Plan for the week

- **This afternoon**: Surface resistance at high RF fields
- **Tuesday afternoon**: TM-class cavity design
- **Wednesday morning**: Computer Lab + cavity limitations
- **Thursday afternoon**: Cavity fabrication
- **Friday morning**: Surface preparation

SURFACE RESISTANCE AT HIGH RF FIELDS

Gianluigi Ciovati

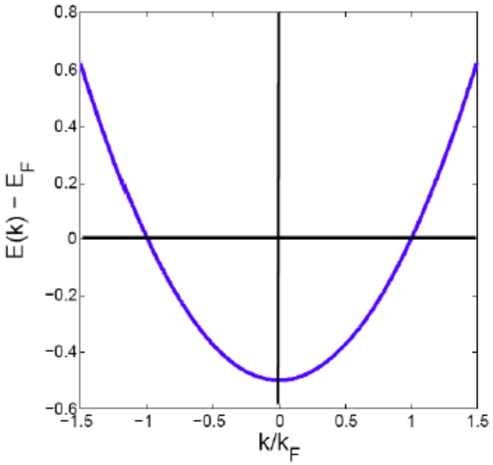
Thomas Jefferson National Accelerator Facility

Outline

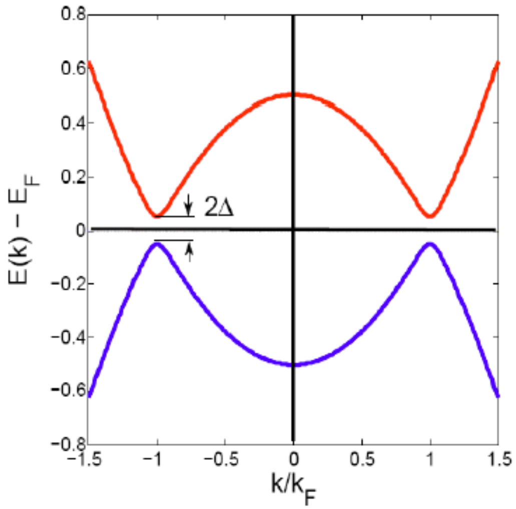
- Surface resistance in strong RF fields
  - Thermal feedback model
  - Non-linear BCS
  - Hot-spots

- RF losses due to vortices' motion
  - Oscillation of pinned vortices
  - Vortex penetration
  - Hotspot generation

BCS Surface Resistance



Normal state for T > Tc



Superconducting state for T < Tc

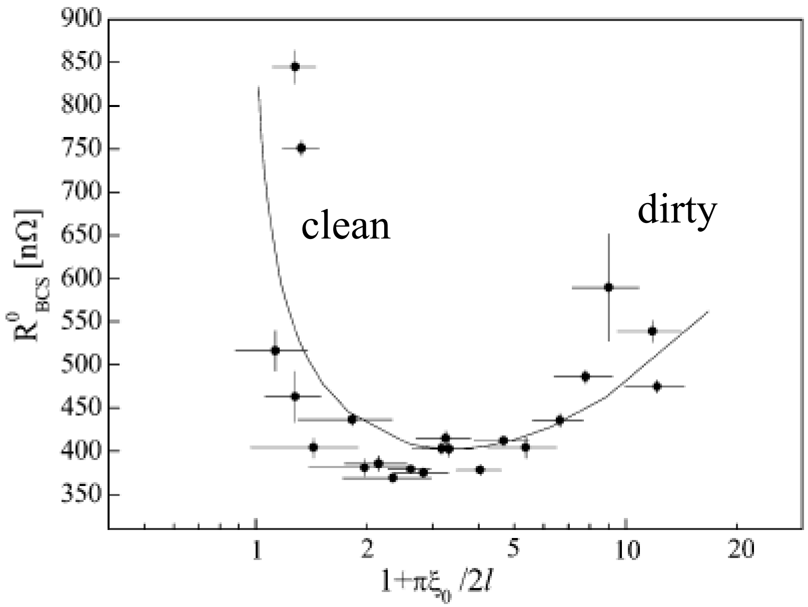
- Superconducting gap Δ on the Fermi surface
- For T<< T_c, a small fraction of electrons are unbound due to thermal dissociation of the Cooper pairs
- Thermally-activated quasiparticles define the exponentially small BCS surface resistance

Clean limit ($l \ll \xi_0$) BCS surface resistance at low field ($H \ll H_c T/T_c$)

$$R_s \cong \frac{\mu_0^2 \omega^2 \lambda^4 \Delta n_0}{k_B T p_F} \ln\left(\frac{\Delta}{\hbar\omega}\right) \exp\left(-\frac{\Delta}{k_B T}\right)$$

The higher $T_c = 0.57\Delta$ the smaller the BCS surface resistance

R_{BCS}: Impurities Dependence



BCS resistance of Nb at 1.5 GHz, 4.2 K

- Scattering mechanisms and normal state conductivity: $\sigma_n = e^2 n_0 l / p_F$, $p_F = \hbar(3\pi^2 n_0)^{1/3}$

$$\lambda = \lambda_L \sqrt{1 + \frac{\xi_0}{l}}$$

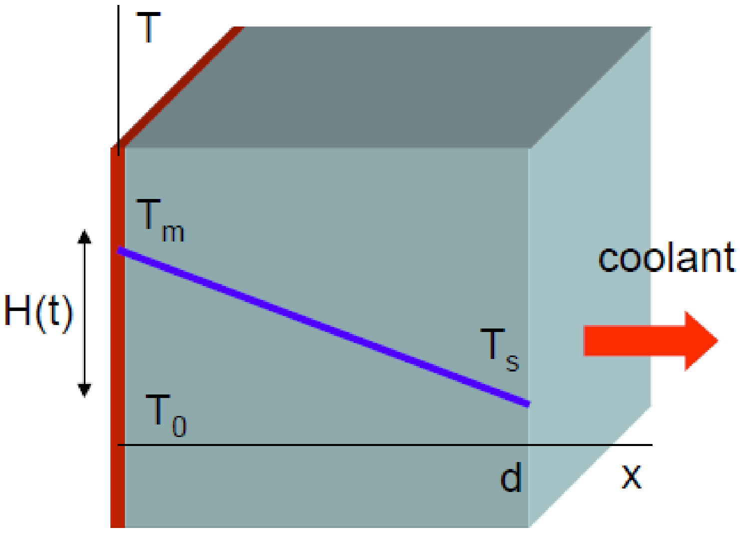
- Dirty limit ($l \ll \xi_0$):

$$R_{BCS} \propto \lambda^3 \sigma_n \propto \frac{1}{\sqrt{l}}$$

- Clean limit ($l \gg \xi_0$):

$$R_{BCS} \propto l \qquad \sigma_n \to \sigma_{eff} \approx e^2 n_0 \lambda / p_F$$

$R_S(H_0)$: Thermal Feedback



$$\Delta T = T_m - T_0 = \left(\frac{d}{\kappa} + \frac{1}{h_K}\right)\dot{q}_{RF} = \left(\frac{d}{\kappa} + \frac{1}{h_K}\right)\frac{1}{2} R_s H_0^2$$

$$R_s(T, H_0) \cong R_s(T_0 + \Delta T, H \approx 10\mathrm{mT}) = R_s(T_0, 10\mathrm{mT}) + \sum_n \frac{1}{n!} \left.\frac{\partial^n R_s(T)}{\partial T^n}\right|_{T=T_0} (\Delta T)^n$$

$$\frac{\partial R_s}{\partial T} \approx R_{BCS} \frac{\Delta}{k_B T^2}$$

To first order we obtain:

$$R_s(T, H_0) = R_s(T_0, 10\mathrm{mT})\left[1 + \gamma^*\left(\frac{H_0}{H_c}\right)^2\right]$$

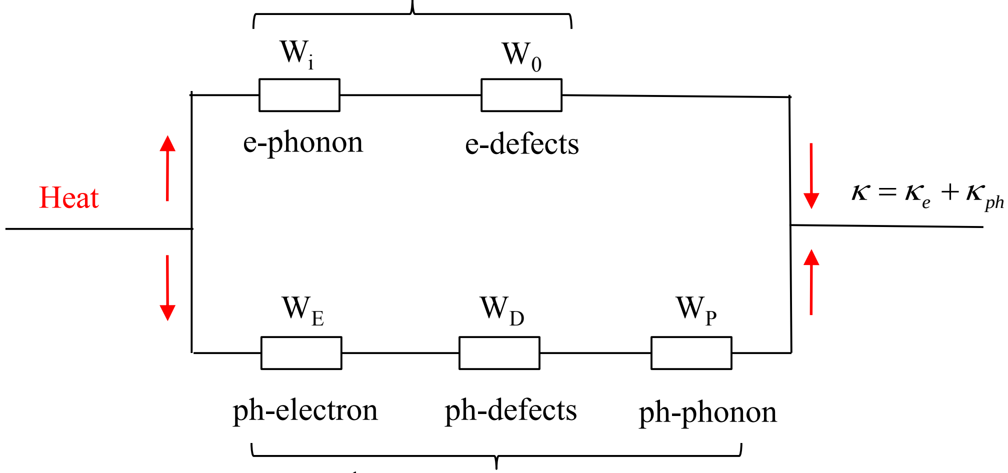
$$\gamma^*(T_0) = R_{BCS}(T_0)\frac{H_c^2 \Delta}{2 k_B T_0^2}\left(\frac{d}{\kappa} + \frac{1}{h_K}\right) \approx 0.2 - 1 \text{ at } 2\text{ K}$$

$\kappa(T)$ Thermal conductivity

$h_K(T)$ Kapitza conductance

Thermal Conductivity of Nb

$$\frac{1}{\kappa_e} = W_e = W_i + W_0$$

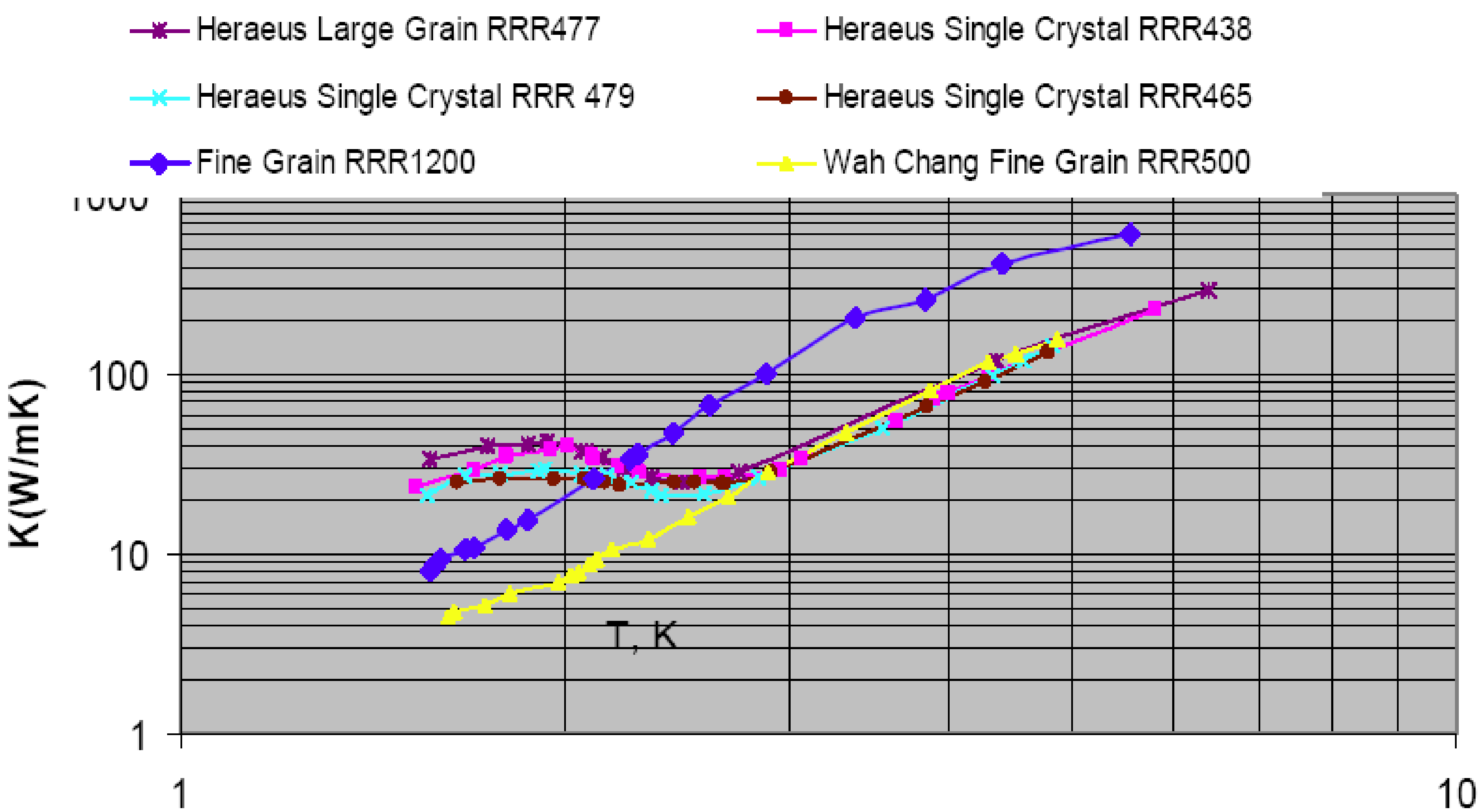


$$\frac{1}{\kappa_{ph}} = W_{ph} = W_E + W_D + W_P$$

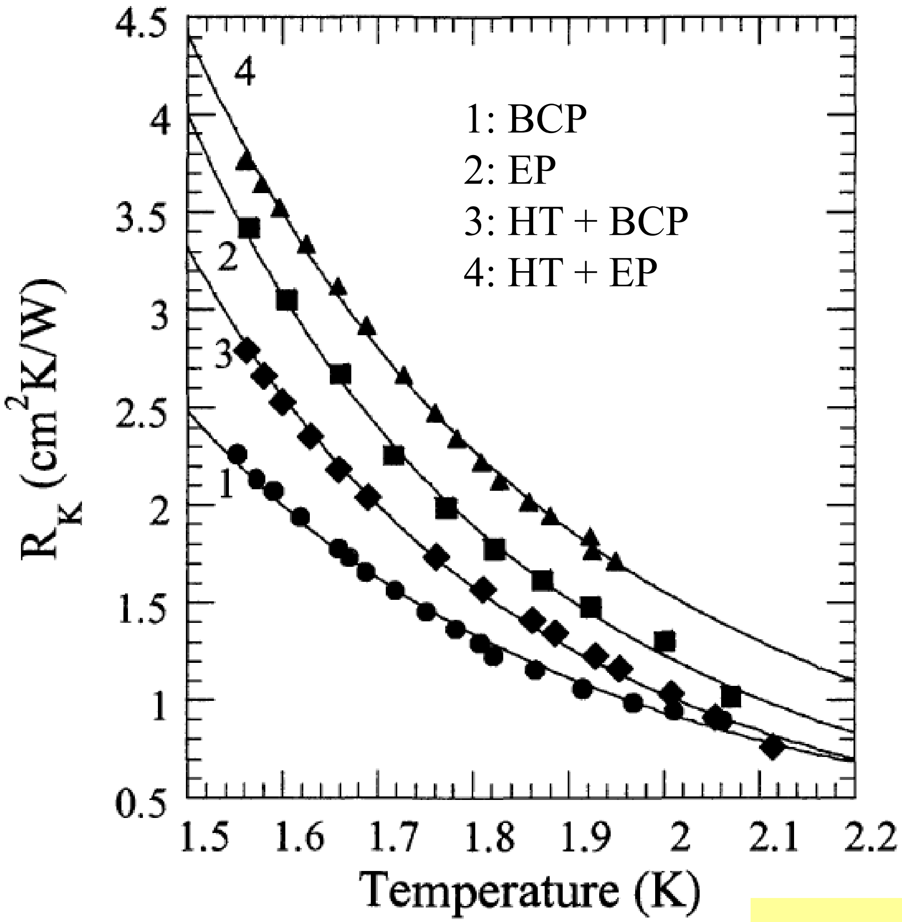
$$\lambda(T, RRR, G) = \underbrace{R(y) \cdot \left[\frac{\rho_{295K}}{L \cdot RRR \cdot T} + a \cdot T^2 \right]^{-1}}_{\text{Electron term}} + \underbrace{\left[\frac{1}{D \cdot \exp(y) \cdot T^2} + \frac{1}{B \cdot G \cdot T^3} \right]^{-1}}_{\text{Lattice term}}$$

$y = \Delta / k_B T$
$R(y) = \lambda_{es} / \lambda_{en}$
$L \cong 2.45 \times 10^{-8}$ W/K^2
$a = 2.3 \times 10^{-5}$ m/(W K)
$B = 7.0 \times 10^{3}$ W/(m^2 K^4)
$1/D = 300$ m/(W K^3)
G: phonon mean free path ~ grain size

Thermal Conductivity of Nb



Kapitza Resistance

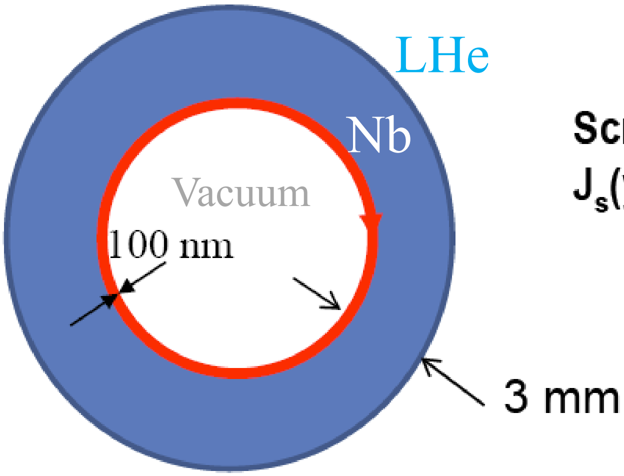


$R_K = 1/h_K$

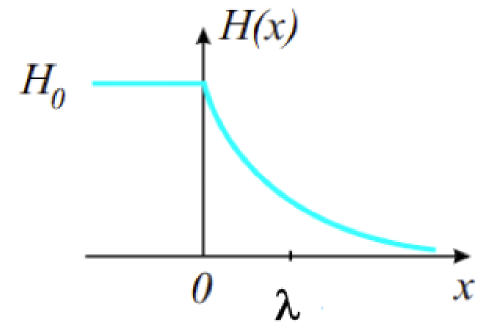
- Determines the heat exchange at the outer surface of the cavity wall in contact with He bath
- Arises from non-unity phonon transmission coefficient across the interface

$$h_K(T_s, T_0) = 200T^{4.65}\left[1 + 1.5\left(\frac{T_s - T_0}{T_0}\right) + \left(\frac{T_s - T_0}{T_0}\right)^2 + 0.25\left(\frac{T_s - T_0}{T_0}\right)^3\right] \left(\frac{\mathrm{W}}{\mathrm{K\,m^2}}\right)$$

Meissner Current at Inner Cavity Surface



Screening surface current density $J_s(y)$ in the λ-thick inner layer

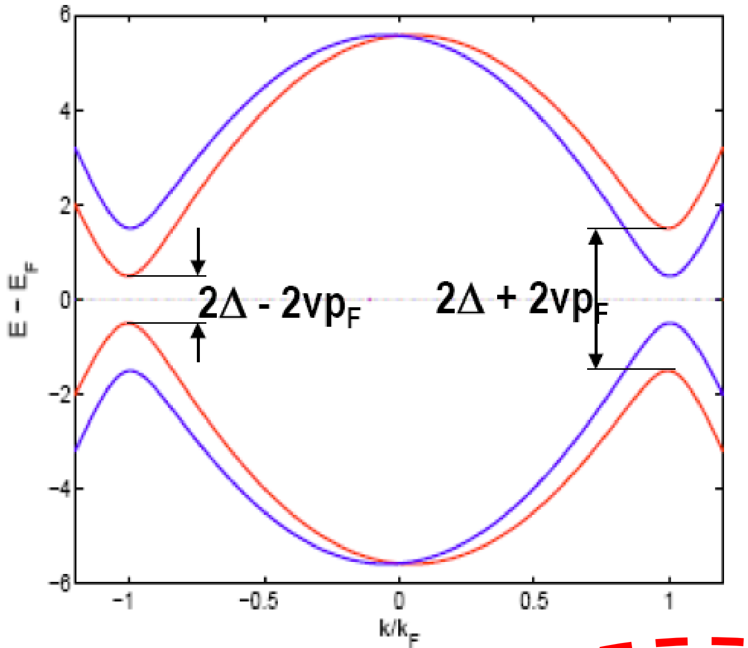


$$H(y) = H_0 e^{-y/\lambda}, \qquad J_s(y) = \frac{H_0}{\lambda} e^{-y/\lambda}$$

- Surface current density cannot exceed the depairing current density $J_d = H_s/\lambda \sim 5\text{MA/cm}^2$
- London penetration depth

$$\lambda = \left(\frac{m}{e^2 n_s(T) \mu_0} \right)^{1/2} \cong 40\ nm$$

Effect of Current on Thermal Activation



Rocking "tilted" electron spectrum under rf current $J = J_0 \cos\omega t$

$$E(p) = \pm\sqrt{\Delta^2 + (p^2/2m - E_F)^2} \pm \vec{p}_F \vec{v}_s(t)$$

Doppler shift due to superfluid velocity $v_s(t) = J/n_s e$

Valid in the clean limit

- Reduction of the gap $\Delta(v_s) = \Delta - p_F|v_s|$ increases the density of thermally-activated normal electrons. Nonlinearity of surface resistance R_s (J).

- Critical pairbreaking velocity:

$$v_c = \frac{\Delta}{p_F}$$

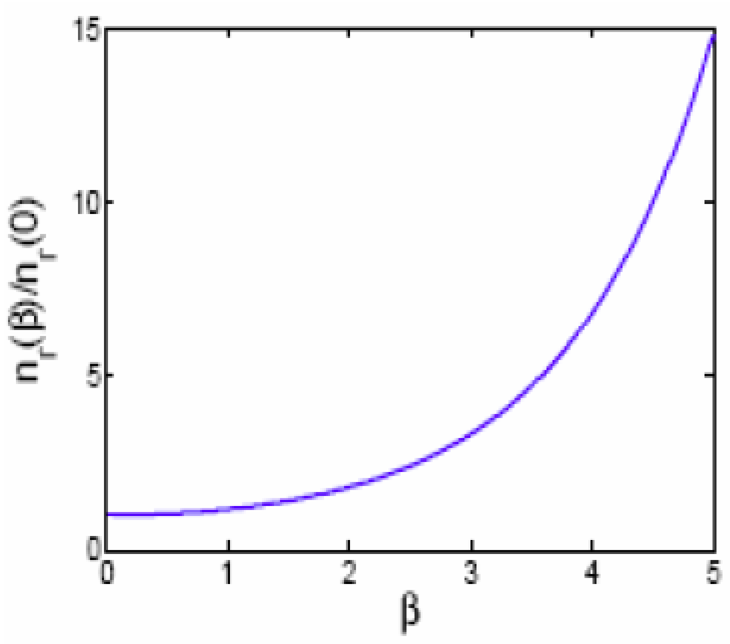
Non-Linear R_{BCS}

$$n(J) = \frac{n_{eq}}{2}\int_0^{\pi} \exp(p_F v_s \cos\theta / k_B T)\sin\theta\, d\theta$$

$$= n_{eq}\frac{\sinh\beta}{\beta} \qquad \text{where } \beta(x,t) = \beta_0 \exp(-x/\lambda)\cos\omega t$$

$$\beta_0 = \frac{v_s p_F}{k_B T} = \frac{\pi}{2^{3/2}}\frac{H_0}{H_c}\frac{\Delta}{k_B T} \qquad \text{Current driving parameter}$$

For T = 2 K and T_c = 9.2K, the parameter β_0 varies from 0 at H_0 = 0 to 8 at H_0 = 160 mT



Non-Linear R_{BCS}

$q(t) = \int \sigma(v_s) E^2(x,t)\mathrm{d}x$, where $\sigma(v_s) = \sigma_{\mathrm{BCS}} n(v_s)/n_{\mathrm{eq}}$ $\quad E(x,t) = -\lambda \dfrac{\partial\left(B_0 e^{-x/\lambda}\cos\omega t\right)}{\partial t}$

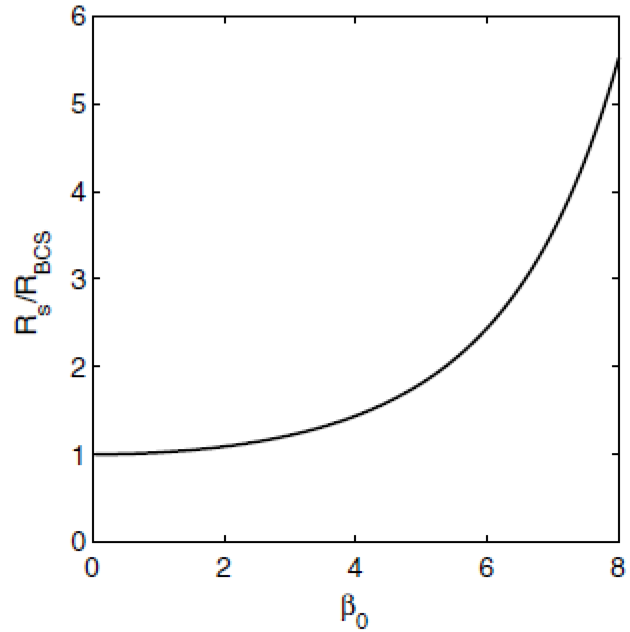
$$\frac{q}{H_0^2} = 2R_{\mathrm{BCS}} \sin^2 \omega t \int_0^{\infty} \mathrm{e}^{-2x/\lambda} \frac{\sinh(\beta_{\mathrm{a}} \mathrm{e}^{-x/\lambda})}{\lambda \beta_{\mathrm{a}} \mathrm{e}^{-x/\lambda}} \mathrm{d}x$$

Integrating and averaging over the RF period we obtain $R_{BCS}^{nl}(H_0) = 2\langle q\rangle / H_0^2$

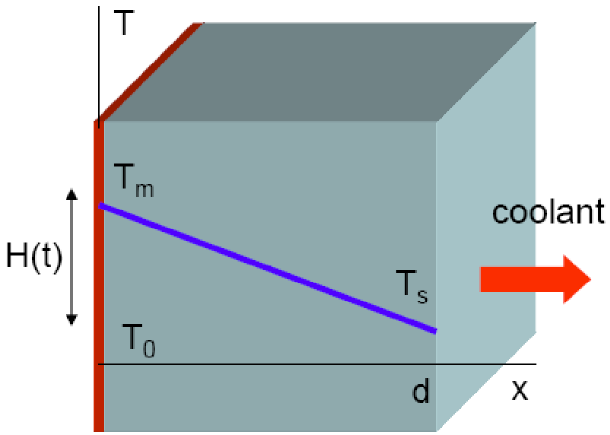
$$\boxed{R_{BCS}^{nl} = \frac{8R_{\mathrm{BCS}}}{\pi\beta_0^2} \int_0^{\pi} \sinh^2\left(\frac{\beta_0}{2}\cos\tau\right) \tan^2\tau \,\mathrm{d}\tau} \qquad \tau = \omega t$$

For $\beta_0 \ll 1$

$$\boxed{R_{BCS}^{nl} \cong \left[1 + \frac{\pi^2}{384}\left(\frac{\Delta}{T}\right)^2 \left(\frac{H_0}{H_c}\right)^2\right] R_{\mathrm{BCS}}}$$



$R_S(H_0)$: Thermal Feedback



$$\frac{\partial}{\partial x}\kappa(T)\frac{\partial T}{\partial x}+\frac{1}{2}H_\omega^2 R_s(T_m)\delta(x)=0$$

Instead of numerically solving this ODE, one can solve much simpler equations for T_m and T_s

$$h_K(T_s,T_0)(T_s-T_0)d=\int_{T_s}^{T_m}\kappa(T)$$

$$\frac{1}{2}R_s(T_m,H_0)H_0^2=h_K(T_s,T_0)(T_s-T_0)d$$

- Numerically solve for $T_m(H_0)$ and $T_s(H_0)$
- Calculate $R_s[T_m(H_0),H_0]$

$$\frac{1}{2}R_s(T_m,H_0)H_0^2=\frac{(T_m-T_0)h_K\kappa}{\kappa+d\,h_K}$$

- Solve for $T_m(H_0)$
- Calculate $R_s[T_m(H_0),H_0]$

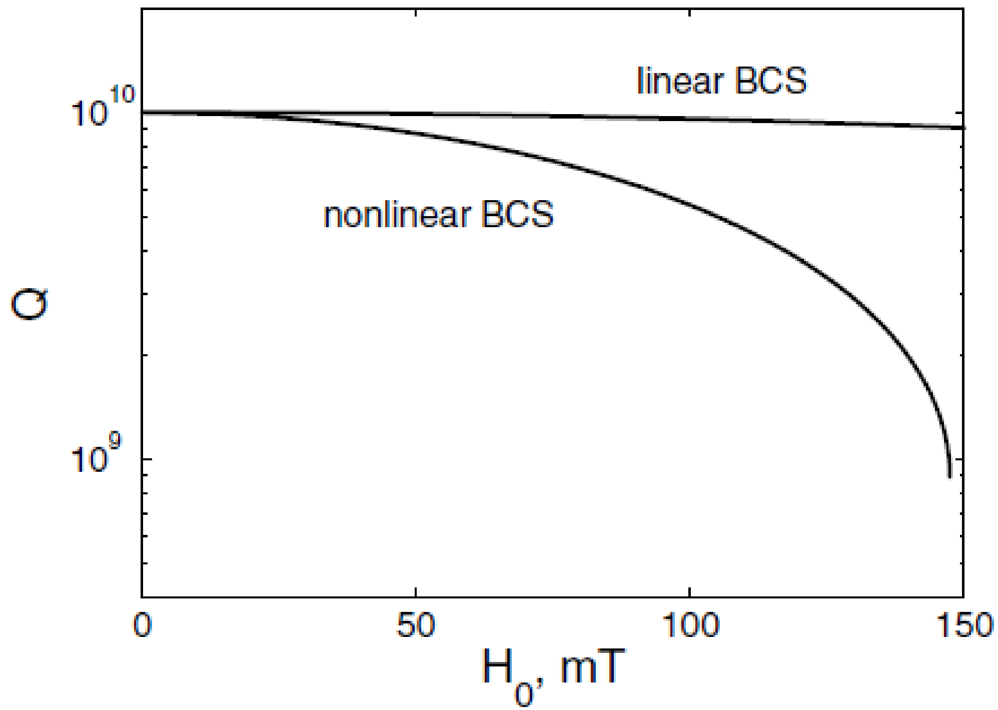
$R_S(H_0)$: Thermal Feedback

$$R_{BCS}(T_m) = \frac{A\omega^2}{T_m} e^{-\Delta/k_B T_m}$$

$$R_s = R_{BCS} + R_{res}$$

or

$$R_s = R^{nl}{}_{BCS} + R_{res}$$



$T_0 = 2$ K, d = 3 mm, κ = 20 W/m K, $h_K = 5$ kW/m^2 K, $R_{BCS} = 10$ nΩ and $Q(0) = 10^{10}$

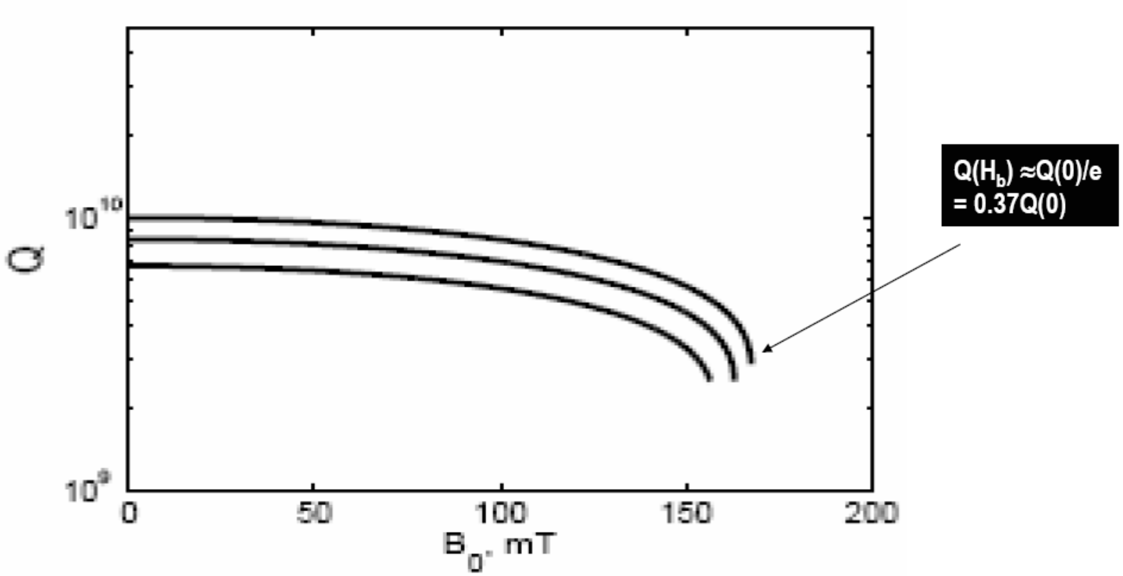
Breakdown Field

Quench will occur at $T_m = T_b$ at which $H_0(T_m)$ is maximum

Assuming linear R_{BCS}

$$T_m - T_0 \approx \frac{T_0^2}{\Delta} = \frac{T_0^2}{1.86\,T_c} = 0.23\ K$$

$$H_b^2 \cong \frac{2 h_K \kappa k_B T_0^2}{e(\kappa + d\,h_K)\Delta R_s(T_0)}$$



$H_b \cong 200$ mT

For T_0=2K, d = 3 mm, κ = 20 W/m K, h_K = 5 kW/m² K, Δ/k_B = 17.7K, R_s(2K) = 20 nΩ

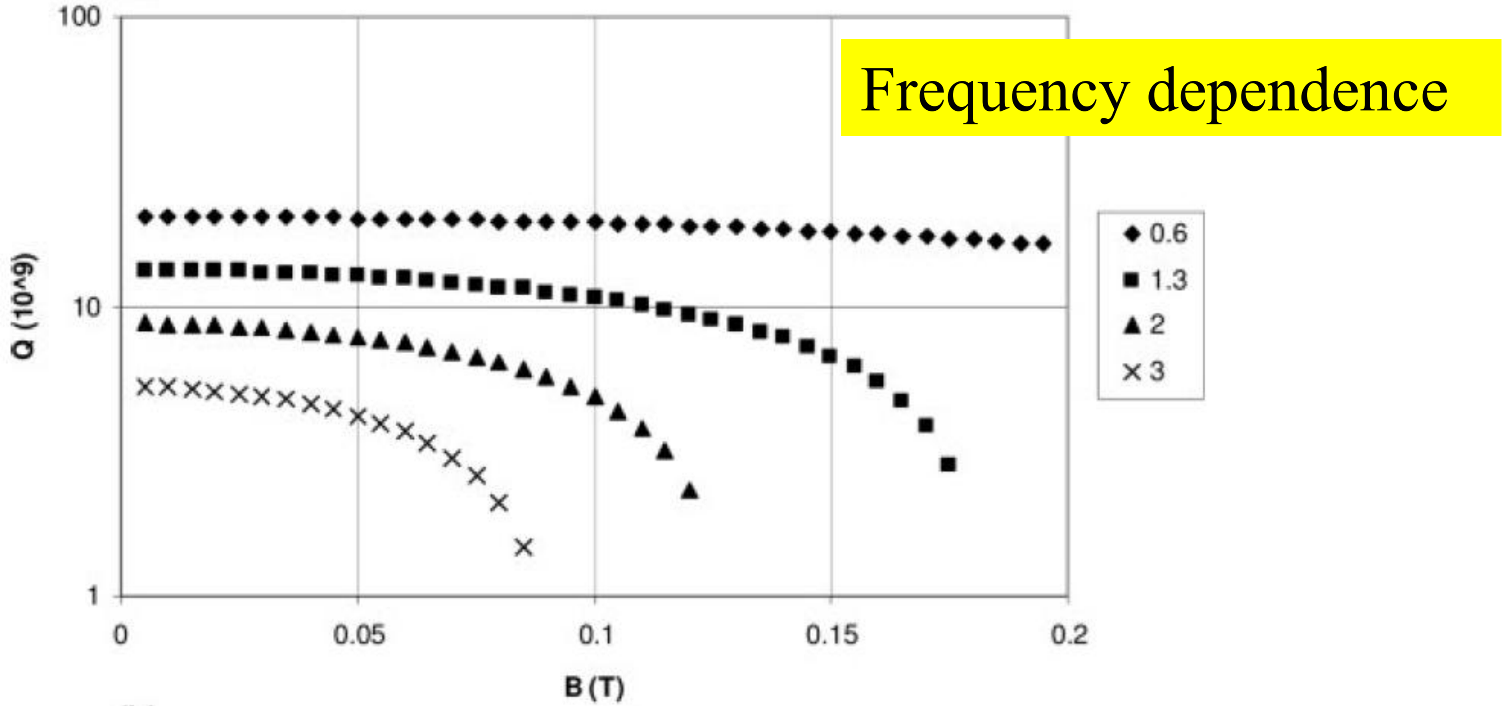
$R_S(H_0)$: Parameters Dependence

Linear R_{BCS}

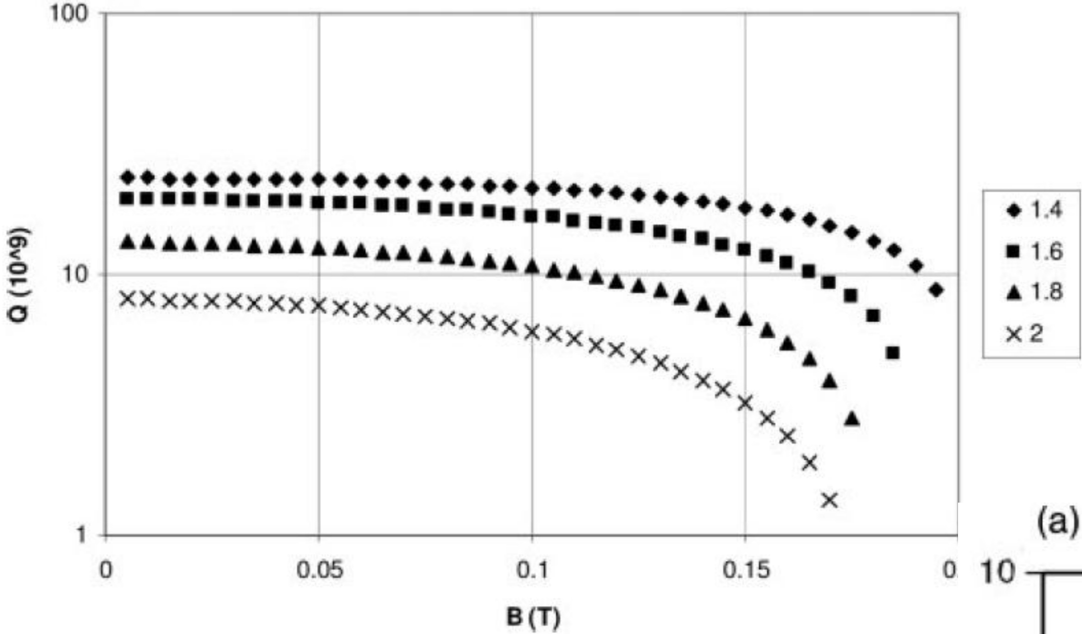
Baseline parameters

RF frequency, f	1.3 GHz
Helium bath temperature, T_b	1.8 K
Residual resistance, R_0	10 nΩ
Wall thickness, d	3 mm
Residual resistivity ratio, RRR	300
Phonon mean free path, l	0.1 mm
Kapitza resistance	Annealed Nb

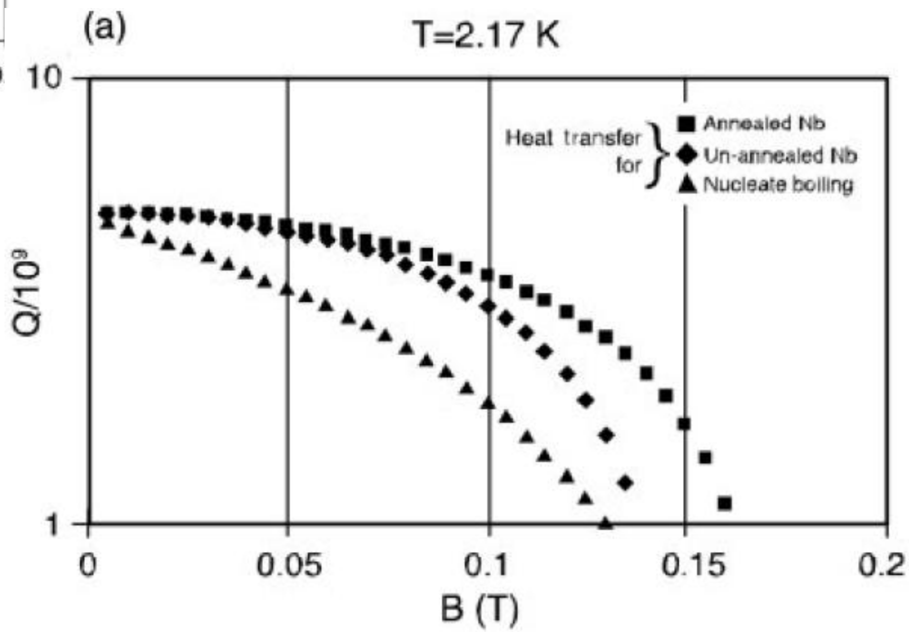
determine κ(T)



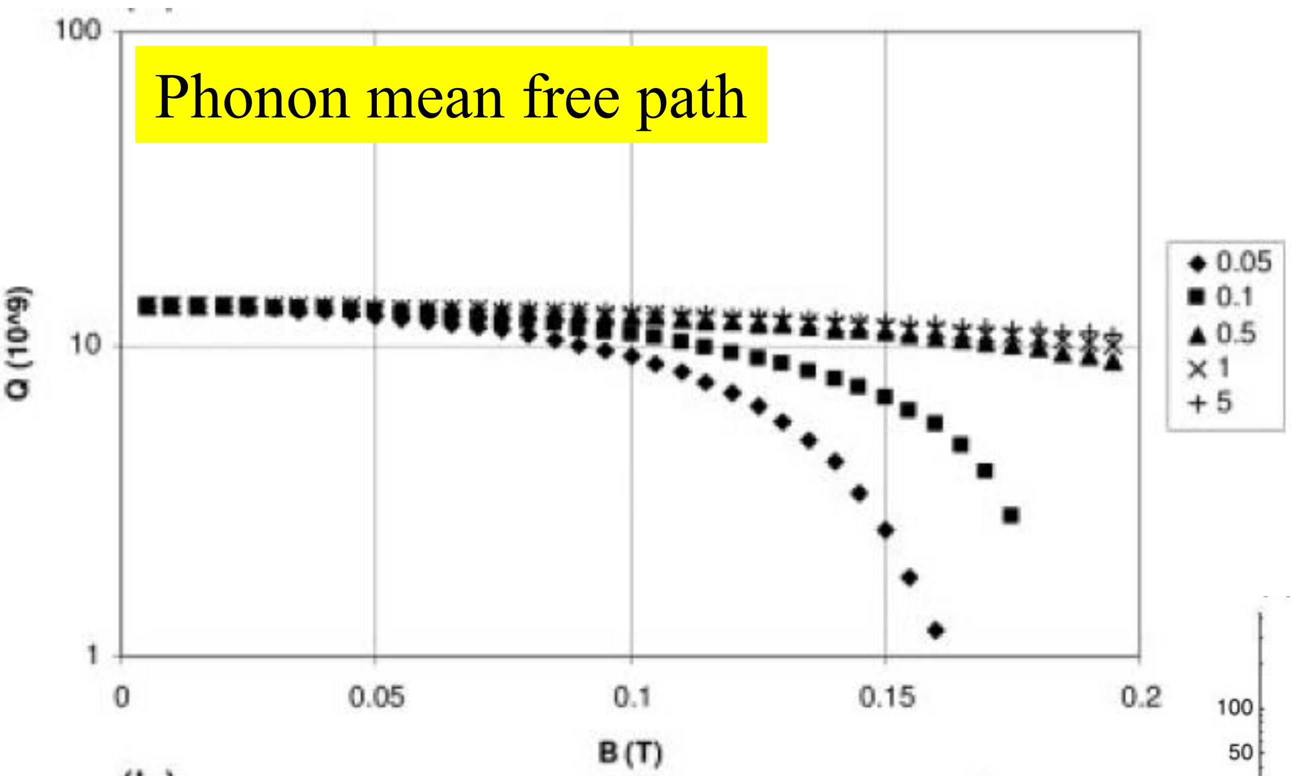
$R_S(H_0)$: Parameters Dependence

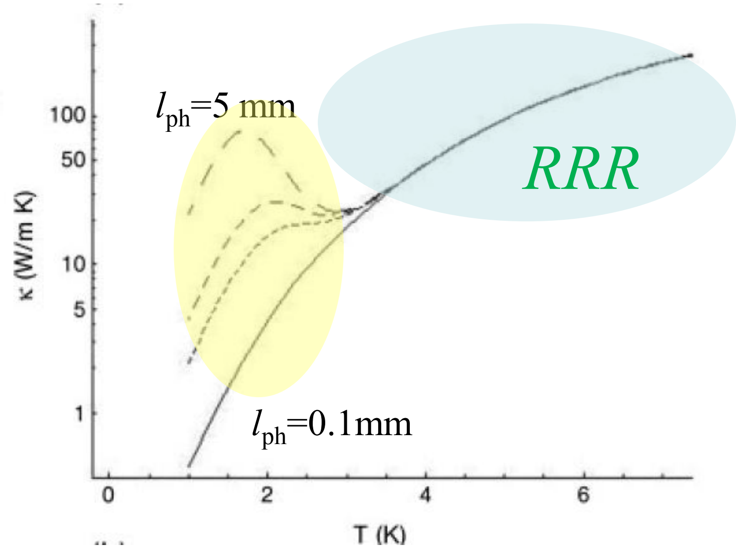


He bath temperature

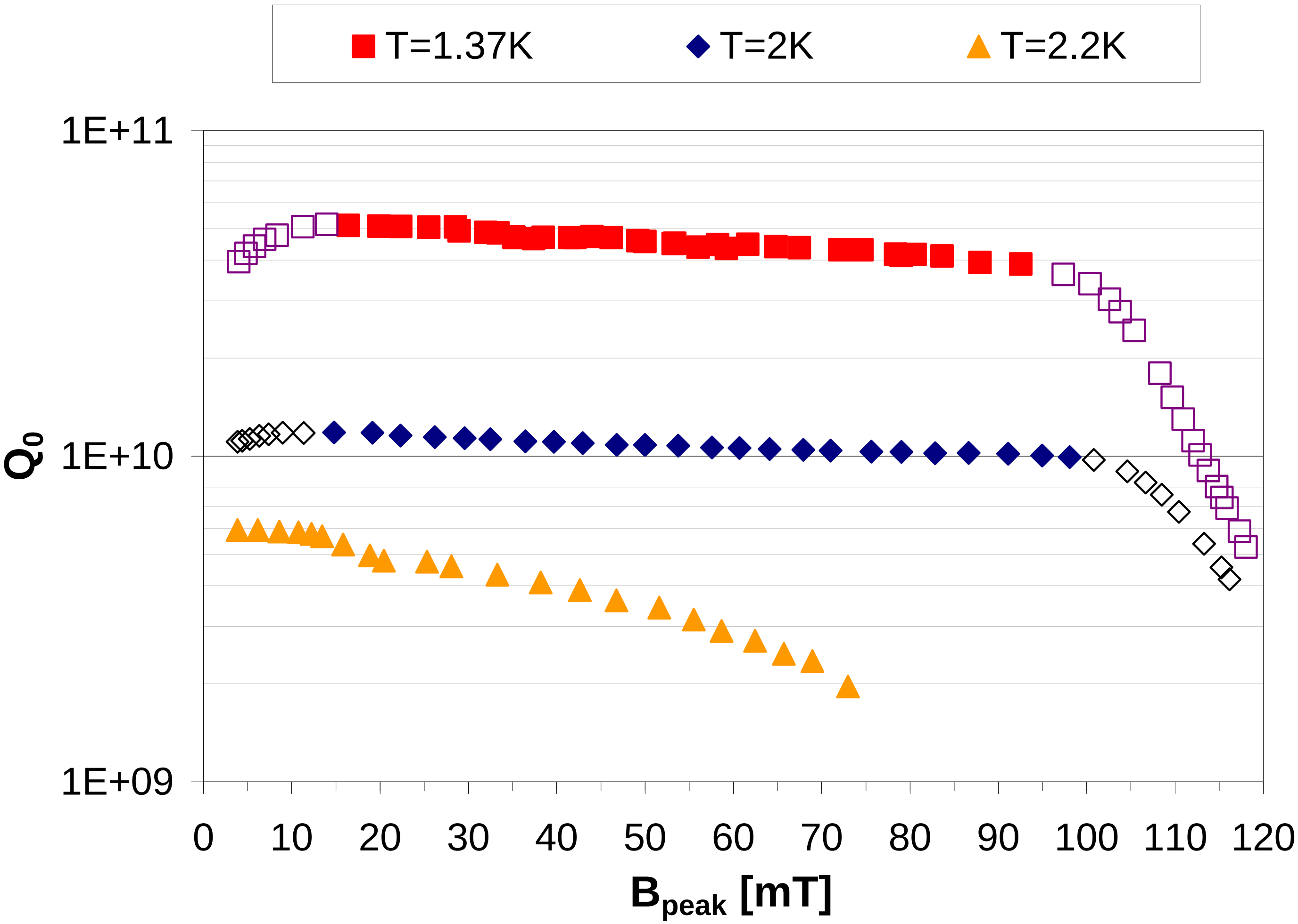


$R_S(H_0)$: Parameters Dependence



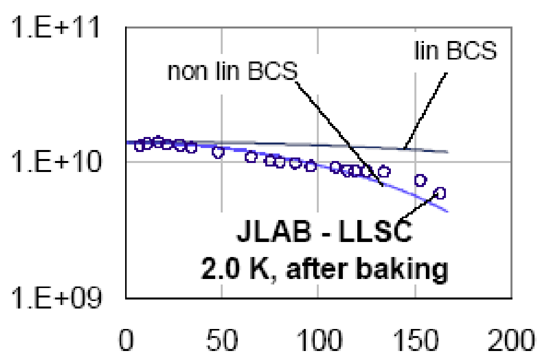


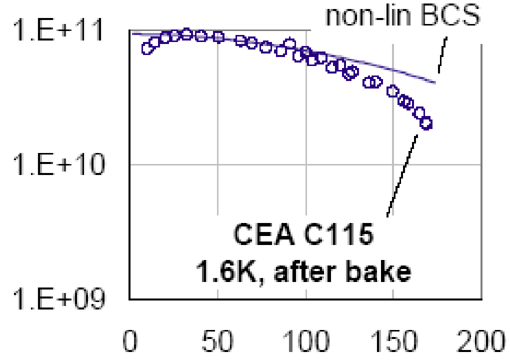
$R_S(H_0)$: Data

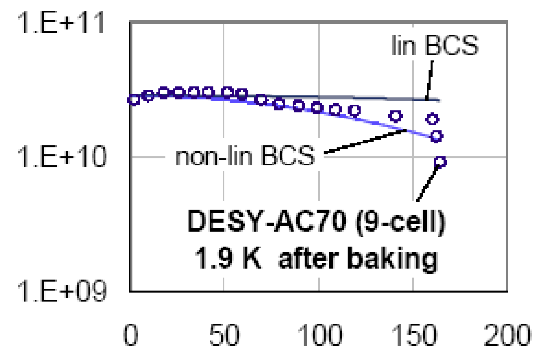


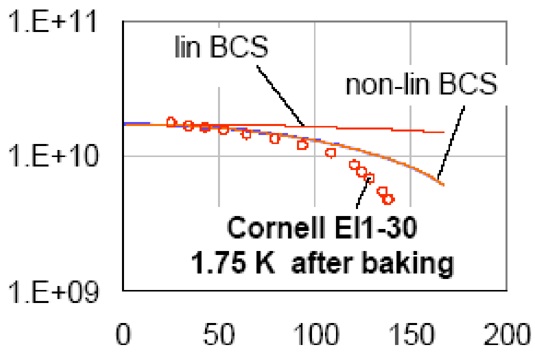
	γ
1.37 K	1.85
2 K	0.94
2.2 K	13.7

Models Comparison with Data



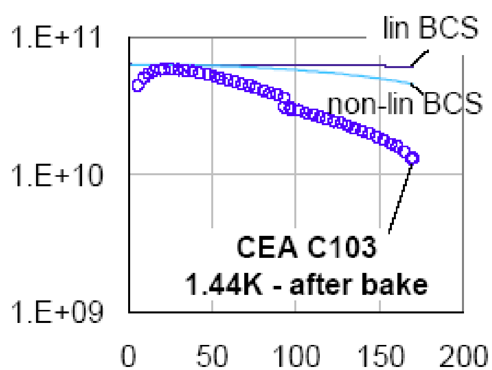


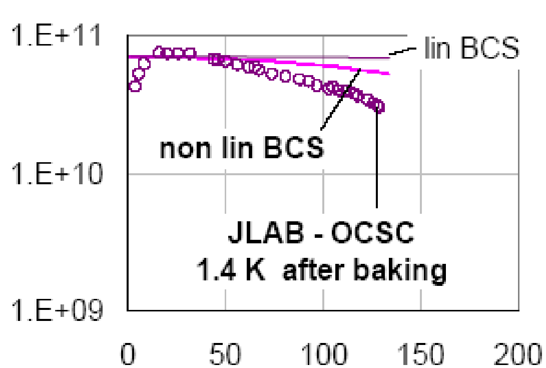


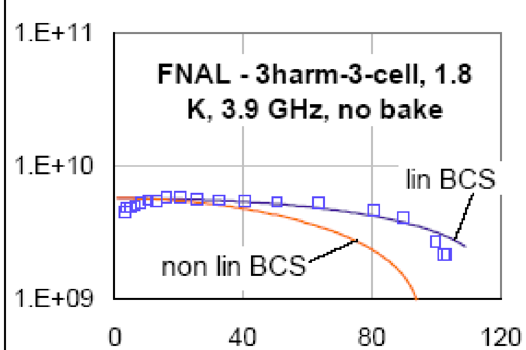


Q-data after baking for Jlab (1.5 GHz, BCP, single cell), CEA (1.3 GHz, EP, single cell), Cornell (1.3 GHz, BCP, single cell) and DESY AC70 (1.3 GHz, 9-cell, EP) are better fitted with the non-linear BCS.

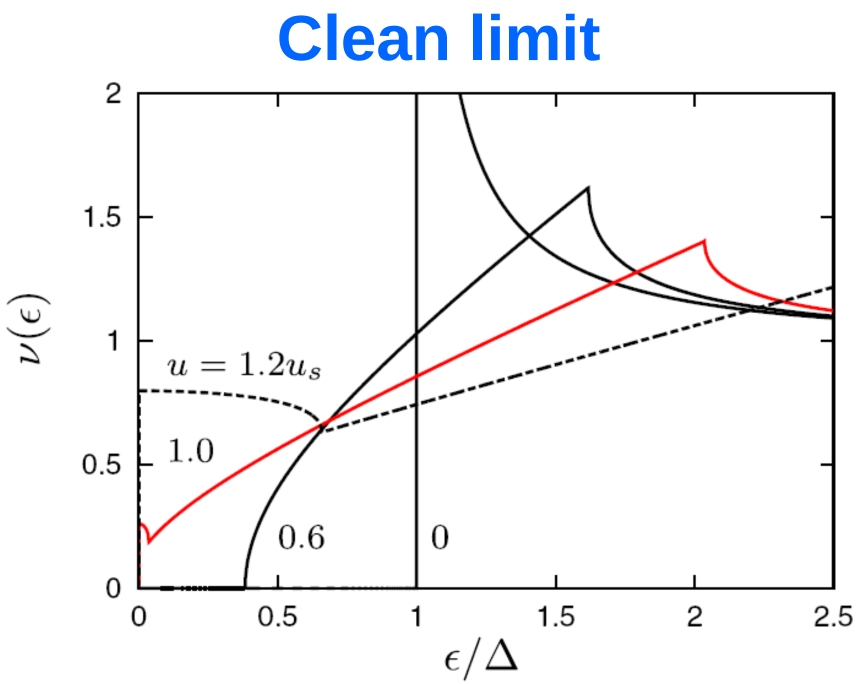
Exceptions: very low temp, high f Fnal (3.9 GHz, BCP)

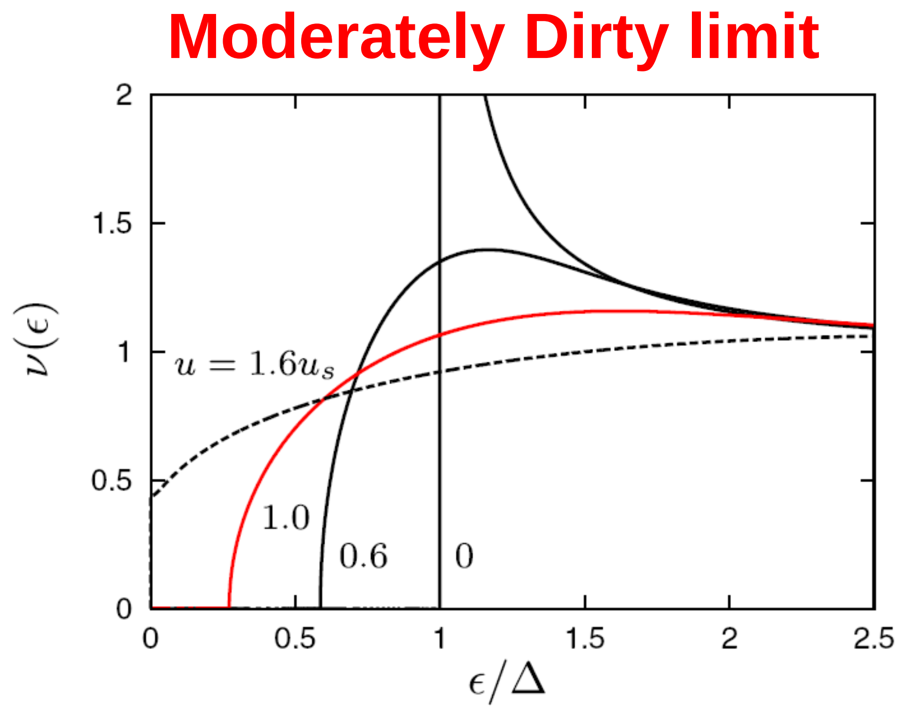






Effect of Impurities on R_s at High Field

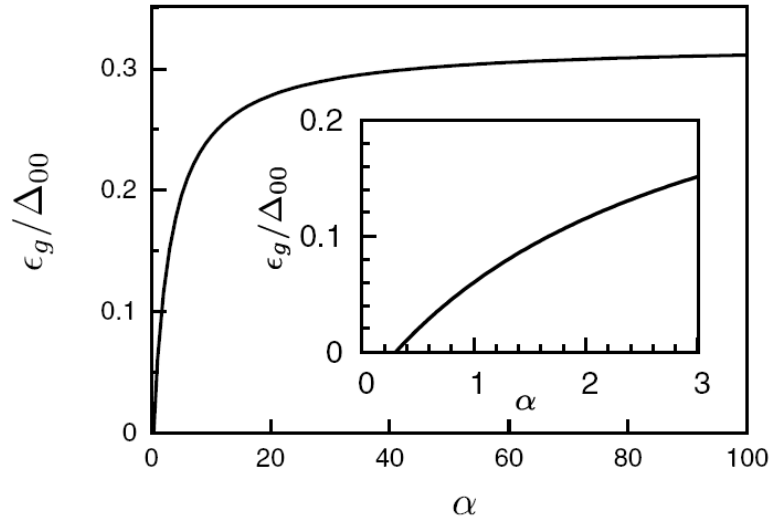




- Unlike in the moderately dirty limit, in a clean SC the quasiparticle density of states become that of a normal-conductor (gapless) at $H < H_{sh}$

Effect of Impurities on R_s at High Field

$\varepsilon_g(\alpha)$ at $H=H_{sh}$

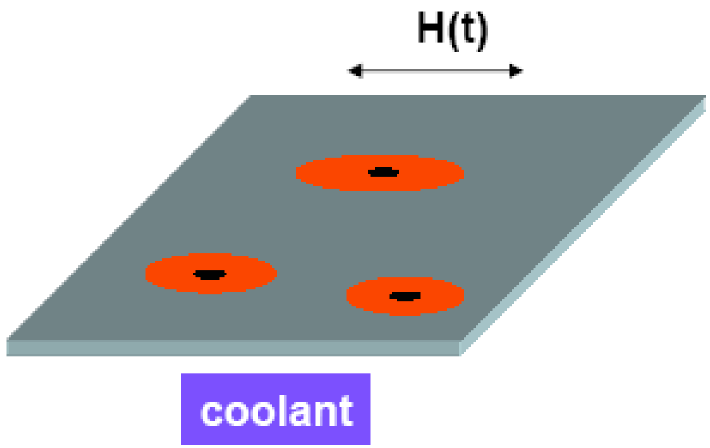


$$\alpha = \frac{\pi\xi_0}{\ell}$$

$$R_s(H) \propto \exp(-\varepsilon_g(H)/kT)$$

Impurities in the top ~40nm layer of Nb can decrease the non-linearity of R_s at high fields

Effect of Hot-Spots



Regions of radius r_0 where A(x,y) or H(x,y) is locally enhanced (impurities, GBs, thicker oxide patches, field focusing near surface defects, local vortex penetration, etc.)

$$\kappa\nabla^2 T - \tilde{\alpha}(T)(T - T_0) + q(T, H, r) = 0$$

T(x,y) = T_s+ δT(x,y), where T_s satisfies the uniform heat balance $\alpha(T_a)(T_a - T_0) = q_0(T_s, H)$, and δT(x,y) is a disturbance due to defects:

$$\tilde{\alpha} = \frac{h_K}{1 + d\,h_k/\kappa}$$

$$\kappa\nabla^2 \delta T - \left(\tilde{\alpha} - \frac{\partial q}{\partial T}\right)\delta T + \delta q = 0$$

Excess heat generation $\delta q = H^2 \delta R/2 + R\delta H^2/2$ in the region of radius r_0

Temperature Distribution

$$\delta T(r) = \frac{\Gamma}{2\pi\kappa d} K_0\left(\frac{r}{L}\right), \qquad r > r_0 \qquad \Gamma = \int \delta q(x,y)\,dxdy$$

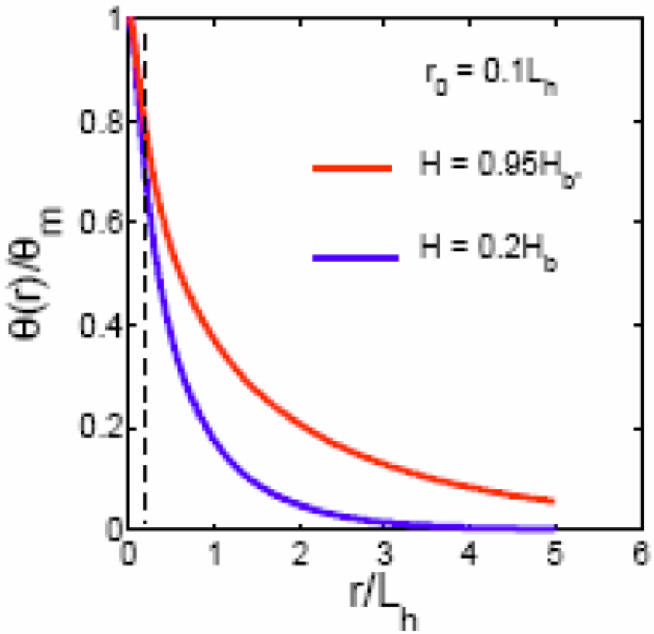
A hotspot produces a temperature disturbance $\delta T(r)$, which spreads along the cavity wall over the distance $L \gg r_0$ greater than the defect size

$$L = \frac{L_h}{\sqrt{1 - f(H/H_b)}}, \qquad L_h = \sqrt{\frac{d\kappa}{\tilde{\alpha}}}$$

Where $f(H/H_b) = (\partial q/\partial T)/\alpha \to 1$ at $H \to H_b$

$$L \cong \frac{L_h}{\sqrt{1-(H/H_{b0})^2}}$$

L increases with H and diverges at the uniform breakdown field, $H = H_b$



Averaged Surface Resistance

$$\eta = \frac{r_0^2}{L_h^2}\left(\frac{\delta A}{A} + \frac{\delta H^2}{H^2}\right)$$

Quantifies extra power generated by the defect due to
- local enhancement of BCS factor A
- local field enhancement

Considering weak hot-spots ($\eta \ll 1$):

Extra dissipation in a hotspot:

$$\tilde{\alpha}\int \delta T(x,y)dxdy = \frac{\pi}{2}L^2H^2\eta_s R_s(T_s)$$

Global surface resistance with the account of non-overlapping hotspots:

$$\tilde{R}_s(T,H) = R_s(T,H)\left[1 + \frac{g}{1-(H_0/H_{b0})^2}\right]$$

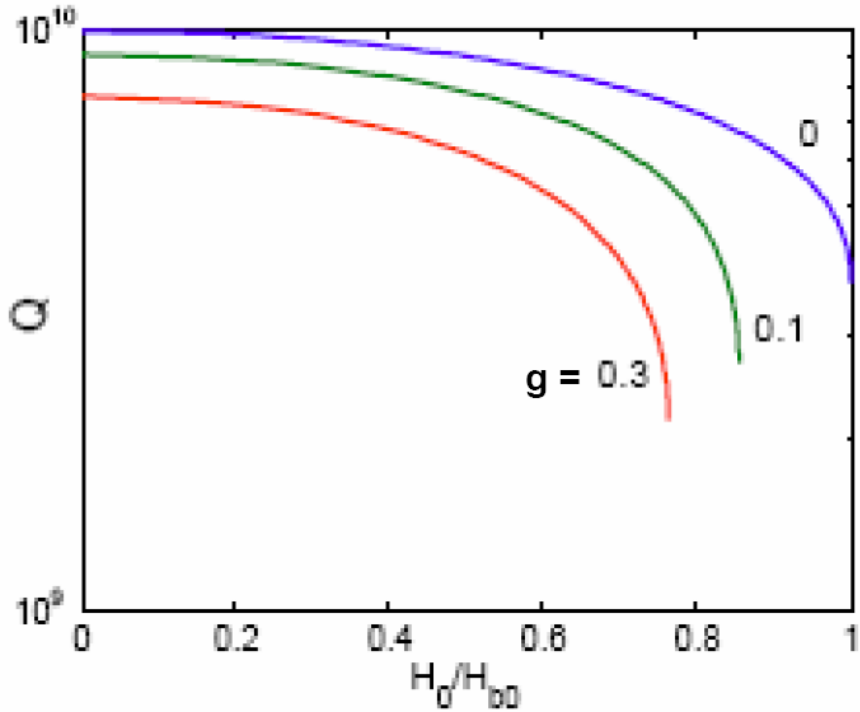
$$g = <\eta> \frac{\pi L_h^2}{\ell_s^2}$$

$R_s(H)$ is the uniform surface resistance, ℓ_s is the mean spacing between hotspots, H_{b0} is the uniform breakdown field,

Nonlinear contribution to the global R_s due to expansion of hotspots with H.

Q_0(H) for Linear BCS + Hot-Spots

Inserting the expression of R_s(H) from the previous slide in the "analytical" thermal balance equation we obtain:



$$\frac{2H_0^2}{H_{b0}^2} = 1 + g + u - \sqrt{(1+g+u)^2 - 4u}$$

$$Q = \frac{Q(0)\exp(-\theta)}{1 + g/[1-(H_0/H_{b0})^2]}$$

$$\theta = (T_m - T_0)\Delta/k_b T_0^2$$

$$u(\theta) = \theta\exp(1-\theta)$$

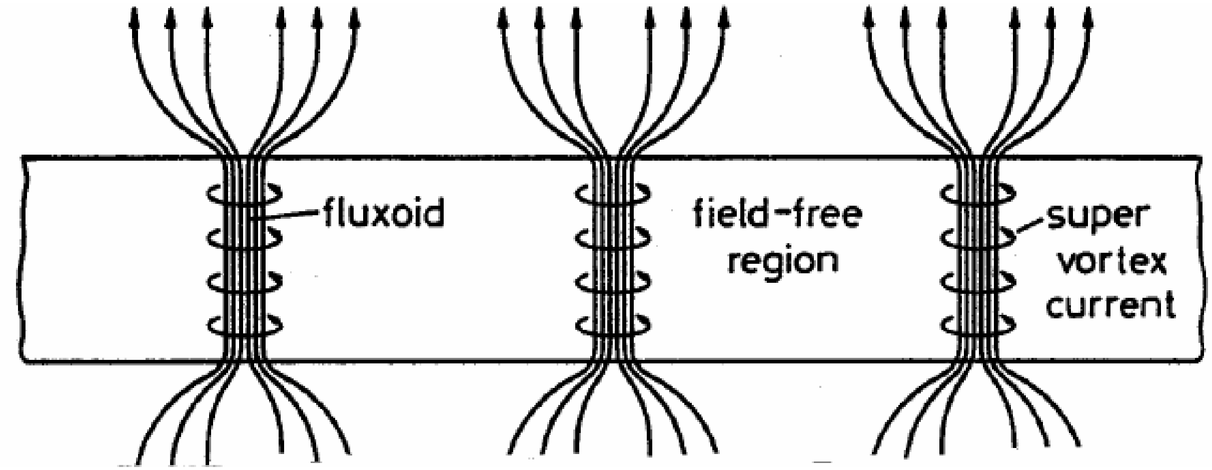
$H_0(\theta)$ is maximum at $\theta = 1$, which defines the global breakdown field H_b reduced by weak hotspots ($g \ll 1$):

$$H_b \cong \left(1 - \frac{\sqrt{g}}{2}\right) H_{b0}$$

Summary

- Several mechanisms lead to a similar increase of R_s for increasing RF fields:
  - Thermal impedance
  - Hot-spots
  - Intrinsic R_{BCS} non-linearity
- All this mechanisms cause the increase of $R_s(H)$ by thermal feedback
- Experiments should be designed to test the influence of each contribution

Vortices in Type-II Superconductors

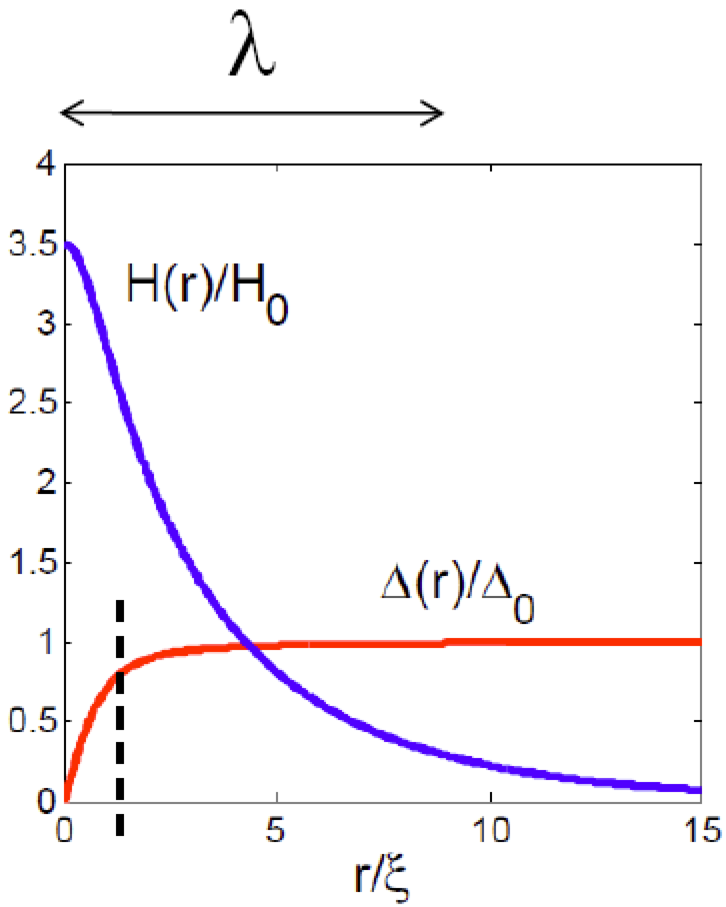


- The magnetic field inside a type-II superconductor is quantized. The unit flux quantum (*fluxon*) is

$$\Phi_0 = \frac{h}{2e} = 2.07 \times 10^{-15}\,\mathrm{Wb}$$

- Supercurrent flows around the fluxon to shield the sc (current vortex)
- The fluxon with the associated current vortex is called *fluxoid*

Vortices in Type-II Superconductors



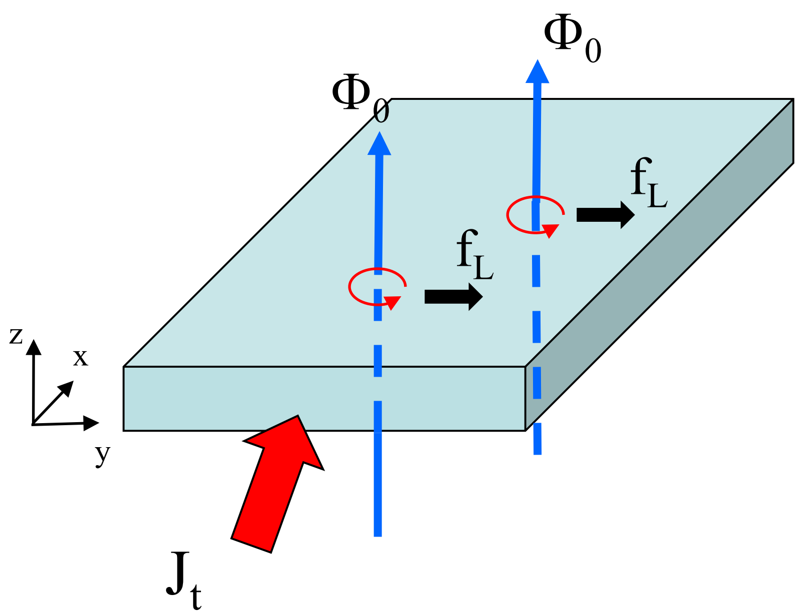
Non-superconducting core of radius ~ξ surrounded by circulating currents spread over ~λ.

Nb: $\lambda \cong 40$ nm, $\xi \cong 32$ nm

Nb_3Sn: $\lambda \cong 90$ nm, $\xi \cong 3$ nm

Energy decrease due to loss of diamagnetism > Energy increase due to loss of condensation energy

Lorentz Force



$$\mathbf{f}_L = \mathbf{J}_s \times \Phi_0$$

$\mathbf{J}_s$: total supercurrent density due to all other vortices and any net transport current at the location of the core of the vortex

Flux lines tend to move transverse to the current

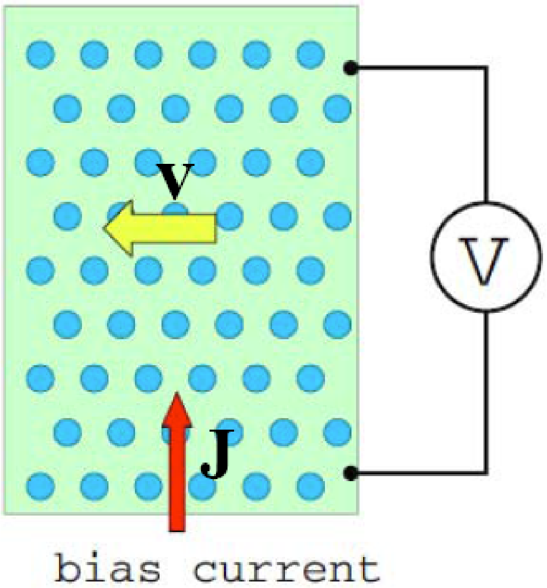
Flux lines moving with velocity **v** induce an electric field parallel to **J**: acts like a resistive voltage

$$\mathbf{E} = \mathbf{B} \times \mathbf{v}$$

Flux line motion causes dissipation

$$P = \mathbf{J} \cdot \mathbf{E}$$

Flux Flow



- Viscous flux flow of vortices driven by the Lorentz force

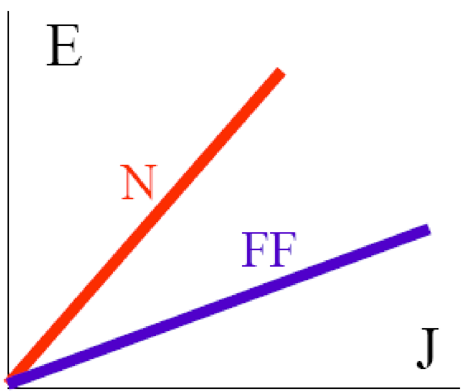
$$\eta\vec{v} = \phi_0[\vec{J}\times\hat{n}], \qquad \vec{E} = [\vec{v}\times\vec{B}] \quad \textbf{Faraday law}$$

This yields the liner flux flow E-J dependence:

$$\vec{E} = \rho_f\vec{J}, \qquad \rho_f = \rho_n B/B_{c2}$$

Volume fraction of normal vortex cores

Vortex viscosity η is due to dissipation in the vortex core and can be expressed in terms of the normal state resistivity ρ_n:



$$\boxed{\eta = \phi_0 B_{c2}/\rho_n}$$

For E = 1μV/cm and B = 1T, the vortex velocity

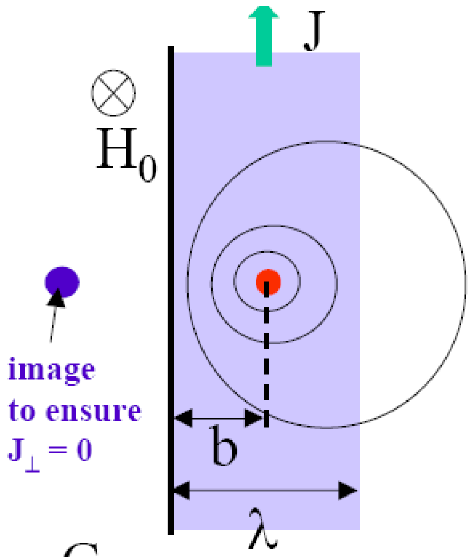
v = E/B=0.1 mm/s

Vortices in Type-II Superconductors

- Vortex penetration
- Vortex pinning

Implications on RF losses

Surface Barrier

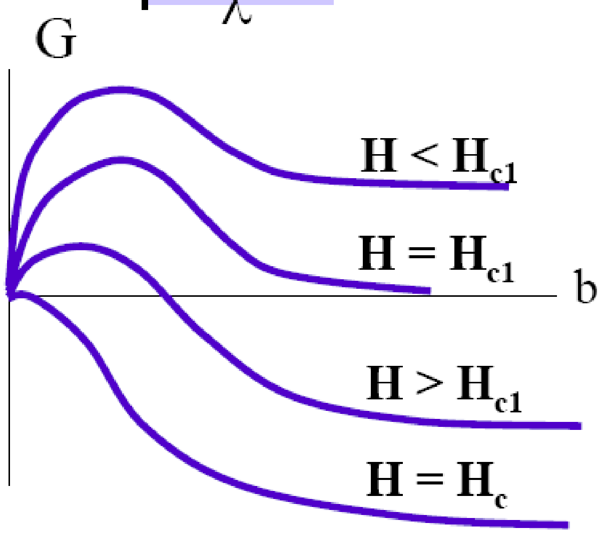


Two forces acting on the vortex at the surface:

- Meissner currents push the vortex in the bulk
- Attraction of the vortex to its antivortex image pushes the vortex outside

Thermodynamic potential G(b) as a function of the position b:

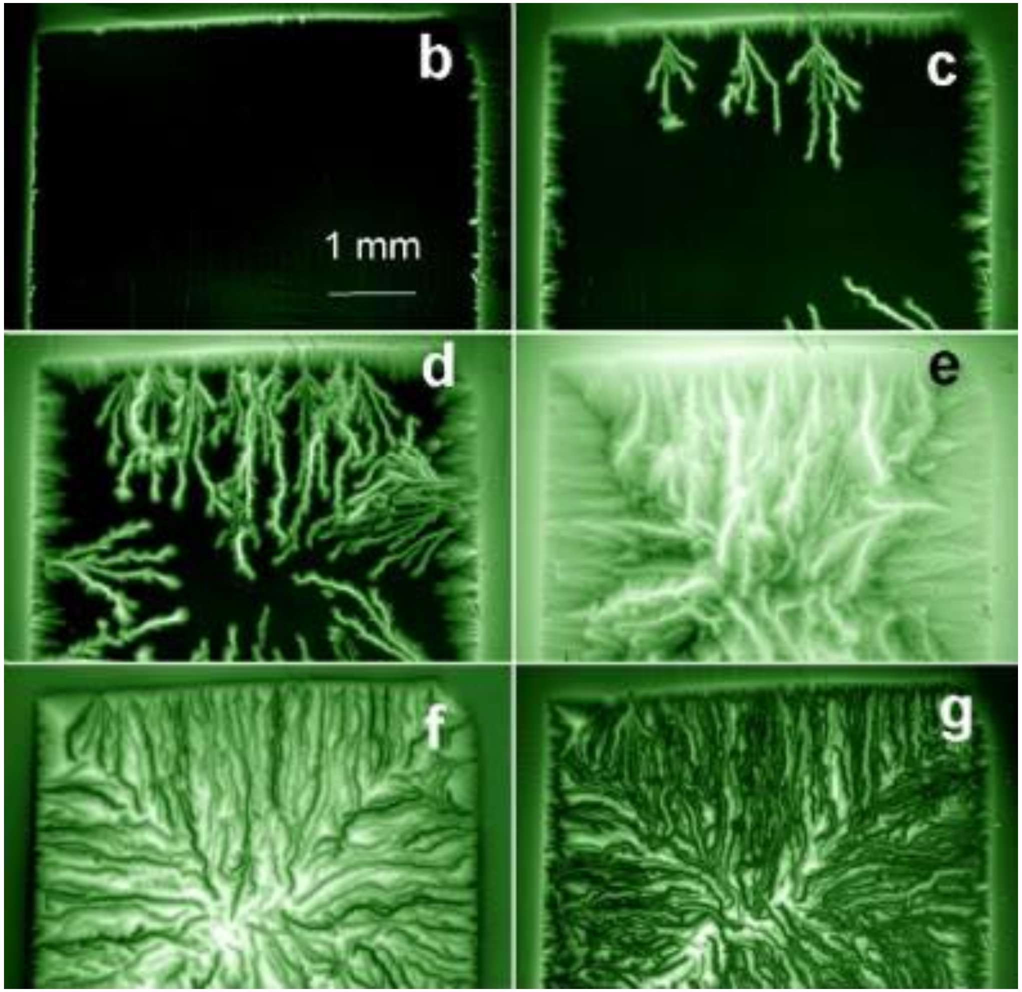
$$G(b) = \phi_0 [\underbrace{H_0 e^{-b/\lambda}}_{\text{Meissner}} - \underbrace{H_v(2b)}_{\text{Image}} + H_{c1} - H_0]$$



Vortices have to overcome the surface barrier even at $H > H_{c1}$ (Bean & Livingston, 1964)

Surface barrier disappears only at the overheating field $H = H_c$ at which the surface J reaches the depairing current density

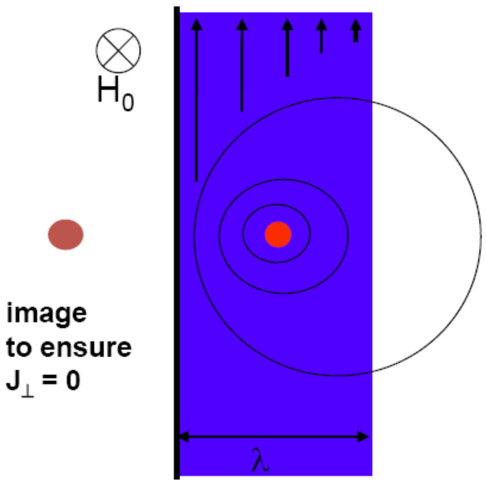
Vortex Penetration into SC



Magneto-optical studies of a c-oriented MgB_2 film show that below 10 K the global penetration of vortices is dominated by complex dendritic structures abruptly entering the film.

Figure shows magneto-optical images of flux penetration (image brightness represents flux density) into the virgin state at 5 K. The respective images were taken at applied fields (perpendicular to the film) of 3.4, 8.5, 17, 60, 21, and 0 mT.

Vortex Penetration in RF Field



Vortex entry at the local penetration field $B_v = \phi_0/4\pi\lambda\xi_s \approx 0.71B_c$

$$\eta\dot{u} = \frac{\phi_0 H_0}{\lambda} e^{-u/\lambda} \sin\omega t - \frac{\phi_0^2}{2\pi\mu_0\lambda^3} K_1\left(\frac{2\sqrt{u^2+\xi^2}}{\lambda}\right)$$

Vortex eq. of motion

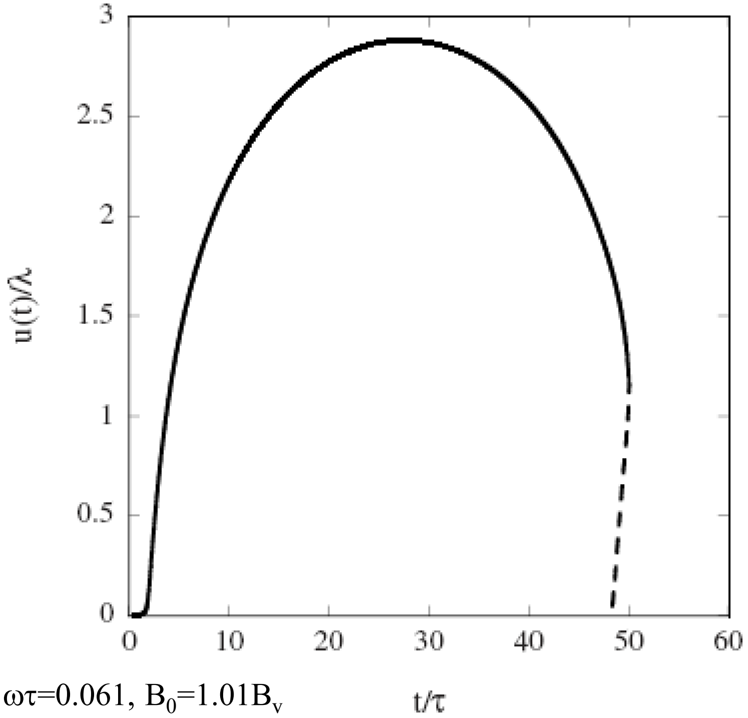
Vortex enters the surface at t=0 when $B > B_v$

At time $t = t_c$, $|B(t_c)| > |-B_v| \rightarrow$ an anti-vortex enters the surface while the vortex is at position u_c

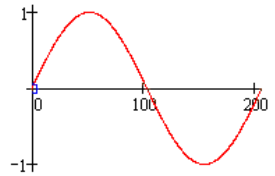
- Eqs. of motion of vortex and anti-vortex

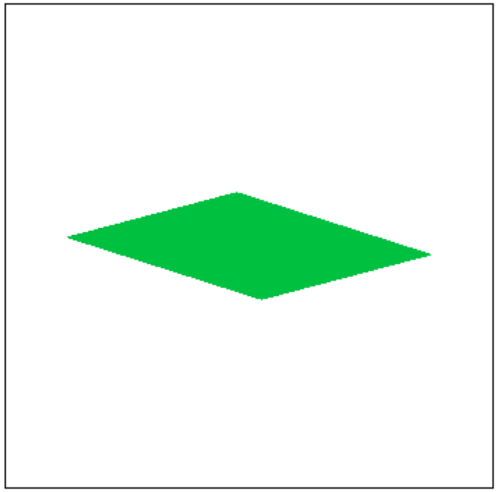
At time $t = t_a$ vortex and anti-vortex annihilate, $u_+(t_a) = u_-(t_a)$

Vortex Trajectory



ωτ=0.061, B_0=1.01B_v





M

Vortex Trajectory

- Only single-vortex penetration was considered: $B_v < B_0 < B_2$

$$B_2 \sin \omega t_2 - \frac{\phi_0}{\pi \lambda^2} K_1 \left(\frac{u_2}{\lambda} \right) = B_v$$

Defines the rf amplitude B_2 above which a second vortex would enter

- The time it takes for a vortex to move by a distance $\sim\lambda$ from the surface is:

$$\tau = \frac{\mu_0 \lambda^2 \eta_0}{\phi_0 B_v} \simeq \frac{2\mu_0 \lambda^3}{\rho_n \xi}$$

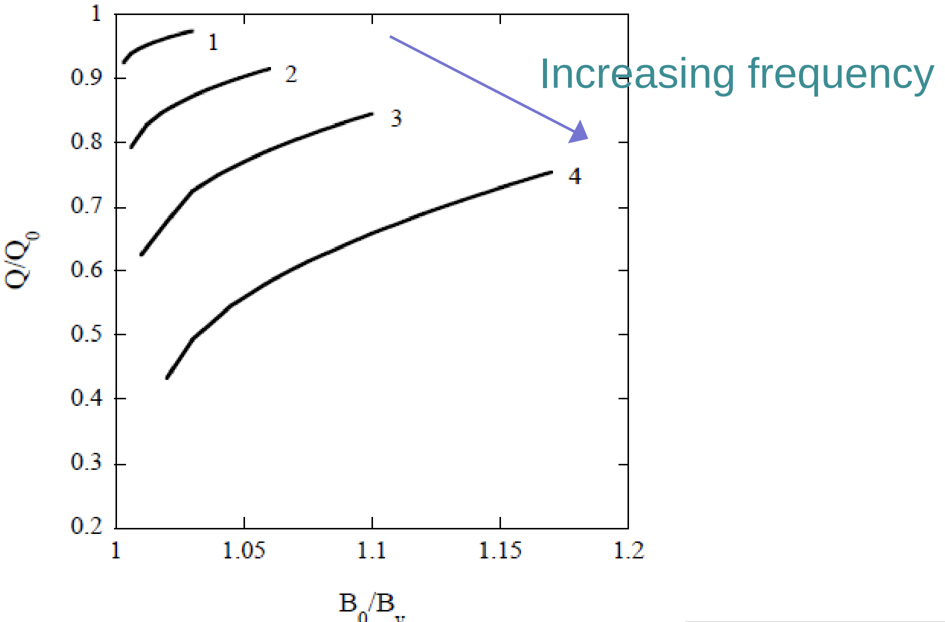
$\sim 4\times10^{-12}$ s for Nb

Dissipation due to vortex motion

The power $Q = (\omega\eta/2\pi)\oint v^2 dt$ dissipated due to the work of the viscous drag force is given by:

$$Q = \frac{\omega\eta}{\pi}\left[\int_{t_0}^{t_c} \dot{u}^2 dt + \int_{t_c}^{t_a} (\dot{u}_+^2 + \dot{u}_-^2) dt\right]$$

In the low-frequency limit: $\boxed{Q = 2\omega\phi_0 B_v/\pi\mu_0}$ ~1 W/m at 1 GHz



How fast is vortex entry?

$$v_m \sim \frac{\lambda}{\tau}$$

- For Nb: $v_m \sim 10$ km/s is greater than the speed of sound and exceeds the BCS pairbreaking velocity $v_c = 0.8$ km/s

- Radical change of the vortex due to nonequilibrium effects in the vortex core)
- Nonlinear viscous drag depending on the vortex velocity
- Dynamically generated vortex mass
- Vortex becomes very hot

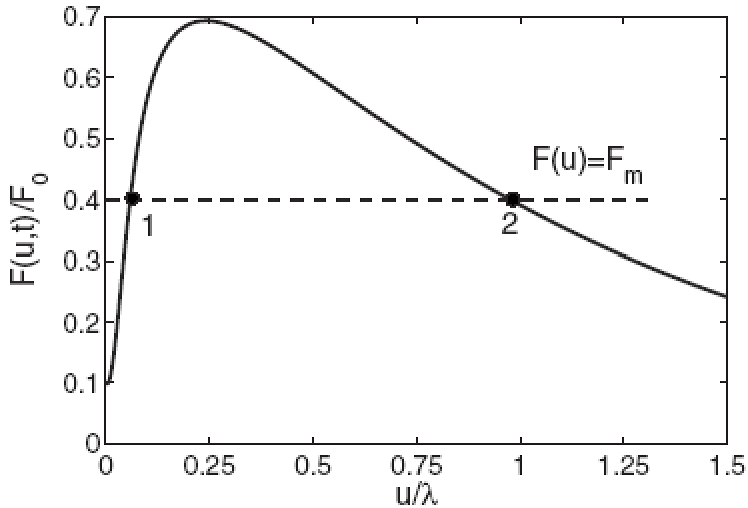
Larkin and Ovchinnikov: nonequilibrium effects in the vortex core decrease the drag coefficient η as v increases:

$$\eta(v) = \frac{\eta_0}{1 + v^2/v_0^2}$$

v_0: critical velocity < v_m

Jumpwise Vortex Penetration

$$\frac{\eta\dot{u}}{1+\dot{u}^2/v_c^2}=\underbrace{\frac{\phi_0 H_0}{\lambda}e^{-u/\lambda}\sin\omega t-\frac{\phi_0^2}{2\pi\mu_0\lambda^3}K_1\left(\frac{2\sqrt{u^2+\xi^2}}{\lambda}\right)}_{F(u,t)\,=\,\text{net e.m. force}}$$



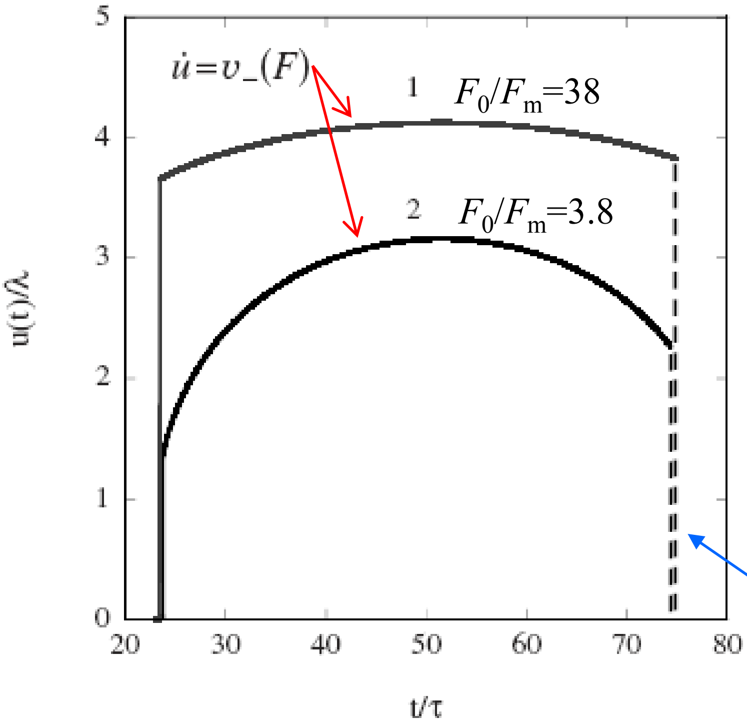
$F_m = \eta_0 v_0/2$ maximum viscous force

$F_0 = \phi_0 B_v/\mu_0\lambda$ maximum Lorentz force

$$\dot{u}=\frac{v_c F_m}{F(u,t)}\left[1\pm\sqrt{1-\frac{F^2(u,t)}{F_m^2}}\right]$$

The vortex first moves from $u=0$ to $u=u_1$, then jumps from point 1 to point 2, after which it moves continuously until the next jump and annihilation with the antivortex on the way back

Jumpwise Vortex Penetration



The positions of the jumps u_j at the corresponding times $t=t_j$ are determined by the equation $F(u_j, t_j)=F_m$

$$\pm\frac{\eta_0 v_0}{2}=\frac{\phi_0 B_0 e^{-u_j/\lambda}}{\mu_0\lambda}\sin\omega t_j-\frac{\phi_0^2}{2\pi\mu_0\lambda^3}K_1\left(\frac{2u_j}{\lambda}\right)$$

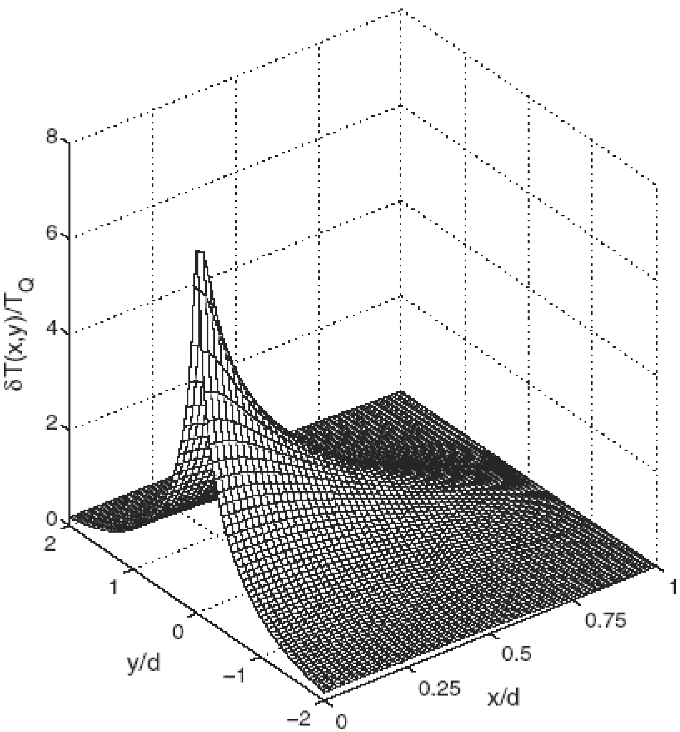
the antivortex jumps in and annihilates with the outgoing vortex before the vortex reaches the critical velocity $-v_0$

Temperature Around the Oscillating Vortex

The distribution of *T(r, t)* around a moving vortex is described by a thermal diffusion equation

$$C\dot{T} = k\nabla^2 T - \alpha(T - T_0) + \underbrace{\eta(T_m)v^2(t)f(x - u(t), y)}_{\text{dissipation in the vortex core}}$$

$f(r) = \pi^{-1}\xi_1^{-2}\exp(-r^2/\xi_1^2)$ core form factor



Steady-state solution for weak overheating:

$$\boxed{\begin{aligned} \delta T(\vec{r}) &= \frac{Q}{2\pi k}\ln\frac{\cosh(\pi y/2d) + \cos(\pi x/2d)}{\cosh(\pi y/2d) - \cos(\pi x/2d)}, \quad |r| >> r_0 \\ \delta T_m &= \frac{Q}{\pi k}\ln\frac{4d}{\pi r_0}, \qquad |\vec{r}| < r_0 \end{aligned}}$$

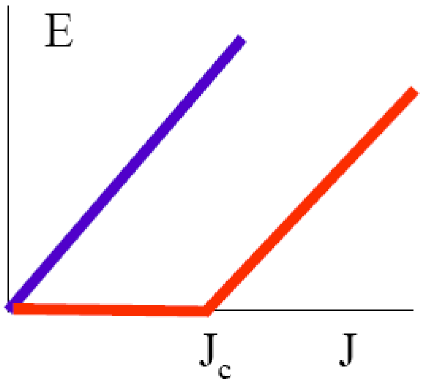
Q: average dissipated power due to vortex motion

- **For Nb: with k = 10 W/mK, f = 2 GHz, $Q \approx 4B_v\phi_o/\mu_o$ d = 3mm, r_0 = 100 nm, B_v = 150 mT, we get**

$\delta T_m \approx 0.6K$

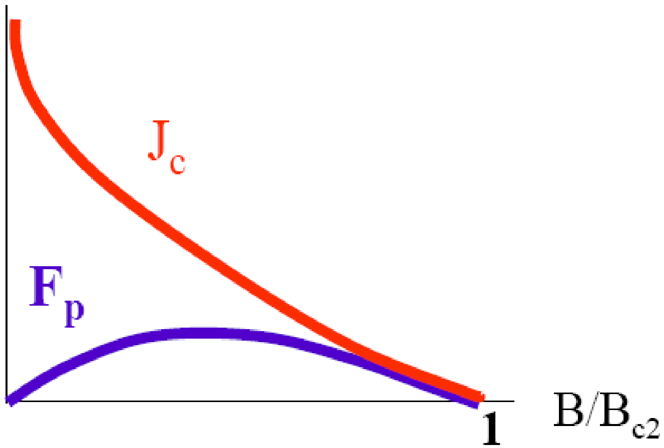
Pinning

- Ideal single crystals without defects have a finite flux flow resistivity and partial Meissner effect



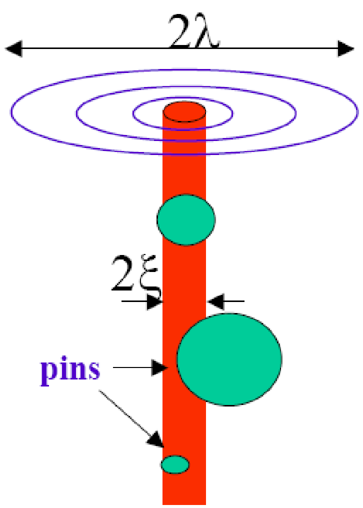
- Defects pin vortices restoring almost zero resistivity for J smaller than the critical current density J_c
- Unlike the thermodynamic quantities (T_c H_{c1}, H_{c2}), the value of J_c can be strongly sample dependent.
- Force balance condition per unit volume, where the pinning force F(T,B) vanishes at B_{c2}

$$BJ_c(T,B) = F_p(T,B)$$



- Pinning of vortex lines is determined by defect microstructure on different length scales $\xi < l < \lambda$.
- Competition between vortex-pin attraction and vortex-vortex repulsion

Core Pinning

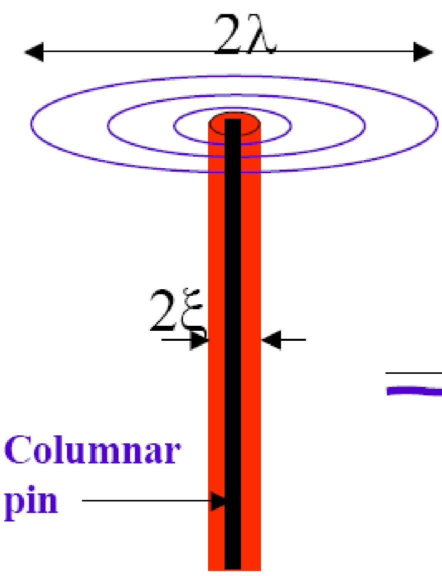


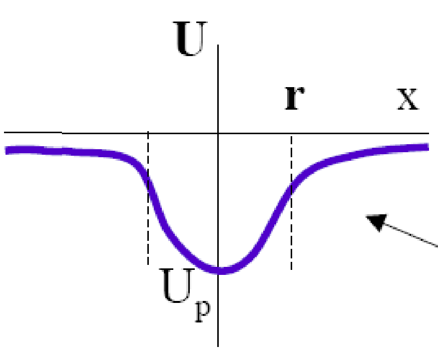
- Nonsuperconducting precipitates, voids, oxygen vacancies in HTS, etc.
- Columnar defects (radiation tracks, dislocations)
- Gain of a fraction of the vortex core line energy, $\varepsilon_0 = \pi\xi^2\mu_0 H_c^2/2$, if the core sits on a defect

- Pinning energy U_p and force f_p for a columnar pin of radius r:

$$U_p \approx \varepsilon_0 \frac{r^2}{\xi^2}, \qquad f_p \approx 2\varepsilon_0 \frac{r}{\xi^2}, \qquad r \ll \xi,$$

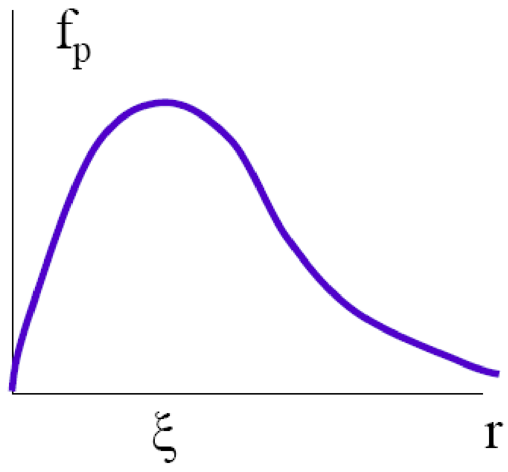
$$U_p \approx \varepsilon_0, \qquad f_p \approx \frac{\varepsilon_0}{r}, \qquad r > \xi$$





- For r << ξ, only a small fraction of the core energy is used for pinning, f_p is small
- For r >> ξ, the whole ε_0 is used, but the maximum pinning force $f_p \sim \varepsilon_0/r$ is small

Optimum Core Pin Size and Maximum J_c

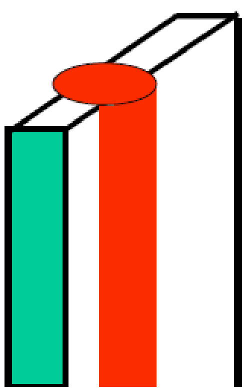


- Because $f_p(r)$ is small for both $r \ll \xi$ and $r \gg \xi$ the maximum pinning force occurs at $r \cong \xi$.
- The same mechanism also works for precipitates.

What is the maximum J_c for the optimum columnar pin?

- Optimum pin allows to reach the depairing current density!

$$J_{\max} \cong \frac{f_p(\xi)}{\phi_0} = \frac{\phi_0}{8\pi\mu_0\xi\lambda^2} \cong J_d$$

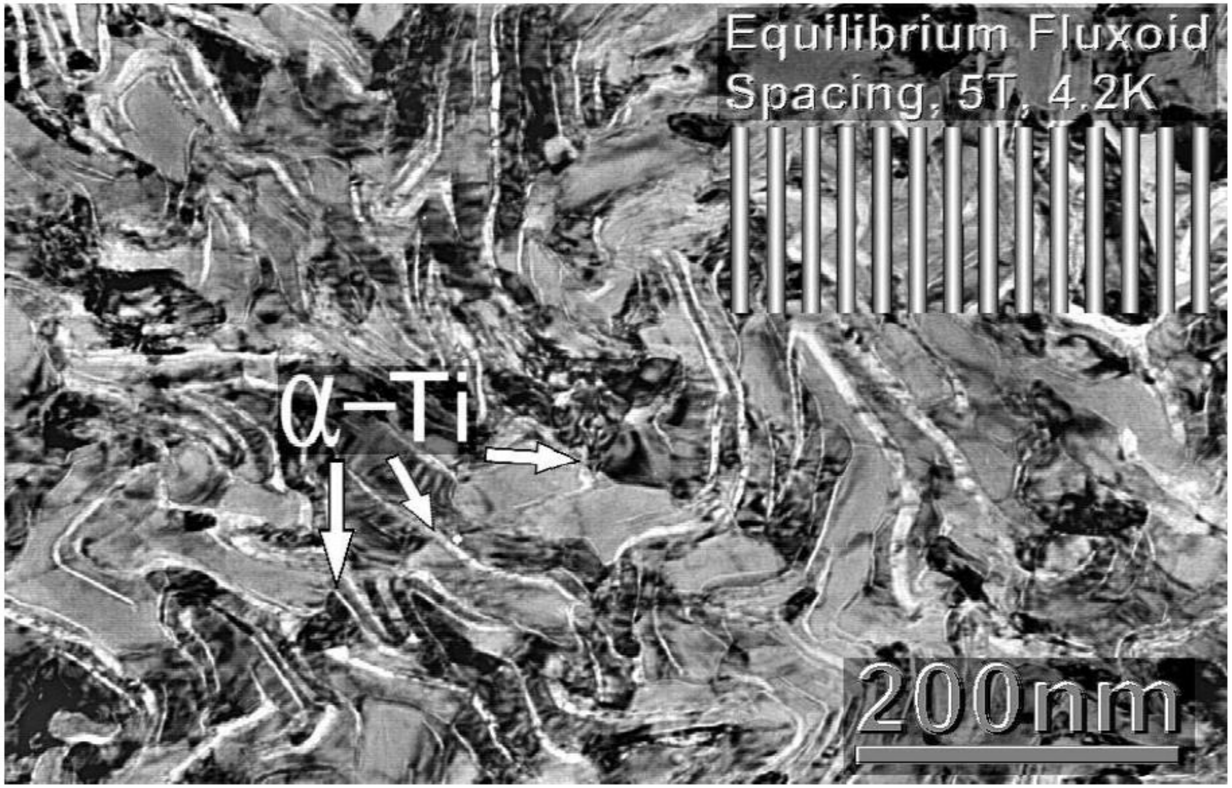


Core pinning by a planar defect of thickness $\approx \xi$ is also very effective

- Core pinning by small precipitates of size $\approx \xi$ yields smaller J_c reduced by the factor $\approx r/l_p$ (fraction of the vortex length taken by pins spaced by l_p)

High J_c-Values for DC Transport

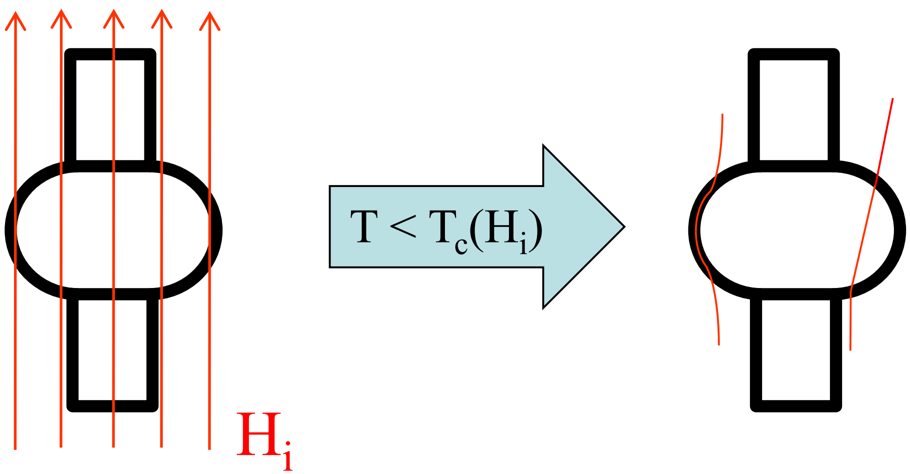
Real pinning microstructure in NbTi



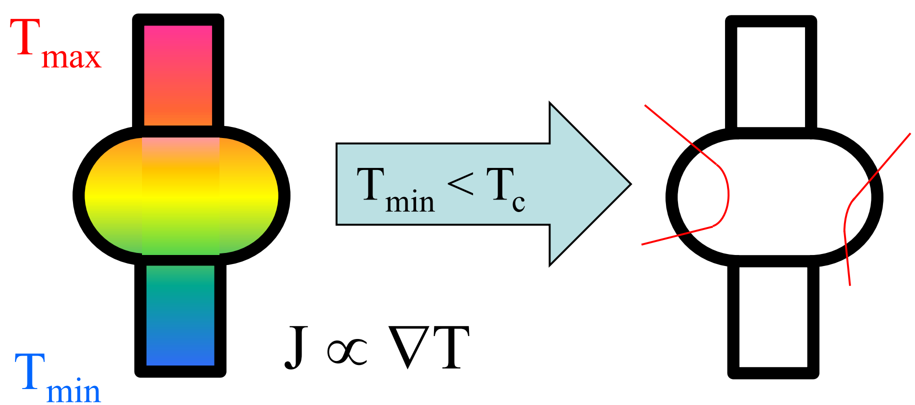
- Pinning effectively prevents vortex motion in presence of a **DC** current: maintains R=0 up to high $\mathbf{J_c}$-values

Trapped vortices in SRF cavities (1)

- Field cooling in residual Earth field

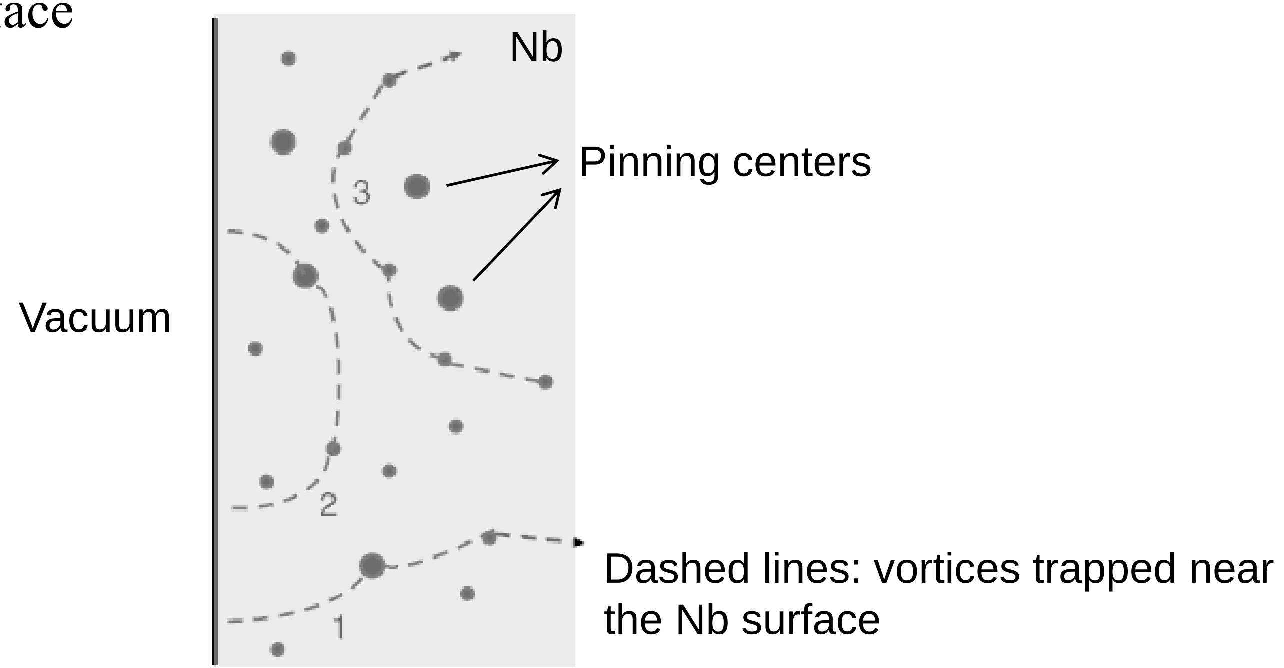


- Thermoelectric currents during cooldown across T_c

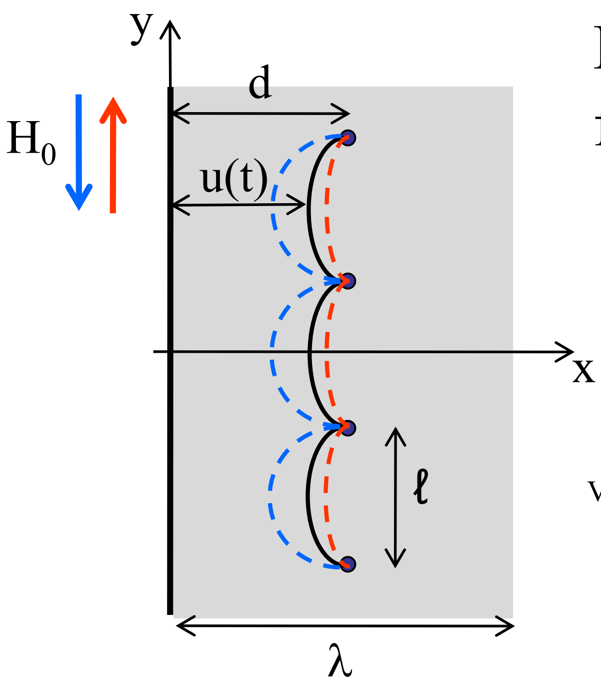


Trapped vortices in SRF cavities (2)

- The resulting distribution of trapped vortices in the cavity can be highly inhomogeneous and the trapping efficiency depends on the material treatment (Post-purification annealing, strong oxidation, large amount of chemical etching)
- Due to the random nature of pinning centers within the wall thickness, we do not know the orientation of the trapped vortices with respect to the cavity surface



Effect of Pinning



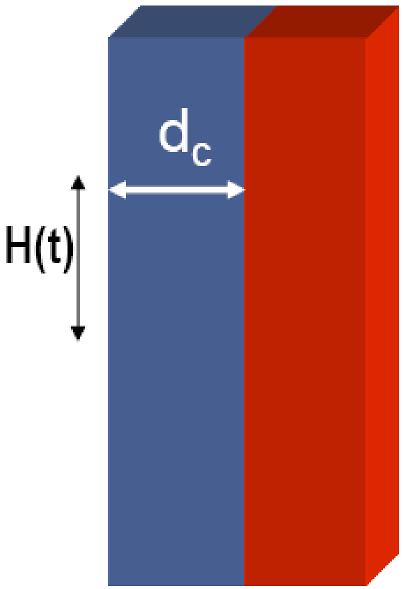
Eq. of motion of the pinned vortex in an RF field:

$$\underbrace{\eta_0 \dot{u}}_{\text{Viscous drag force}} = \underbrace{\varepsilon u''}_{\text{Bending stress}} + \underbrace{\frac{\phi_0 B_0}{\mu_0 \lambda} e^{-u/\lambda} \sin \omega t}_{\text{Lorentz force}} - \underbrace{\frac{\phi_0^2}{2\pi\mu_0\lambda^3} K_1\left(\frac{2u}{\lambda}\right)}_{\text{image force}} + \underbrace{\sum_m f_p(u, y - m\ell)}_{\text{pinning force}}$$

The Eq. of motion can be solved for small oscillations and the dissipated power can be calculated

Vortex oscillations under RF field cause losses

Vortex Free Layer



- Vortex-free layer of thickness $d_c \sim \lambda$ at the surface

$$\frac{\phi_0^2}{2\pi\mu_0\lambda^3} K_1\left(\frac{2d_c}{\lambda}\right) = \frac{f_p}{\ell}, \qquad \varepsilon \cong \frac{\phi_0^2}{4\pi\mu_0\lambda^2} \ln\frac{\lambda}{\tilde{\xi}}$$

- Pinning time constant: $\boxed{\tau_p = \frac{\eta\ell^2}{2\varepsilon} = \frac{\tau\kappa}{\ln\kappa}\left(\frac{\ell}{\lambda}\right)^2}$

$\tau_p \sim 10^{-8}$ s
for $\ell \sim 4$ μm

RF Dissipation

- Seek a solution of the eq. of motion of the form: $u(y,t) = u_0(y) + \delta u(y,t)$
- Solve linearized eq. of motion for the Fourier component $\delta u_\omega(y) = \int \delta u(y,t) e^{-i\omega t} dt$

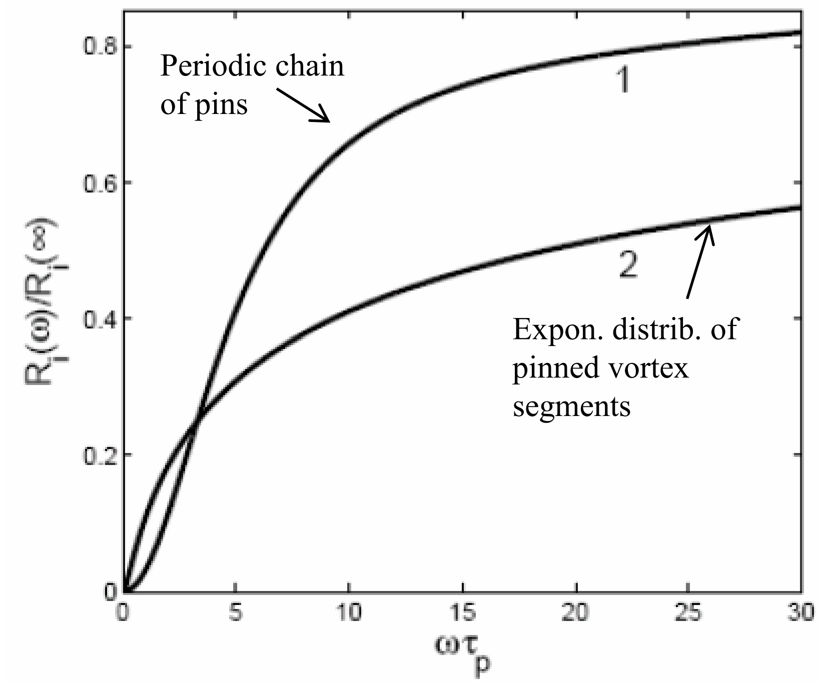
$$Q_v = \frac{\eta_0 \omega^2}{2\ell} \int_{-\ell/2}^{\ell/2} |\delta u_\omega(y)|^2 dy$$ dissipated power per unit vortex length

$Q_v / a = B_0^2 R_i / 2\mu_0^2$ dissipated power per unit area resulting in additional surface resistance

$$R_i = \frac{\phi_0^2 \langle e^{-2d/\lambda} \rangle}{\lambda^2 \eta a} \left[1 - \frac{\sinh\sqrt{\omega\tau_p} + \sin\sqrt{\omega\tau_p}}{\sqrt{\omega\tau_p}\left(\cosh\sqrt{\omega\tau_p} + \cos\sqrt{\omega\tau_p}\right)} \right]$$

High-frequency limit

RF Dissipation



R_i depends strongly on the distribution of the pinning centers (and therefore on the size ℓ of the pinned vortex segments)

- $R_i \propto \omega^\beta$ $\beta \cong 0.5\text{-}0.7$ in agreement with experiments on Nb cavities[2]
- $\tau_p \sim 10^{-8}$ s from experiments on Nb cavities[2] $\Longrightarrow$ ℓ ~ 4 μm

$$R_\infty = \frac{\rho_n \Phi_0 \left\langle e^{-2d/\lambda} \right\rangle}{\lambda^2 a B_{c2}} \cong \frac{\rho_n \Phi_0}{\lambda a^2 B_{c2}} \cong \frac{\rho_n H_i}{\lambda H_{c2}}$$

Strong pinning

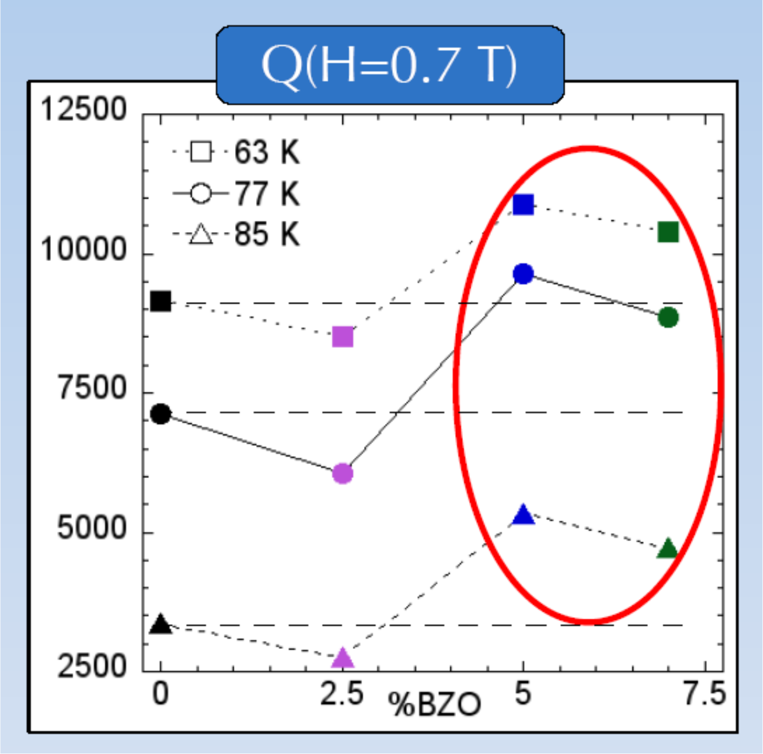
100% flux trapping

~ 2.5 nΩ/mG

RF Dissipation

- Stronger pinning ($\downarrow \tau_p$) reduces R_i

Experimental evidence: Reduced RF losses in YBCO (47.7 GHz) by adding 7% BZO pinning centers !

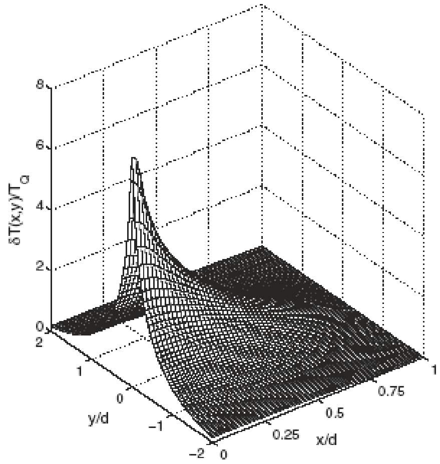


N. Pompeo et al., ASC'08, Chicago, paper 4MC05

Vortex Hotspot

$$R_{BCS}(\mathbf{r}) \propto \frac{\omega^2}{T}\exp\left[-\frac{\Delta}{T_0}+\frac{\delta T(\mathbf{r})\Delta}{T_0^2}\right]$$

even weak variations $\delta T(\boldsymbol{r}) << T_0$ can produce strong variations in $R_{BCS}(\mathbf{r})$ at low temperatures

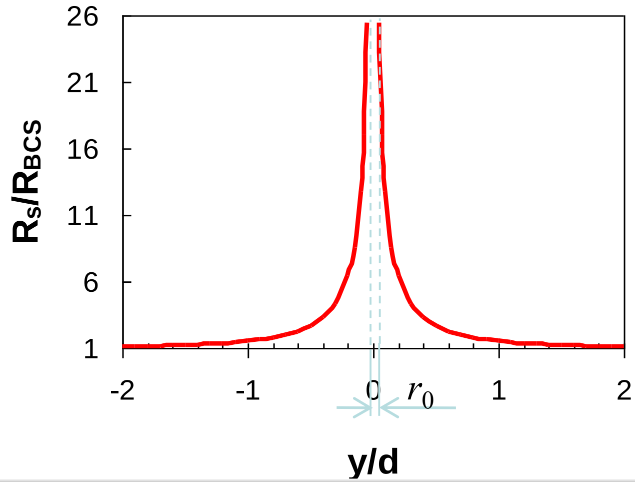


$$\delta T(0,y) = (Q/\pi k)\ln\coth(\pi y/4d)$$

$$R_s(x=0,y) = R_{BCS}(T_0,\omega)\coth^{\sigma}\left(\frac{\pi y}{4d}\right) \qquad |y| > r_0$$

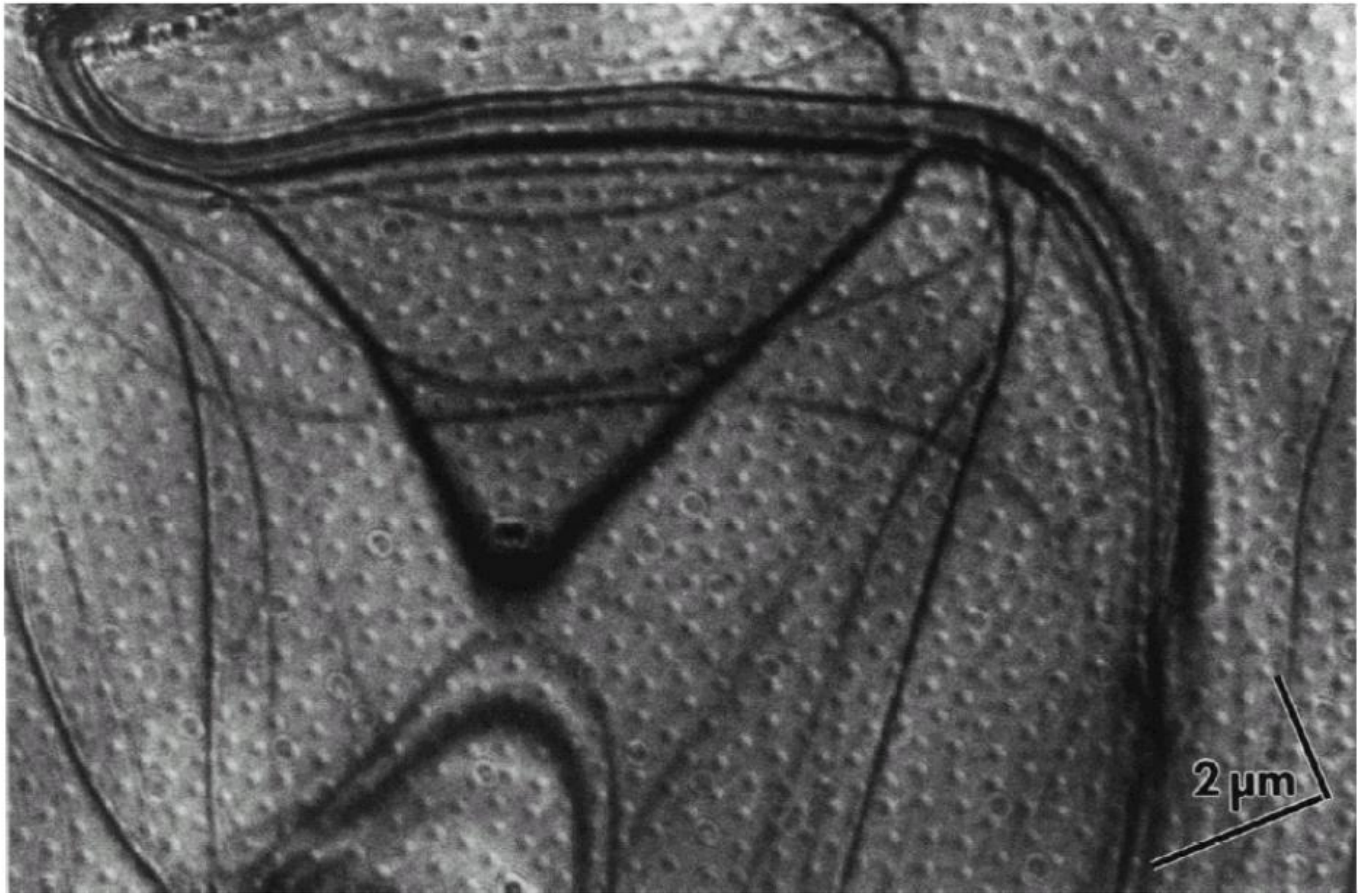
$$\sigma(B_0,T_0,\omega) = \underbrace{Q(B_0,T_0,\omega)}_{\text{Dissipated power}}\Delta(T_0)/\pi\kappa(T_0)T_0^2$$

d: wall thickness



Local dissipation due to vortex motion produces a long-range hot-spot

Lorentz Electron Microscopy of Vortex Structures



Fascinating vortex movies at:
http://www.hitachi.com/rd/research/em/movie.html