Longitudinal space charge and the microbunching instability.

MV

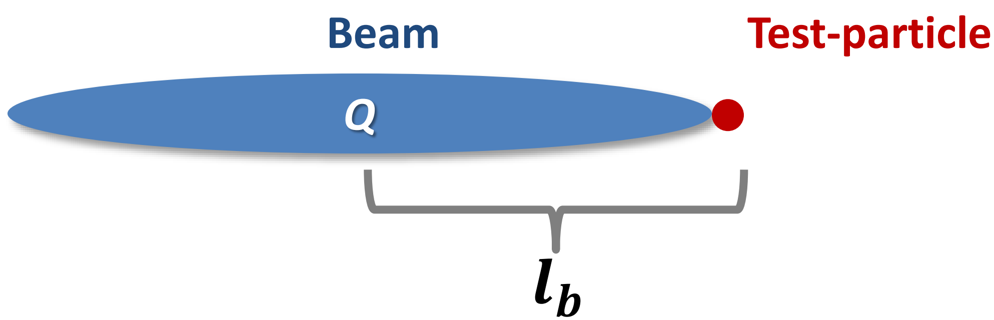
last revised 19-June-2015

Outline

1. **Longitudinal Space-Charge (LSC)**
    1. Short-scale effects.
    2. Long-scale effects
2. **The microbunching instability**
    1. The physical picture
    2. Simplified linear theory for the instability gain
    3. The laser heater as a remedy

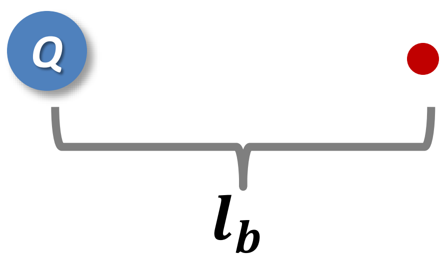
On-the-spot exercise: Estimate effect of longitudinal space-charge on ultrarelativistic beam

- Consider a beam of length $2l_b$, with charge $Q = -eN$ and a test electron $q = -e$ close to the beam head. The beam is in relativistic motion with respect to the lab.



The physical system

- Model the beam as a point charge.



The simplified model

- **Exercise**: Write the expression for the Coulomb E'_z field on the test particle in the beam co-moving frame. Lorentz-tranform field to lab frame. Estimate the work done by the space-charge force on the test particle over a distance $\boldsymbol{L = 1m}$. Assume $\boldsymbol{Q = 1nC}$, $\boldsymbol{E_b = 500\ \mathrm{MeV}}$ beam energy, and $\boldsymbol{l_b = 1\mathrm{mm}}$.

On-the-spot exercise: Answer.

Space charge vs. rf wakefields

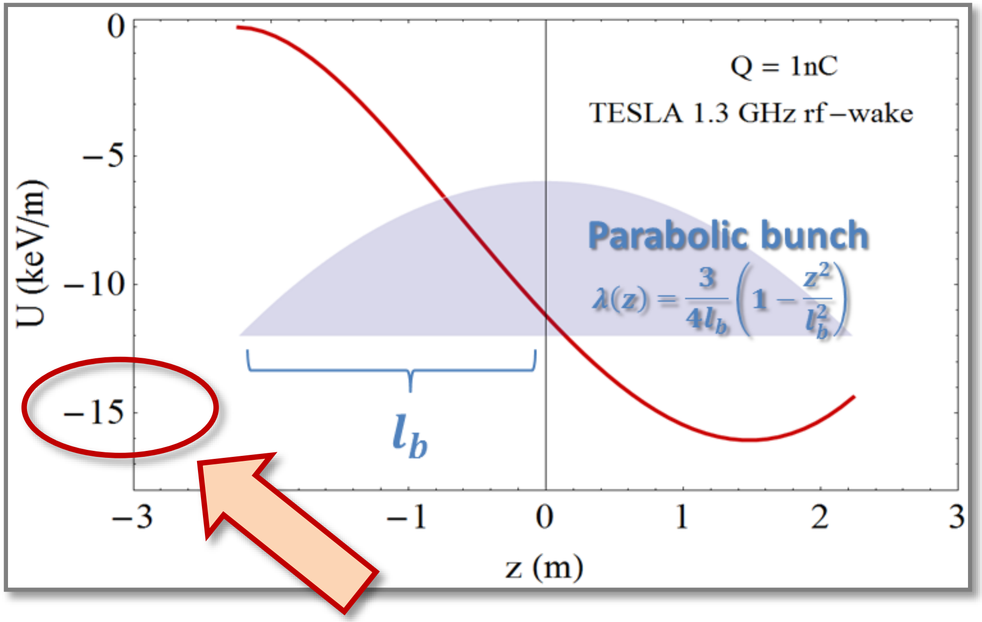
Result from exercise shows:

$\Delta U_{sp.ch.} \simeq 9\ eV/m$ @ $E = 500MeV$

@ $E = 100MeV$?

$\Delta U_{sp.ch.} \simeq 9 \times 25 = 0.23keV/m$

Still much smaller than ~10's keV/m associated with typical rf wakefields



- Only at 10s of MeV energy or lower (i.e. in the injector) space charge effects over **bunch-length scale** are significant

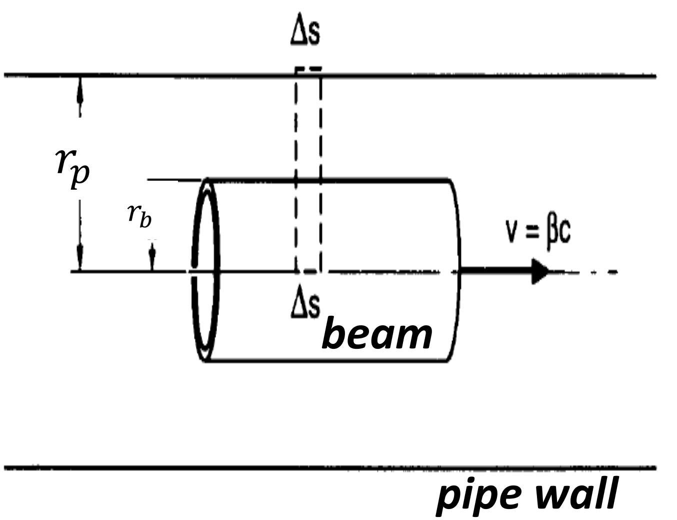
- **Q**: Can we then forget about space charge altogether in the Linac($\gtrsim 100\ MeV$)?
- **A**: Not quite…

$$U = \frac{Z_0 c}{4\pi l_b^2} \frac{e|Q|}{\gamma^2} L$$

Space charge can become relatively large (and dominant) either for very short bunches or on short **length scales**

A more refined model for longitudinal space-charge LSC (in the presence of metallic boundaries)

- Discussed in A. Chao's "Instabilities" book
- **Assumptions:**
  - Ultrarelativistic approximation: (the fields from a point charge are a 'pancake' with a small opening angle $\frac{1}{\gamma}$)
  - Beam with cylindrical charge density with radius r_b
  - Infinitely conducting cylindrical pipe with radius r_p
  - Bunch density is smooth and length in co-moving frame is long compared to radius of beam pipe $\gamma L_b \gg r_b$



$$E_z(r,z) \simeq -\frac{2qN}{4\,\pi\varepsilon_0\gamma^2}\frac{d\lambda(z)}{dz}\left(\log\frac{r_p}{r_b}+\frac{r_b^2-r^2}{2r_b^2}\right)$$

Space-charge suppression at high energy

Field is proportional to derivative of bunch profile (can be large if density varies significantly over short length $\ll L_b$)

Note: in this formula's following A. Chao's convention **bunch head is 'to the right' at $z > 0$**

Analysis of LSC effects on micro-scale is most conveniently done in frequency domain (Impedance)

- Suppose we have a high frequency perturbation with wavenumber $\boldsymbol{k = 2\pi/\lambda}$ on a beam with local unperturbed current $I_0 > 0$
  - I_0 is a slow-varying function of z, over a distance $\sim\lambda$ can be taken as constant

$$I(z) = I_0[1 + \ A\, cos(\mathrm{kz})]$$

- Density wave induces energy modulation $\Delta\gamma = \Delta E/mc^2$ over a distance L_s (rigid bunch; ultra-relativistic approx.)

Impedance per unit length

$$\Delta\gamma(z) = -4\pi \frac{I_0}{I_A} L_s \frac{A}{2}\left[\frac{Z(k)}{Z_0} e^{ikz} + c.c\right]$$

Alfven current

$I_A = ec/r_c \simeq 17kA$

Vacuum impedance

$Z_0 = 120\pi$ *ohms*

- For LSC, the impedance turns out to be purely imaginary:

$$\Delta\gamma(z) = 4\pi \frac{I_0}{I_A} L_s A \frac{|Z(k)|}{Z_0} sin(kz)$$

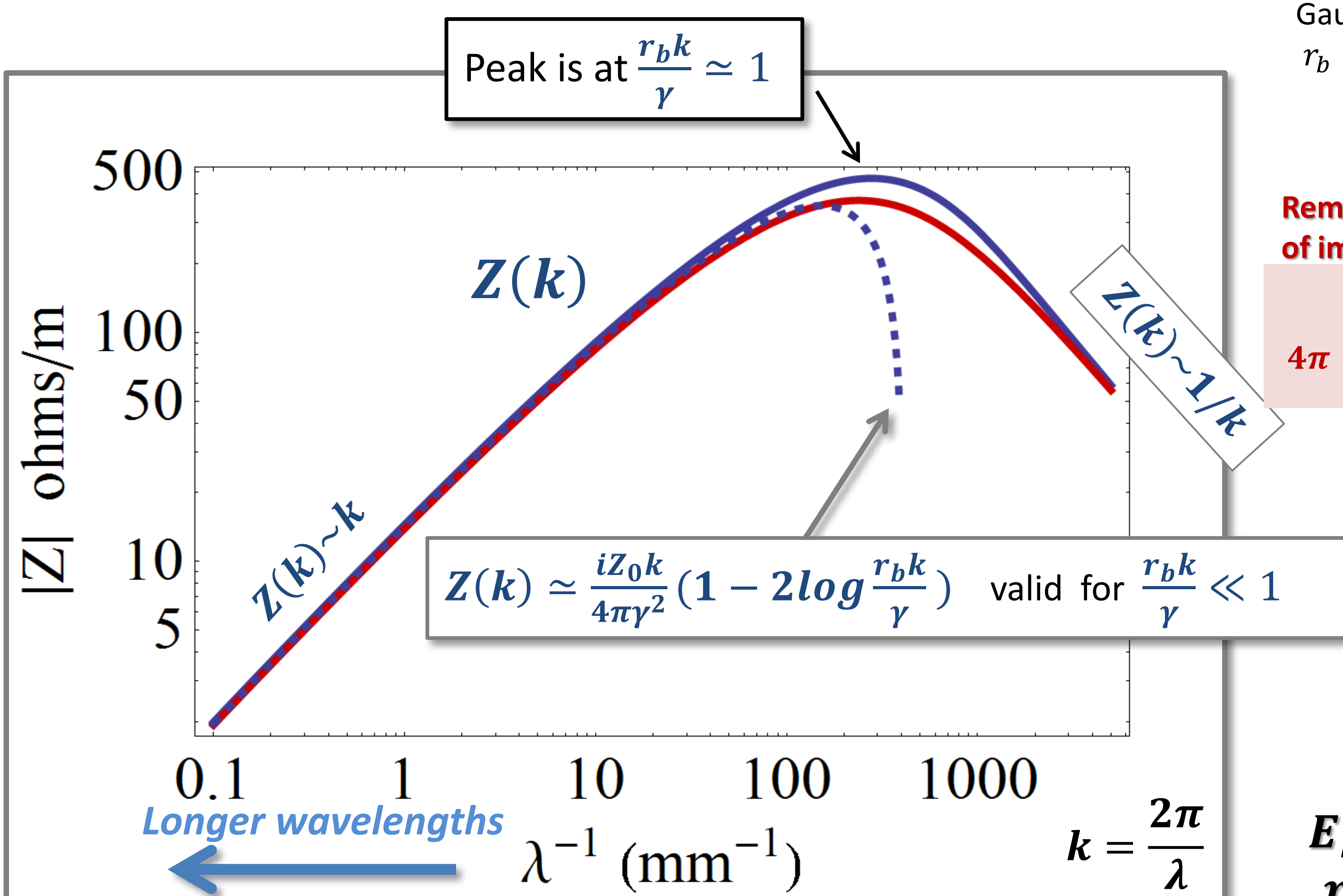
Behavior of LSC impedance (free space)

$$Z(k) = \frac{iZ_0}{\pi\gamma r_b}\frac{1-\xi_b K_1(\xi_b)}{\xi_b}$$

Bessel function

$\xi_b = kr_b/\gamma$

Effective radius for Gaussian bunches:
$r_b \simeq 1.7(\sigma_x + \sigma_y)/2$



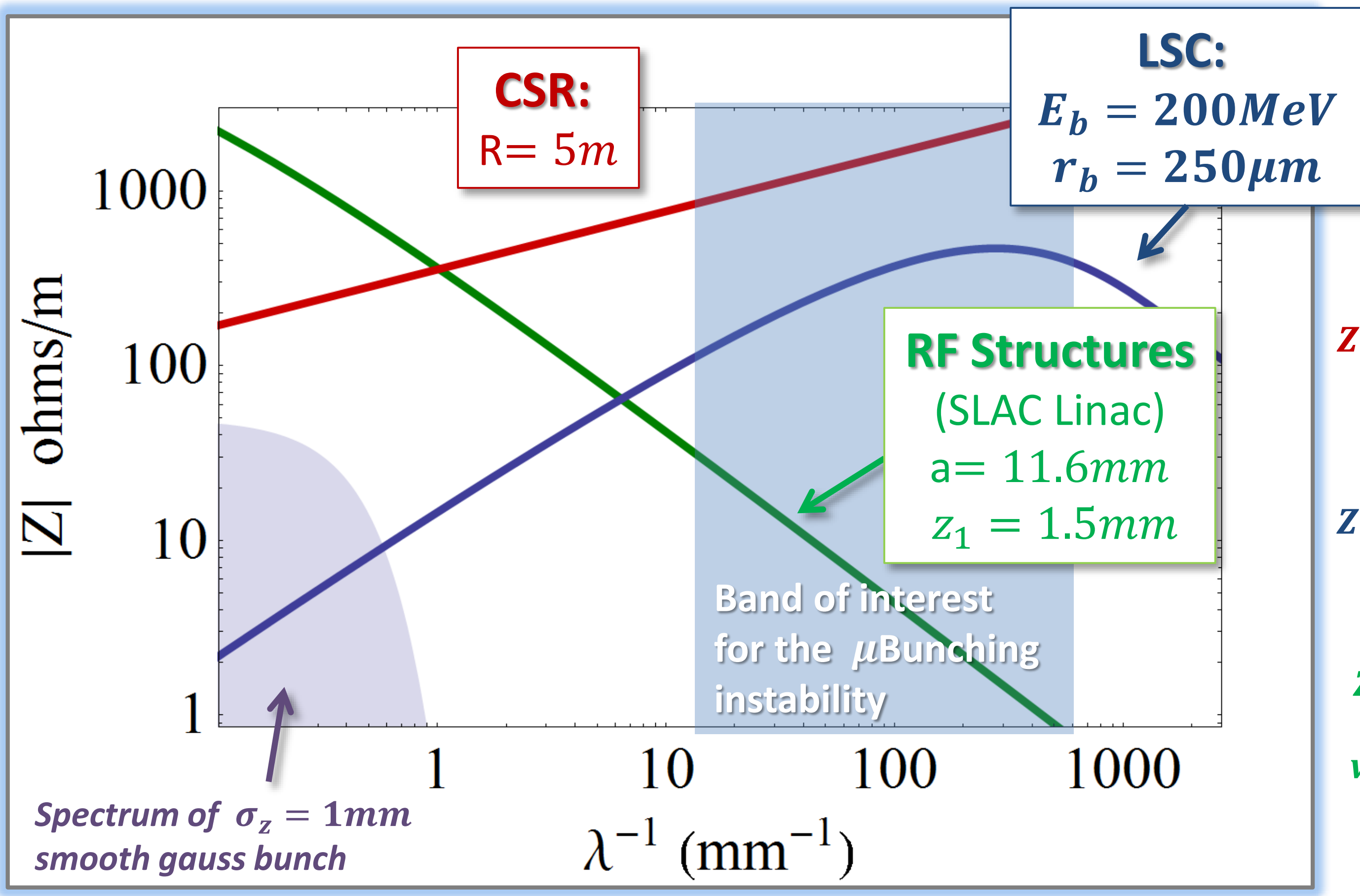
Remember meaning of impedance:

$$\Delta\gamma(z) = 4\pi\frac{I_0}{I_A}L_s A\frac{|Z(k)|}{Z_0}sin(kz)$$

$$E_b = 200MeV$$
$$r_b = 250\mu m$$

Comparison of main Linac Impedances (per m): LSC, CSR, & rf structures wakefields

- CSR impedance is the largest at high frequencies but overall CSR effect is smaller than LSC (dipoles are short compared to rest of machine)



$$Z_{CSR} = \frac{Z_0}{\pi R}(0.41 + i0.23)(kR)^{1/3}$$

$$Z_{LSC} = \frac{iZ_0}{\pi\gamma r_b}\frac{1 - \xi_b K_1(\xi_b)}{\xi_b}$$

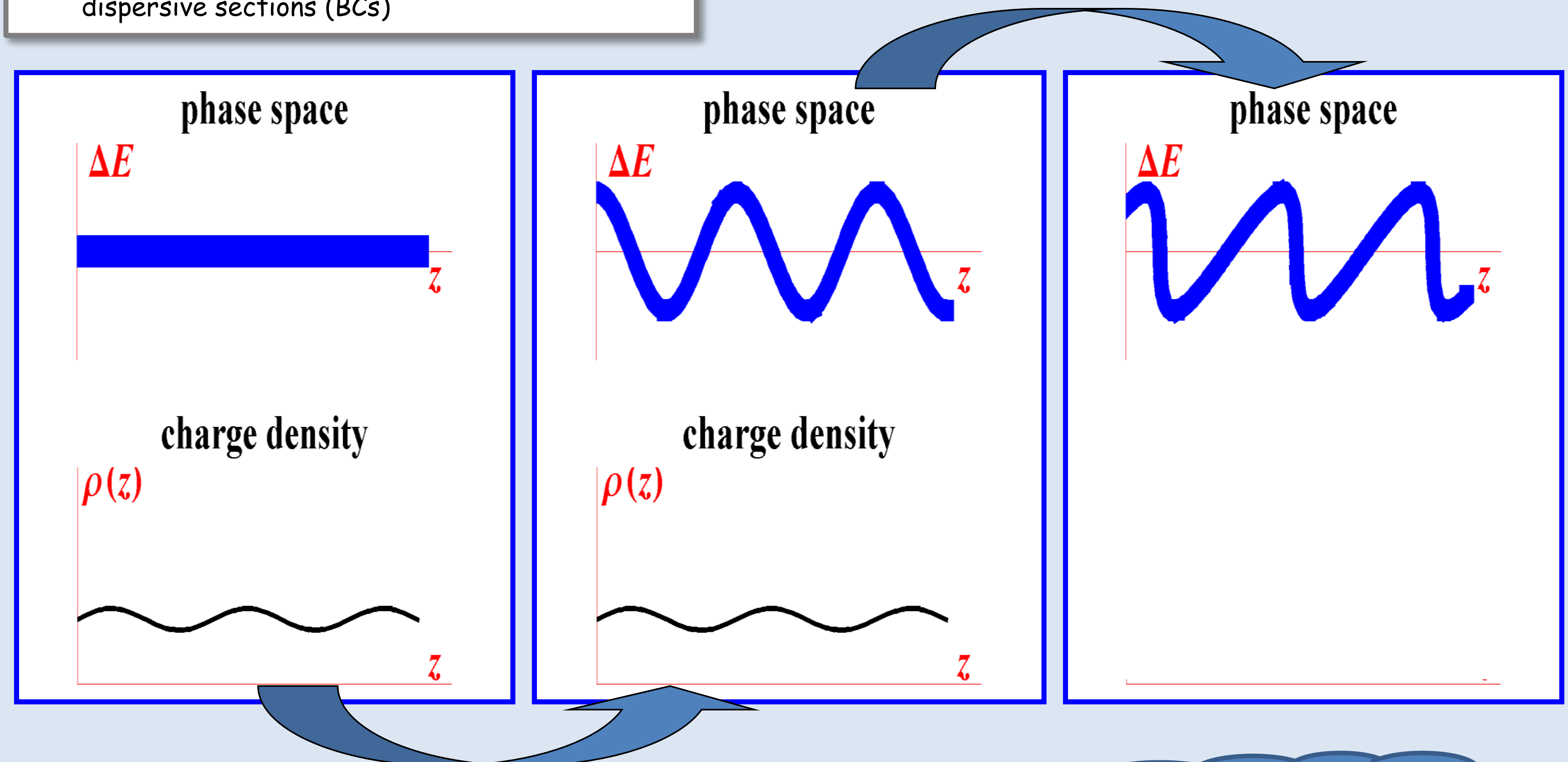
Z_{RF} associated with:

$$w_z = \frac{Z_0 c}{\pi a^2}\exp(-\sqrt{z/z_1})$$

The microbunching instability: The physical picture

- First observed in simulations (M. Borland); Importance pointed out by Saldin et al.. Early 2000s
- Seeded by irregularities in longitudinal beam densities
- Caused primarily by LSC + presence of dispersive sections (BCs)

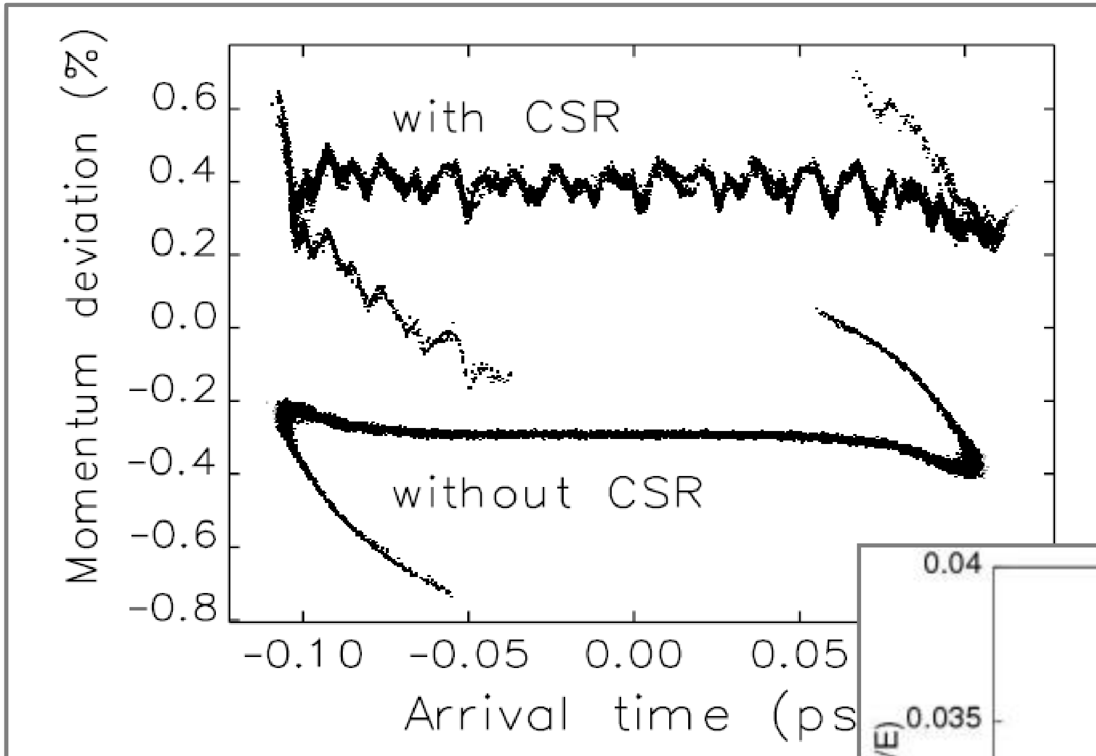
Dispersion turns energy modulation into larger charge-density ripples



Collective effects turn ripples of charge-density into energy modulation

Reminiscent of FEL process

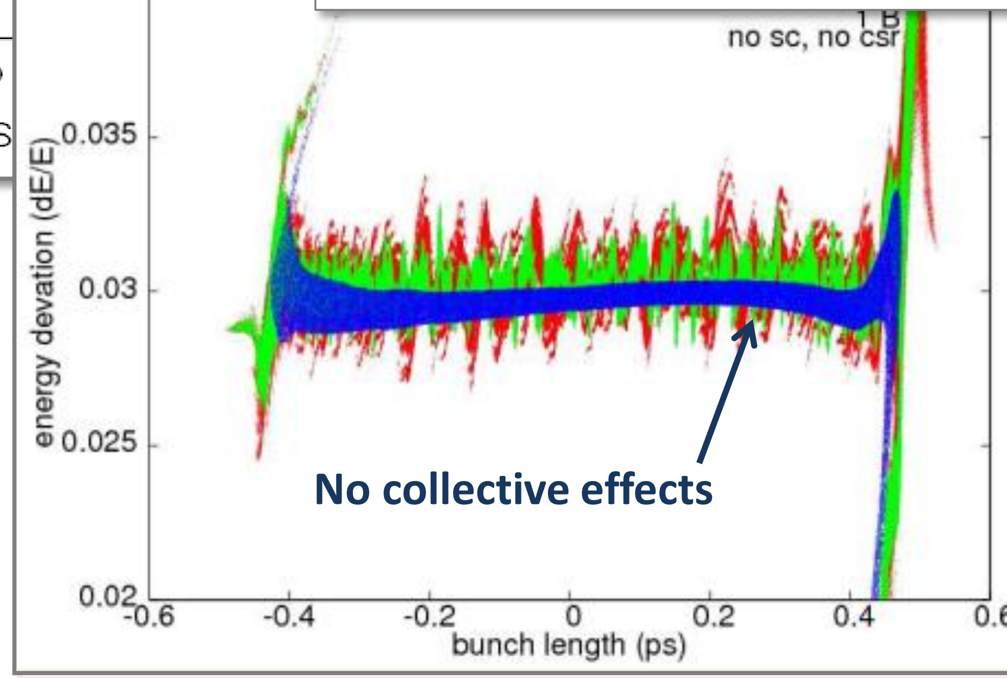
The instability as observed in simulations



LCLS longitudinal-phase space in first start-to-end simulations for LCLS *(M. Borland, 2001)*

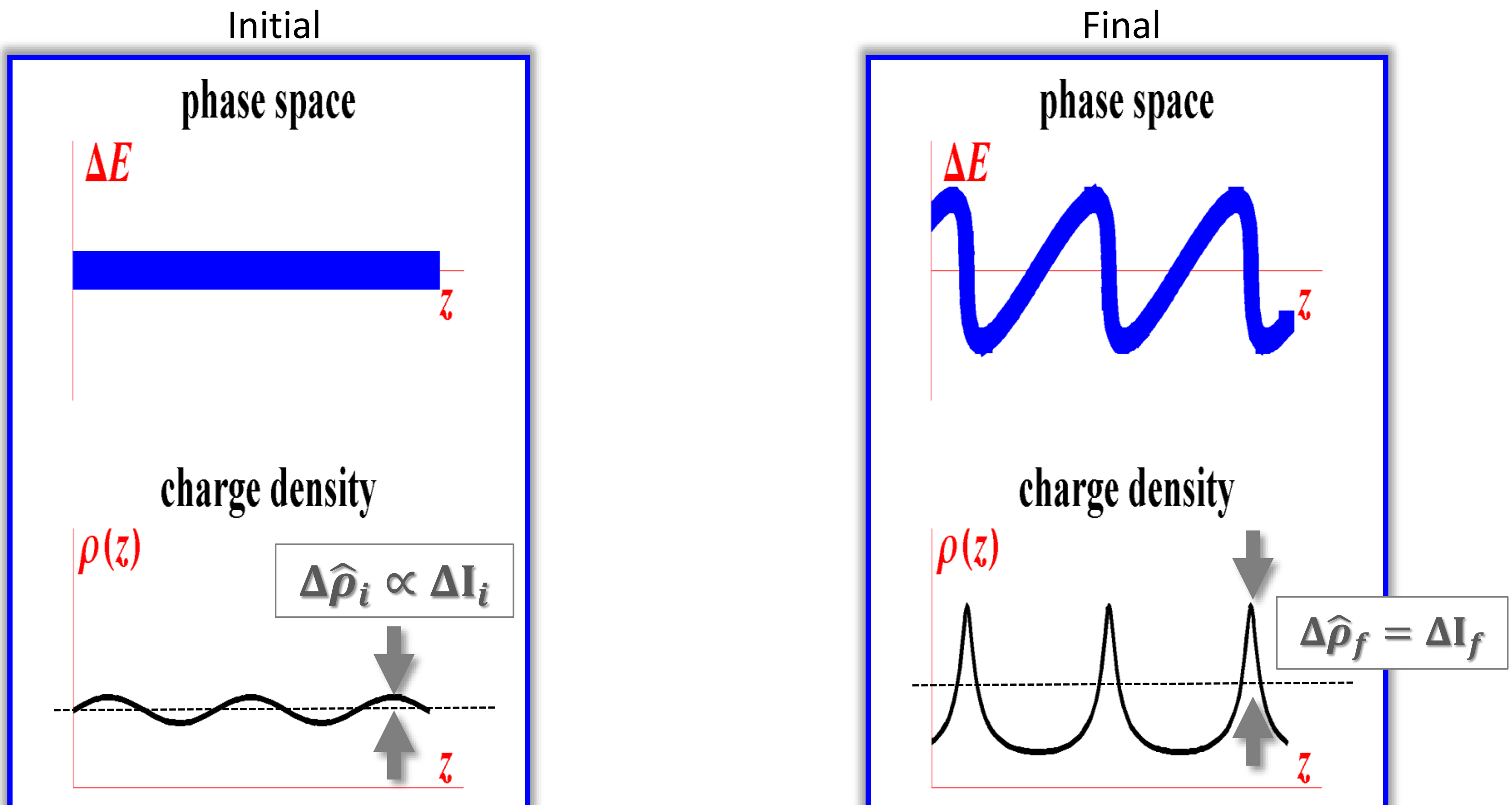
Early physics model included CSR, not LSC (which is actually more relevant)

Linac simulations including LSC
(J. Qiang, IMPACT)



Main adverse effect of micro-bunching instability is growth in energy spread *(limits SASE performance; degrades HG in seeding methods and reduces longitudinal coherence of radiation)*

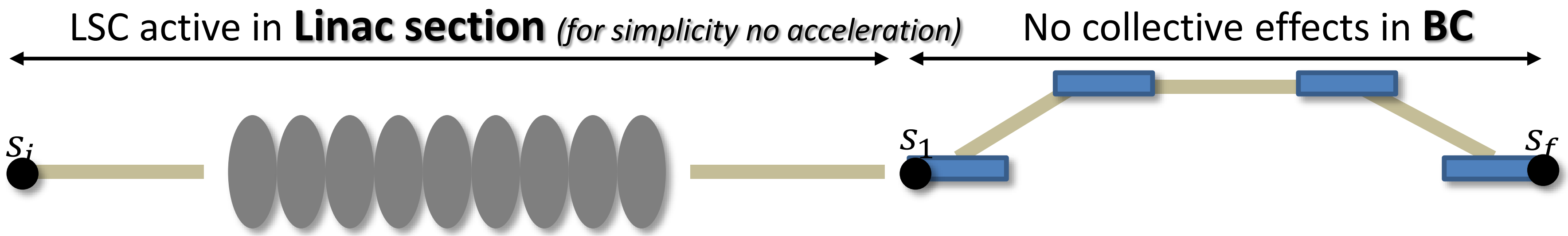
Characterize the instability in terms of gain

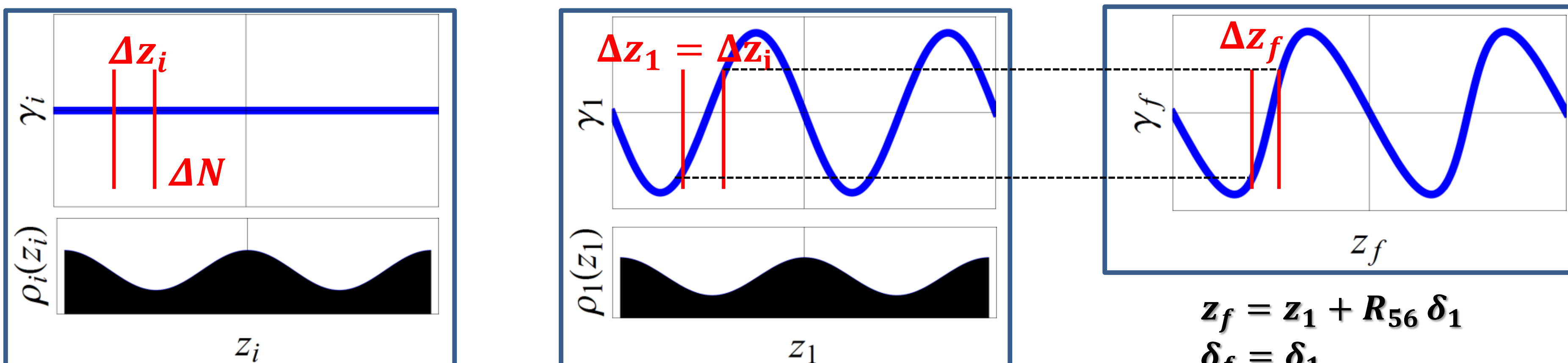


$$G = \frac{relative\ amplitude\ of\ final\ density\ perturbation}{relative\ amplitude\ of\ initial\ density\ perturbation} = \frac{\Delta\hat{\rho}_f/\rho_f}{\Delta\hat{\rho}_i/\rho_i}$$

Analytical model for linear gain through chicane (1)

(no compression, linear and cold-beam approx., ultrarelativistic approx.)





$$I_i(z) = I_0[1 + \ A\cos(kz_i)]$$

uniform beam with
small cos perturbation

There are ΔN particles in interval Δz_i

$$\mathbf{z_1 = z_i}$$
$$\boldsymbol{\gamma_1 = \gamma_i + \Delta\hat{\gamma}\sin kz_i}$$

$$\Delta\hat{\gamma} = 4\pi \frac{AI_0}{I_A} L_s \frac{|Z(k)|}{Z_0}$$

$$\boldsymbol{\delta_1 \equiv \frac{\gamma_1 - \gamma_i}{\gamma_{BC}} = \frac{\Delta\hat{\gamma}}{\gamma_{BC}} \sin kz_i}$$

Note: the same ΔN particles are still in same interval $\Delta z_1 = \Delta z_i$

$$\boldsymbol{z_f = z_1 + R_{56}\,\delta_1}$$
$$\boldsymbol{\delta_f = \delta_1}$$

$$\boldsymbol{z_f = z_i + R_{56}\frac{\Delta\hat{\gamma}}{\gamma_{BC}}\sin kz_i}$$

The same ΔN particles are now in a shorter interval $\Delta z_f < \Delta z_i$. Differentiate to find new density:

$$\boldsymbol{1 = \frac{\Delta z_i}{\Delta z_f} + R_{56}\frac{\Delta\hat{\gamma}}{\gamma_{BC}}\frac{\Delta z_i}{\Delta z_f} k\cos kz_i}$$

Analytical model for linear gain through chicane (2)

$$\rho_f = \frac{dN}{dz_f} = \frac{dN}{dz_i}\frac{dz_i}{dz_f} \simeq \frac{\rho_0}{1 + kR_{56}\frac{\Delta\hat{\gamma}}{\gamma_{BC}}\sin kz_i} \simeq \rho_0\left[1 - kR_{56}\frac{\Delta\hat{\gamma}}{\gamma_0}\sin kz_f\right]$$

Linear expansion in $\Delta\hat{\gamma}$

Use $\frac{dN}{dz_i} \simeq \rho_0$, and $\frac{dz_i}{dz_f}$ from last slide

Gain is ratio of initial and final amplitudes of density modulation

$$G = \left|\frac{\Delta\hat{I}_f}{\Delta\hat{I}_i}\right| = \left|\frac{\Delta\hat{\rho}_f}{\Delta\hat{\rho}_i}\right| = \frac{\frac{k|R_{56}|\Delta\hat{\gamma}}{\gamma_{BC}}}{A} = 4\pi\frac{I_0}{I_A}L_s\frac{|Z(k)|}{Z_0}k\,|R_{56}|$$

Generalizations

- In the presence of **compression C**

$$G \simeq 4\pi\frac{I_0}{I_A}L_s\frac{|Z(k)|}{Z_0\gamma_{BC}}|R_{56}|Ck$$

- In the presence of **finite slice energy spread σ_δ** (*e.g.* gaussian energy spread distribution model) gain is reduced

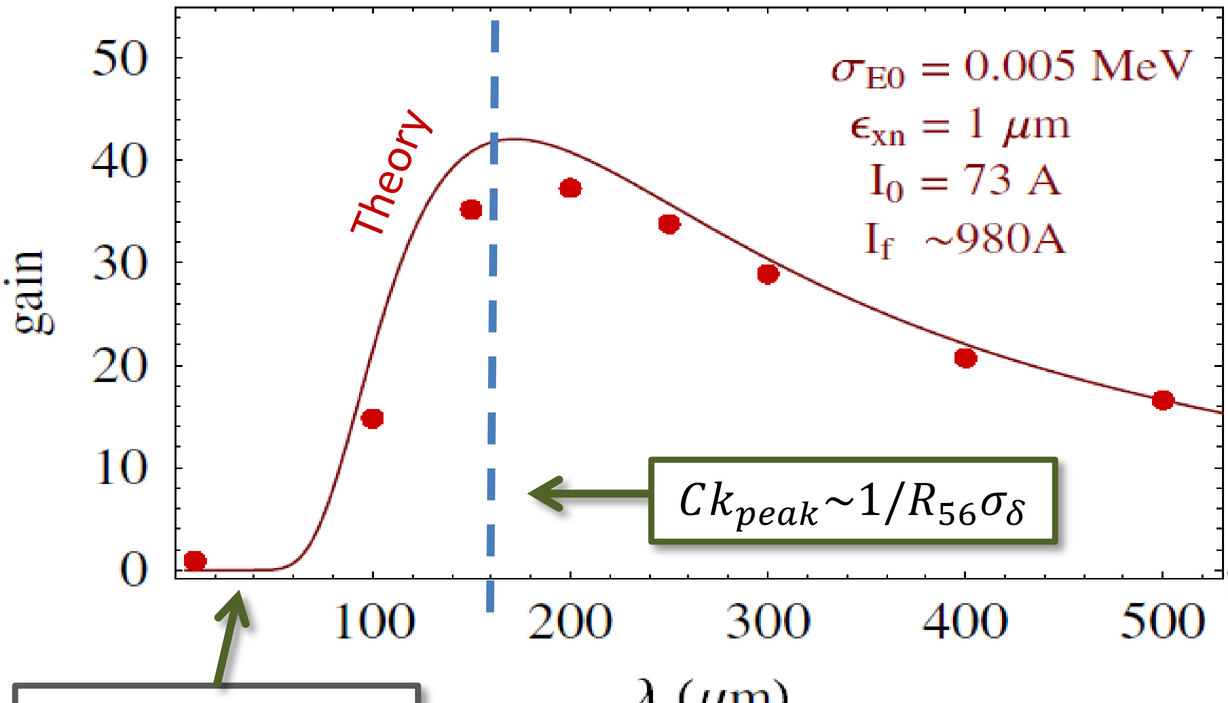
$$G \simeq 4\pi\frac{I_0}{I_A}L_s\frac{|Z(k)|}{Z_0\gamma_{BC}}|R_{56}|Ck\,e^{-(CkR_{56}\sigma_\delta)^2/2}$$

Note: here k is the wavenumber before compression

Gain function: theory vs. macroparticle simulations

$$G \simeq 4\pi \frac{I_0}{I_A} L_s \frac{|Z(k)|}{Z_0 \gamma_{BC}} (R_{56} C k) e^{-(C k R_{56} \sigma_\delta)^2/2}$$

Theory vs. macroparticle simulations

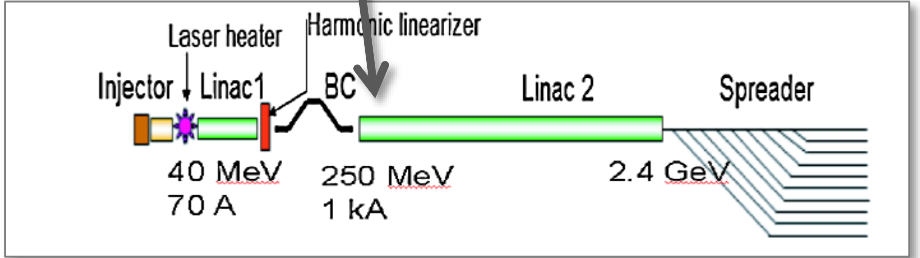


Wavelength of modulation
Before compression

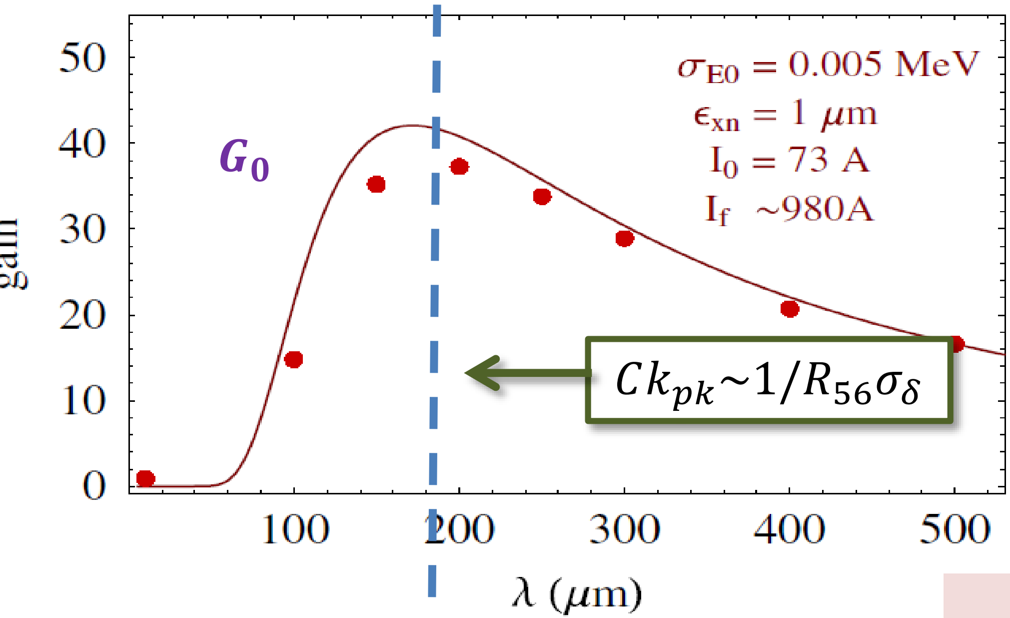
Gain is exp suppressed at short wavelengths

Gain has form of low(frequency)-pass filter

Gain curve is from end of Inj. through BC



Microbunching instability induces an energy modulation downstream of compressor



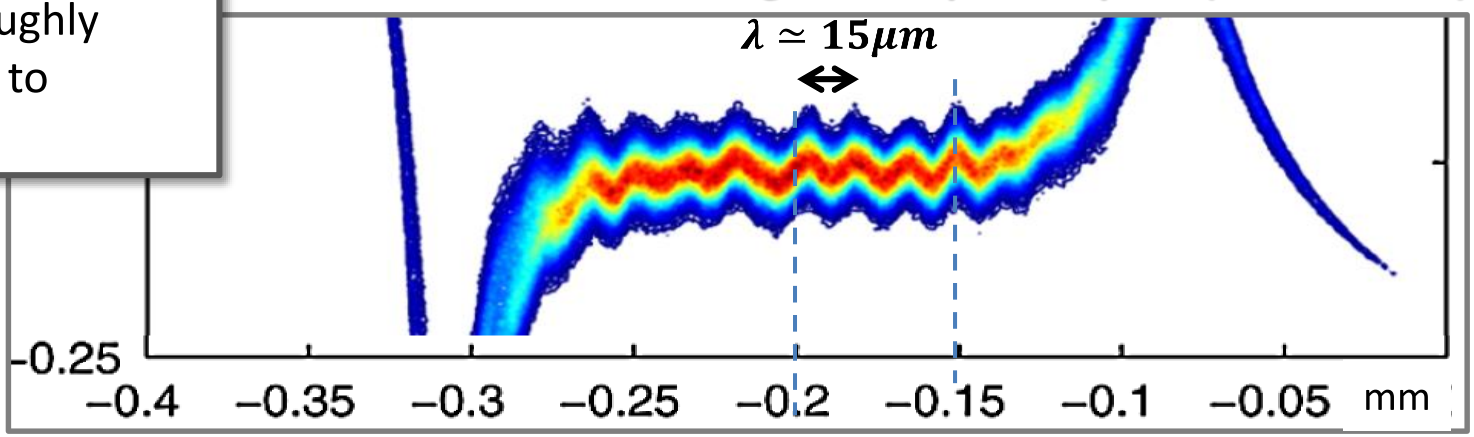
- At the very least, the electron bunch carries shot noise (uniform power spectrum)
- Additional noise may be present due e.g. to noisy laser in photo-gun injector.
- Because of the microbunching instability Spectral component of noise at $k \simeq k_{pk}$ will dominate after compression.
- These, in turn, will seed energy modulation in the linac section downstream of the compressor

A=relative density perturb.

$$\Delta\gamma(z) \simeq -4\pi \frac{I_0}{I_A} L_s A \frac{|Z(Ck_{pk})|}{Z_0} \cos(Ck_{pk}z)$$

Phase-space shows energy modulation with wavelength roughly corresponding to Gain peak

Longitudinal phase-space (exit of Linac)

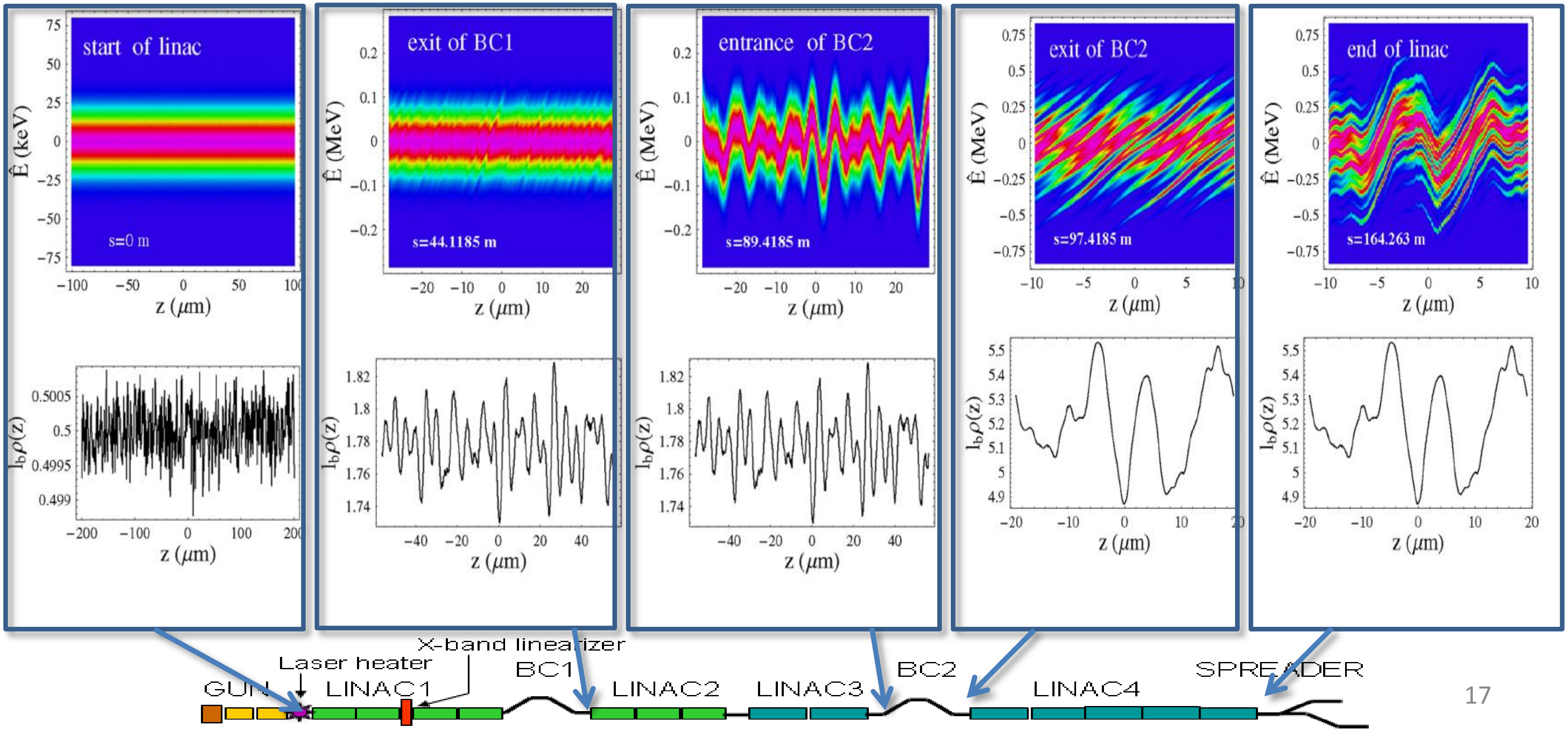


Numerical simulations by code IMPACT

Multiple-stage bunch compression enhances instability

- Effect compounded by repeated compression through bunch compressors. In first approx.:
- $$G_{tot} \simeq G_{BC1} \times G_{BC2} \times \cdots$$
- If instability is large effects beyond the linear approximation used here can become important.

Study of μB-instability for FERMI: *Longitudinal phase space, current profile at selected points*



Possible cure for the μB-I: "Heat" the beam or "fight fire with fire"

$$G \simeq 4\pi \frac{I_0}{I_A} L_s \frac{|Z(k)|}{Z_0 \gamma_{BC}} (R_{56} C k) e^{-(CkR_{56}\sigma_\delta)^2/2}$$

- **Finite uncorrelated (slice) energy spread σ_δ helps with reducing the instability gain** ("Landau damping").
- **Why?**
  - Through chicane, particles separated in energy by σ_δ move away from each other:

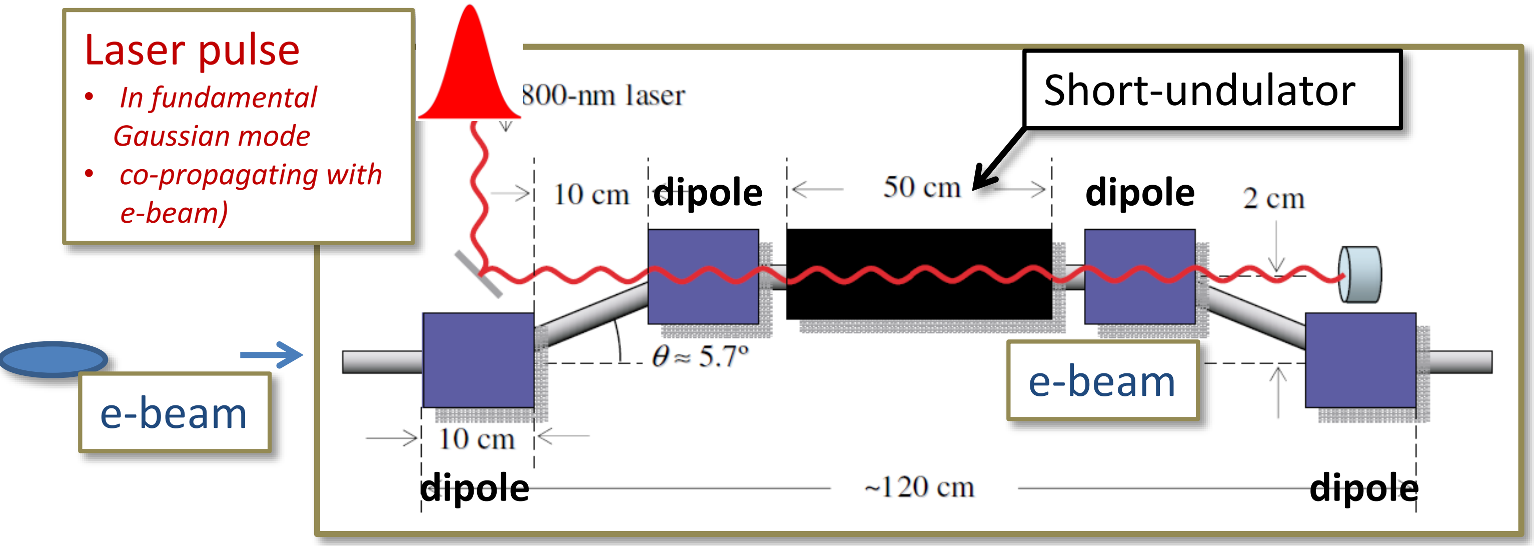
$$\Delta z = R_{56}\sigma_\delta$$

  - This washes away clumps of charge (bunching) on the scale λ if $\Delta z > \frac{\lambda}{2}$
  - Leads to condition $CkR_{56}\sigma_\delta \gtrsim 1$ (exponential suppression in above Eq.).

- **Generally, beam out of injector is longitudinally cold** (colder than needed for FEL).
  - We can afford to increase slice energy spread if this helps to reduce damage later on.
- **How can we "heat" the beam?**

An ingenious solution: the "Laser Heater"

- **Exploit the principle of the Inverse Free Electron laser**
  - *conventional-laser & e-beam interact in short undulator placed in the middle of small magnetic chicane*

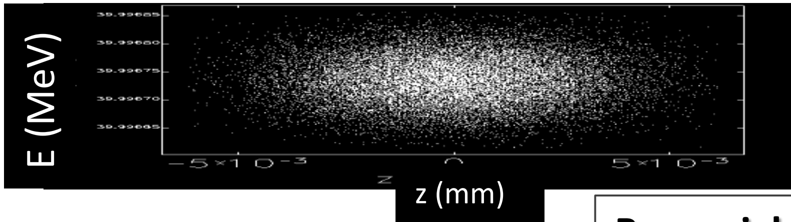


- **Energy exchange** is possible between **laser pulse** and **electrons** interacting in a wiggler/undulator when the laser wavelength meets our familiar **FEL resonance condition**:

$$\lambda(K, \lambda_u, \gamma) \equiv \frac{\lambda_u}{2\gamma^2}\left(1 + \frac{K^2}{2}\right) = \lambda_L$$

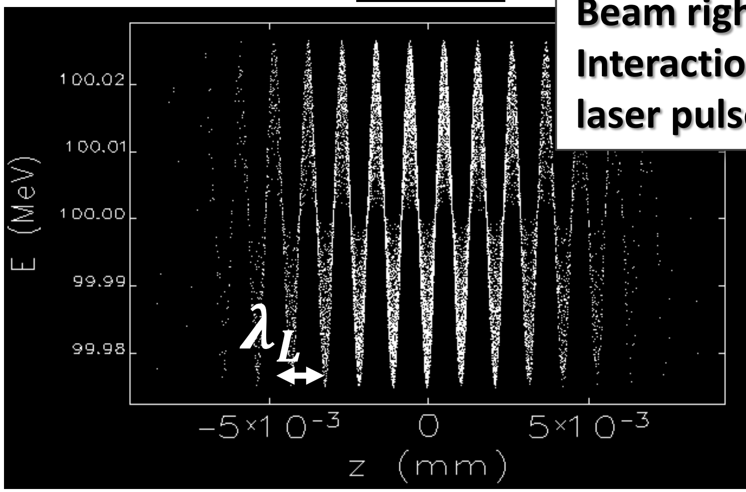
Recall: undulator parameter: $K = 0.934 \times B[T] \times \lambda_u[cm]$

The Laser Heater in action

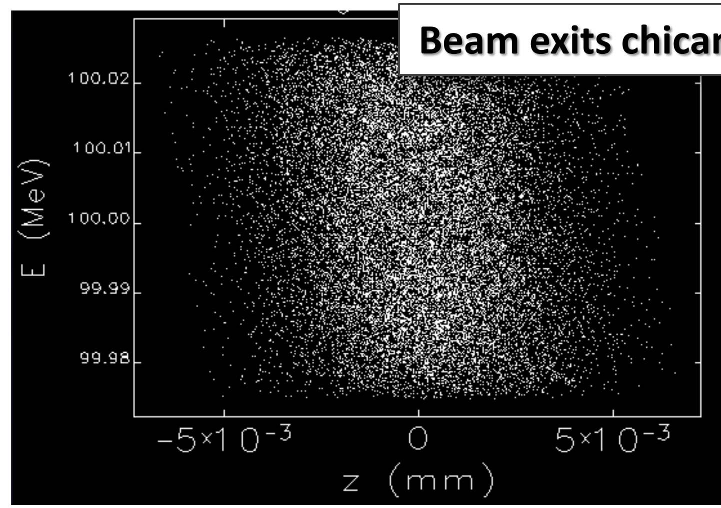


Beam injected into LH with very small slice energy spread.

Beam right after Interaction with laser pulse



Beam exits chicane



Desired e-beam rms energy spread

$$P_L = 2P_0 \left(\frac{\sigma_E}{m_e c^2}\right)^2 (\sigma_x^2 + \sigma_r^2) \left(\frac{\gamma}{K[JJ]N_u \lambda_u}\right)^2$$

Required laser pulse peak-power

e-beam rms size

Laser rms spot size

Eq. is valid for round e-beam with $\sigma_x = \sigma_y = \sigma_r$ (optimal)

$$P_0 = \frac{mc^3}{r_c} \simeq 8.7GW$$

$[JJ] = J_0(\xi) - J_1(\xi) \simeq 1 - \frac{K^2}{8} + \frac{3K^4}{64} + \cdots$ (for K $\leq$1)

with $\xi = K^2/(4 + 2K^2)$,

$$z' = z + R_{51}x + R_{52}x' + R_{56}\delta$$

Entries of transfer matrix from undulator to exit of chicane

$R_{51} = 0$,

$|R_{52}| = \eta_u$ =**dispersion in middle of chicane**

If angular spread is large the phase-space randomizes and energy spread becomes truly uncorrelated

$$|R_{52}|\sigma_{x'} \gg \lambda_L/2\pi$$

Generally, the $R_{56}\delta$ term is negligible

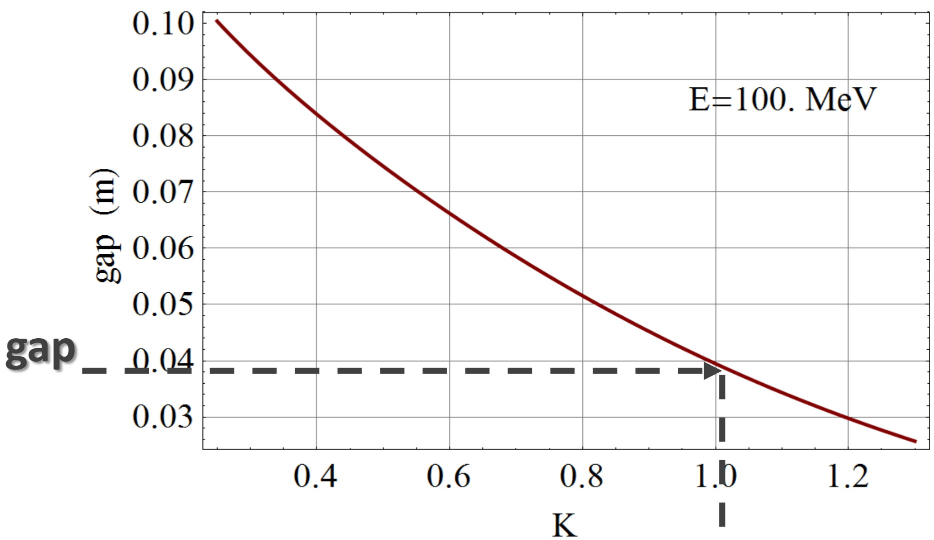
Designing a laser heater

- **Step 1: Choose no. of undulator periods N_u**
  - $N_u \sim 10$ is a reasonable choice (should not be too large to keep width $\sim 1/2N_u$ of u-resonance condition wide enough)
- **Step 2: Choose e-beam energy.**
  - Can't be too large or else the resonance condition will demand too-short laser wavelength. Typically LH is placed right after injector. Say $E_b = 100\ MeV$
- **Step 4: Choose laser wavelength λ_L**
  - Based on commercially available high-power lasers, *e.g.* $\lambda_L = 1064nm$
- **Step 5: Choose undulator period λ_u (see next slide)**

On choice of undulator period

At this point laser wavelength and beam energy have been set

A. Select desired undulator min. gap



$$\begin{cases} \lambda_L = \dfrac{\lambda_u}{2\gamma^2}\left(1 + \dfrac{K^2}{2}\right) \\ K = 0.934 \times b[T] e^{-a\left(\frac{g}{\lambda_u}\right)} \times \lambda_u[cm] \end{cases}$$

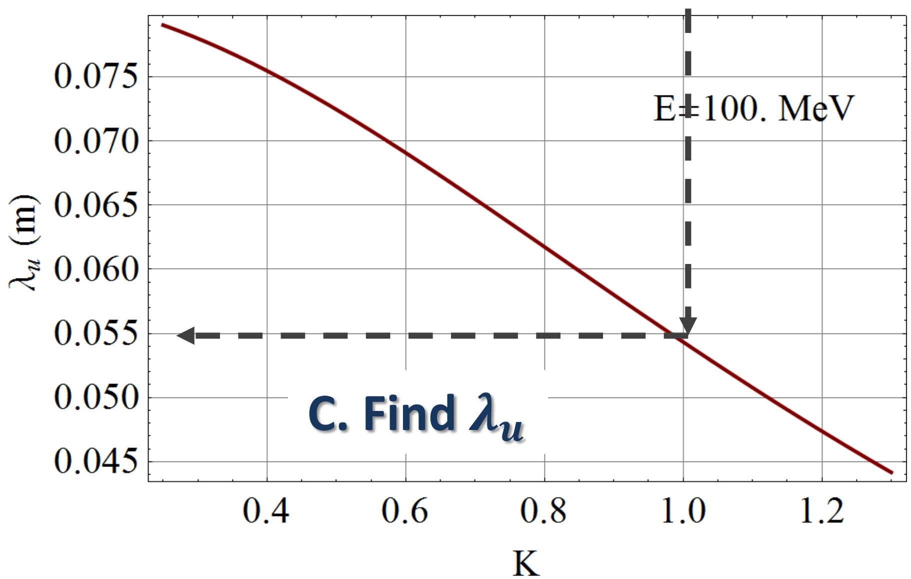
Solve above two equations (eliminate λ_u) to get $gap\ vs.\ K$

(for PM undulator, e.g. ***b=2.08 T*** *and* ***a=3.24****)*

B. Find corresponding *K*

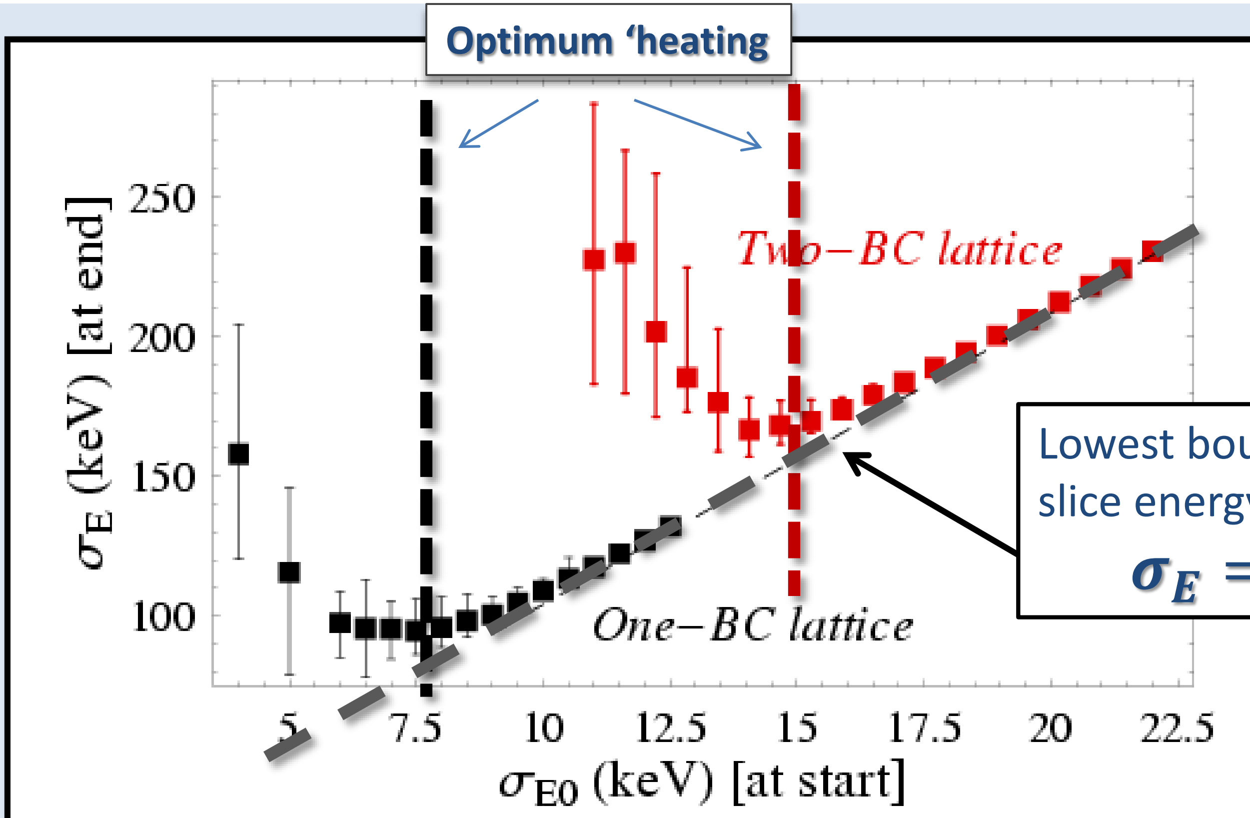
Plot λ_u vs. K

$$\lambda_L = \frac{\lambda_u}{2\gamma^2}\left(1 + \frac{K^2}{2}\right)$$



There is an optimum initial slice rms energy spread

Numerical study for FERMI



Lowest bound to final slice energy spread is

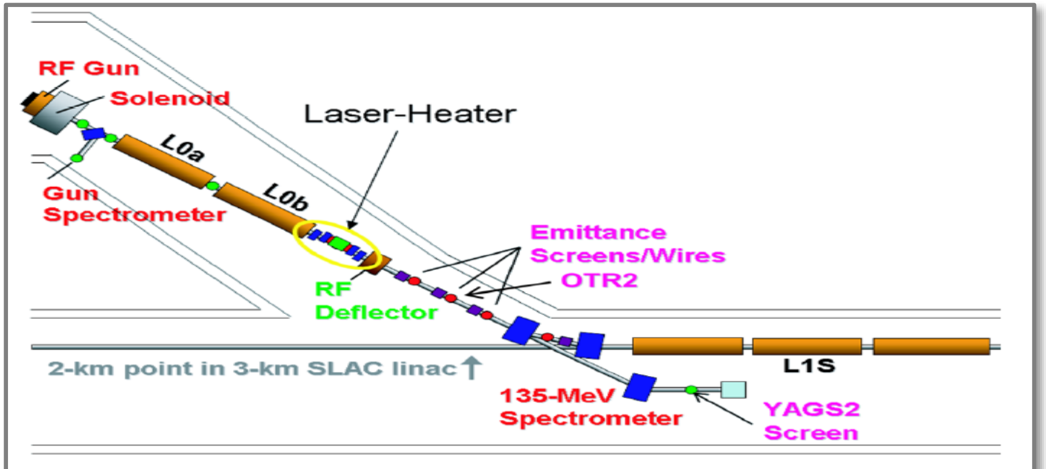
$$\sigma_E = C\sigma_{E0}$$

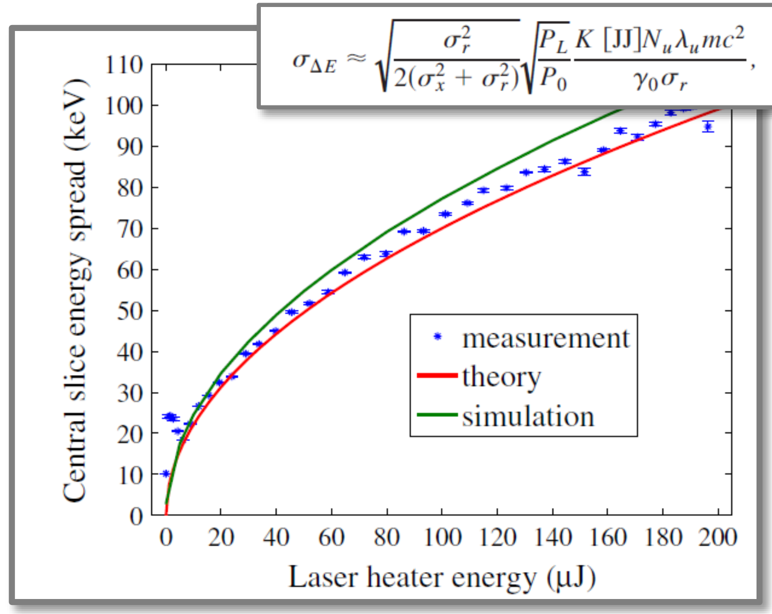
C= Compression factor

Stronger instability

Effectiveness of the laser heater: LCLS experiments

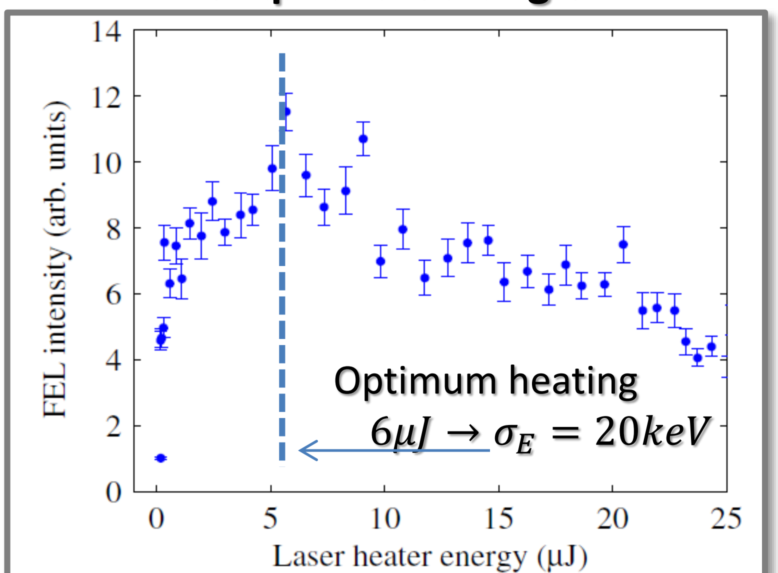
- First Laser Heater installed in LCLS and tested during commissioning



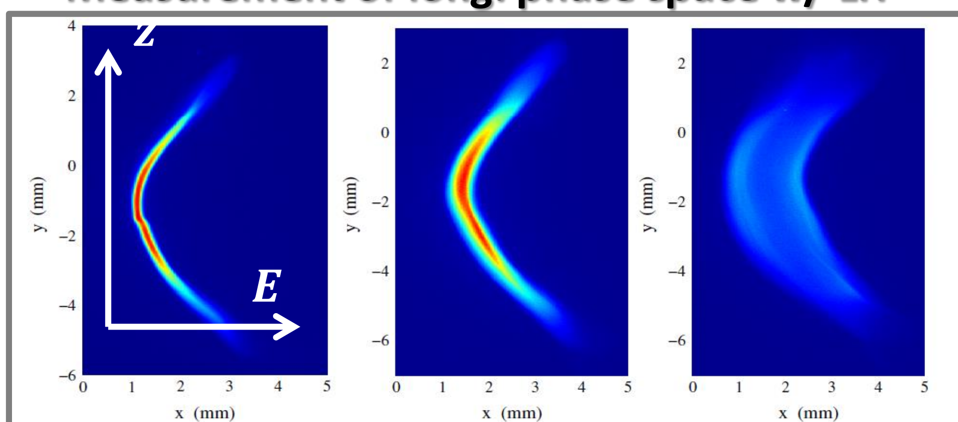


$$\sigma_{\Delta E} \approx \sqrt{\frac{\sigma_r^2}{2(\sigma_x^2+\sigma_r^2)}}\sqrt{\frac{P_L}{P_0}}\frac{K\,[\mathrm{JJ}]N_u\lambda_u mc^2}{\gamma_0\sigma_r},$$

FEL output vs. setting of LH



Measurement of long. phase space w/ LH

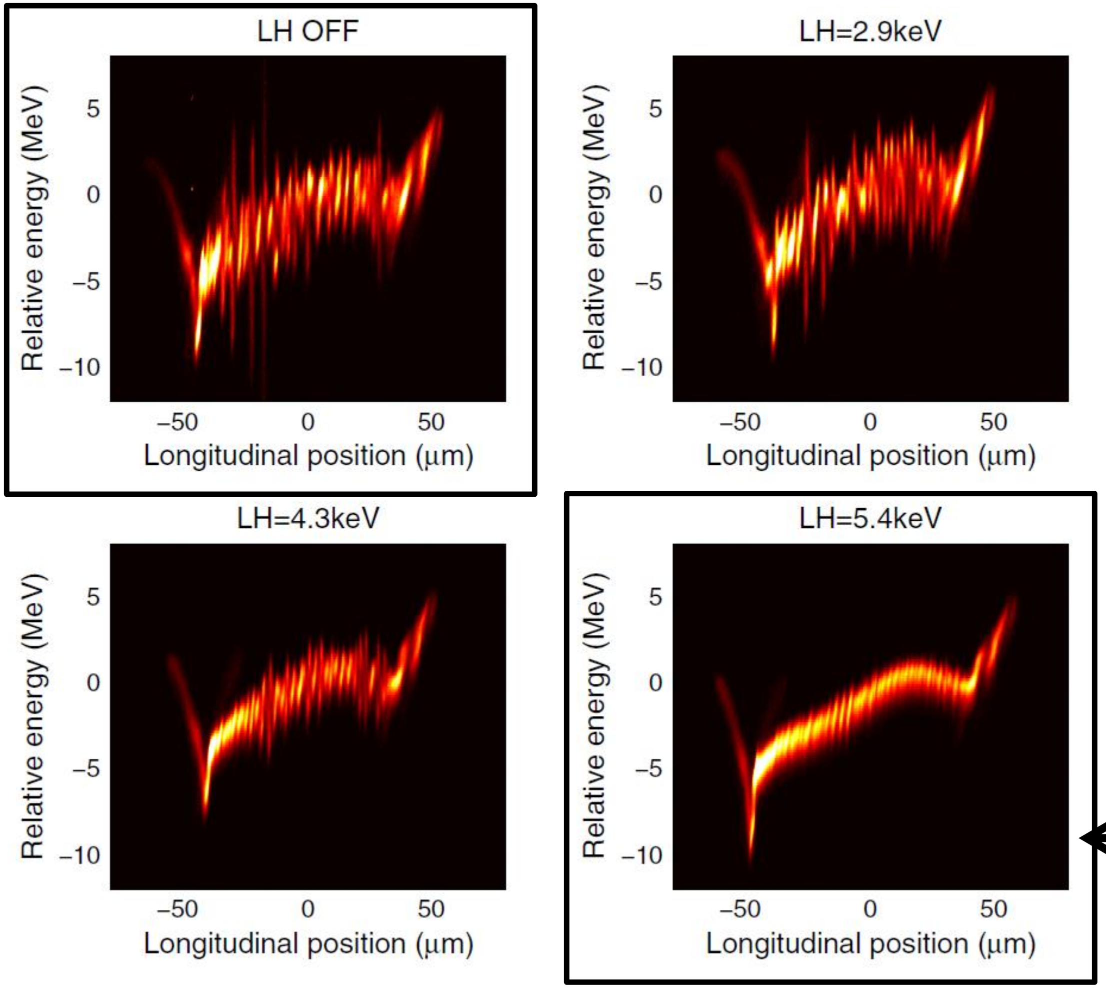


(a) no heating (b) nominal heating (c) maximum heating

Very recent measurements of microbunching instability at LCLS

- Pictures of longitudinal phase space are from screen measurements downstream of X-band transverse RF deflector (positioned after the FEL)
- First direct measurement of effect of LH on instability

LH is turned off



LH is suppressing the instability

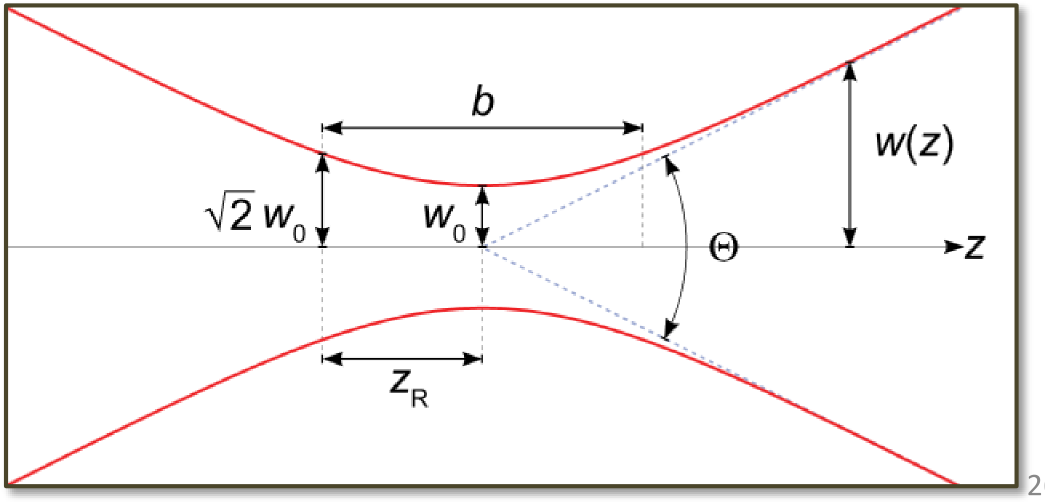
D. Ratner et al., PRST-AB ***18*** *030704 (2015)*

The fine print

- **Make sure transverse beam emittance does not suffer:**
  - Dispersion should not be too large (usually not an issue)

$$\frac{\Delta\varepsilon_{nx}}{\varepsilon_{nx}} \simeq \frac{1}{2}\left(\frac{\eta_u \sigma_E}{\sigma_x E}\right)^2 \ll 1$$

- **Formula for laser power is valid when the Rayleigh range $Z_R = \pi w_0^2/\lambda_L$, long compared to undulator length $L_u = N_u \lambda_u$** (i.e. laser cross section doesn't vary significantly)
  - $\boldsymbol{w_0 = 2\sigma_r}$ with σ_r being the laser *intensity* rms transverse size



Schematic of laser-pulse envelope with Rayleigh range

Summary highlights

- Model of LSC impendance

$I(z) = I_0[1 + \; A\,cos(\mathrm{kz})]$

$$Z(k) \simeq \frac{iZ_0 k}{4\pi\gamma^2}\left(1 - 2log\frac{r_b k}{\gamma}\right) \quad \text{valid for } \frac{r_b k}{\gamma} \ll 1$$

- Energy modulation seeded current modulation

$$\Delta\gamma(z) = 4\pi\,\frac{I_0}{I_A}\,L_s A\,\frac{|Z(k)|}{Z_0}\,sin(kz)$$

- Bunching resulting from μB-I, seeded by shot-noise, through system with G_0 peak-gain.

$$b = \frac{\langle(\Delta I_{exit})^2\rangle^{1/2}}{I_{exit}} \simeq G_0\sqrt{\frac{2}{N_{\lambda min}}}$$

- Laser pulse peak power requirement for Laser Heater

$$P_L = 2P_0\left(\frac{\sigma_E}{mc^2}\right)^2(\sigma_x^2 + \sigma_r^2)\left(\frac{\gamma}{K[JJ]N_u\lambda_u}\right)^2$$

Elegant uses this input → $w_0 = 2\sigma_r$

Supplemental material

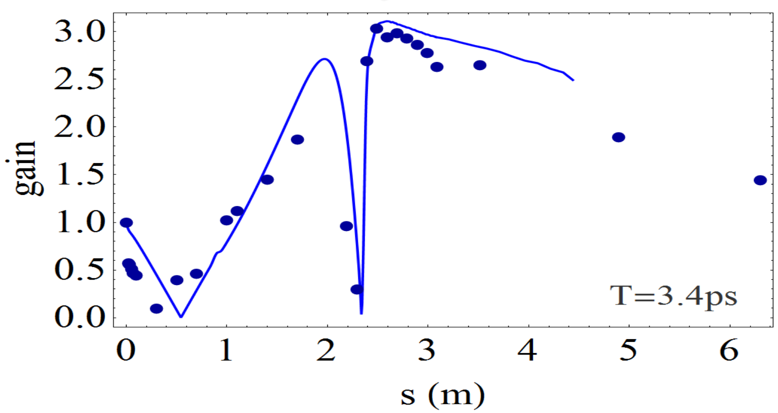
Final comments:

- **Simple model of linear theory discussed neglects collective effects (CSR, LSC) within chicane**
- **A more general theory of linear gain is available**
  - *Yielding instability gain as a solution of a certain integral equation*
- For proper numerical simulation no. of macroparticles should ideally equal no. of physical electrons to avoid overestimating shot noise

- **In addition to shot noise instability can be seeded by disturbances at the photocathode (e.g. temporal non-uniformity of photo-laser)**
  - Analytical modeling is trickier. High-resolution macroparticle-modeling is the way to go, but these too require good care.

Fresh from the presses:
Evolution of amplitude of
Small current perturbation at
cathode (3.4ps period).
Ref. plasma oscillations.

LBNL APEX-injector simulations



Impedance model for LSC (in free-space)

E_z field (lab-frame) at $\vec{x} = (x, y, z)$ due to a single electron at $\vec{x}'$, with charge $q = -e$

$$E_z(\mathrm{x,y,z}) = \frac{q}{4\pi\varepsilon_0} \frac{(\mathrm{z}-\mathrm{z}')\gamma}{[(\mathrm{x}-\mathrm{x}')^2 + (\mathrm{y}-\mathrm{y}')^2 + (\mathrm{z}-\mathrm{z}')^2\gamma^2]^{3/2}}$$

- Beam with cylindrical charge density with radius r_b; transverse uniform density
- Look for field E_z on axis $x = y = 0$ generated by a thin disk of charge at z' of radius r_b
  - Normalized transverse density: $\int \lambda_r(x', y'; s)\, dx'dy' = 1$

$$\frac{E_z(0,0,\mathrm{z}-\mathrm{z}';s)}{q_{disk}} = \frac{1}{4\pi\varepsilon_0} \int \frac{(\mathrm{z}-\mathrm{z}')\gamma\lambda_r(x',y';s)\, dx'dy'dz'}{[(\mathrm{x}-\mathrm{x}')^2 + (\mathrm{y}-\mathrm{y}')^2 + (\mathrm{z}-\mathrm{z}')^2\gamma^2]^{3/2}}$$

Definition of Wakefield potential

$$w_z(\Delta z) = -\frac{1}{q_{disk}} \int_0^L ds\, E_z(s, \Delta z)$$

...or $\widehat{w}_z(\Delta z) \equiv \frac{w_z(\Delta z)}{L}$

Wake-field potential per unit length

Modified Bessel function

$$\hat{Z}(k) = \frac{1}{c}\int_{-\infty}^{\infty} d\Delta z\, \widehat{w}_z(\Delta z) e^{-ik\Delta z}$$

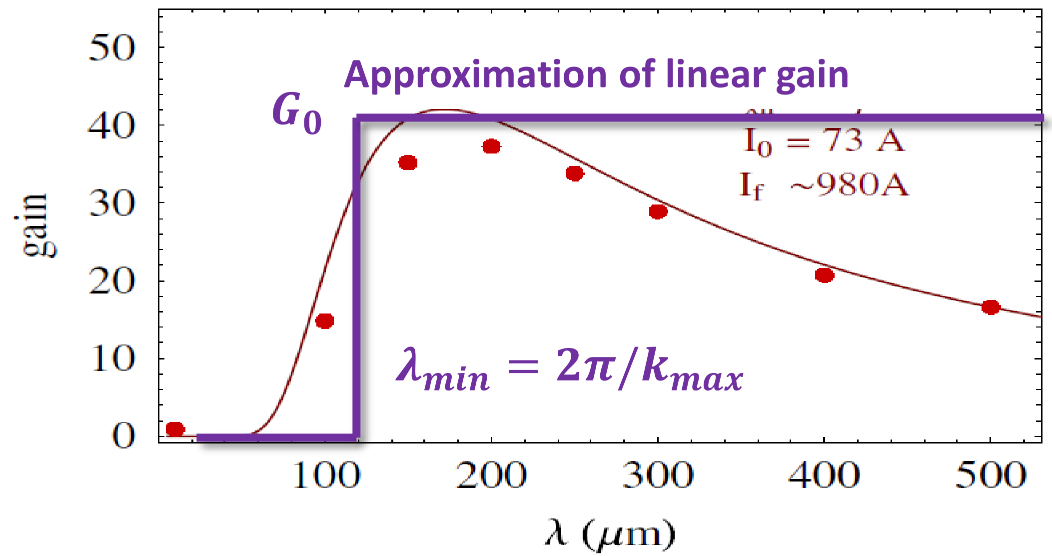
Impedance per unit length

$$\hat{Z}(k) = \frac{iZ_0}{\pi\gamma r_b} \frac{1 - \xi_b K_1(\xi_b)}{\xi_b}$$

$$\xi_b = kr_b/\gamma$$

Note: from now on for simplicity we drop the hat: "⌢"

Estimating amplification of shot-noise: the difficulty with macroparticle-simulations



Cut-off wavelength

$$N_{\lambda min} = N_b \frac{\lambda_{min}}{L_b}$$

No. of electrons/bunch

Bunch length (model assumes flat-top)

- Estimate of bunching (at exit of last bunch compressor)

$$b = \frac{\langle(\Delta I_{exit})^2\rangle^{1/2}}{I_{exit}} \simeq G_0\sqrt{\frac{2}{N_{\lambda min}}}$$

Assuming $L_b \gg \lambda_{min}$

- Macroparticle simulation that uses N_{mp} macroparticles/bunch overestimates bunching by: $\sqrt{N_b/N_{mp}}$

$E.g.\ N_{mp} = 10^6, N = 6.25 \times 10^9 (1nC) \rightarrow \sqrt{N_b/N_{mp}} \sim 80$