Collective effects in longitudinal dynamics (single-bunch)

MV

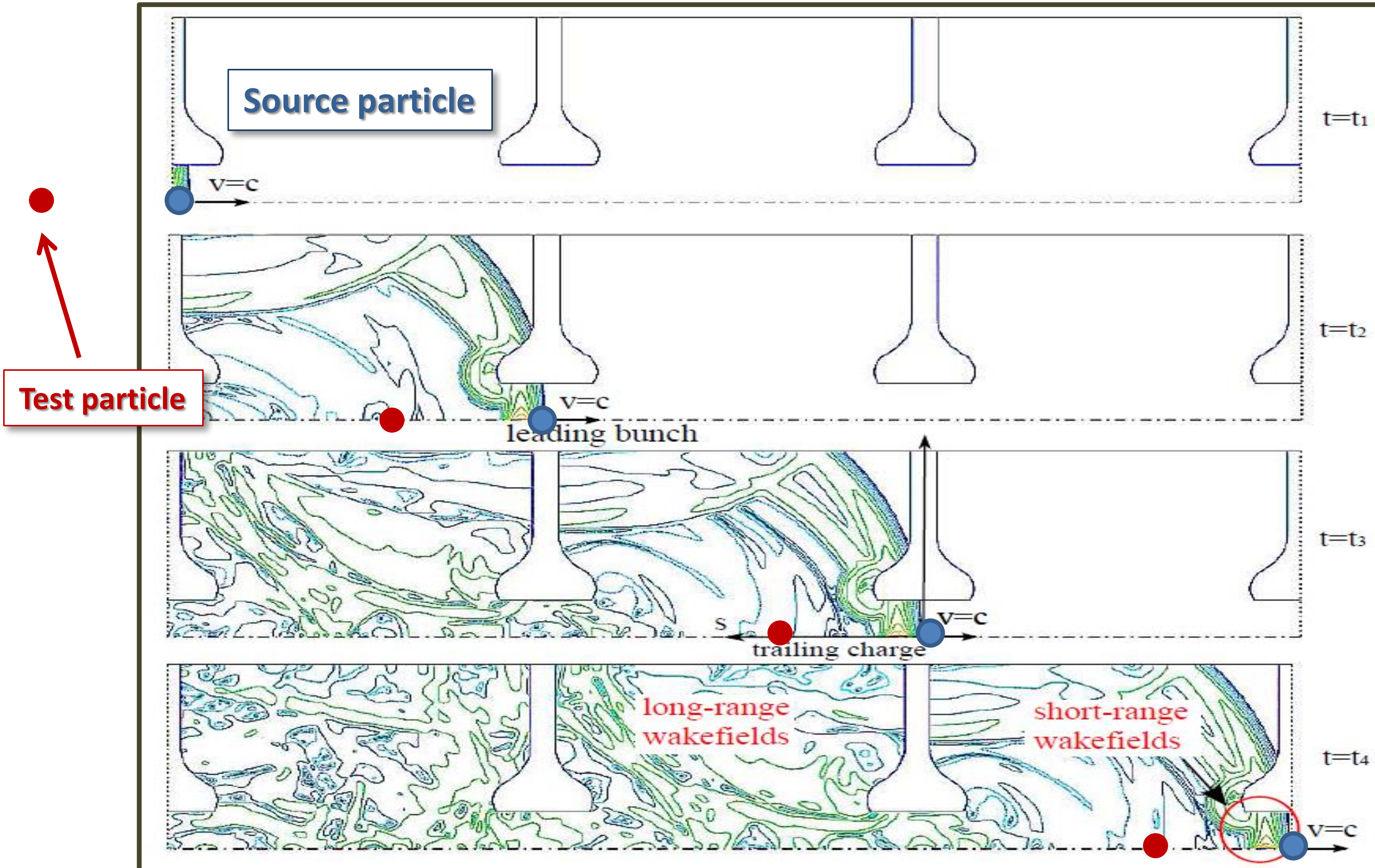
last revised 23-June-2015

Outline

1. **Refresher on general concept of wakefields;**
   1. "time-domain " description
   2. "frequency domain" description, impedance.
2. **Short-range rf wakefields in the accelerating structures**
   1. Models for short-range rf wakefields
3. **Impact of short-range longitudinal rf wakefields on beam dynamics**
   1. Energy loss
   2. Energy chirp
   3. Bunch profile shaping
4. **Coherent Synchrotron Radiation (CSR)**

RF Wakefields: the pictorial view.

Electric fields induced by passage of bunch through rf structure: four snapshots in time



P. Craievich, Dissertation (2010)

Wakefields: a brief intro

- **Two aspects of the problem**
  1. Wakefields (i.e. E&M fields) generation?
  2. **Effects of fields on beam dynamics**.

- **How do wakefields affect the beam dynamics?**
  - kicking a particle transversely $E_x, E_y, \left(\vec{v}_z \times \vec{B}\right)_x, \left(\vec{v}_z \times \vec{B}\right)_y$
  - Changing a particle energy E_z

Our focus

- **Wakefield are complicated functions of time and space** (picture in previous slide)
  - Usually what counts is the integrated field, as seen by a test particle at fixed distance from the source particle (ultrarelativisitc approx.) -> concept of wakefield potential
  - Integration (over length of the structure inducing the fields) washes out some of the complicated behavior
  - Here we are interested in **short-range** fields (generated by and acting on particles in the **same** bunch).
  - Bunch-to-bunch interactions (long-range wakefields) can also be important

Concept of rf wake potential (or point-charge wakefield potential)

Longitudinal wakefield potential
(space-integrated e-field over charge)

$$w_z(|\boldsymbol{\Delta z}|) = -\frac{1}{q_S}\int_0^L ds\, \boldsymbol{E_z}\left(s\,, t = \frac{s + |\Delta z|}{c}\right)$$

E-field experienced by test particle (along direction of motion)

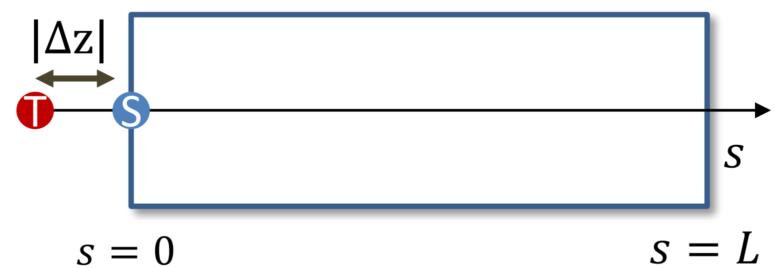
Long. separation between source and test particles

"-" sign: a matter of convention

Source-particle charge

***Source** particle enters rf structure at time t=0*

***Test** particle trails behind at distance $|\Delta z|$*



Similarly: Transverse wake potential
(more on this later today)

$$w_\perp\left(\vec{r},\vec{r'},\Delta z\right) = -\frac{1}{q}\int_0^L ds\,[\vec{E_\perp} + c\left(\hat{x}\times\vec{B}\right)_\perp]$$

(In this example the bunch head is to the right)

"Bunch" wakefield potential

$$w_z(|\Delta z|) = -\frac{1}{q}\int_{s_1}^{s_2} ds\, \boldsymbol{E_z}(s\,, t = \frac{s + |\Delta z|}{c})$$

MKS units: $\frac{1}{C} \times m \times \frac{V}{m} = \frac{V}{C}$ (voltage over charge)

Note : dependence on transverse coordinates is usually weak and negligible.

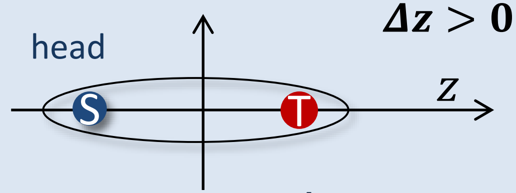
Energy change by test particle with charge $q_T = q_S = q$

$$\Delta U = q_T \int ds\, E_z = -q_T q_S w_z = -q^2 w_z$$

Note on sign convention: $w_z > 0$ means energy loss i.e. $\Delta U<0$

$\Delta z > 0$ when test particle is behind source

Once again, we are adopting the convention that the head of the bunch is $z < 0$



$$\Delta z \equiv \boldsymbol{z} - \boldsymbol{z'}$$

Coordinate of test particle

Coordinate of source particle

Because of causality:

$w_z(\boldsymbol{\Delta z}) = 0$ if $\Delta z < 0$

(a test particle ahead of source is not affected)

"Bunch" wakefield potential is the voltage drop experienced by a test particle in the bunch

$$\boldsymbol{V(z) = Q\int_{-\infty}^{z} dz' w_z(z - z')\lambda(z')}$$

MKS units: of $C \times m \times \frac{V}{C} \times \frac{1}{m} = V$ (voltage)

Bunch charge $= Nq$

Longitudinal bunch density (no. part/m) normalized to unity $\int dz'\, \lambda(z') = 1$

Loss factor and wakefield (diffraction) model for array of cavities

Energy loss (gain) by particle along bunch as beam travels through structure:

$$U(z) = -qV_z(z) = -Nq^2 \int_{-\infty}^{z} dz' \, w_z(z-z') \, \lambda(z')$$

Total energy loss by bunch:

$$U_{tot} = \int U(z) \, \lambda(z) dz$$

The loss factor is characteristic of the rf structure:

$$k_l = \frac{U_{tot}}{Nq^2}$$

- Example of wakefield potential: *infinitely long array of rf cavities with cylindrical symmetry (very relevant for FEL linacs)*

$$Z_0 = 120\pi \ \Omega$$

(vacuum impedance)

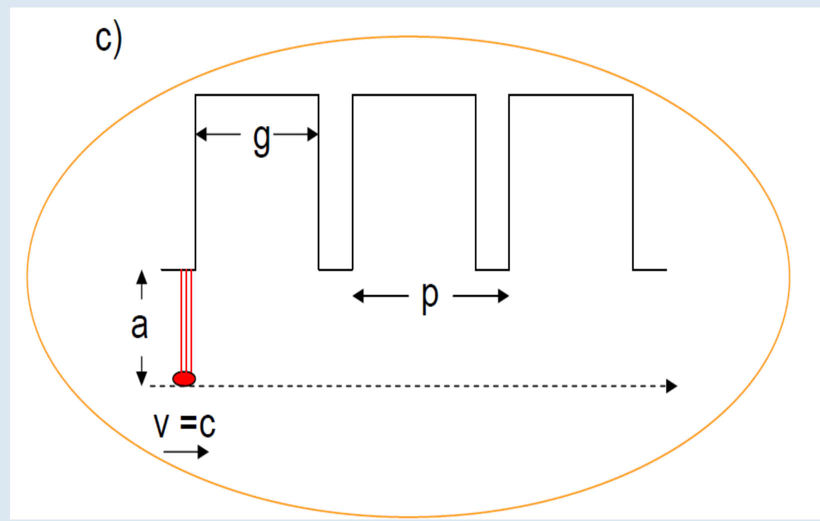
$$w_z(\Delta z) \simeq \frac{Z_0 c}{\pi a^2} e^{-\sqrt{\Delta z/d}}$$

Actually a wake-field potential per unit of rf structure length (V/C/m)

Giving good approx.

over wide range of parameters

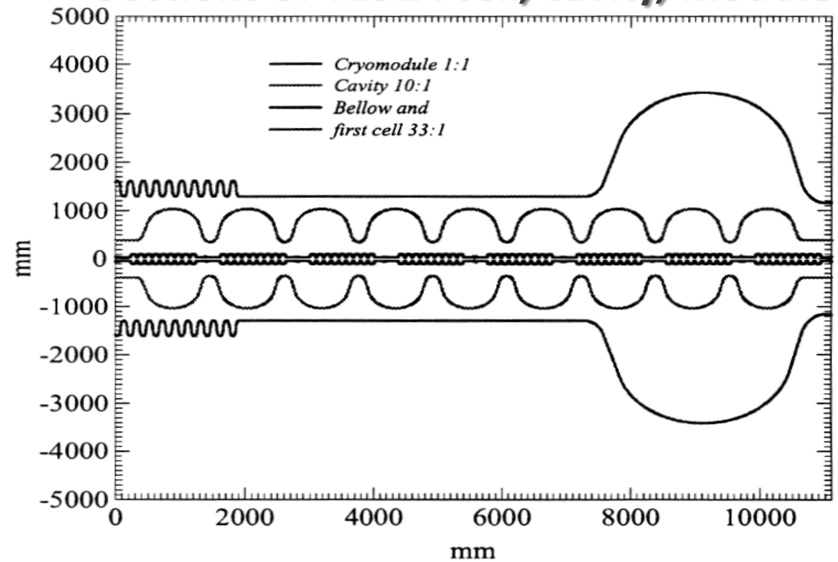
$$d = \frac{0.41 a^{1.8} g^{1.6}}{p^{2.4}}$$



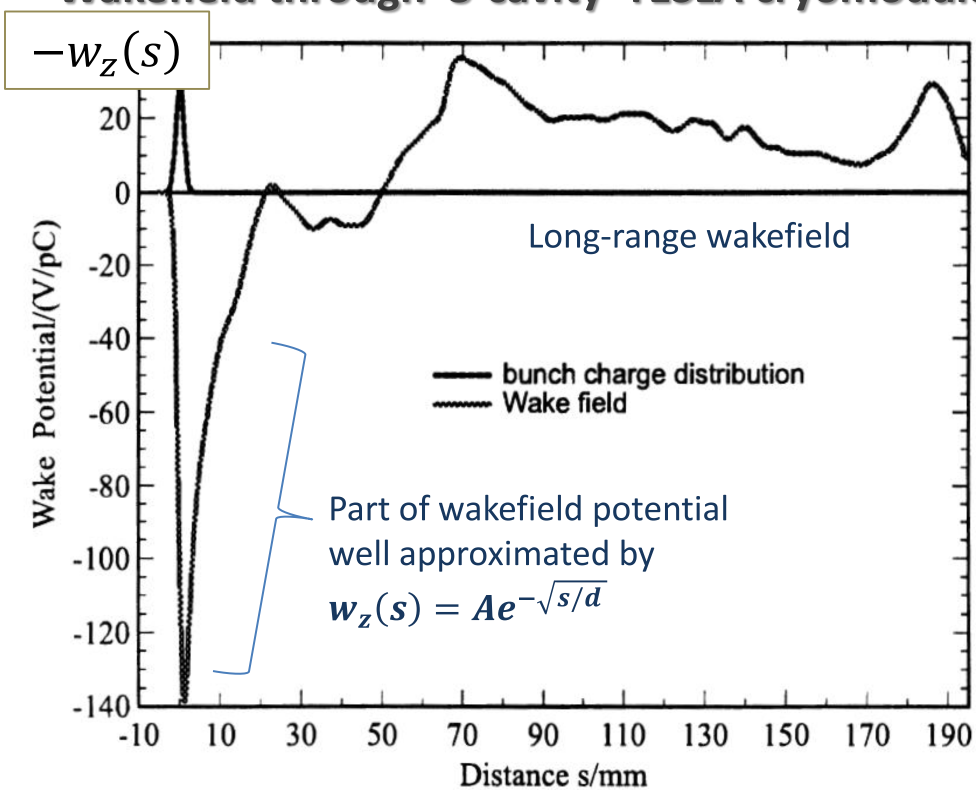
Numerical modeling of longitudinal wake potentials: E.g. the TESLA structures

- Accurate determination of wakefield potential for actual structure design can be done with specialized E&M numerical codes (e.g MAFIA by T. Weiland et al.)
- Fit numerical result with analytical model

Sections of TESLA cell, cavity, module:



Wakefield through 8-cavity TESLA cryomodule



A. Novokhastski et al. TESLA Report 99-16

Short-range numerical wake is fitted against analytical model:

$$w_z(z) = Ae^{-\sqrt{z/d}}$$

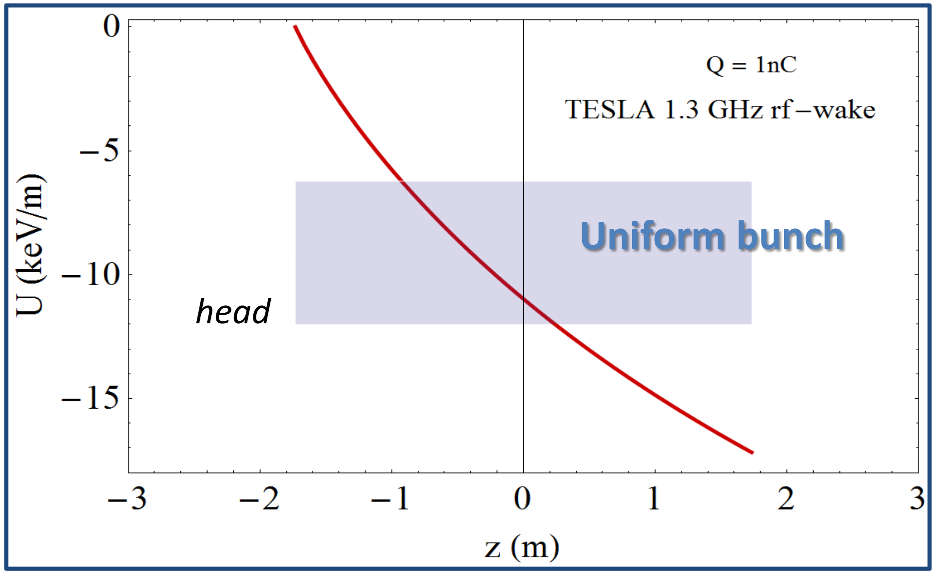
$$A = 41.5\frac{V}{pC\ m} \qquad d = 1.74\ mm$$

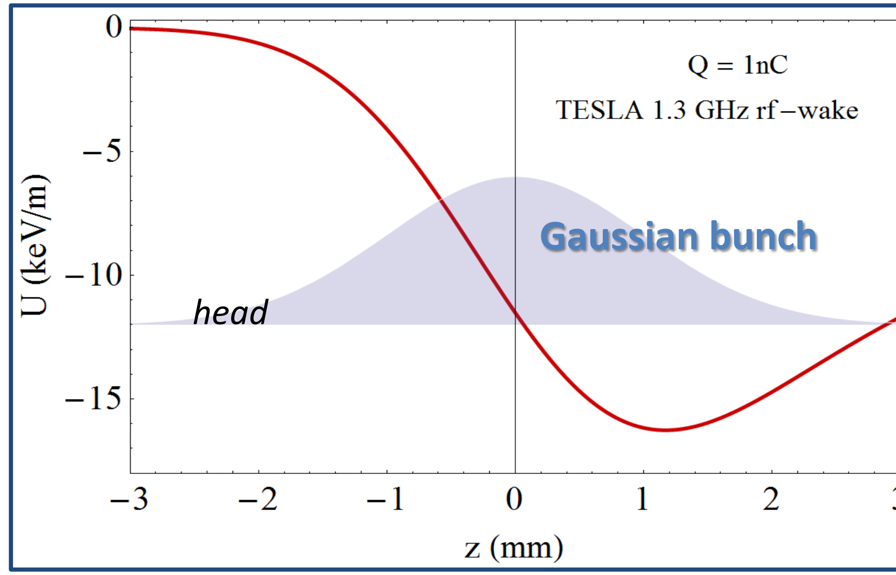
8-cavity cryomodl. active length L=8.29m

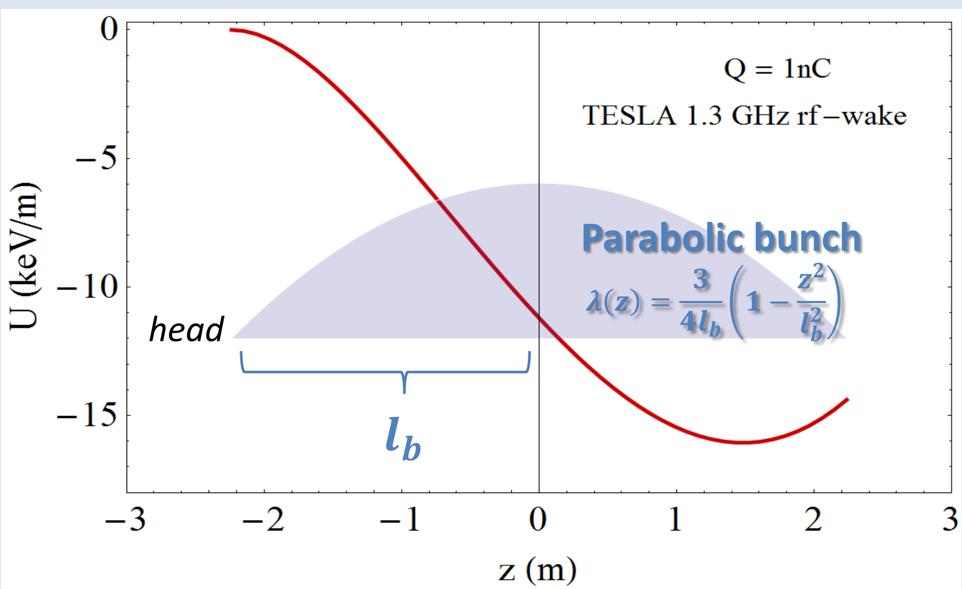
Weiland at al., TESLA Report 2003-19

Energy loss induced by wakefields depends on current profile along

Energy loss through 1m active length of 1.3GHz TESLA rf structures ($\sigma_z = 1mm$ bunches)







Wakefield potential model $w_z(z) = Ae^{-\sqrt{z/d}}$

$$U(z) = U_0 + U_1\ z + U_2\ z^2 + U_3\ z^3 + \cdots$$

$$x = \sqrt{l_b/d}$$
$$Q > 0$$

$$U_0 = -eQ\frac{A}{2}\left(1 - \frac{4}{7}x + \frac{3}{16}x^2 + \cdots\right)$$

$$U_1 = -eQ\frac{3A}{4l_b}\left(1 - \frac{4}{5}x + \frac{1}{3}x^2 + \cdots\right)$$

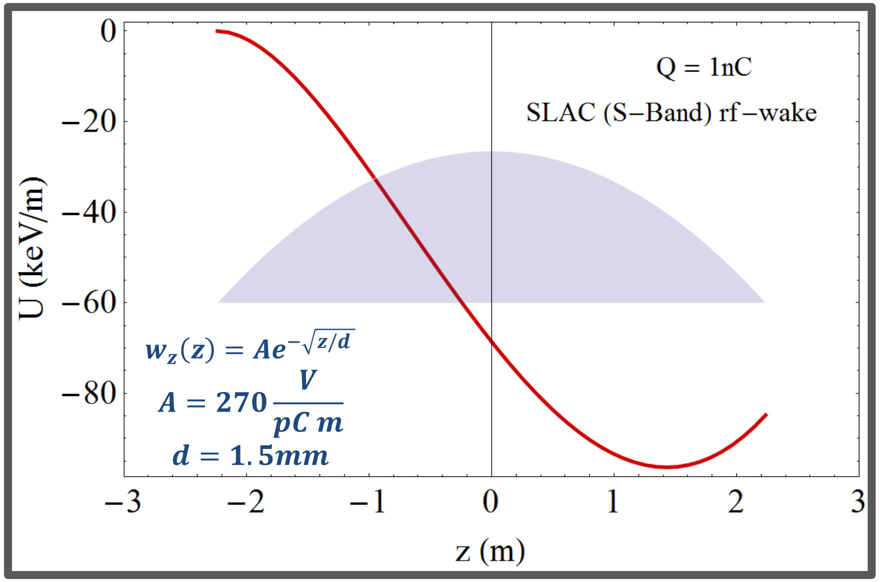
$$U_2 = eQ\frac{A}{4l_b^2}\left(x - \frac{3}{4}x^2 + \cdots\right)$$

$$U_3 = eQ\frac{A}{8l_b^3}e^{-\sqrt{x}}(2 + x) \qquad \text{(exact)}$$

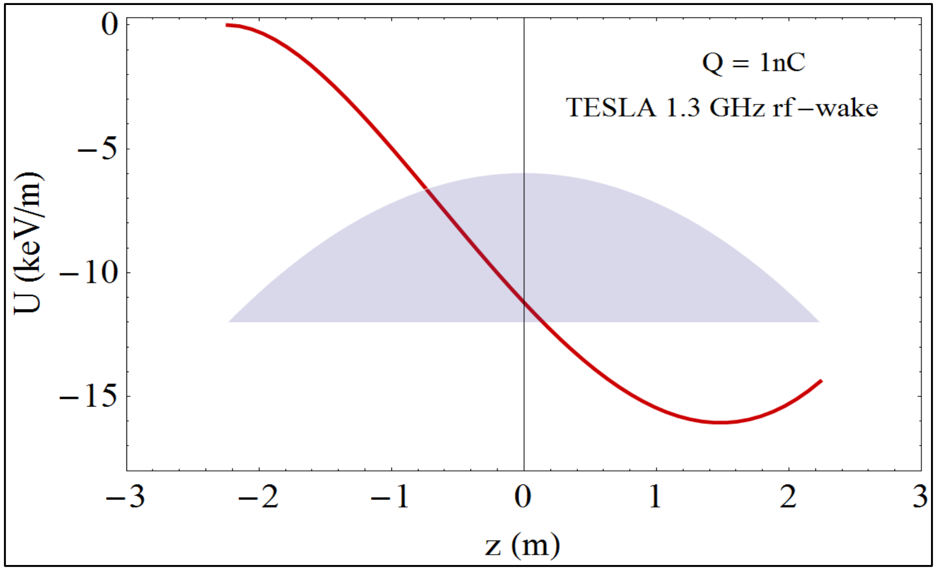
$$U_{tot} = -eQ\frac{A}{2}\left(1 - \frac{128\sqrt{2}}{273}x + \frac{18}{70}x^2 + \cdots\right)$$

Other example: Longitudinal wakefield potential for SLAC (LCSL) Linac

SLAC S-Band (2.856 GHz)



Tesla L-Band (1.3 GHz)



- Notice difference in magnitude
- SLAC S-band structure iris radius $a{\sim}11.mm$; TESLA 1.3GHz $a{\sim}35\ mm$
  - Ratios of energy drop not quite the same as in the ratio $\left(\frac{a_{Sband}}{a_{1.3GHz}}\right)^2$ as predicted by diffraction model . TESLA structures deviate more from model.

- Note: steady state regime (expression above) is reached after a transient length on the order $L \sim \frac{a^2}{2\sigma_z}$

The frequency-domain view: rf wakefield potential expressed in terms of an impedance

- The wake-field impedance is the Fourier transform for the wakefield potential.

$$Z(k) = \frac{1}{c}\int_0^\infty dz\, w_z(z) e^{-ikz}$$

MKS: units $\frac{1}{m/s} \times m \times \frac{V}{C} = \frac{V}{A} = \Omega$

starting from 0 because of causality

FT Inversion: $\mathrm{w}_z(z) = \frac{c}{2\pi}\int_{-\infty}^{\infty} dk\, Z(k) e^{ikz}$

- Aide note: it can be easily seen that it is the ratio of the FT of voltage drop $V(k)$ and FT of instantaneous current $I(z) = Nq\lambda(z)$

$$Z(k) = -\frac{V(k)}{I(k)}$$

Effect of rf wake-fields on longitudinal dynamics

Wakefield-induced energy change along bunch:

$$U(z) = U_0 + U_0' z + \frac{U_0'' \, z^2}{2} + \frac{U_0''' \, z^3}{6} + \cdots$$

- 0-order: Total energy loss by bunch
  - Usually not an important issue for the dynamics, it can be compensated by adjusting RF structure voltage. However, it can add significantly to heat load in SC structures ☹

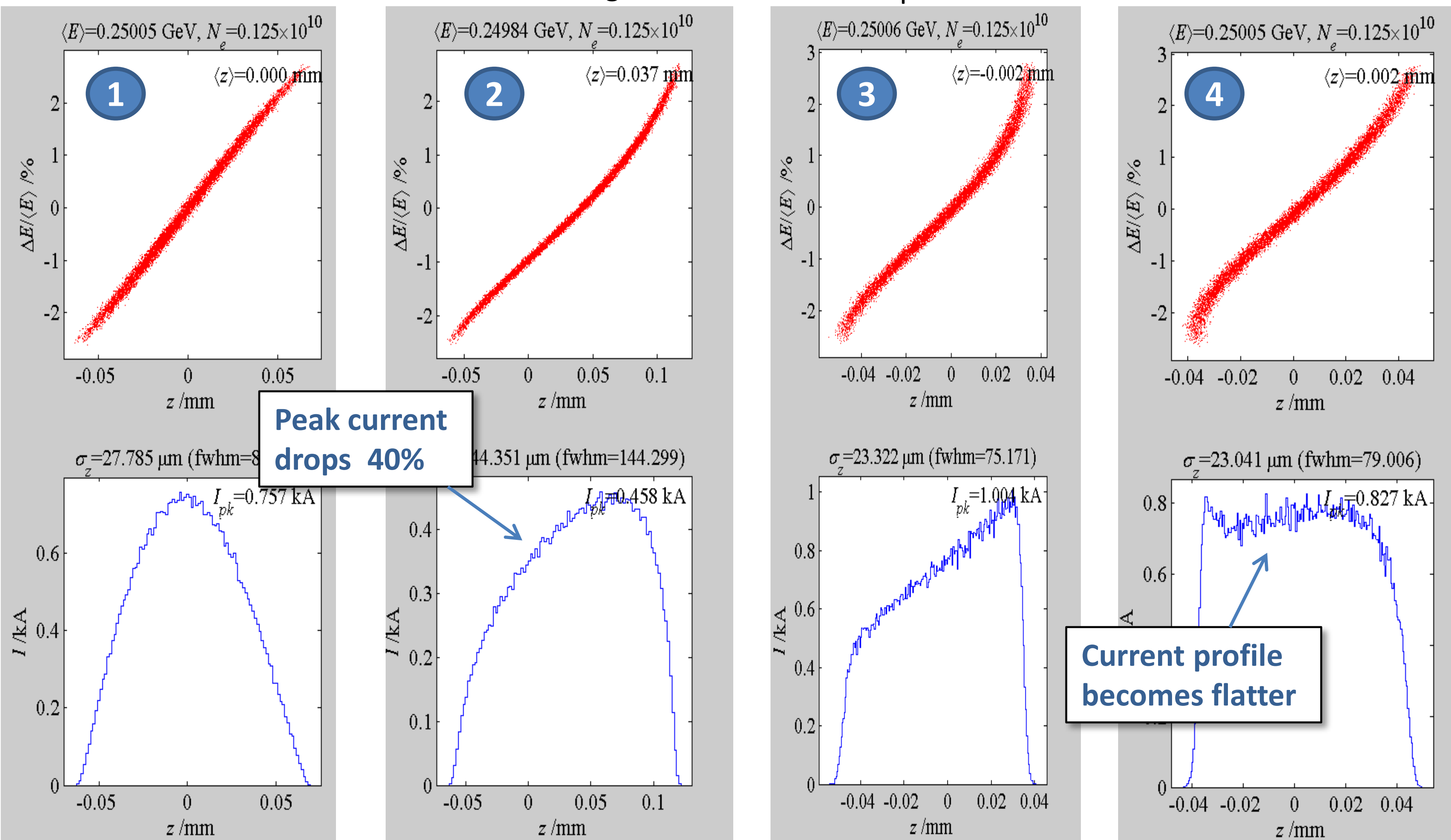
- 1st-order
  - Affects the linear energy chirp of bunch (and hence compression). Can be compensated by adjusting the RF structure phases. Typically not a problem; it can actually be helpful with the removal of the energy chirp left after the last bunch compressor.

- 2nd order
  - Affects the quadratic energy chirp. Can be corrected with small adjustment of linearizer setting.

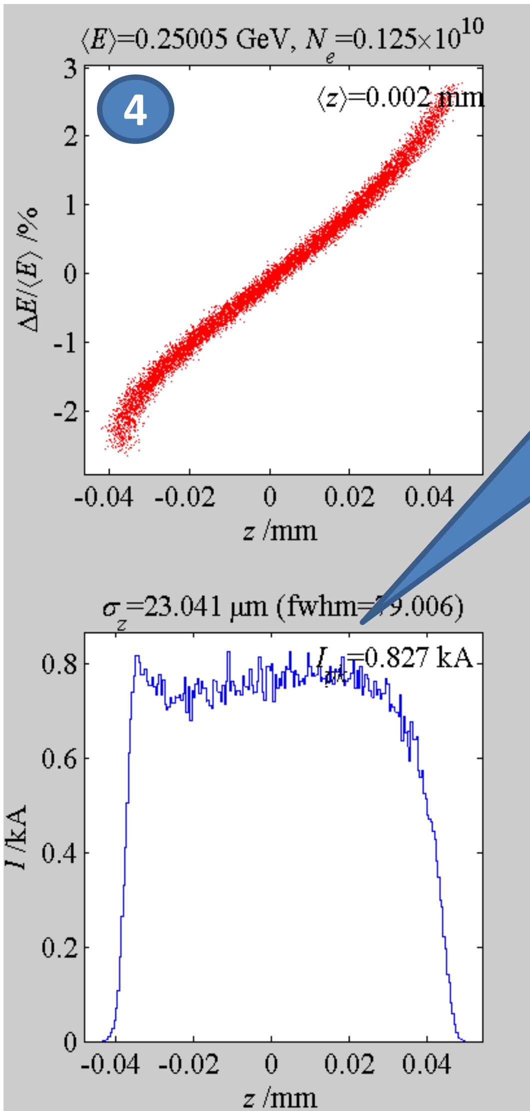
- 3rd order and higher
  - Usually much stronger than cubic (and higher order) nonlinearities from the RF-structure waveform, it cannot be compensated. Contributes to shaping current profile; will cause current spikes when pushing for large compression factors.

Effect of RF wakefields: Inspect beam at exit of Linac Section + BC

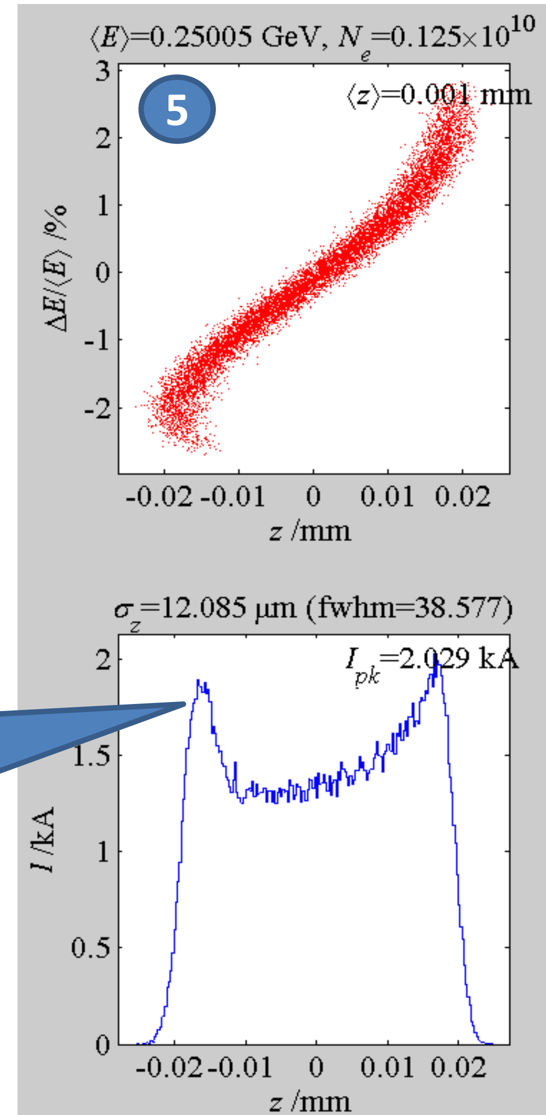
Wakefields OFF	Wakefields ON	Wakefields ON	Wakefields ON
		Modify RF phase:	
	Machine settings as in 1.	Same compression as in 1	Retune linearizer



Highlighting the 3rd -order effect of RF wakefields: appearance of current spikes



Moderate S-shaped phase-space is associated with an almost flat-top ☺ current profile



Strongly S-shaped phase-space is associated with current spikes at the edges for large compression.

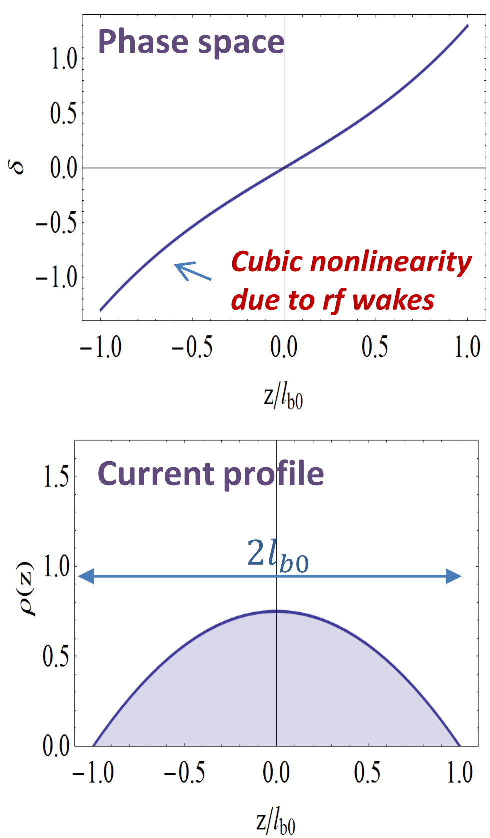
(See Elegant exercise)

Simple analytical model showing the condition for appearance current spikes (details left as an exercise)

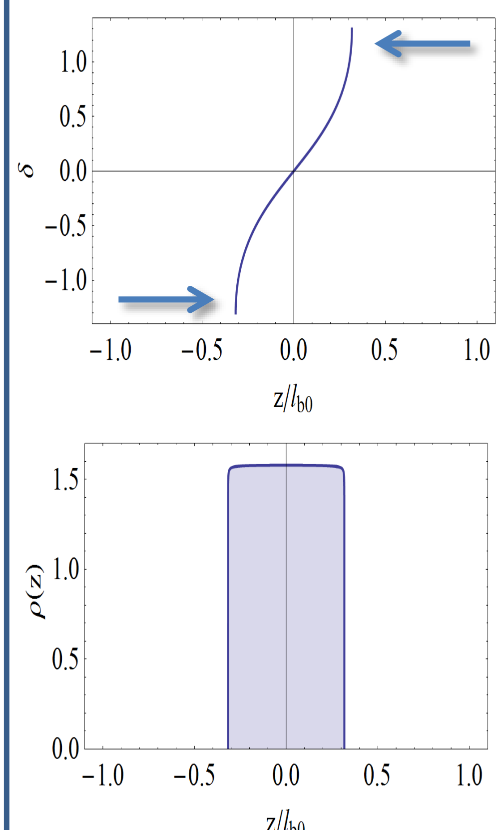
1. Start with beam model having
   - parabolic charge density
   - vanishing slice energy spread and cubic chirp before entering

$$f(z,\delta) = \frac{3}{4l_b}\left(1 - \frac{z^2}{l_{b0}^2}\right)\delta_D(\delta - h_1 z + \mathbf{h_3} z^3)$$

Entrance of BC



Exit of BC



2. Propagate beam phase-space density through compressor
   - Neglect 2nd order effects (assume linearizer has been set so as to compensate them)

$$\begin{cases} z' = z' + R_{56}\delta \\ \delta' = \delta \end{cases}$$

3. Find charge density of compressed beam

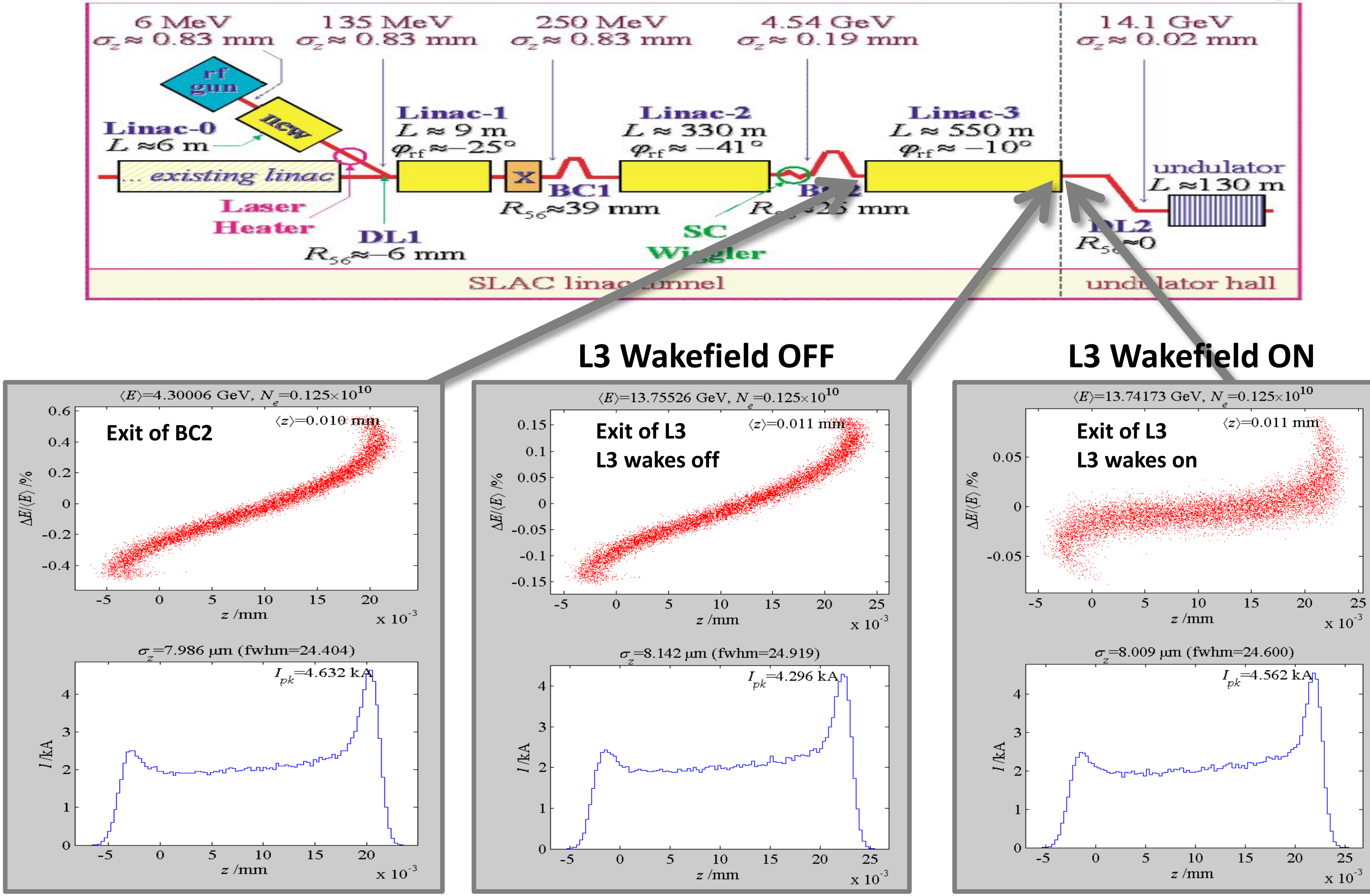
4. Find condition for flat charge density

$$\boxed{C_{crit} = \frac{1}{3\,|h_3 R_{56}|\,l_{b0}^2}}$$

If $C > \frac{1}{3|\,h_3 R_{56}|l_{b0}^2}$ the beam density in phase-space folds over and current spikes will appear.

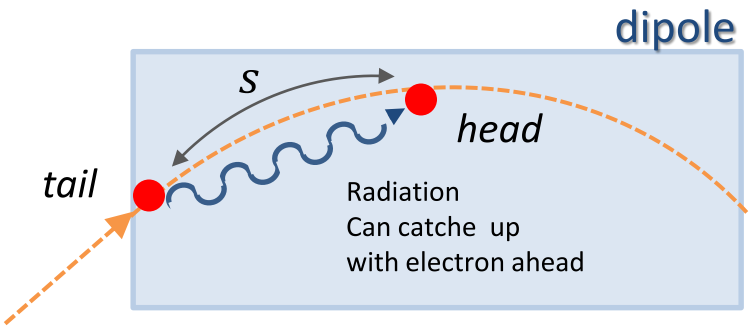
The bright side of RF wakefields: they help with removing the energy chirp after we are done with compression

LCLS Example



Radiation effects (CSR) on longitudinal dynamics

- Radiation in the bunch-compressor dipoles is responsible for the largest longitudinal collective effects besides the rf wakes



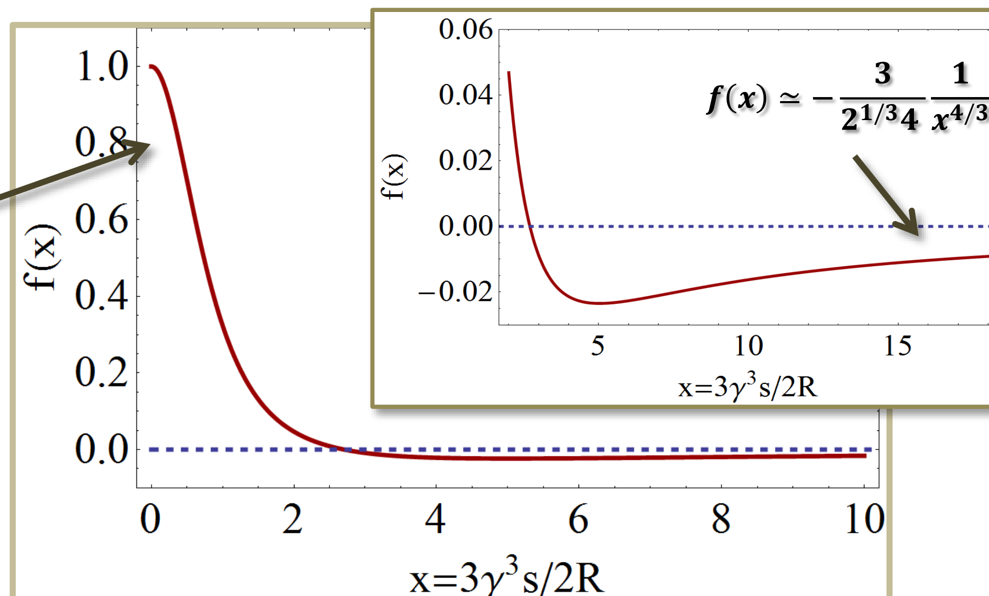
- Closed-form analytical expression for electric field (along direction of motion)
  - Two particles on same trajectory path
  - Uniform circular motion (steady-state)
  - Use expressions for retarded-fields

$$E_\varphi[rad.] = -\frac{4}{3}\frac{e}{4\pi\varepsilon_0}\frac{\gamma^4}{R^2}\times \boldsymbol{f(x)}$$

$$E_\varphi = E_\varphi[Coulomb] + E_\varphi[\text{radiation}]$$

Scales as $1/\gamma^2$
More on this tomorrow

$$\boldsymbol{f(x) \simeq 1 - \frac{14}{9}x^2}$$



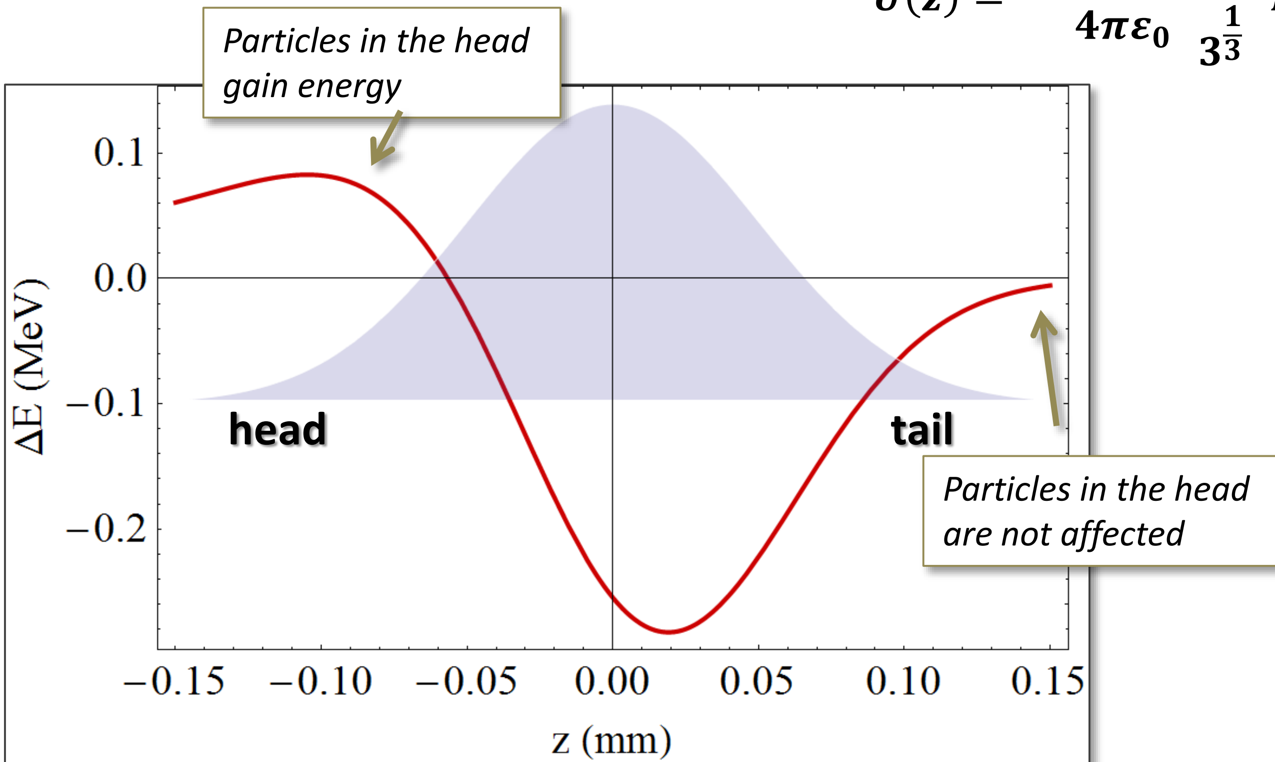
Important properties:

$f(x) = 0$ for $x < 0$

$$\int_0^\infty f(x)dx = 0$$

Example of CSR-induced energy loss along gaussian bunch (steady state)

$$U(z) = -\frac{Ne^2}{4\pi\varepsilon_0}\frac{2}{3^{\frac{1}{3}}}R^{1/3}\theta_B\int_z^\infty \frac{dz'}{(z'-z)^{1/3}}\frac{d\lambda(z')}{dz'}$$



$$\begin{aligned} \sigma_z &= 50\mu m \\ Q &= 300pC \\ L_B &= 1m \\ \theta_B &= 10^o \\ R &= 5.7m \\ I_{pk} &= 715\ A \end{aligned}$$

Total energy loss by gaussian bunch through dipole of length L_B

Proportional to 1/3 power of bend radius

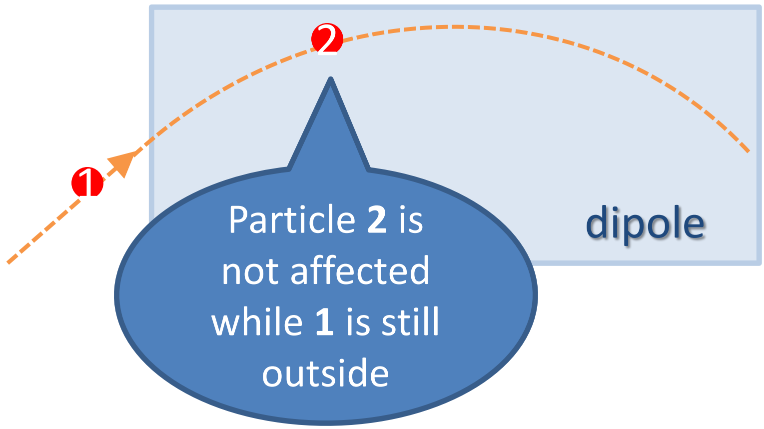
Proportional to bend angle

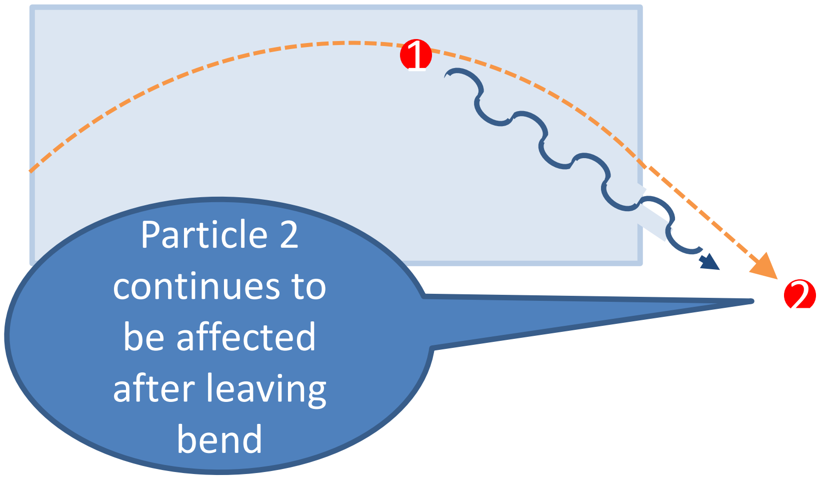
$$U_{tot} = -0.028 \times Ne^2 Z_0 c \times \frac{R^{1/3}\theta_B}{\sigma_z^{\frac{4}{3}}} = -0.16\text{MeV}$$

Inversely proportional to 4/3 power bunch length

Limitations to the steady-state CSR model and other remarks

- Steady-state model doesn't account for transient effects (entering, and downstream of the dipoles)





Overtake length $L_o \simeq (24\ R^2\ \sigma_z)^{1/3}$ *(distance traveled by bunch before steady-state regime sets in)*

- There are analytical 1D models accounting for transient effects, and implemented e.g. in the Elegant. Usually these are important and cannot be neglected (see Elegant ex.)
- CSR effects on longitudinal dynamics are generally non-negligible but not as large as those from the RF wakefields
- In practice, the rf settings will have to be adjusted slightly to compensate for linear and nonlinear effects induced by CSR

- The most notable effect of the <u>longitudinal</u> CSR wake is on the transverse dynamics → see tomorrow's lecture

Summary highlights

- Longitudinal wakefield potential

$$w_z(|\Delta z|) = -\frac{1}{q_s}\int_0^L ds\, \boldsymbol{E_z}(s, t = \frac{s + |\Delta z|}{c})$$

- Energy change induced by wakefields along bunch

$$U(z) = -qV_z(z) = -Nq^2 \int_{-\infty}^{z} dz'\, w_z(z - z')\, \lambda(z')$$

- Diffraction model for periodic array of cavities

$$w_z(\Delta z) = \frac{Z_0 c}{\pi a^2} e^{-\sqrt{\Delta z / z_1}} \qquad z_1 = \frac{0.41 a^{1.8} g^{1.6}}{p^{2.4}}$$

- CSR-induced energy change along bunch (steady state model)

$$U(z) = -\frac{Ne^2}{4\pi\varepsilon_0} \frac{2}{3^{\frac{1}{3}}} \frac{L_B}{R^{\frac{2}{3}}} \int_z^{\infty} \frac{dz'}{(z' - z)^{1/3}} \frac{d\lambda(z')}{dz'}$$

  - Energy loss by gaussian bunch passing through bend (steady state model)

$$U_{tot} = -0.028 \times Ne^2 Z_0 c \times \frac{R^{1/3}\theta_B}{\sigma_z^{\frac{4}{3}}}$$