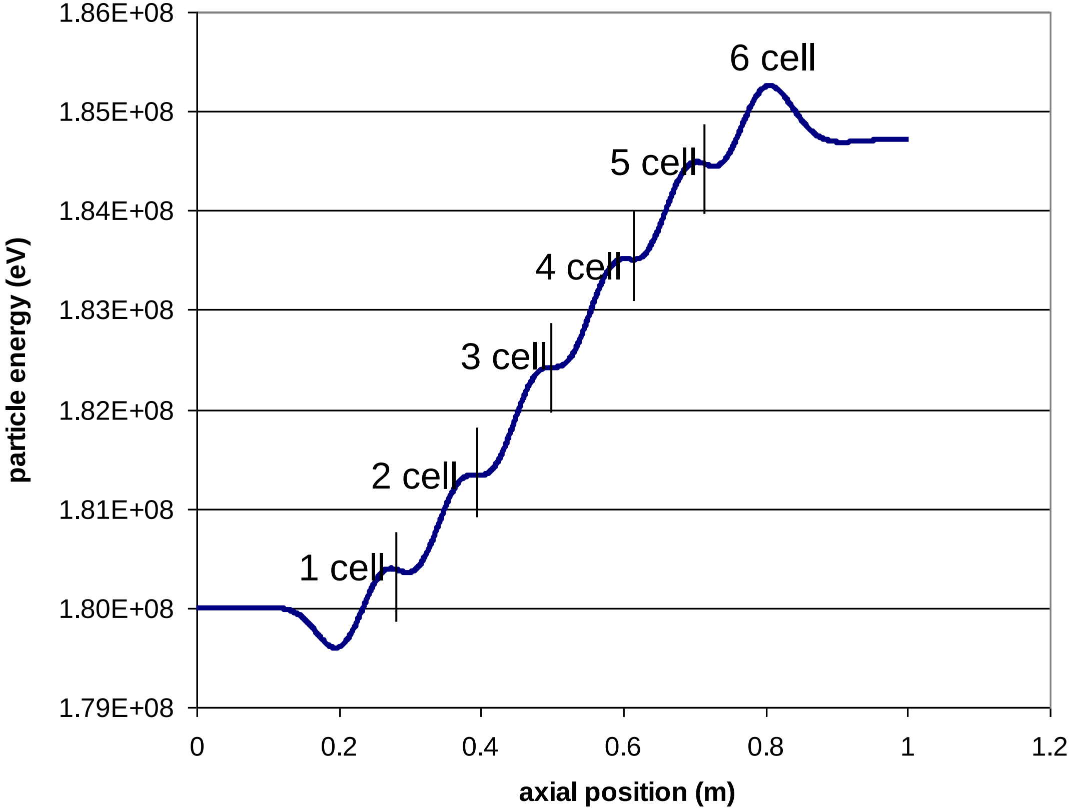
Explains why TTF is lower and shifted to the higher beta

2.3 Acceleration in multi-cell cavity

In a multi-cell cavity the energy gain is not monotonous.

Maximum energy gain at input beam energy 180 MeV

Using the field data in the previous example at $E_0T(\beta_g=0.61)=10$ MV/m

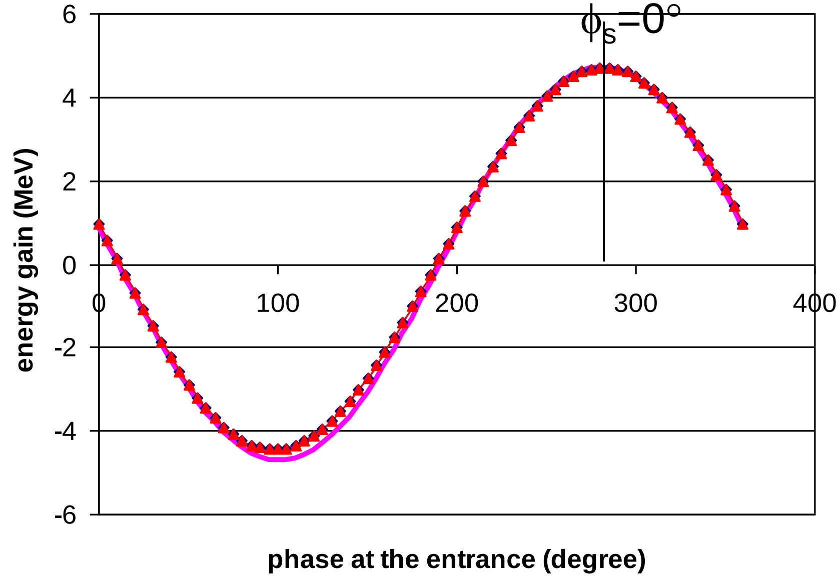


tracking1_med_3.xls

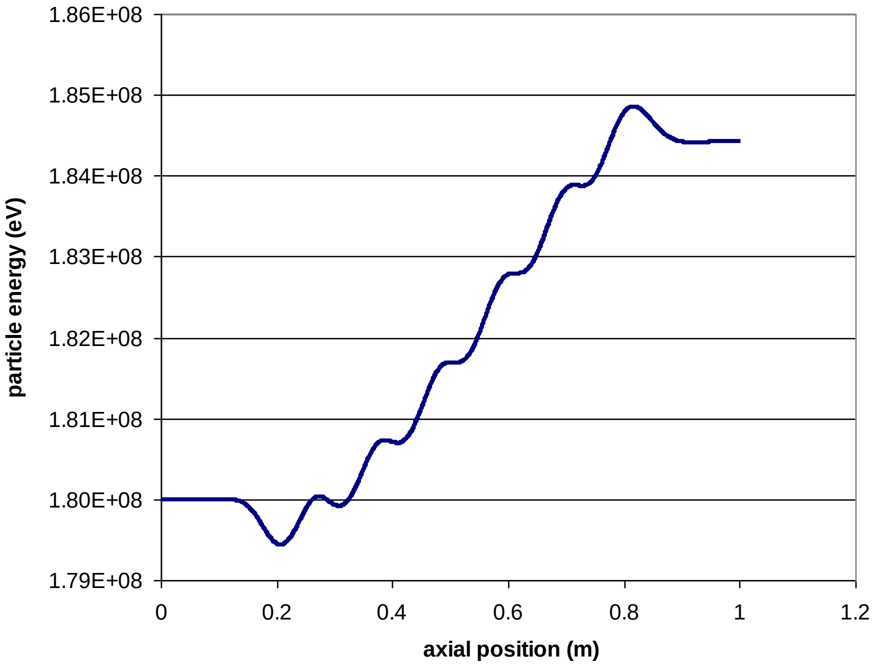
Acceleration/deceleration in the first and the last cell;

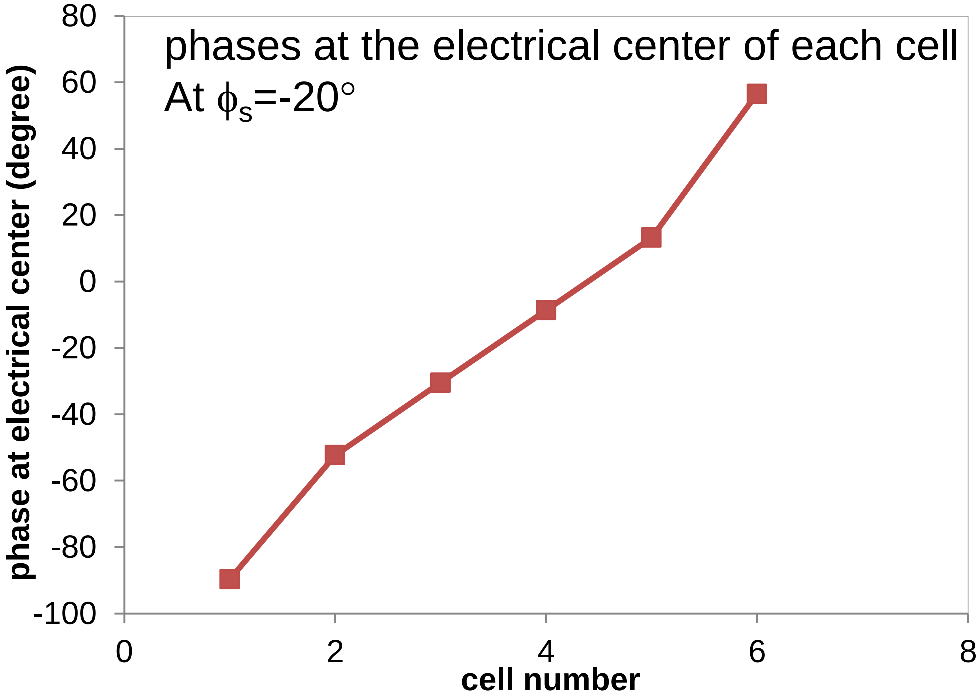
input beam energy 180 MeV
(β=0.544)

Maximum energy gain
ϕ_s=0°

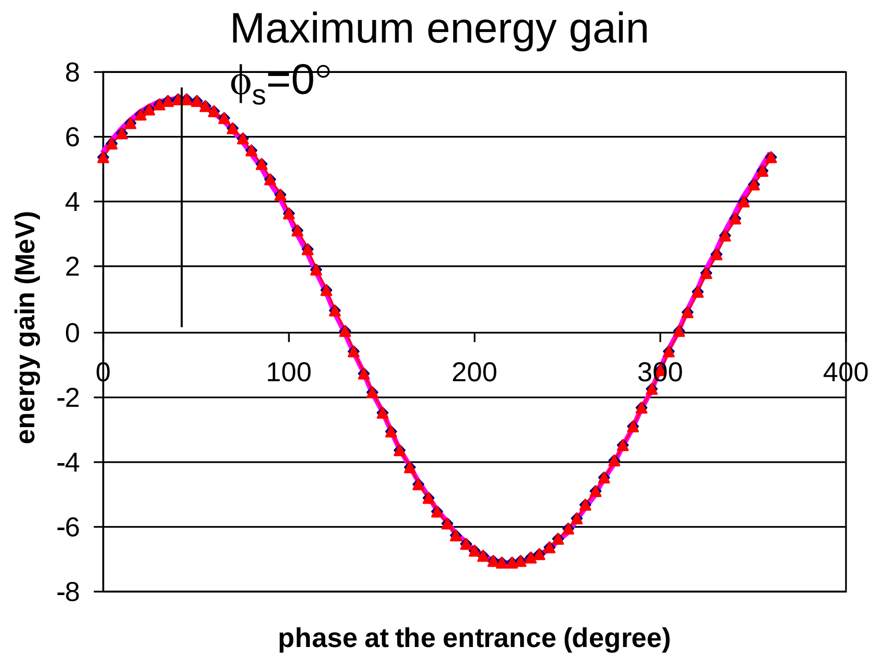


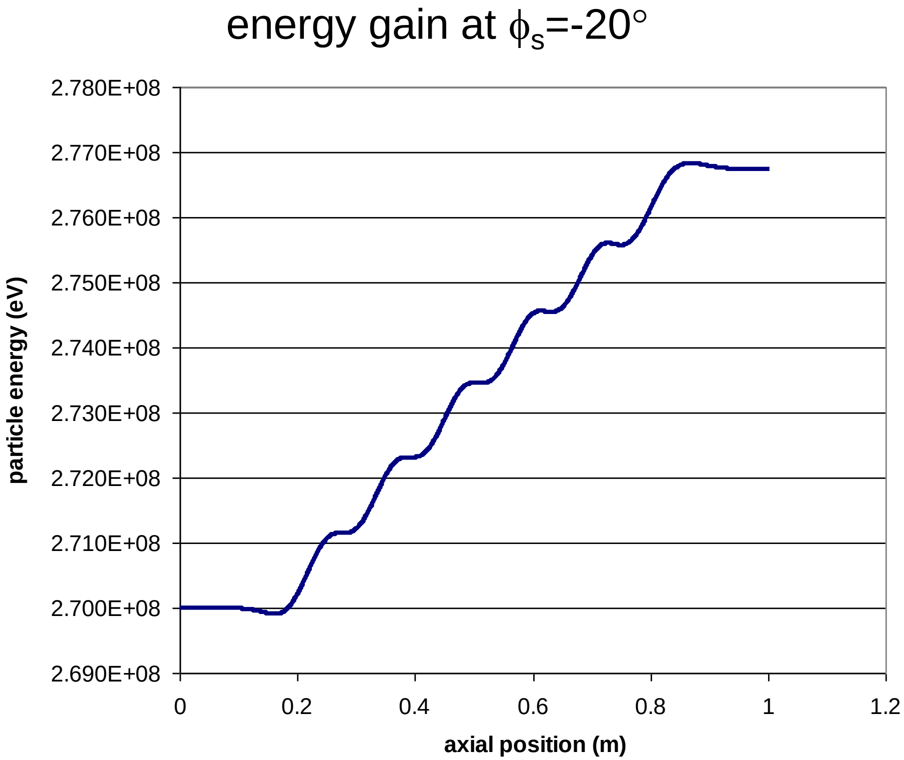
energy gain at ϕ_s=-20°

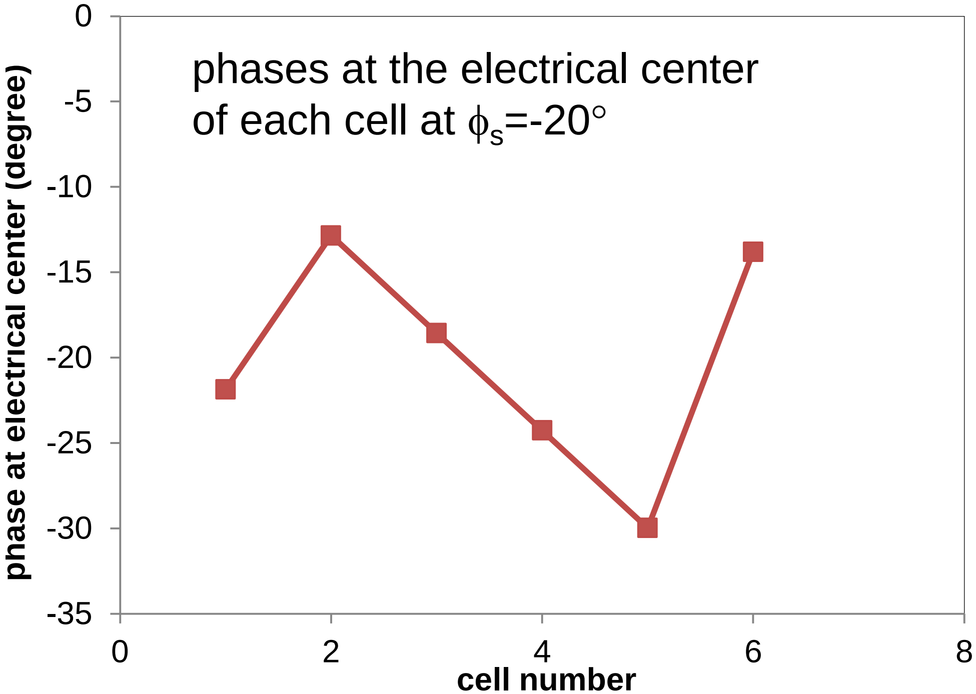




input beam energy 270 MeV (β=0.63)

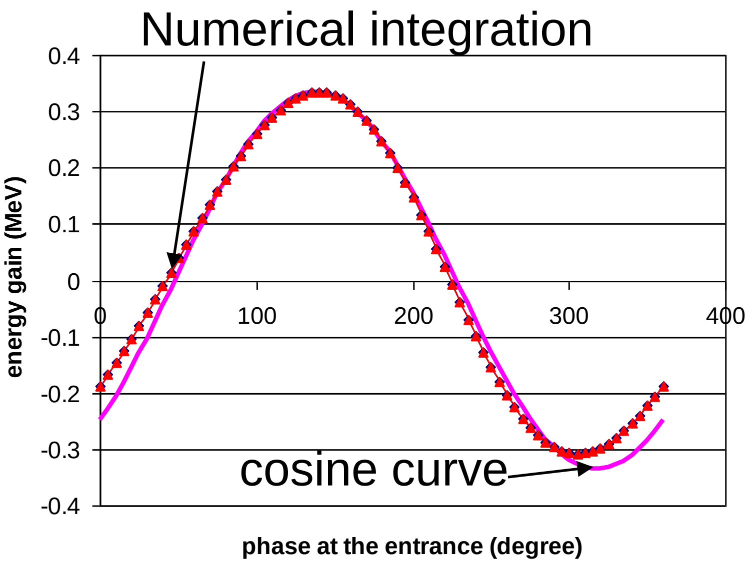


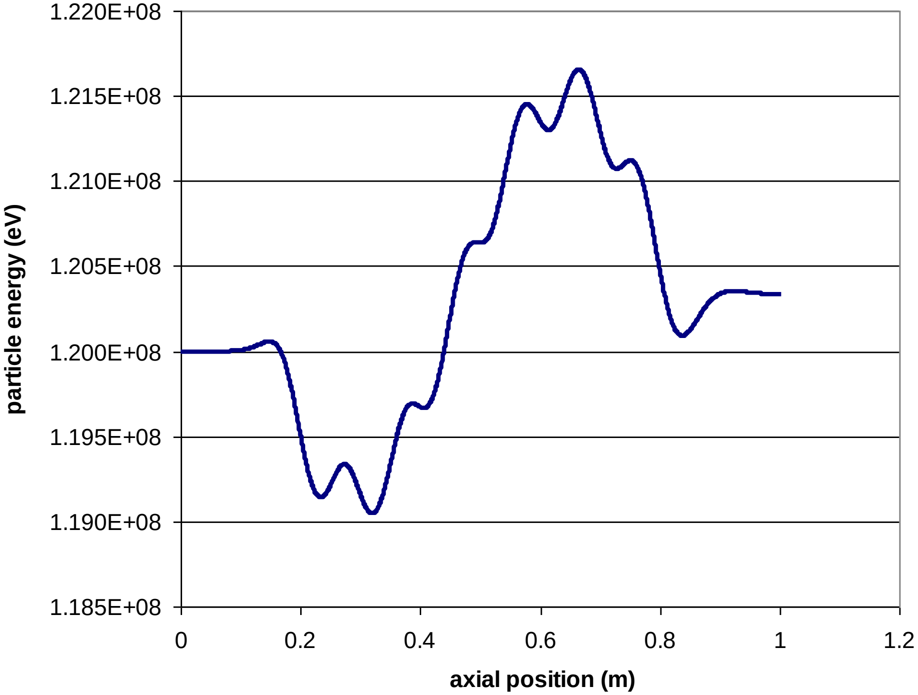


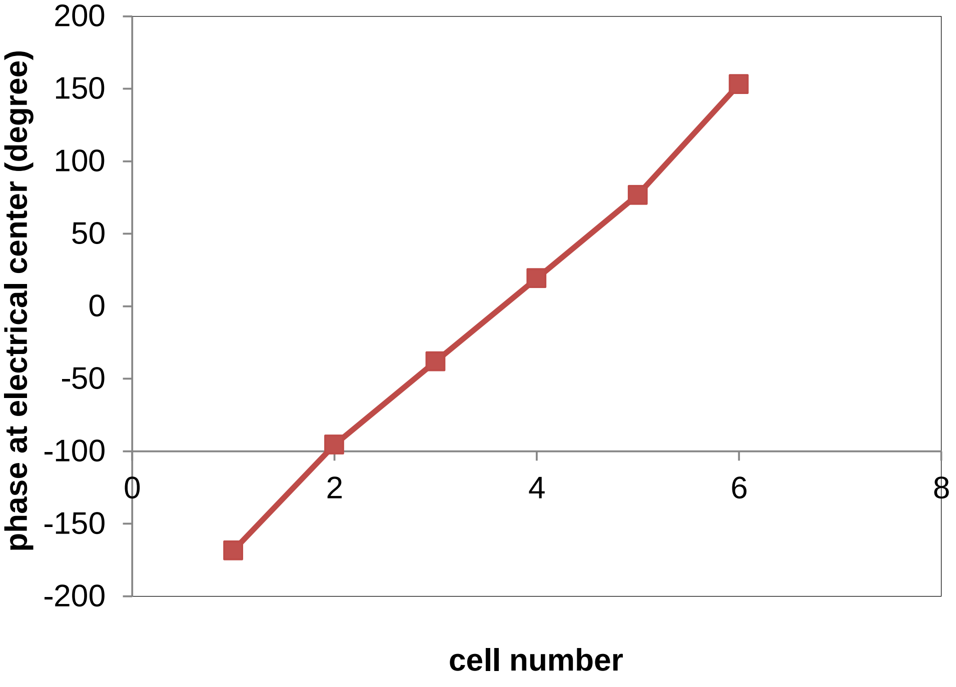


When input beam energy is too low or too high at a given structure:
Acceleration is inefficient.

Ex.) input beam energy 120 MeV (β=0.46)
→ large phase slip,

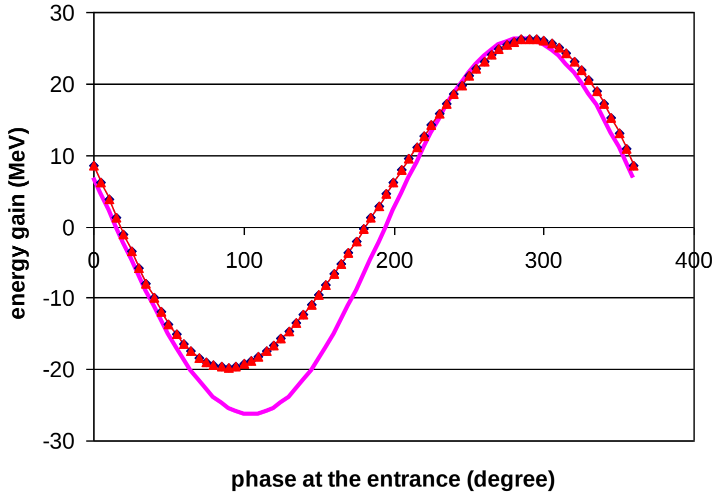


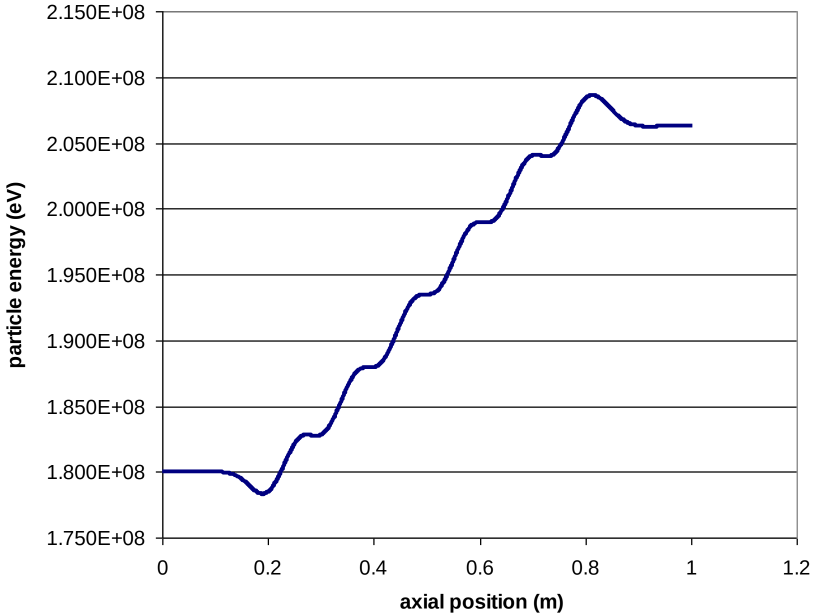


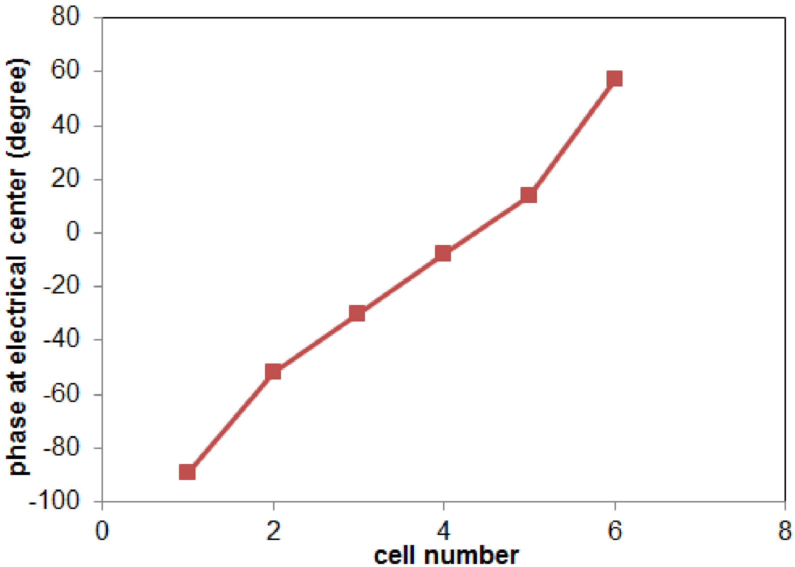


When beta changes a lot, simple 'cosine' approximation may not be accurate.
Let's test it with an extreme example.

Maximum energy gain at input beam energy 180 MeV
Using the field data in the previous example at $E_0T(\beta_g=0.61)=50$ MV/m





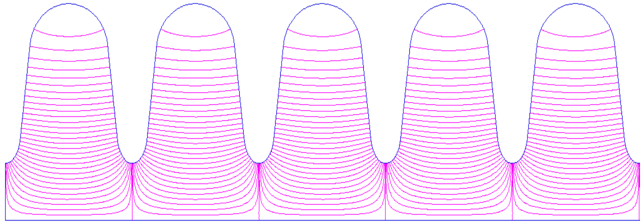


HOMEWORK 2-3

Using SUPERFISH, design 700 MHz elliptical cavity (inner cell only)
Geometrical beta=0.48, Eo=12 MV/m, Ri (iris radius)=4cm, α=5 degree

1. Do some optimization works
Epeak, Bpeak, r/Q at beta=0.48, QRs….

2. Generate 5 cell cavity like



- Get TTF values for 0.4~0.65
- (extra credit) calculate the phase of electric center of each gap for 100 MeV proton at ϕ_s=-20 degree

2.4 Higher order mode

-The RF fields inside a cavity are governed by Maxwell's equations subject to boundary conditions.

-A RF cavity is resonant at various frequencies. These are modes of a cavity.

-Since modes are defined by boundaries of a cavity, resonant conditions are discrete.

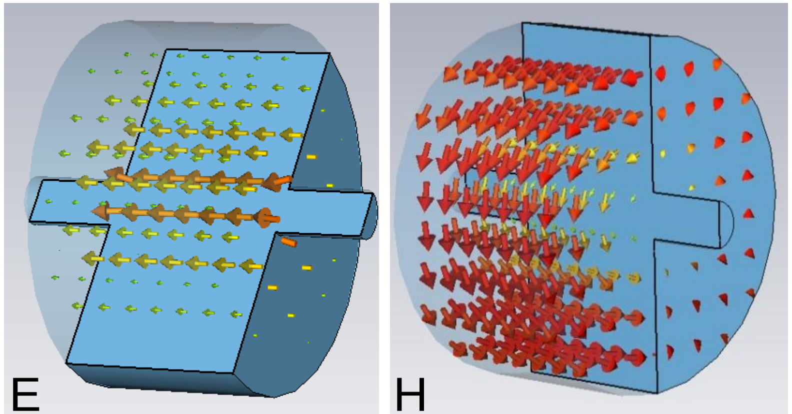
-Any surface and/or part of surface can be either capacitors or inductors there are infinite numbers of combinations.

-In a cavity with hole(s), modes with higher frequencies than the cut-off frequency of the hole(s) can not have resonant condition. Propagation through the hole(s). A finite number of modes can be exited in a RF cavity for particle acceleration.

-Modes except fundamental passbands are called 'Higher-order-modes'.

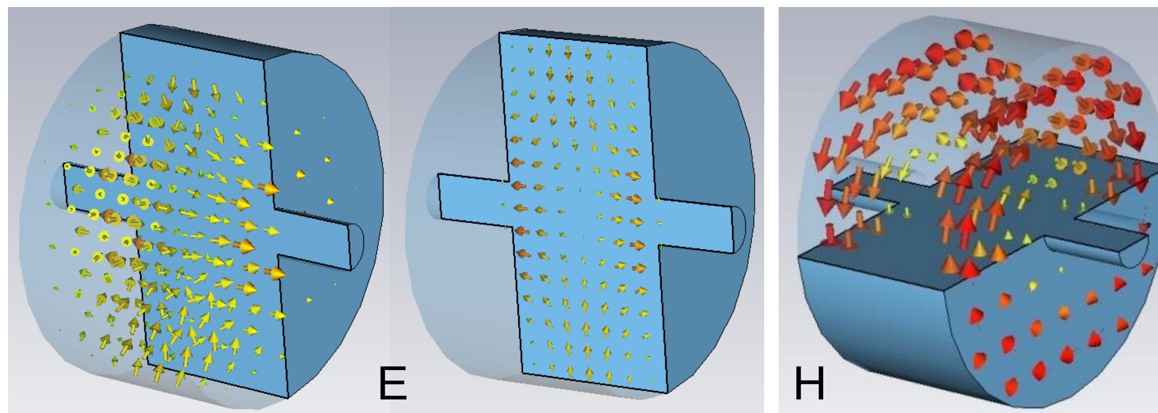
Modes (example; pillbox cavity, r=15cm, l=15cm, r_b=2cm)

TM monopole (fundamental mode)



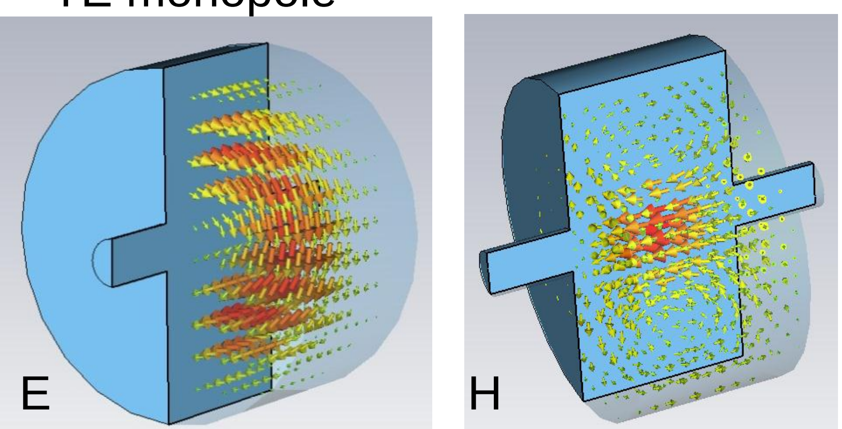
TM010 (762.5MHz)

TM monopole



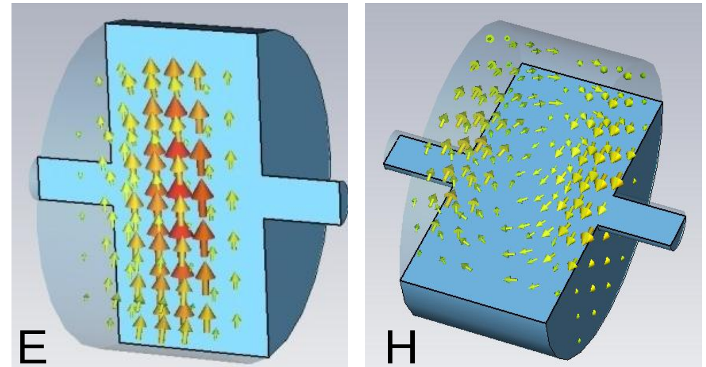
TM011 (1258.1MHz)

TE monopole



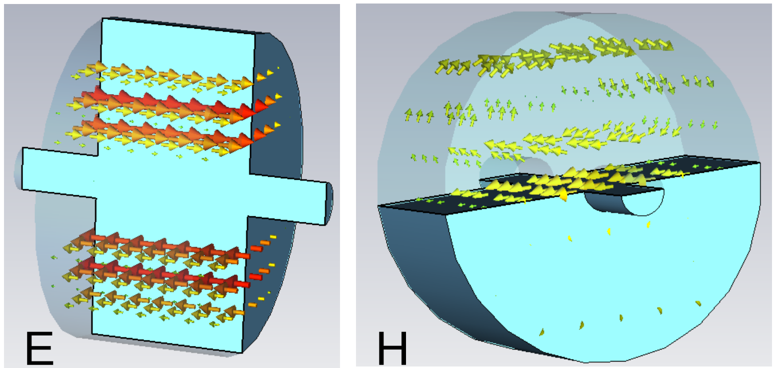
TE011 (1573.1MHz)

TE dipole



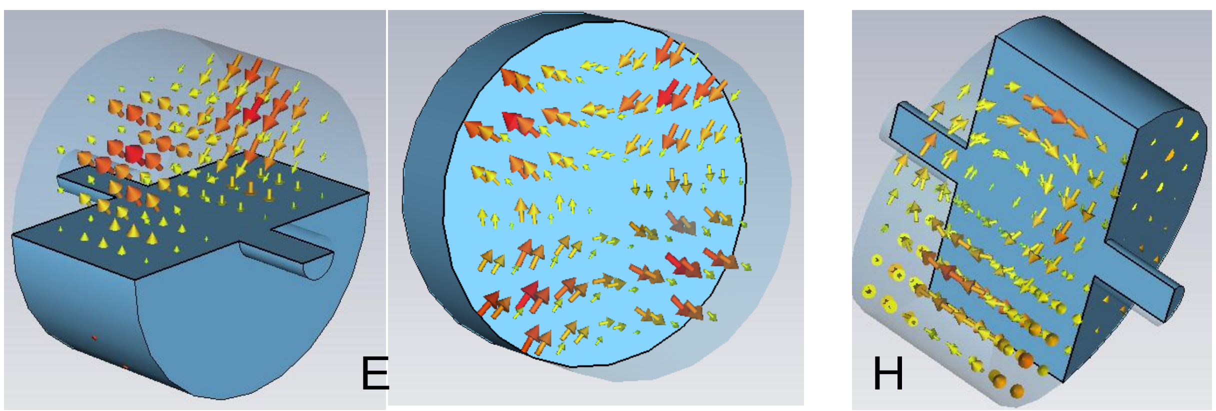
TE111 (1154.5 MHz)

TM dipole



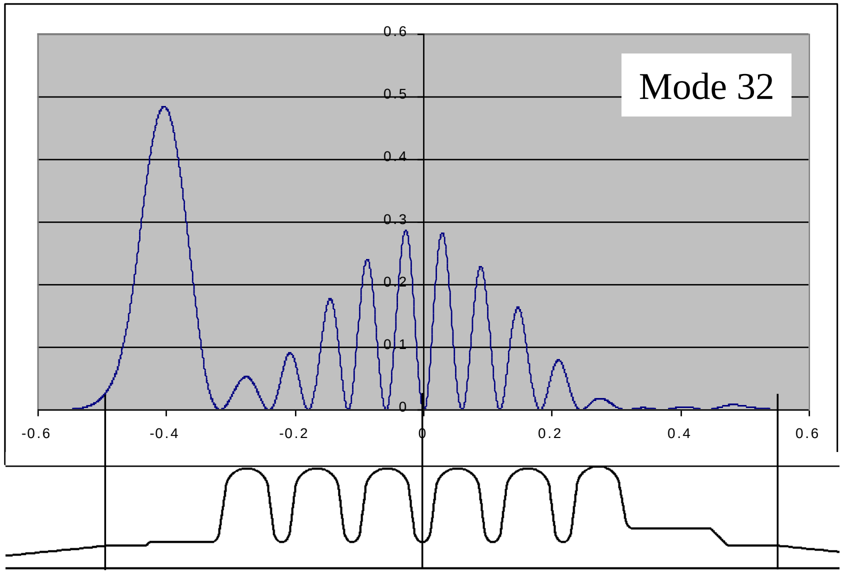
TM110 (1214.1MHz)

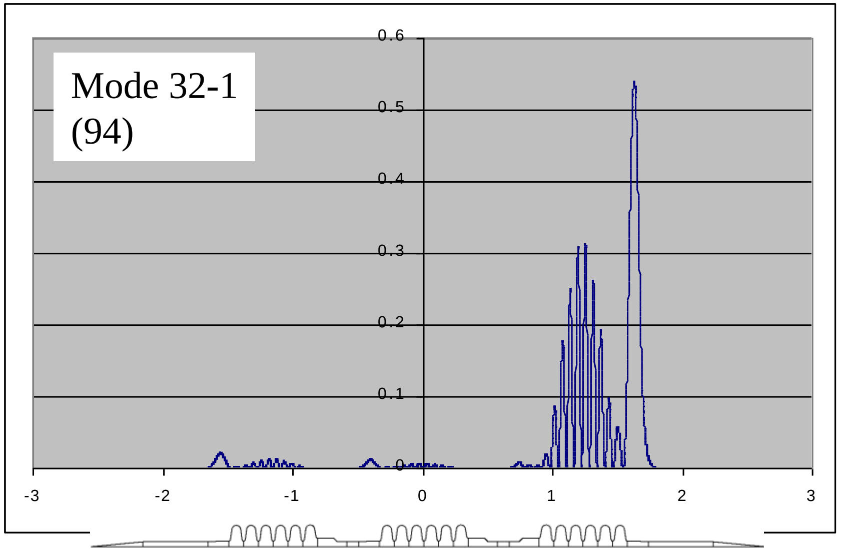
TE quadrupole

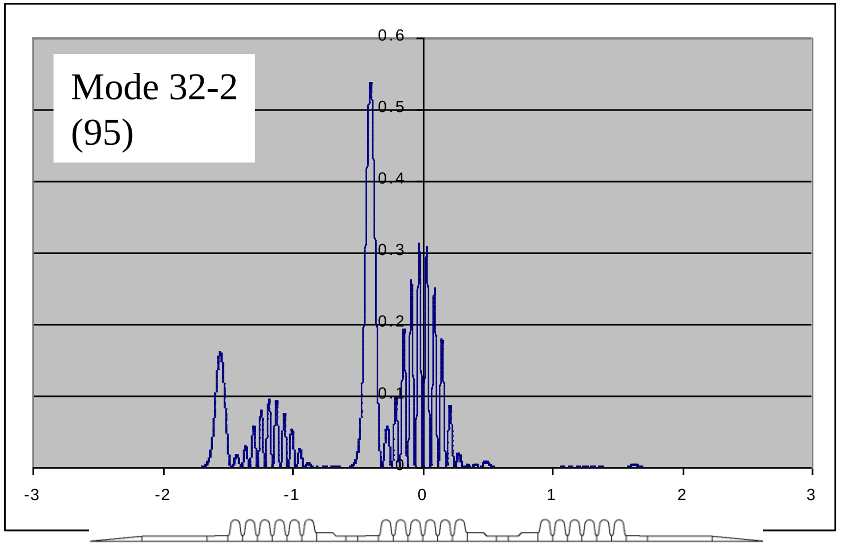


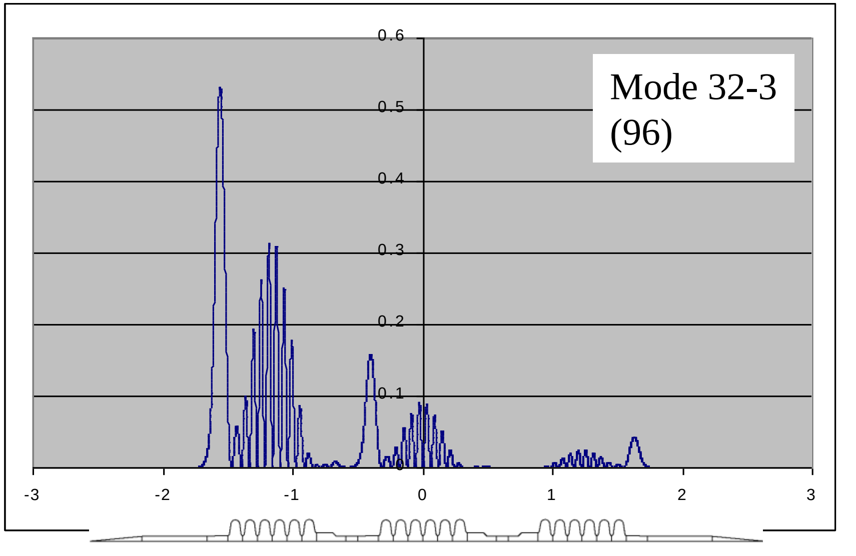
TE211 (1391.3MHz)

TM monopoles



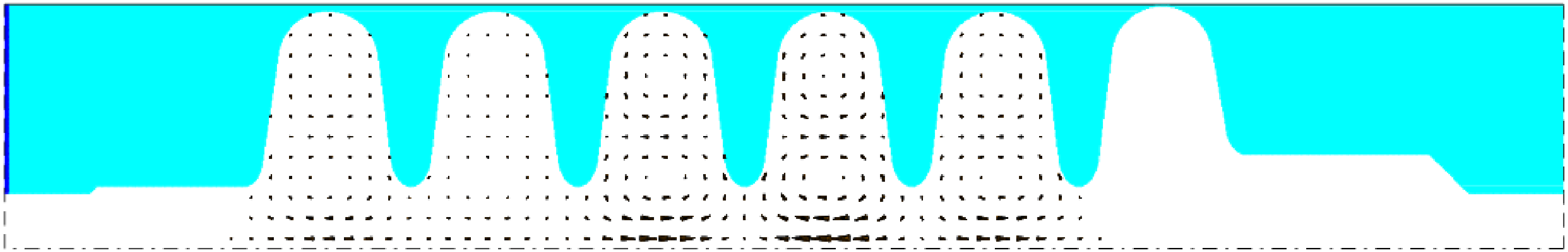




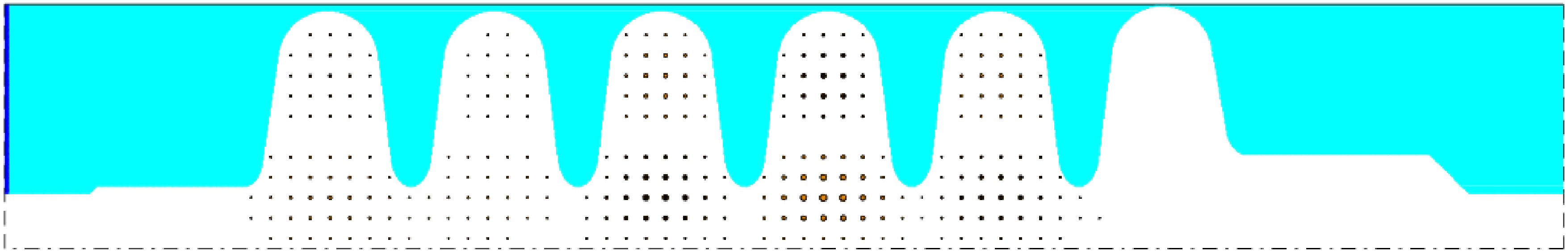


TE monopole (TE021 like)

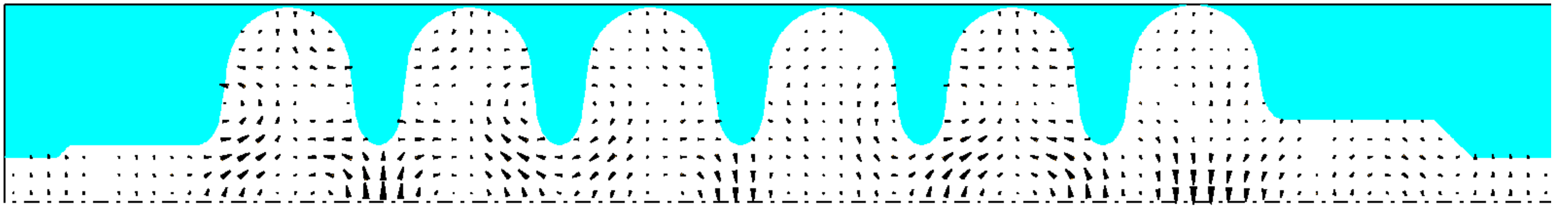
Magnetic field



Electric field



TE dipole



Electric field

Mode excitation by beam

-The field (electro magnetic radiation) from a moving charge induce surface charges (surface current) on the walls.

-When a charge is passing through a cavity (or any other geometrical variations along the structure like bellows, size changes of beam pipes, etc), scattered field (perturbed electromagnetic radiation) is produced. → wake field

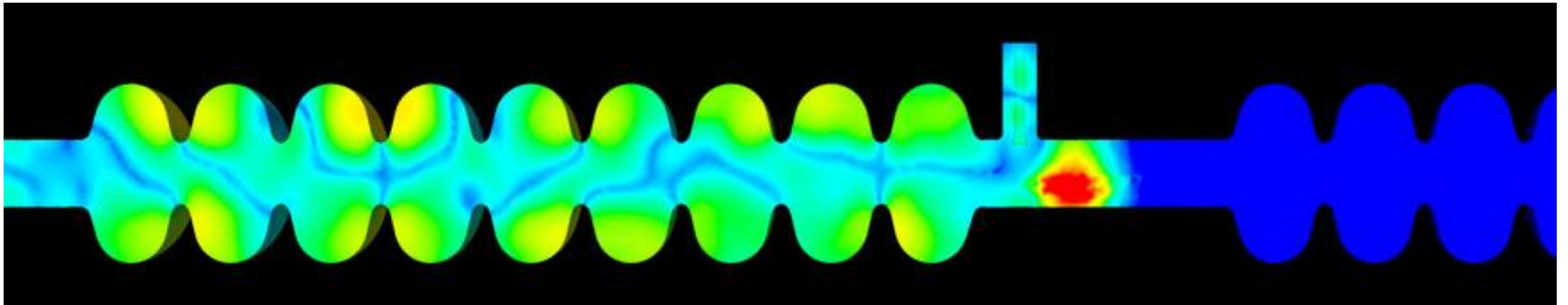
-In electron accelerators (high charge per bunch, and short bunch), SRF cavities are preferred, since large apertures reduce wake fields.

-Beam will loose energy by inducing wake field (single bunch effects)

power loss; need to be treated properly

(issues for high average current, high charge per bunch at CW operation as in electron rings and ERL machine)

energy spread & emittance growth; beam dynamics design should take care of this



Courtesy of SLAC group (wakefield simulation)

Induced voltage by a bunch

The induced voltage can be scaled using energy balance and super-position of field in a cavity for an arbitrary mode.
(following the sequence in 'fundamental theorem of beam loading', P. Wilson)

-A point charge q is passing an empty lossless cavity.

-It will induce a cavity voltage $V_a=-V_b$ (retarding. we don't know yet how much it is).

-Some fraction (f) of this induced voltage will act on the charge itself, fV_b.

-So the charge will loose energy by qfV_b. $\Delta W_1=-qfV_b$

-The stored energy in the cavity by this charge is proportional to square of induced voltage, $\boldsymbol{U=\alpha V_b^2}$. → $qfV_b=\alpha V_b^2$ → $\boldsymbol{V_b=fq/\alpha}$

-Half a period after the first point charge, the second charge q passes the cavity. The induced voltage by the first point charge is now V_b (accelerating) & the second charge will induced a cavity voltage -V_b

→ net cavity voltage becomes 0

→ sum of energy changes of two charges should be zero

-$\Delta W_2=qV_b$ *(field by the first charge)*$-qfV_b$, $\Delta W_1=-qfV_b$ → $\Delta W_1+\Delta W_2=0$ → $\boldsymbol{f=1/2}$ → $\boldsymbol{V_b=q/2\alpha}$

Loss factor

Induced voltage acting on the charge itself, $V_b/2$ *(where* $V_b=q/2\alpha$*)* $=q/4\alpha$

For a convenience, replacing $1/4\alpha$ *with* k → *induced voltage* $V_b=2kq$

Energy loss can be expressed with k → $U= U=\alpha V_b^2 = V_b^2/(4k)=kq^2$

Using the definition of $r/Q=V_a^2/(\omega U)$ & $V_a^2 = V_b^2$

$$k = \frac{\omega}{4}\left(\frac{r}{Q}\right): \text{loss factor in [V/C] or [V/pC]}$$

In SFO file, there's a number for k named 'Wake loss parameter'

This general expression is directly applicable to TM monopole modes.
Induced voltage of each mode:

$$V_{an} = -V_{bn} = -2k_n|q|e^{i\omega_n t},\ k_n = \frac{\omega_n}{4}\left(\frac{r}{Q}\right)_n ;\text{n is the mode number}$$

$$\left(\frac{r}{Q}\right)_n = \frac{\left|\int E_{nz}(z)\exp(i\omega_n z/v)dz\right|^2}{\omega_n U_n}$$

$$U_n = k_n q^2,\ P_n = k_n q I_{bo}$$

If we define,

$$\text{Total loss factor from HOM } k_{l,HOM} = \sum_n k_n - k_0,\ k_0 : \text{loss factor of accelerating mode}$$

$$\text{Total average power loss by this effect} : U_{l,HOM} = k_{l,HOM} q I_{bo}$$

This induced voltage & additional power dissipation only depends on mode frequency, r/Q of modes and beam intensity.

This power loss by the single bunch effect is not related to HOM damping (or $Q_{ex,n}$) since it comes directly from wake field. This is one of big issues in the proposed high current ERL like-machine.

Ex) CW, 10 nC/bunch, 200 mA if a cavity has $k_{l,HOM}$=2V/pC, then P=4kW

Ex) SNS: 100pC/bunch, 26 mA during macro-pulse, high beta cavity
$k_{l,HOM}$<<2V/pC for design beta range
→ <5 W (cf. Pc=53 W at R_S=16 nΩ by accelerating mode)
<<10 % of cavity wall loss of acceleration mode.

Usually beam is not exactly on the cavity's RF axis. Many steering magnets are involved to correct orbit trajectory close to design one.

Off-axis beam can excite dipole, quadrupole, sextupole, and so on which can deflect beam.

For deflection modes (dipole, quadrupole, sextupole, …), a similar expression can developed with equivalent r/Q for deflection force on beam.

The fundamental concept is same as for the TM monopoles described in previous pages.

$$\left(\frac{\mathrm{r}}{\mathrm{Q}}\right)_{\perp} = \frac{\mathrm{c}^2 \left| \int \nabla_r E_z \exp(i\omega_n z/v) dz \right|^2}{\omega_n^3 U} [\Omega],\ \ k_{\perp} = \rho^2 \frac{\omega_n^2}{c^2} \frac{\omega_n}{4} \left(\frac{\mathrm{r}}{\mathrm{Q}}\right)_{\perp} [\mathrm{V/C}]$$

or

$$\left(\frac{\mathrm{r}}{\mathrm{Q}}\right)_{\perp} = \frac{\left| \int \nabla_r E_z \exp(i\omega_n z/v) dz \right|^2}{\omega_n U} [\Omega/\mathrm{m}^2],\ \ k_{\perp} = \frac{\omega_n}{4} \left(\frac{\mathrm{r}}{\mathrm{Q}}\right)_{\perp} [\mathrm{V/C} \cdot \mathrm{m}^2]$$

Mode excitation by beam and build up fields

-Beam has many frequency components:
beam time-structure, beam amplitude fluctuation

-Specific modes can be excited and develop a high field when a beam time-structure hits the cavity HOM.

-**Beam quality** can be affected in both transverse and longitudinal directions if $Q_{ex,n}$ and $(r/Q)_n$ are high, and/or beam is passing the same cavity many times as in the ring. Non-π fundamental passband can also induce energy oscillations (longitudinal).

-**HOM power** can be excessive at around the bunch frequency and its harmonics.

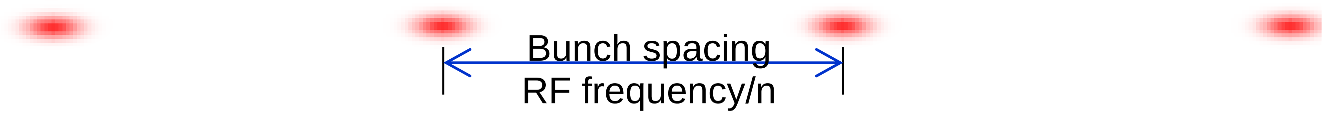
-If needed, it should be damped down to a certain level using a HOM (allowable $Q_{ex,n}$).

-A series of recent studies tells that HOM damping requirements of SRF cavities for recently built or proposed proton/heavy ion accelerators are modest.

Sources of HOM excitation (I)

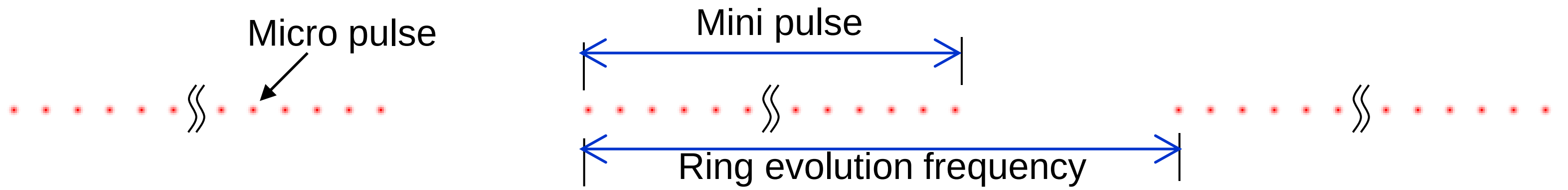
In both CW and Pulsed machine

1. Micro-pulse



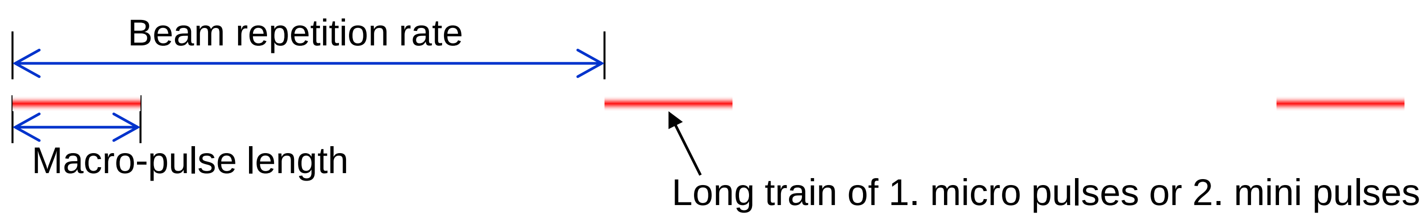
For RING extraction (chopping)

2. Mini-pulse

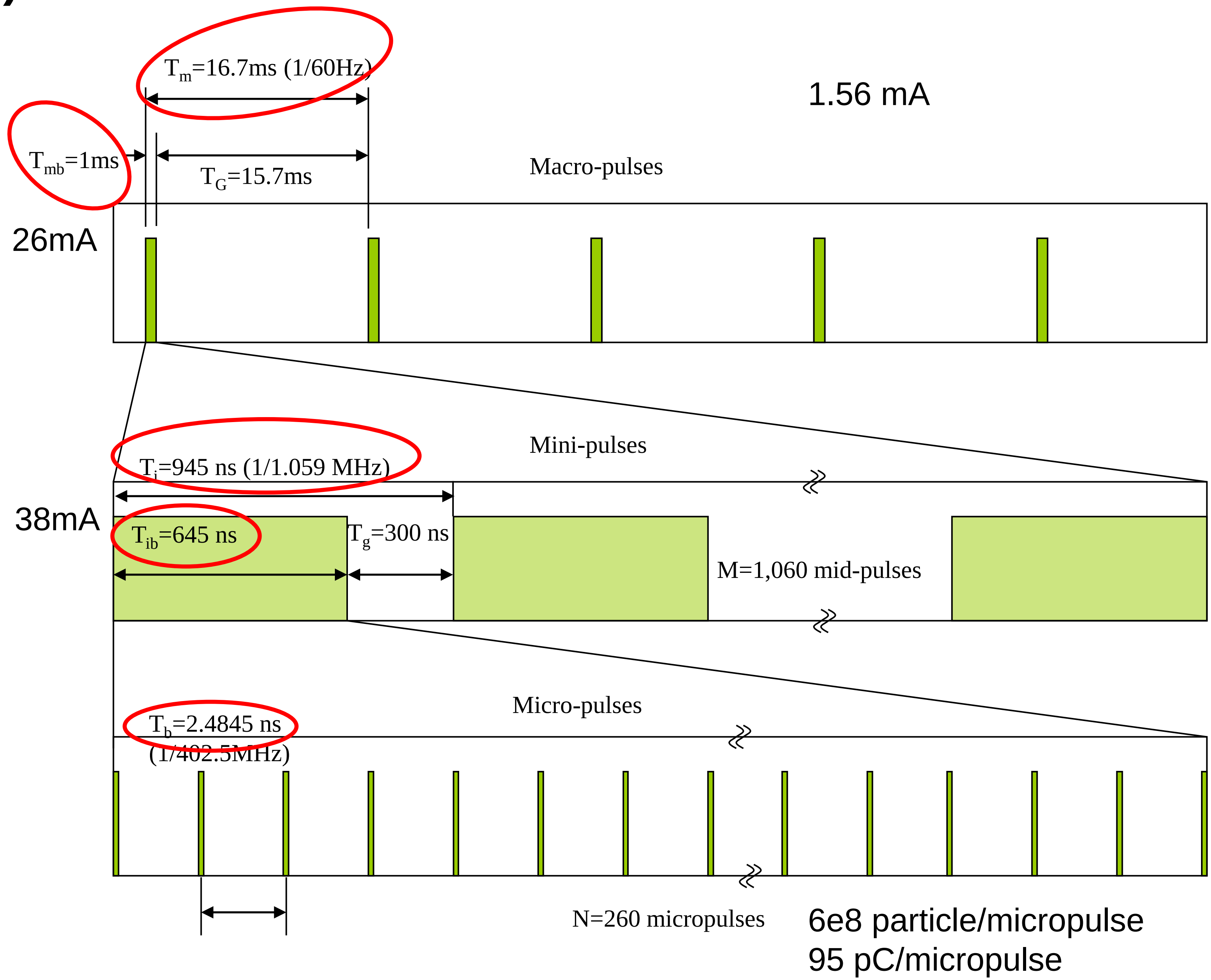


In pulsed machine

3. Macro-pulse



Ex) SNS beam time-structure

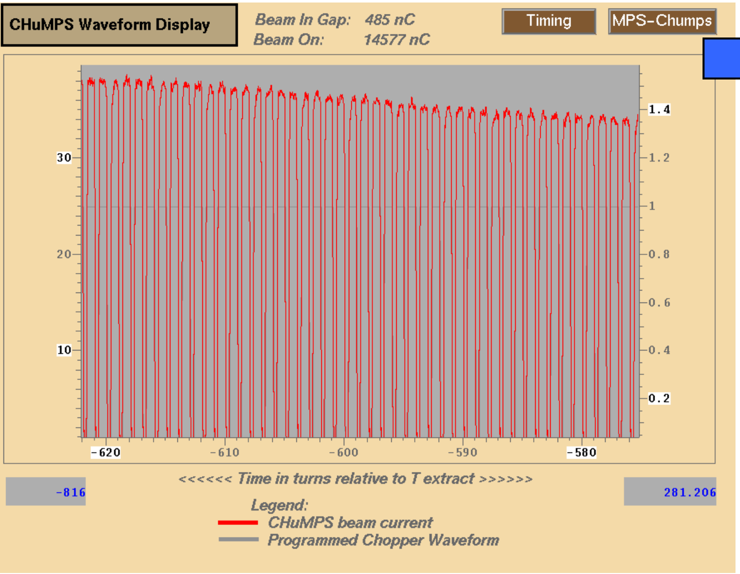


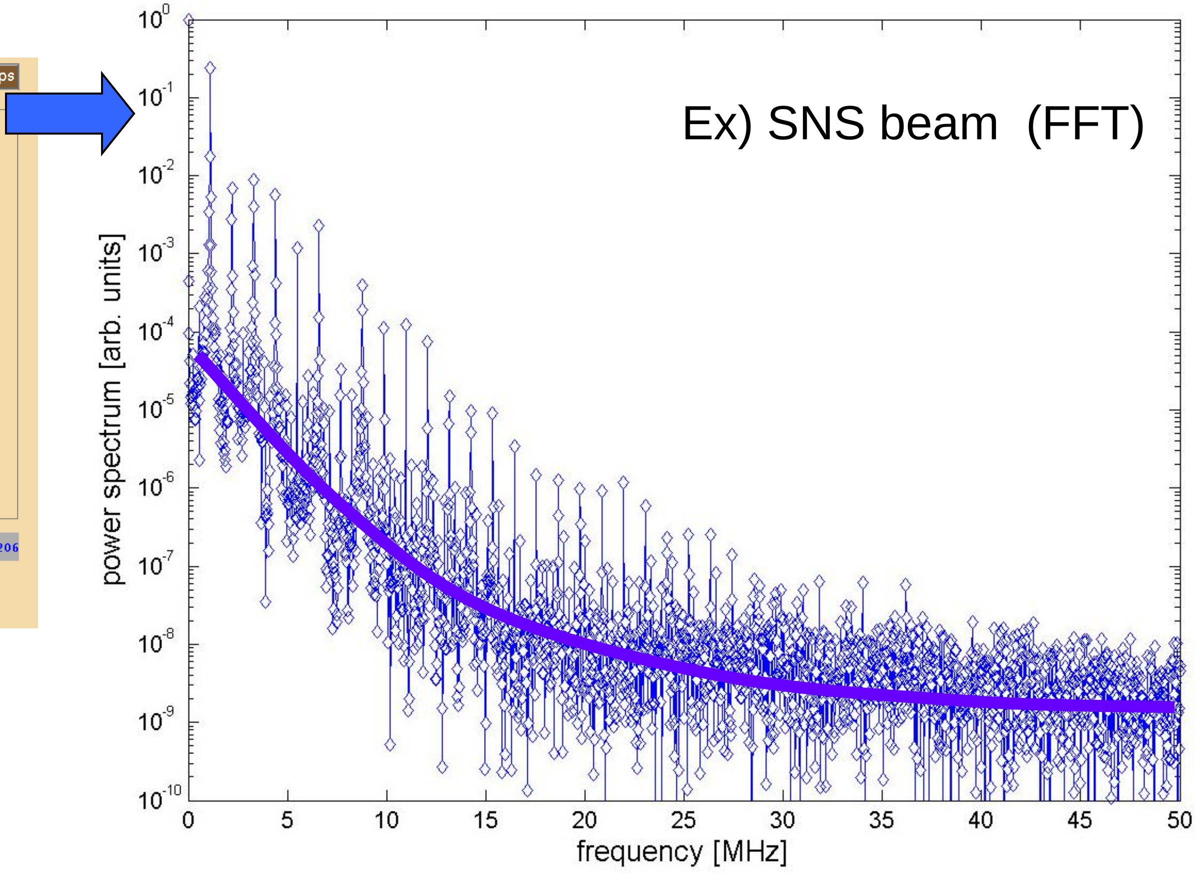
Sources of HOM excitation (II)

Bunch intensity fluctuation:

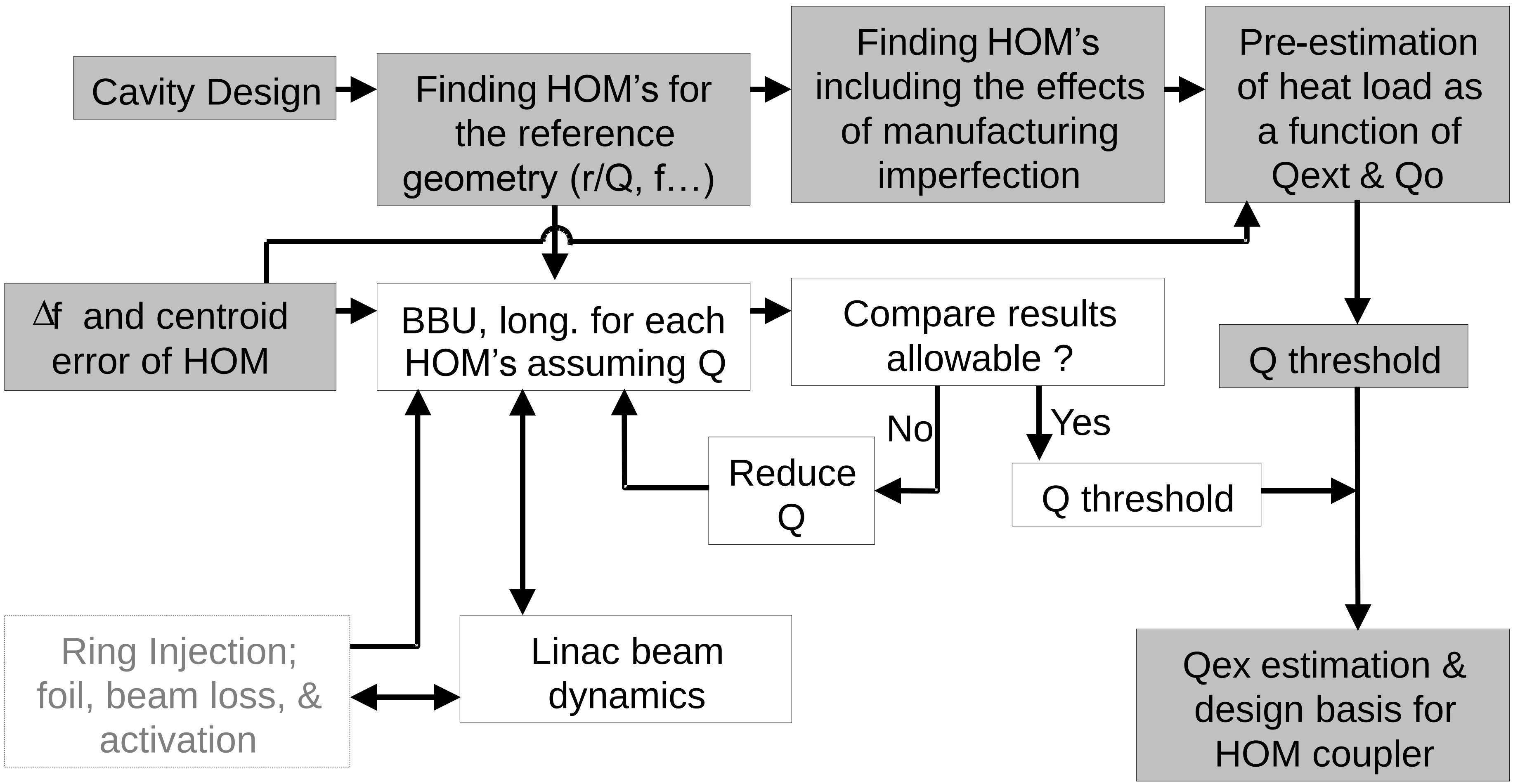
It is not a white noise but can occur at almost any frequency.
Exponentially decaying with frequency.
In linac it is not an issue at a few % fluctuations in total.

Micro-pulse





-In a design stage, HOM damping requirement should be addressed and dangerous trapped mode (coupling to HOM damper is very low) should be eliminated by modifying a cavity geometry.



HOM concerns (in linac single pass effects only)

Transverse

- Cumulative effects; beam break-up, emittance growth
  - Source: off axis beam, beam time-structure
  - True instability; can occur at almost any frequency
  - Error magnification; worst when an HOM frequency differs by of the order of 1 cavity bandwidth from beam spectral lines

Longitudinal

- Instability; energy spread, oscillation
  - Source: Bunch energy error (non-relativistic), bunch-to-bunch charge variation, beam time structure
  - Can occur at almost any frequency
  - Non-pi fundamental passband can excite energy oscillations
- HOM power dissipation; additional heat load
  - Source: beam time structure
  - excessive heat dissipation: worst at beam spectral lines

HOM field build up

Here we will quantify the HOM field build up of TM monopoles only from beam time-structure as a source term.

It needs a numerical calculation through a particle tracking for other source terms such as bunch energy error (for non-relativistic beam) & bunch-to-bunch charge variation.

As a charge q passes a cavity on axis, monopoles are excited and the cavity voltage induced by the charge is:

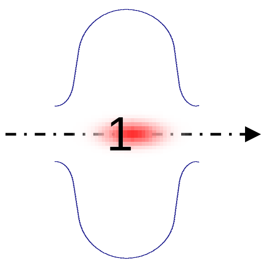
$$\mathbf{V}_{\mathrm{an}} = -\frac{\omega_{\mathrm{n}}}{2}\left(\frac{\mathrm{r}}{\mathrm{Q}}\right)_n |\mathrm{q}| \exp(\mathrm{i}\omega_{\mathrm{n}}\mathrm{t}) = -\mathrm{V}_{\mathrm{bn}} \exp(\mathrm{i}\omega_{\mathrm{n}}\mathrm{t})$$

And the acting voltage back on the charge itself is:

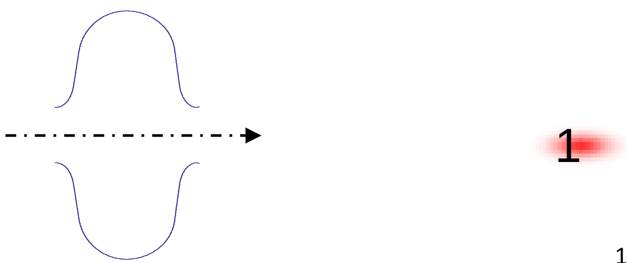
$$\mathbf{V}_{\mathrm{self}} = \mathbf{V}_{\mathrm{an}} / 2$$

Ex) if a cavity has HOM at f_n=3 GHz & q=95pC/bunch is passing this cavity
→normalized voltage $\mathrm{V}_{\mathrm{an}}/(\mathrm{r/Q})_{\mathrm{n}} = 0.9/(\mathrm{r/Q})_{\mathrm{n}}\ [\mathrm{V}/\Omega]$

If we include the decay term (surface dissipation, coupling out to the external devices), induced voltage between pulses will decay exponentially with the time constant $\tau_n = 2Q_{L,n}/\omega_n, Q_{L,n}$: Loaded Q of mode n, ω_n : angular resonance frequency of mode n

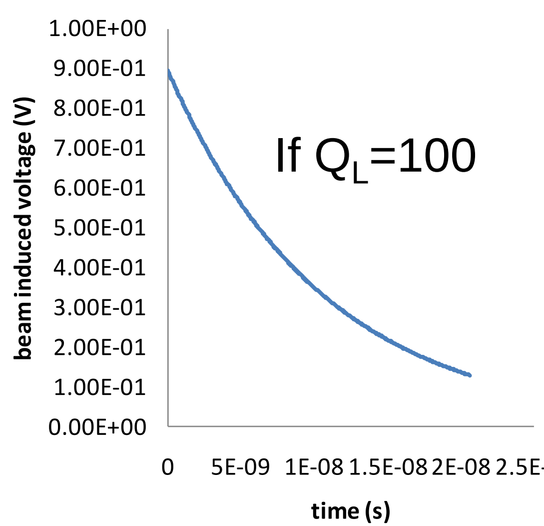


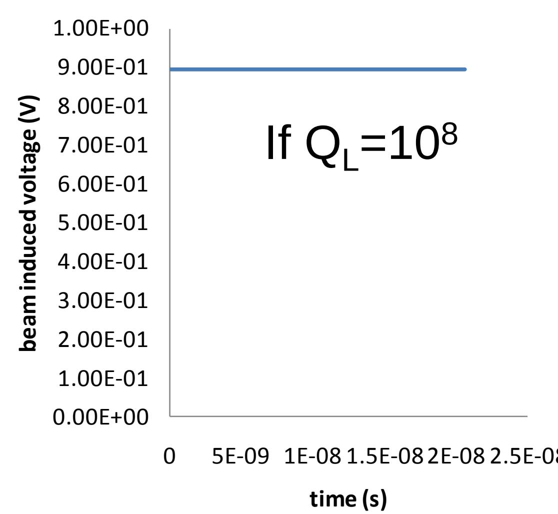
$$\mathbf{V}_{an} = -\frac{\omega_n}{2}\left(\frac{r}{Q}\right)_n |q| \exp(i\omega_n t) = -V_{bn}\exp(i\omega_n t)$$



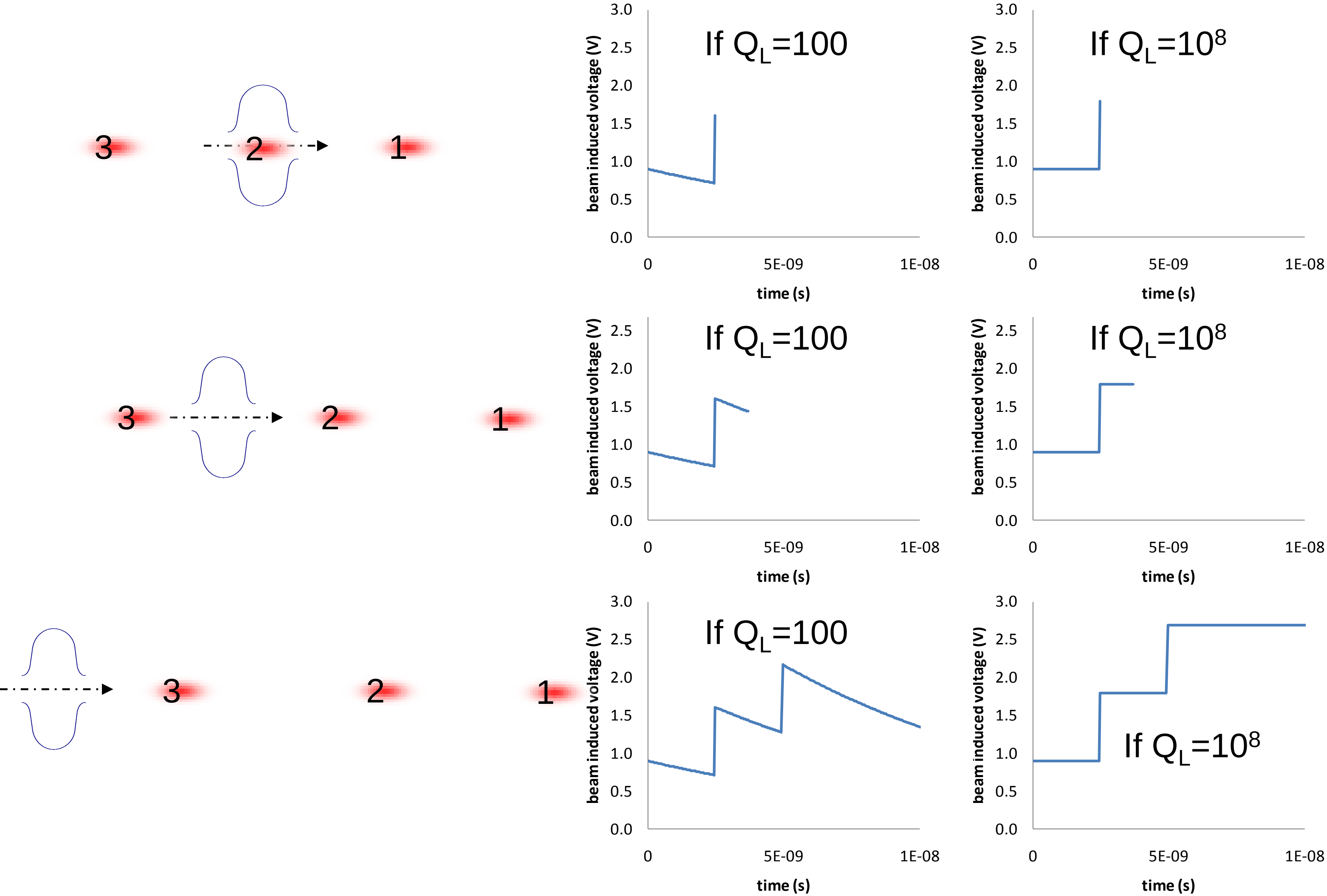
$$\mathbf{V}_{an} = -V_{bn}\exp(i\omega_n t)\exp(-t/\tau_n)$$

Ex)
at f_n=2.8175 GHz
q=95pC/bunch,
(r/Q)n=1 Ω



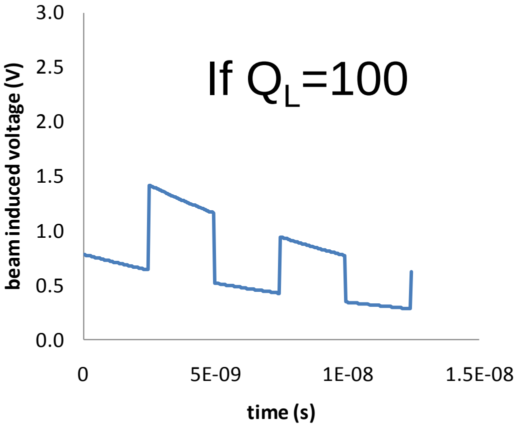


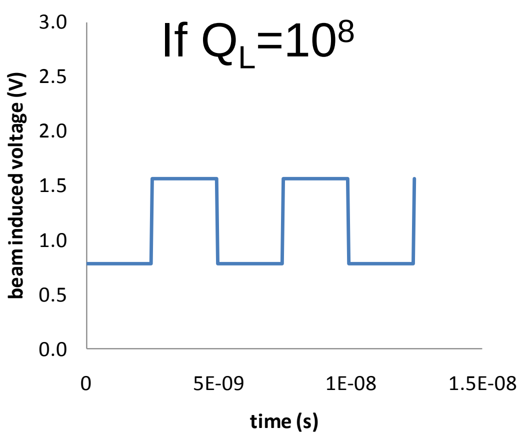
Ex) 3 bunches are passing with a bunch spacing, 1/402.5MHz~2.5ns
Since the HOM frequency is harmonics of bunch frequency → in phase



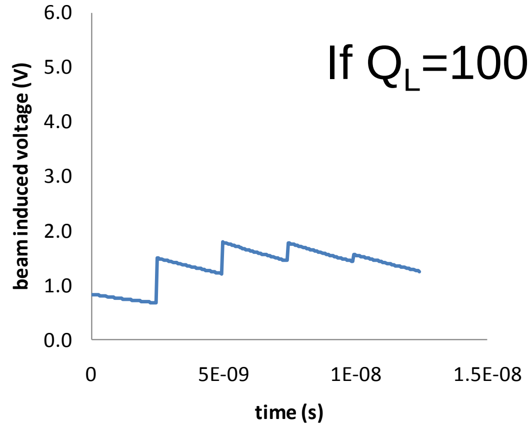
Ex) if HOM frequency is

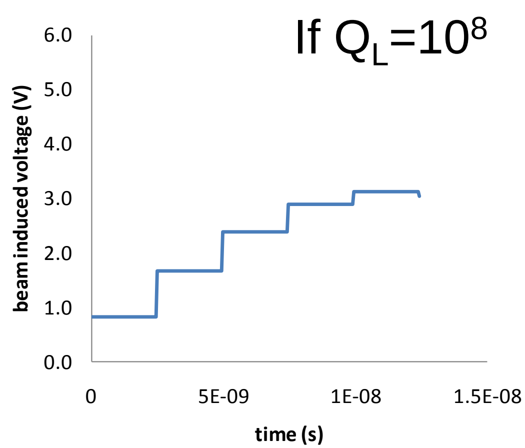
6.5xbunch frequency



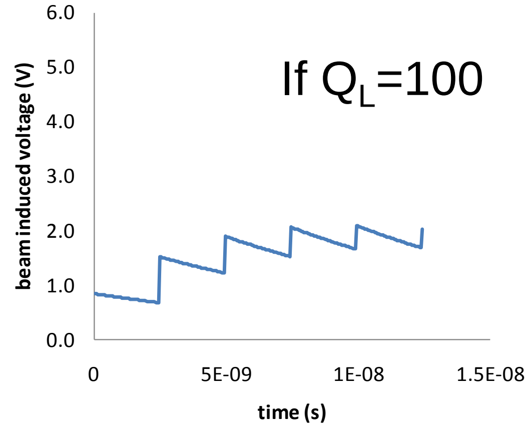


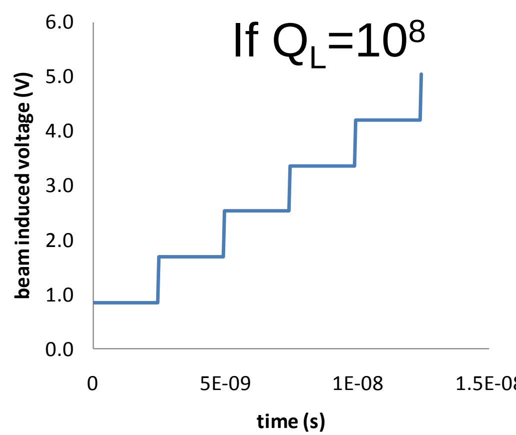
6.9xbunch frequency



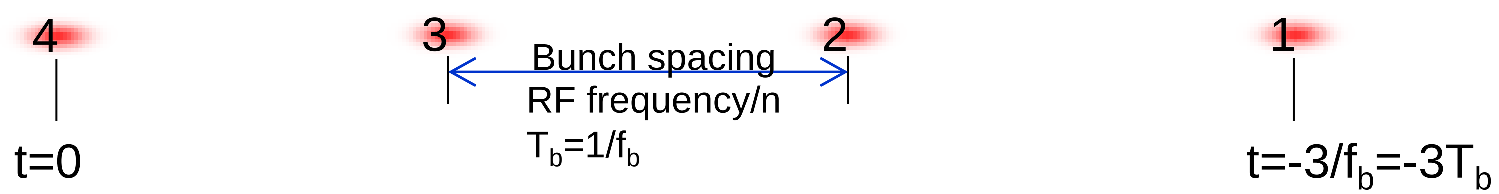


7xbunch frequency





Analytic expression in CW machine : single beam time structure



V_b from bunch 4

$V_b \exp(i\omega_n T_b - T_b/\tau_n)$ from bunch 3

$V_b \exp(i2\omega_n T_b - 2T_b/\tau_n)$ from bunch 2

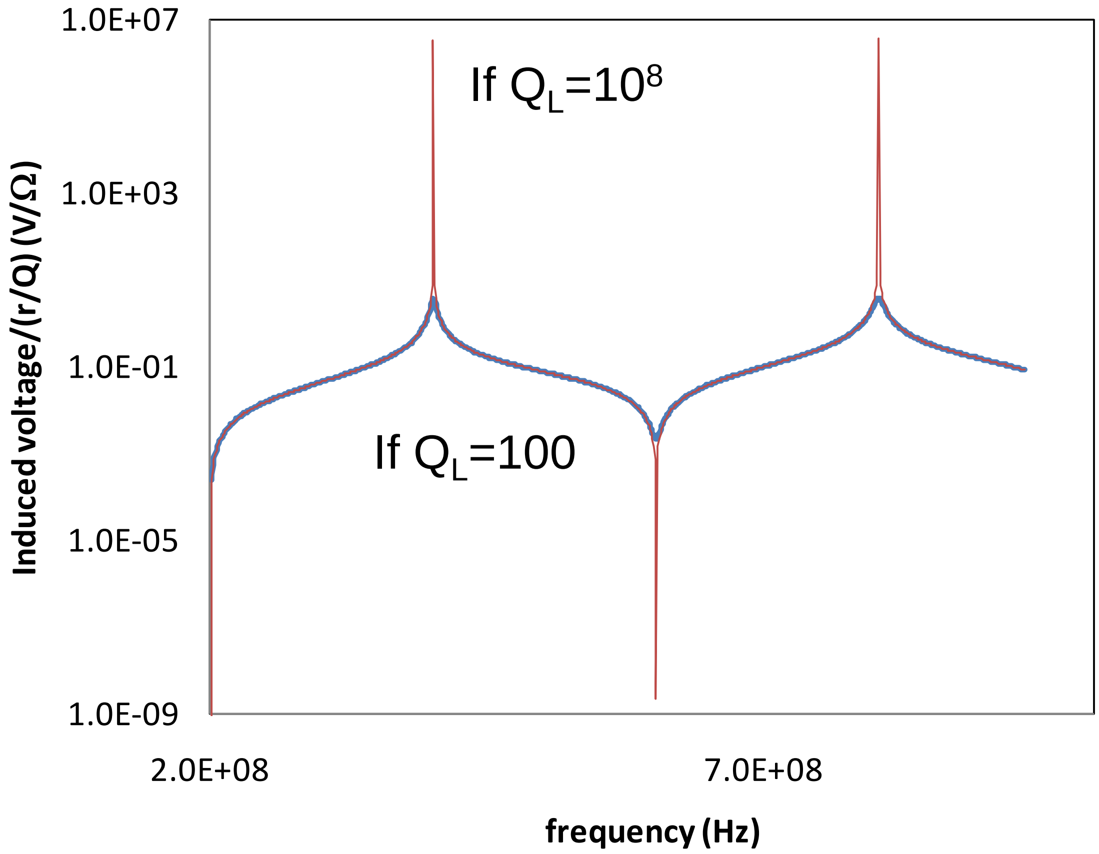
$V_b \exp(i3\omega_n T_b - 3T_b/\tau_n)$ from bunch 1

$$V_{bn} = \frac{\omega_n}{2}\left(\frac{r}{Q}\right)_n |q|$$

The cavity voltage by beam at t=0 (now) is the summation of all.
In CW operation

$$\mathbf{V}_{an} = -V_{bn}\sum_{m=0}^{\infty}\exp(im\omega_n T_b - mT_b/\tau_n) = \frac{-V_{bn}}{1-\exp(i\omega_n T_b - T_b/\tau_n)}$$

Possible induced voltage by beam in the continuous HOM frequency.
After figuring the HOM properties (frequency, r/Q, Q_L), one can calculate HOM voltages induced by beam. (using previous example: q=95pC, f_b=402.5MHz)



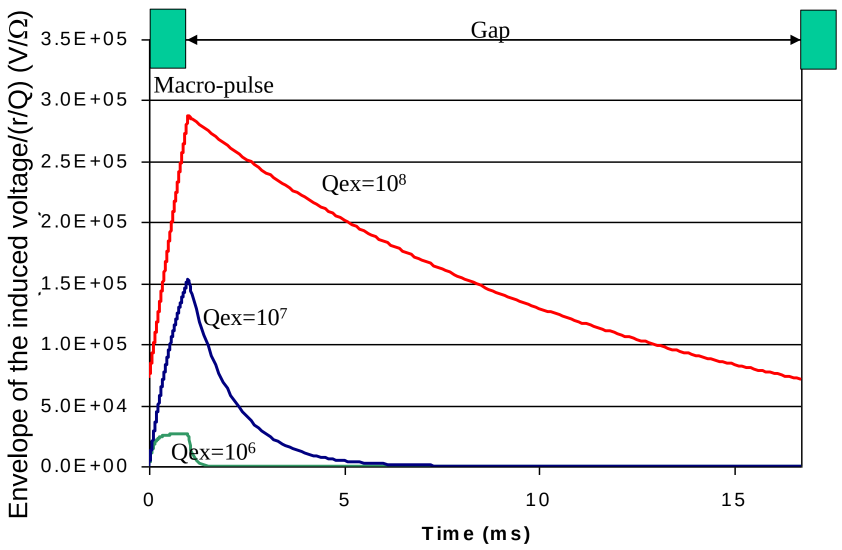
HOM Power

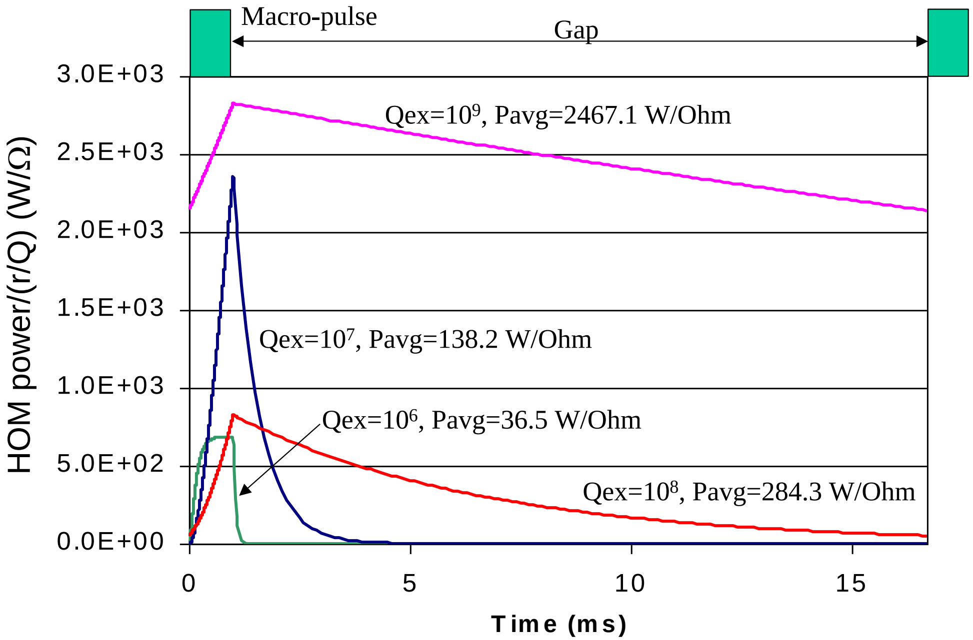
CW beam

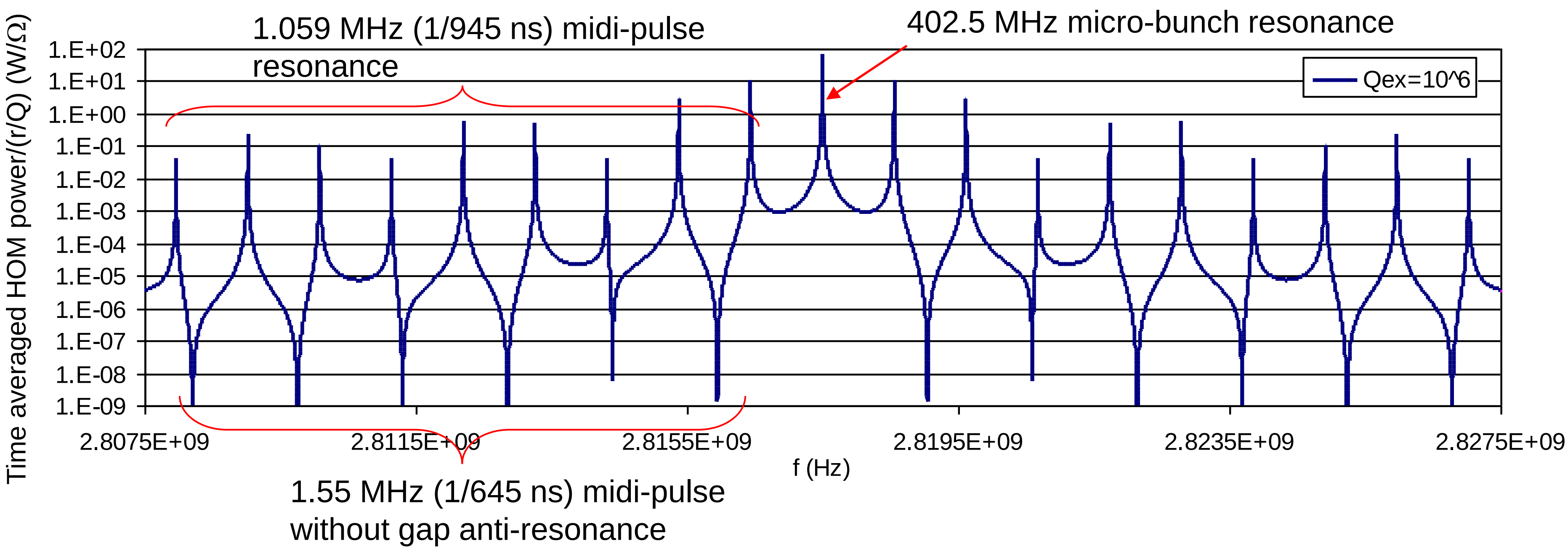
$$\mathbf{V}_{an} = \frac{-V_{bn}}{1-\exp(i\omega_n T_b - T_b/\tau_n)} = V_{bn}C_n \qquad V_{bn} = \frac{\omega_n}{2}\left(\frac{r}{Q}\right)_n |q|$$

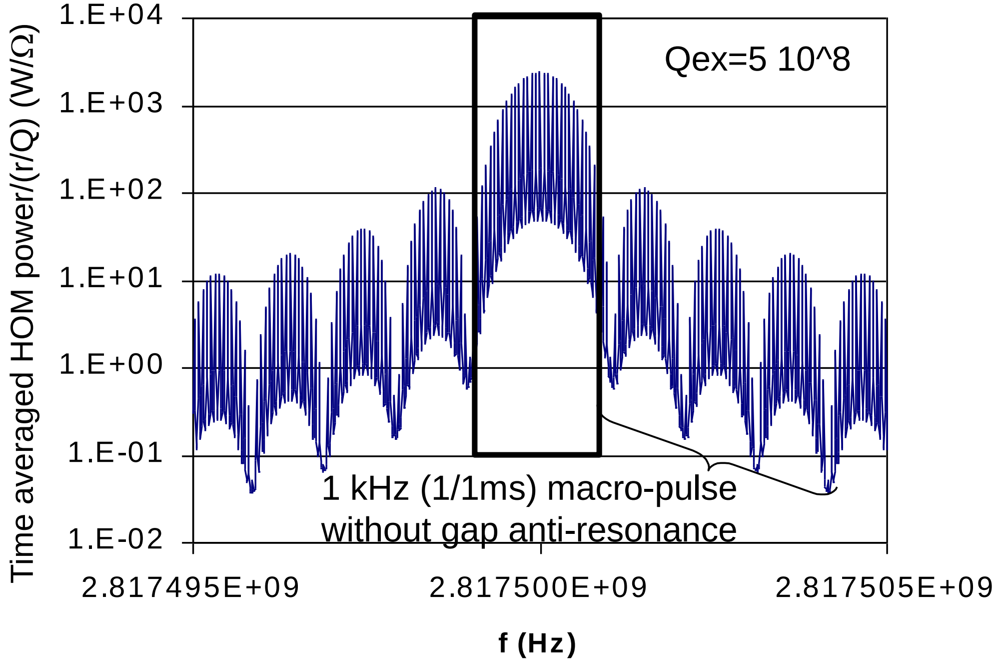
$$P_n = \frac{\mathbf{V}_{an}\cdot\mathbf{V}_{an}^*}{(r/Q)_n Q_{n,L}} = \frac{\omega_n^2}{4}|q|^2 \frac{(r/Q)_n}{Q_{n,L}} C_n \cdot C_n^*$$

From complex beam time-structure (ex. SNS)

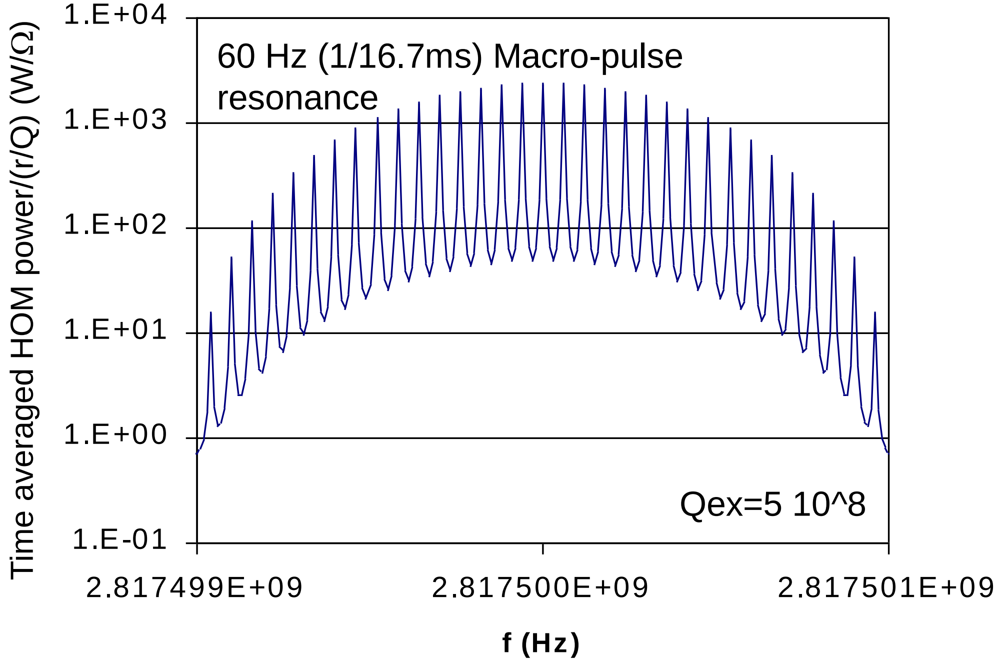






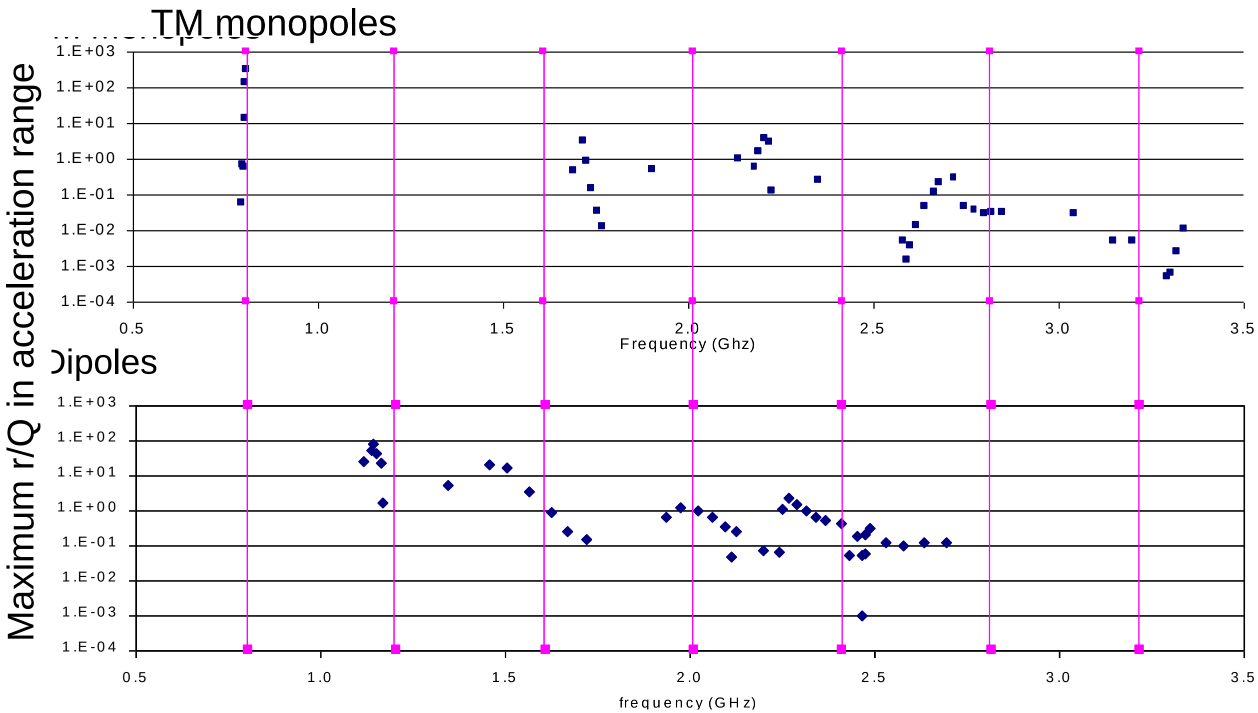


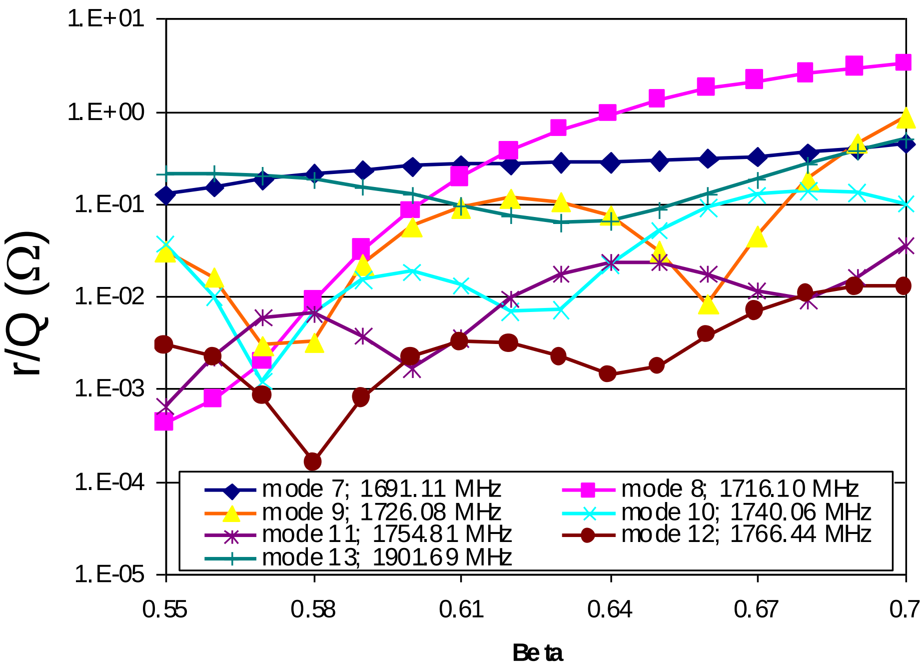
Time averaged HOM power/((r/Q) (W/Ω)
1.E+04
1.E+03
1.E+02
1.E+01
1.E+00
1.E-01
1.E-02
Qex=5 10^8
1 kHz (1/1ms) macro-pulse
without gap anti-resonance
2.817495E+09
2.817500E+09
2.817505E+09
f (Hz)



Time averaged HOM power/((r/Q) (W/Ω)
1.E+04
1.E+03
1.E+02
1.E+01
1.E+00
1.E-01
60 Hz (1/16.7ms) Macro-pulse
resonance
Qex=5 10^8
2.817499E+09
2.817500E+09
2.817501E+09
f (Hz)

HOM in SNS medium beta cavity





In most of cavities only few modes are in concern.

HOM frequency scattering: due to mechanical imperfection

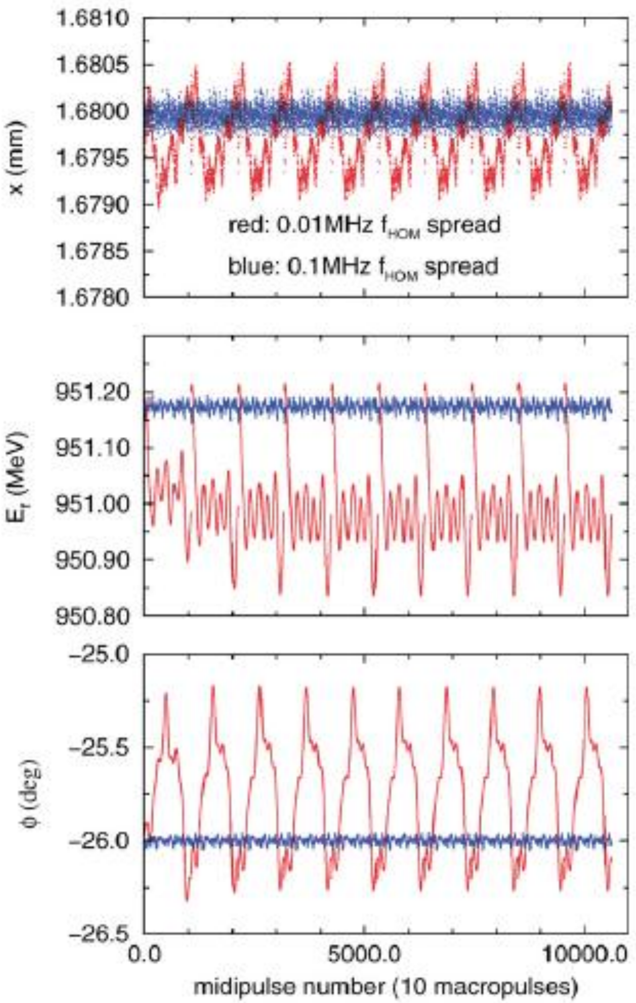
- HOM frequency Centroid Error between analysis & real ones
  - Fractional error; $(f_{analysis}-f_{real,avg})/f_{analysis} < 0.0038$
- HOM frequency spread

$$\sigma = 0.00109 \times \left| f_n - f_0 \right|$$

f_o; fundamental frequency, f_n; HOM frequency

- Non-pi fundamental mode

$$\left| \frac{f_{measured} - f_{calculated}}{f_{calculated}} \cdot \frac{f_{\pi-mode}}{f_{\pi-mode} - f_{calculated}} \right| \le 0.027$$



Trapped mode

Modes that do not have field at around end-cell/beam pipe region.

due to the differences in HOM frequency between inner cell and end cell.
also due to the weak cell-to-cell coupling.
more chance as number of cells increases.

Coupling to the external circuit is about zero.

Q_0 is usually $10^8 \sim 10^{10}$.

If it locates around dangerous frequency region, the modes should be eliminated by re-design the cell shapes.

bigger iris, put the end-cell shape close to the inner cell

In any case, modes around beam spectral lines are the most concern. (HOM field build up).

If r/Q and Q_L of modes are high and/or excessively large other source terms (bunch energy error, bunch-to-bunch charge variation) are assumed, there are always instabilities.

But, overly conservative approach can make a system more complex. All analysis needs a certain amount of margin that should be reasonably conservative.

Other damping mechanism such as stainless steel bellows between cavities, to fundamental power coupler plays important role.

Some practical concerns when using FEM code

When using an FEM code, improper setting of

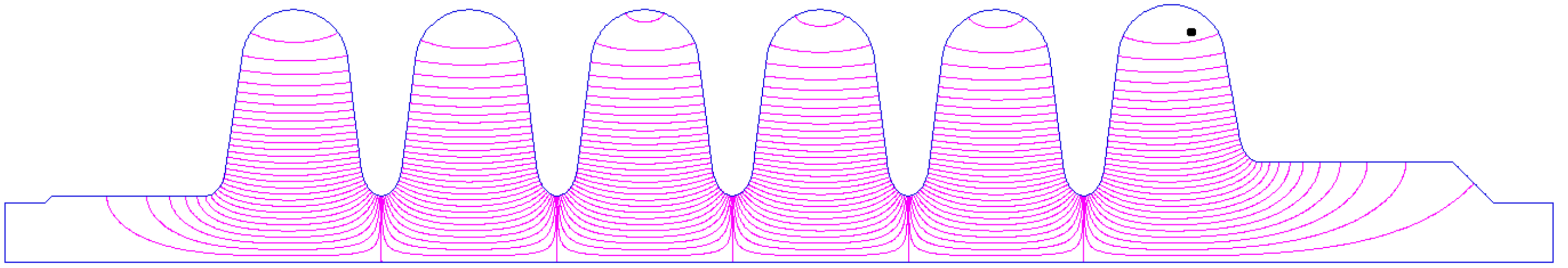
Mesh size,
Boundary condition,
Driving point

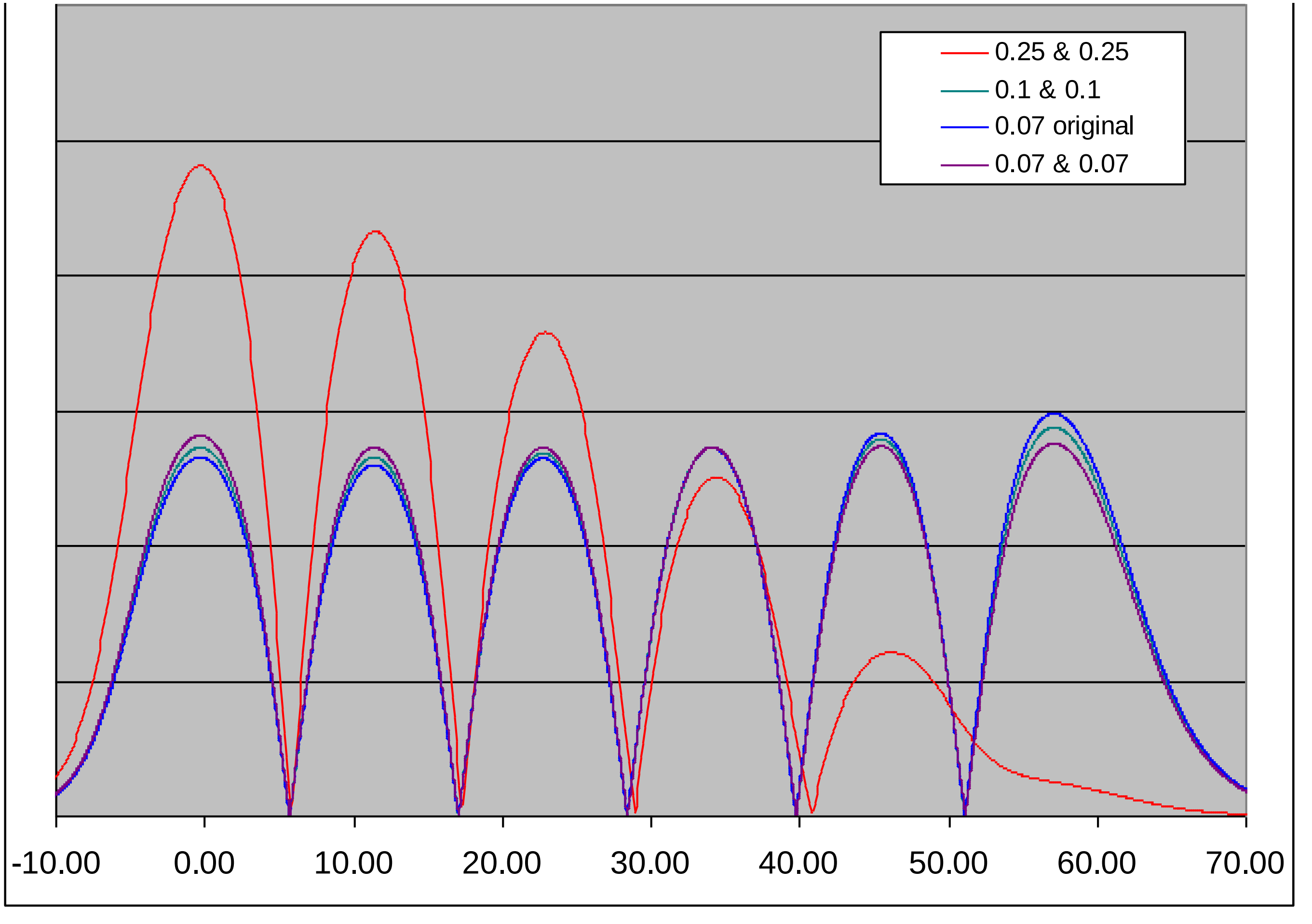
could give rise to large errors.

Could be even worse in 3D simulations.

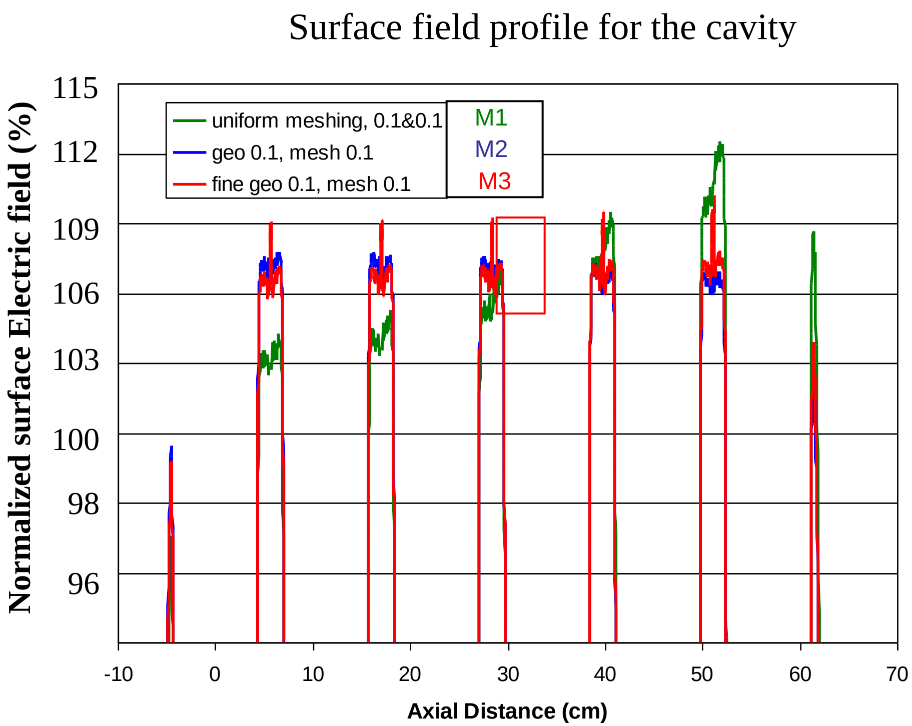
Some examples are followings;

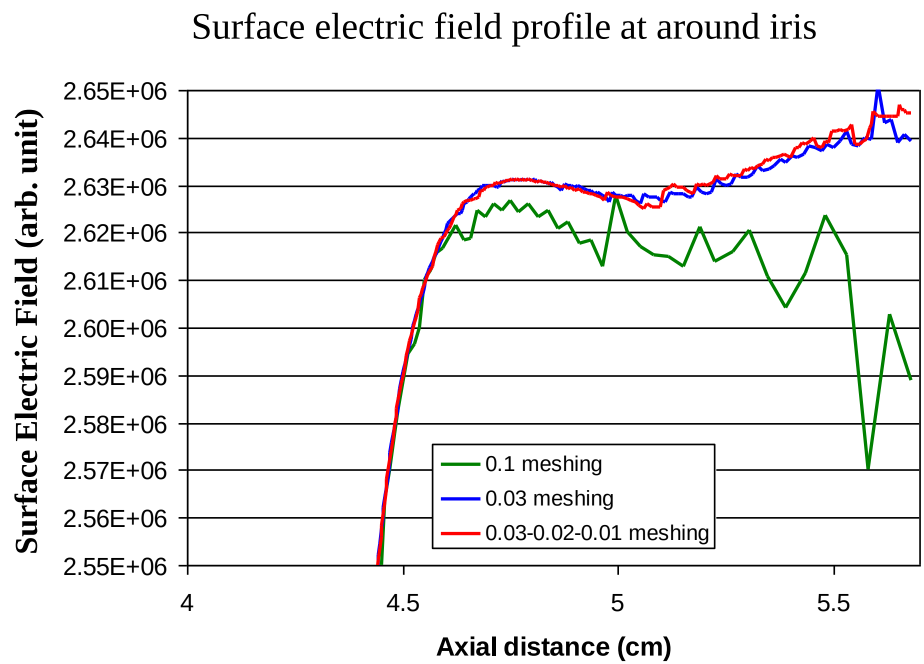
Mesh size: axial field

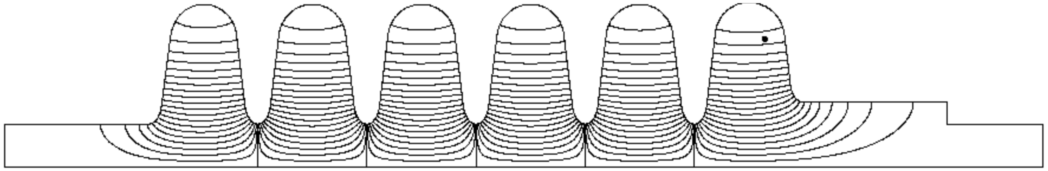




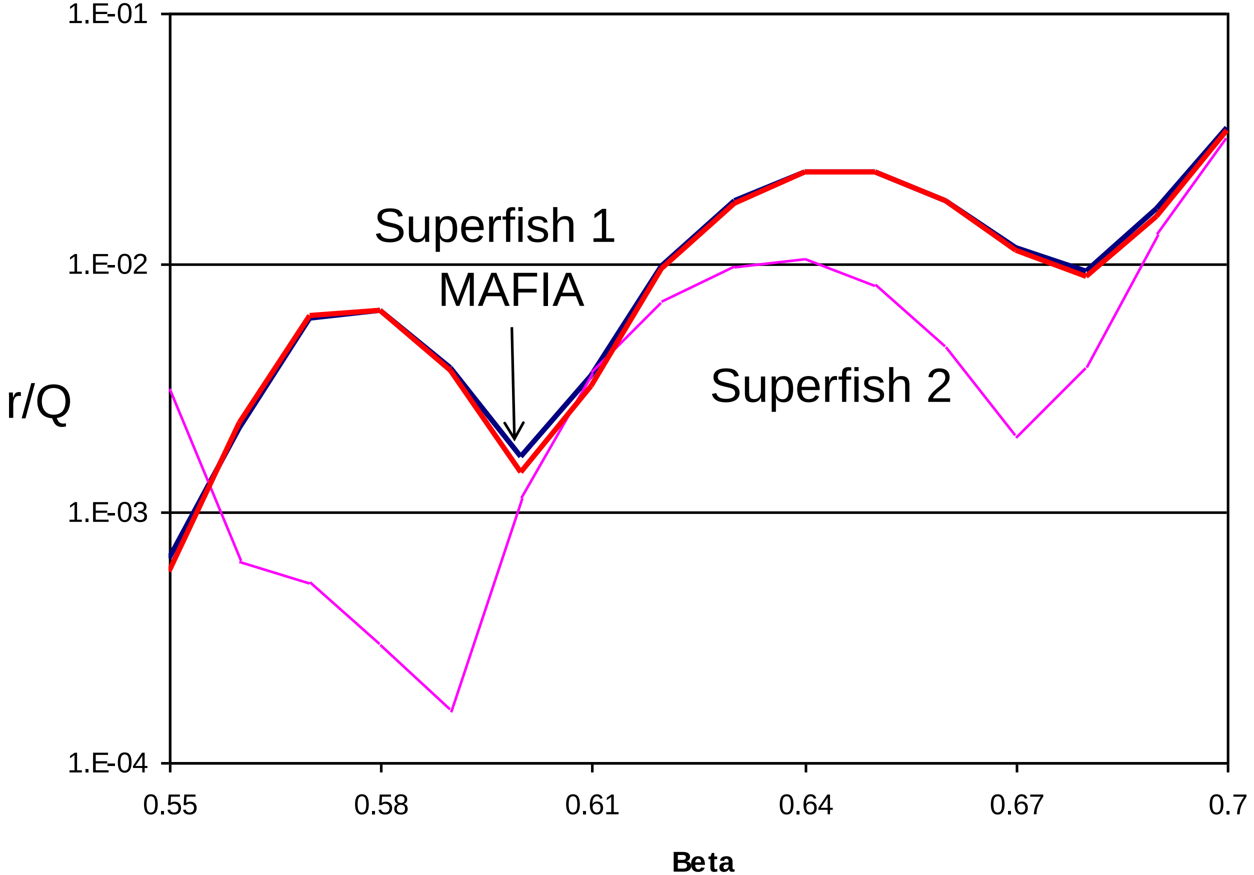
Mesh size: surface field







Driving point settings in superfish



Boundary setting and boundary condition→ misleading r/Q, f