U.S. Particle Accelerator School
Education in Beam Physics and Accelerator Technology

Elettra
Sincrotrone
Trieste

Layout Definition for e-Beam Diagnostics

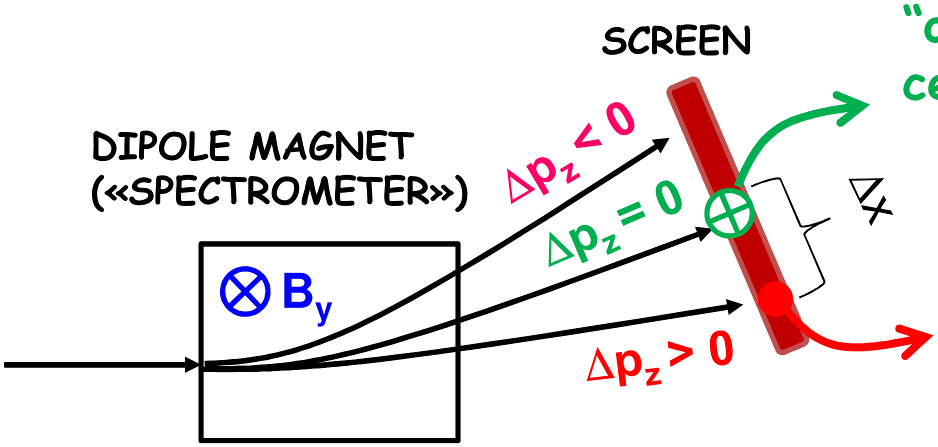
S. Di Mitri (90min.)

Prologue

- ❑ e-Beam diagnostics is a vast and multi-disciplinary field, and necessarily interfacing with e-beam dynamics (a dedicated international conference, IBIC, is devoted to this field only).
- ❑ We will limit to the interplay of beam dynamics and diagnostic resolution power, skipping details on the diagnostic hardware (which is assumed to do what we expect it does…)
- ❑ For reason of space, we will limit to some of the most common diagnosis techniques, and discuss their impact on the machine layout.
- ❑ In the following, we will consider measurement of:

- mean energy,
- energy spread,
- bunch length,
- transverse emittance

Mean Energy ~ Spectrometer



"on-energy" particle centers the screen:

$$\langle p_z \rangle \left[\tfrac{GeV}{c}\right] = 0.3 \cdot B_{y,0}[T] \cdot R[m]$$

"off-energy" particle follows a different (dispersive) path:

$$p_z = \langle p_z \rangle \left(1 + \frac{\Delta x}{\eta_x} \right)$$

- **How to proceed:**
  1. geometry of the ref. trajectory (θ, R) is fixed by the mechanical assembly;
  2. B_y is chosen to center the *beam* onto the detector (screen or BPM);
  3. calculate $\langle p_z \rangle$.

- **Measurement errors:**
  - trajectory distortion before/after the dipole magnet,
  - dipole field calibration errors (vs. the supplying current),
  - misalignment of the dipole/detector.

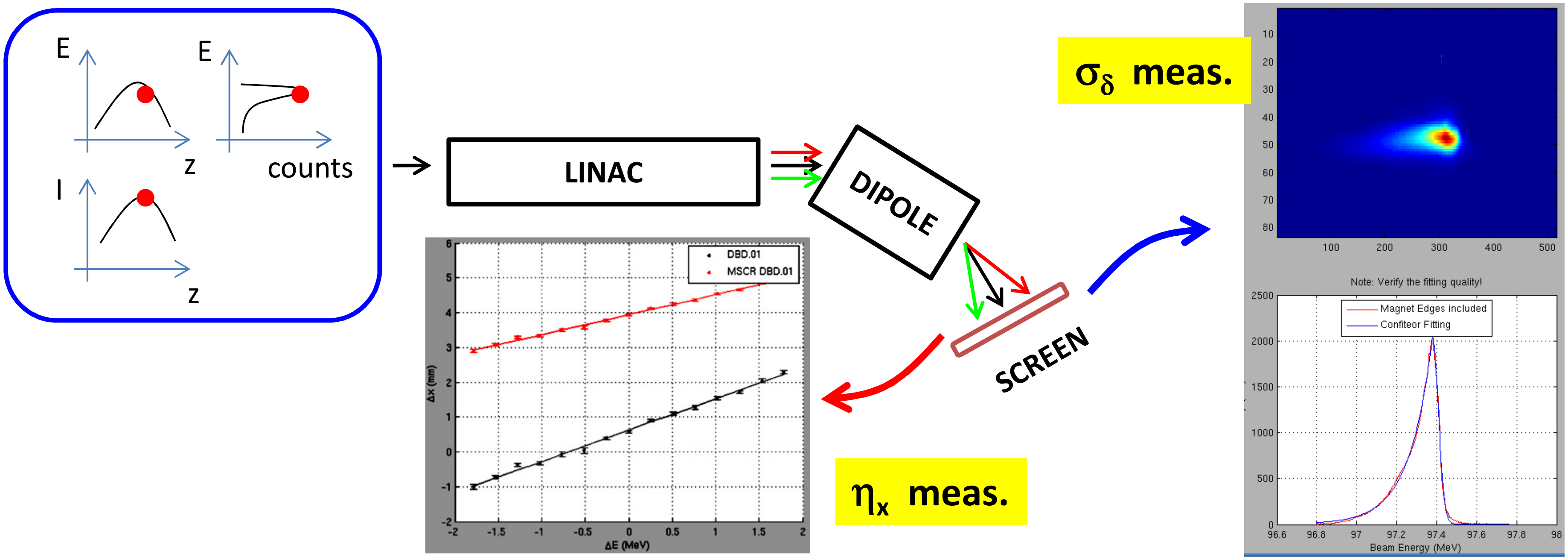
Typical error is of at ~1 MeV level, at energies higher than 10s of MeV.

Energy Spread ~ Spectrometer

- Average over the beam particle ensemble:

$$\langle p_z^2 \rangle = \langle p_z \rangle^2 + \langle p_z \rangle^2 \frac{\langle \Delta x^2 \rangle}{\eta_x^2}; \Rightarrow \boxed{\sigma_E = \langle E \rangle \frac{\sigma_x}{\eta_x}}$$

- The measurement of σ_E depends on the value of η_x. This can be measured in turn as follows:
  1. change the beam *mean* energy (by a few %),
  2. look at the variation of beam centroid position on the detector,
  3. apply a polynominal fit to the curve $\Delta x(\delta)$; the linear term is η_x .



σ_δ meas.

η_x meas.

Energy Resolution

- The **beam spot size** at the screen is the sum of the geometric (betatron) and chromatic (dispersive) particle motion:

$$\sigma_x = \sqrt{\varepsilon_x \beta_x + \eta_x^2 \sigma_\delta^2}$$

- The ***energy resolution*** due to the beam ***geometric optics*** (for an *infinite screen resolution*) is:

$$\boxed{\sigma_{E,res} = \frac{\sqrt{\varepsilon_x \beta_x}}{\eta_x} \langle E \rangle}$$

⇒ minimize $\beta_x / \sqrt{\eta_x}$ by design

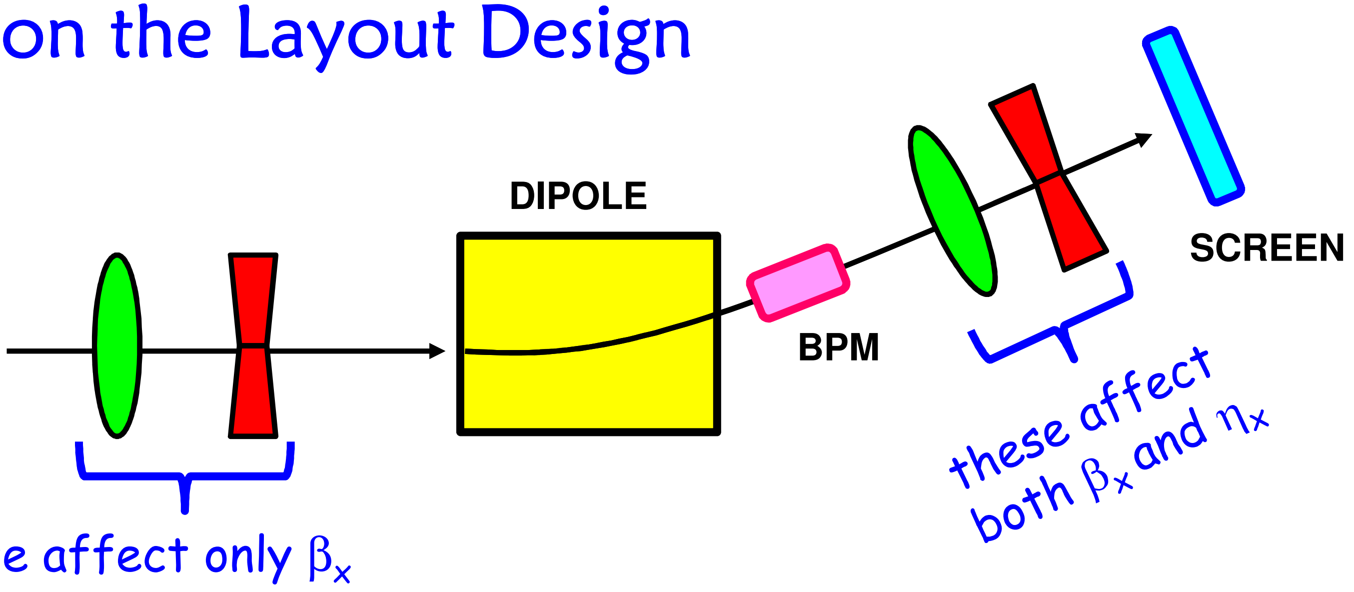
- In reality, the finite ***screen resolution*** (millimeter per pixel) can be neglected if it is *smaller than the geometric beam size*:

$$\sigma_{E,res} = \frac{\sqrt{\varepsilon_x \beta_x}}{\eta_x} \langle E \rangle >> \frac{\sigma_{screen,res}}{\eta_x} \langle E \rangle$$

⇒ $\sigma_{screen,res} << \sqrt{\varepsilon_x \beta_x}$

EXERCISE: assume $\sigma_{screen,res}$ = 30 μm/pixel, $\gamma\varepsilon_x$ = 1 mm mrad at 100 MeV, η_x = 1.5m. Evaluate β_x at the screen to ensure $\sigma_{E,res}$ = 10keV. Is it large enough to overcome $\sigma_{screen,res}$?

Impact on the Layout Design



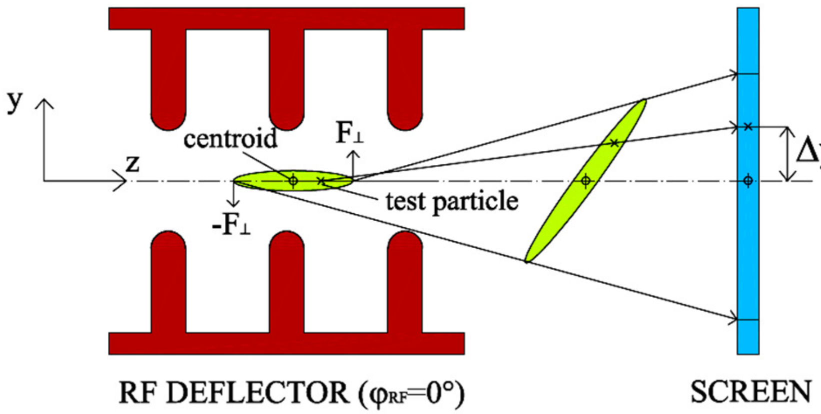
- Make η_x **large** to minimize $\sigma_{E,res}$ ⇒ large bending angle (typically > 20deg)
  ⇒ add quadrupoles *after* the dipole
- Make β_x **small** to minimize $\sigma_{E,res}$ ⇒ use quadrupoles *after* the dipole
  ⇒ use quadrupoles *before* the dipole
- For any given screen resolution, make β_x larger to increase the beam size
  ⇒ tune the **ratio** $\sqrt{\beta_x}/\eta_x$ to keep $\sigma_{E,res}$ fixed

☐ A **BPM** soon **after the dipole** can be easily used for the mean energy measurement, being less sensitive to trajectory distortion occurring between the dipole and the screen.

☐ A **SCREEN** at the **line end** takes advantage of the maximum dispersion and optics tuning. It is devoted to the energy spread measurement.

Bunch Length ~ RF Deflector

Pictures courtesy
of P. Craievich



$$\Delta y_S(z) = R_{34}\Delta y'_D$$

- **RF deflector transverse kick:**

$$\Delta y'_D \approx \frac{eV_\perp}{E}\left[\underbrace{z\frac{\omega_{RF}}{c}\cos\varphi_{RF}}_{\textbf{this applies to the entire bunch length}} + \underbrace{\sin\varphi_{RF}}_{\textbf{this applies to the CM}}\right]$$

- **Transport matrix element:**

$$R_{34} = \sqrt{\beta_D\beta_S}\sin(\Delta\psi_{DS})$$

For $\varphi_{RF}\approx 0°$, the **RMS spot size** on the screen:

$$\boxed{\sigma_{y,S} = \frac{eV_\perp}{E}\sigma_z\left[\frac{\omega_{RF}}{c}\cos\varphi_{RF}\right]\sqrt{\beta_D\beta_S}\sin(\Delta\psi_{DS})}$$

For $\varphi_{RF}\approx 90°$, the ***CM deviation*** onto the screen is:

$$\langle\Delta y_S\rangle = \frac{eV_\perp}{E}\sqrt{\beta_D\beta_S}\sin(\Delta\psi_{DS})\sin\varphi_{RF}$$

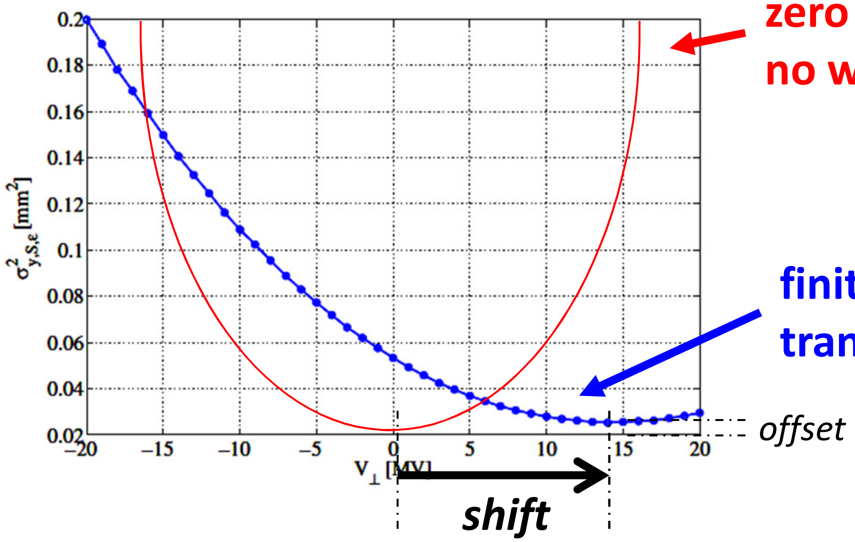
How to proceed:

- vary φ_{RF} and measure $\langle\Delta y_S\rangle$, then fit the curve to compute the "*optics calibration factor*" $\boldsymbol{B = eV_\perp R_{34}/E}$ **[mm/rad]**;
- now measure $\sigma_{y,S}$ and evaluate the **bunch length** as $\boldsymbol{\sigma_z = \sigma_{y,S} / B}$;

Bunch Length Resolution

Pictures courtesy
of P. Craievich

- Beam **non-zero transverse emittance** and residual z-y correlations (e.g., by **transverse wakefield**) affect the ***calibration curve***, thus the σ_z resolution.



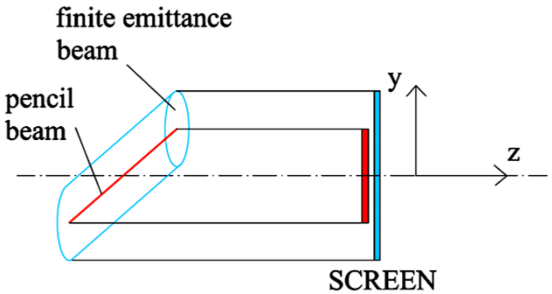
zero emittance, no wakes

finite emittance, transverse wakes

- **Shift of the parabola minimum** reveals an incoming **z-y correlation**, which is removed by a nonzero $V_\perp$.
- **Vertical offset of the parabola minimum** reveals a nonzero vertical **emittance** (finite spot size for $V_\perp$=0).

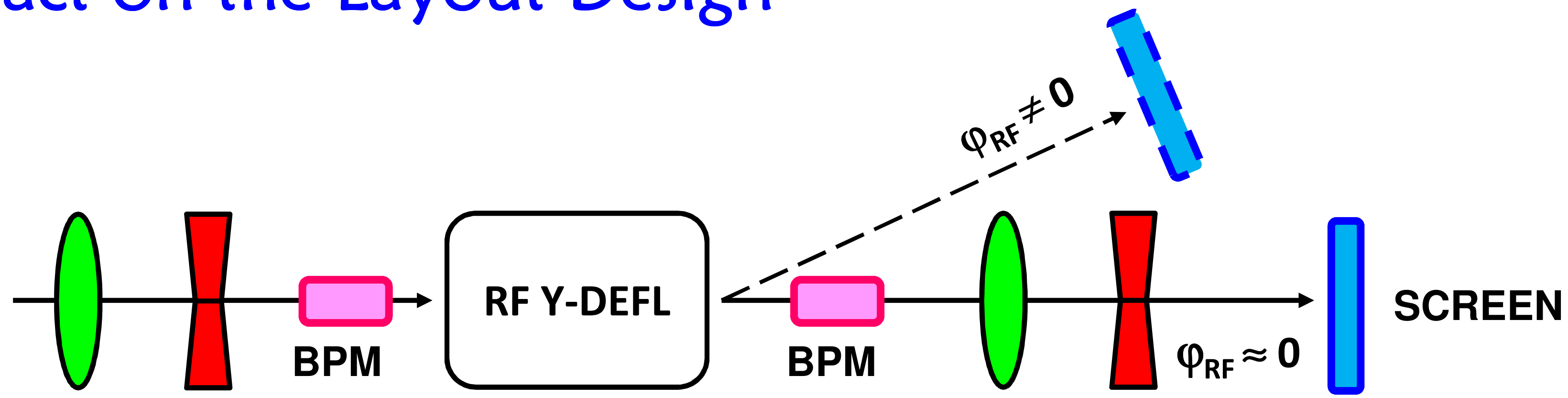
- If we assume no net wakefield effect, the **bunch length resolution** due to the beam ***geometric optics*** (for an *infinite screen resolution*) at the zero-crossing RF phase is:

$$\boxed{\sigma_{z,res} = \frac{\sigma_{y,S}}{T_{RF} R_{34}} = \frac{p_z c\sqrt{\varepsilon_y}}{eV_\perp k_{RF}\sqrt{\beta_D}\left|\sin\Delta\psi_{DS}\right|}}$$



- In practice, $V_\perp$ and $\beta_{y,S}$ should be **large** enough to ensure $\sigma_{y,S} >> \sigma_{screen,res}$.

Impact on the Layout Design

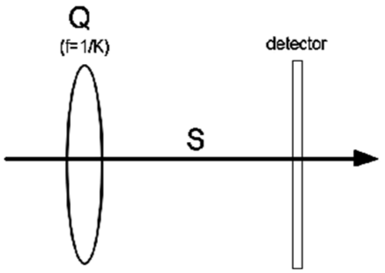


- Make $\beta_{y,D}$ **large** to minimize $\sigma_{z,res}$ $\Rightarrow$ use quadrupoles *before* the deflector
- Make $\Delta\psi_{S,D}$ **close to $\pi/2$** to minimize $\sigma_{z,res}$ $\Rightarrow$ use quadrupoles, possibly *after* the deflector.
- Make $\beta_{y,S}$ **large** to dominate over $\sigma_{screen,res}$ $\Rightarrow$ use quadrupoles, possibly *after* the deflector.

❑ Two **BPMs at the deflector edges** can be used for beam steering onto the deflector electric axis, thus for a correct setting of the RF phase.

❑ A **SCREEN at the end** of the diagnostic line is devoted to the bunch length measurement.

Projected Emittance ~ Quadrupole Scan

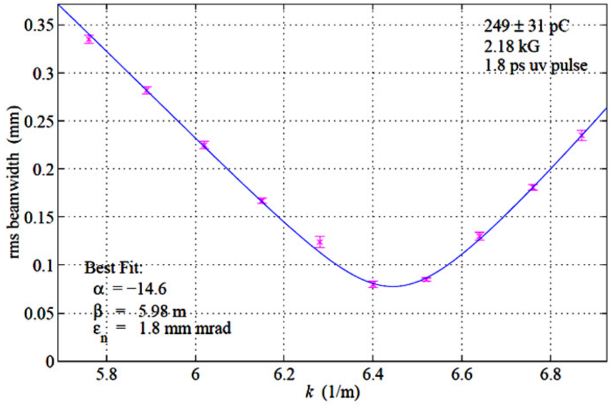


$$Q \cong \begin{pmatrix} 1 & 0 \\ K & 1 \end{pmatrix} \quad S = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix}$$

1. The **transport matrix** from quadrupole (thin lens approximation) to detector (screen) is assumed to be **known:**

$$T = QS = \begin{pmatrix} S_{11} + KS_{12} & S_{12} \\ S_{21} + KS_{22} & S_{22} \end{pmatrix}$$

2. The beam matrix transforms according to $\Sigma_f = T\Sigma_0 T^t$
3. The beam matrix element $<x^2>$ is quadratic in the quadrupole integrated strength, K := kl



$$\Sigma_{11}(=\langle x^2\rangle) = ({S_{11}}^2\Sigma_{11_0} + 2S_{11}S_{12}\Sigma_{12_0} + {S_{12}}^2\Sigma_{22_0}) + (2S_{11}S_{12}\Sigma_{11_0} + 2{S_{12}}^2\Sigma_{12_0})K + {S_{12}}^2\Sigma_{11_0}K^2$$

4. In practice: **vary the quad strength k and measure σ_x** at the screen. Then **fit** with a **parabola:**

$$\begin{aligned}\Sigma_{11} &= A(K-B)^2 + C \\ &= AK^2 - 2ABK + (C + AB^2)\end{aligned}$$

- Emittance and the Twiss parameters from the fitting coefficients:

$$\boxed{\epsilon_x = \sqrt{AC}/S_{12}^2 \qquad \beta_x = \frac{\Sigma_{11}}{\epsilon} = \sqrt{\frac{A}{C}} \qquad \alpha_x = -\frac{\Sigma_{12}}{\epsilon} = \sqrt{\frac{A}{C}}\left(B + \frac{S_{11}}{S_{12}}\right)}$$

Projected Emittance ~ Multiple Screens, Fixed Optics

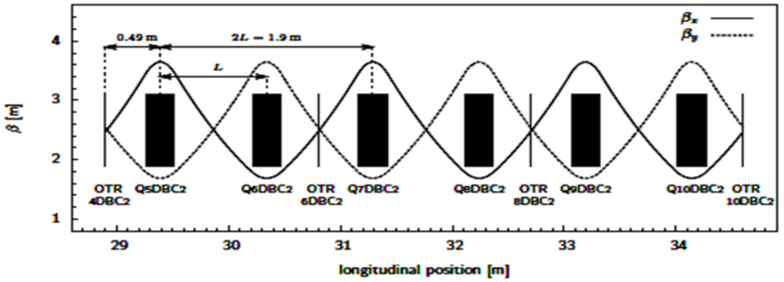
1. **Multiple beam size mesurements** are now done with fixed quadrupole strength, but at **different locations (multi-screen).**
2. **The beam size** at screen *1…n-th* **transforms** according to:

$$\begin{pmatrix} (\sigma_x{}^{(1)})^2 \\ (\sigma_x{}^{(2)})^2 \\ (\sigma_x{}^{(3)})^2 \\ \dots \\ (\sigma_x{}^{n})^2 \end{pmatrix} = \begin{pmatrix} (R_{11}{}^{(1)})^2 & 2R_{11}^{(1)}R_{12}^{(1)} & (R_{12}{}^{(1)})^2 \\ (R_{11}{}^{(2)})^2 & 2R_{11}^{(2)}R_{12}^{(2)} & (R_{12}{}^{(2)})^2 \\ (R_{11}{}^{(3)})^2 & 2R_{11}^{(3)}R_{12}^{(3)} & (R_{12}{}^{(3)})^2 \\ & \dots & \\ (R_{11}{}^{(n)})^2 & R_{11}^{(n)}R_{12}^{(n)} & (R_{12}{}^{(n)})^2 \end{pmatrix} \begin{pmatrix} \beta(s_0)\epsilon \\ -\alpha(s_0)\epsilon \\ \gamma(s_0)\epsilon \end{pmatrix}$$

Beam size measured at screen *1…..n.* *Vector Σ.*

Initial ellipse in phase space. *Vector o.*

Transport elements from the source point to the nth-screen. *Matrix B.*



3. Determine vector *o* by **minimizing the sum** (least square fit):

$$\chi^2 = \sum_{l=1}^{n} \frac{1}{\sigma^2_{\Sigma_x^{(l)}}} \left(\Sigma_x^{(l)} - \sum_{i=1}^{3} B_{li} o_i \right)^2$$

Beam size measurement error

Beam size measured

Transport matrix (quads setting)

System needs *at least 3 screens*, for 3 independent parameters to be determined:

$$\epsilon = \sqrt{o_1 o_3 - o_2^2}\,, \qquad \beta = o_1/\epsilon\,, \qquad \alpha = -o_2/\epsilon\,.$$

Twiss Parameters – Optics Matching

PRSTAB 15
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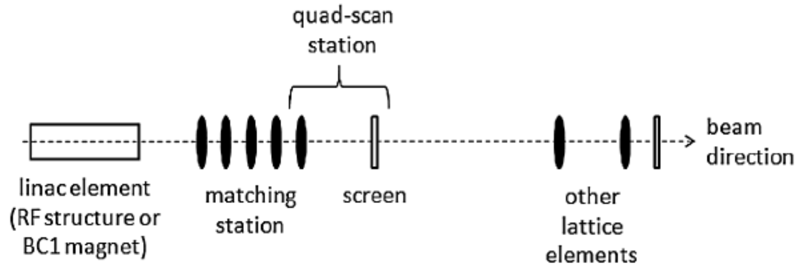
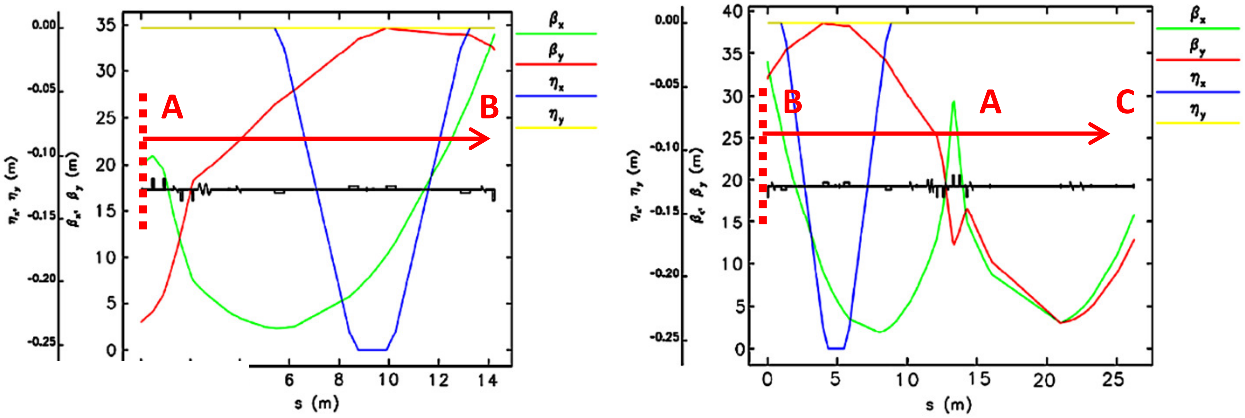
1.



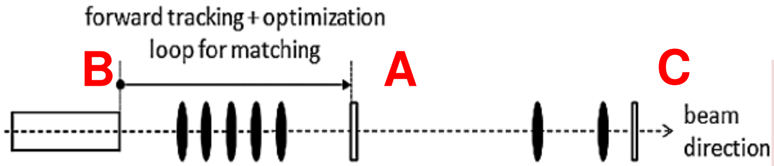
Quad-scan provides the beam Twiss parameters at the entrance of the last quad of the matching station.

2.

The machine is read by a code and the measured Twiss parameters back-tracked to upstream of the matching station.

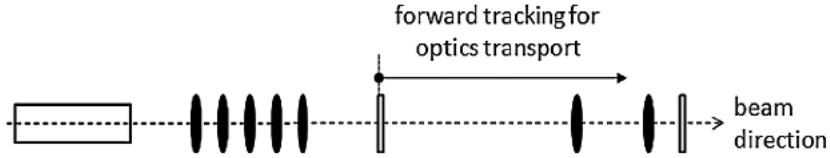


3.



Code optimizes the quad strengths to match the beam to the design Twiss parameters.

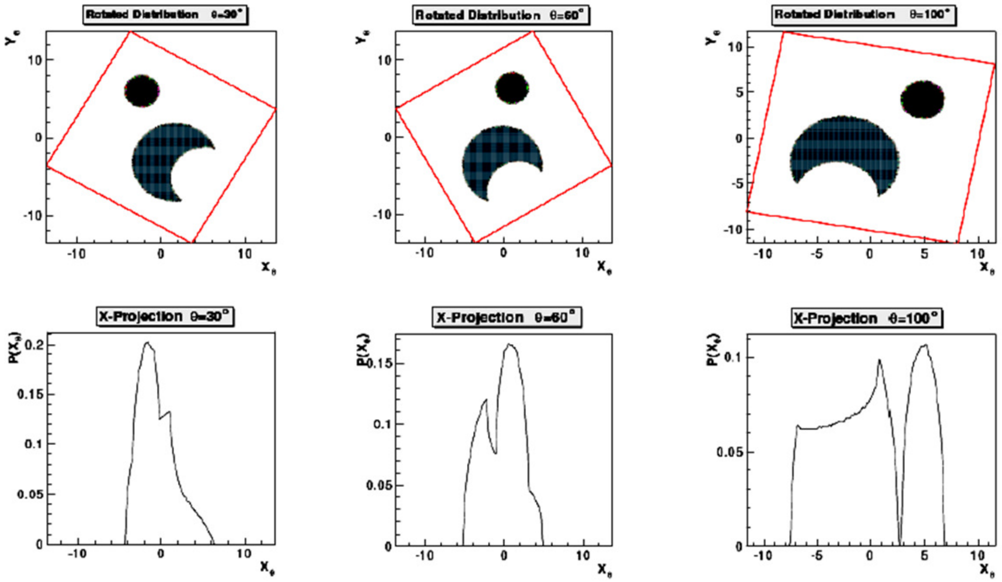
4.



The solution is applied to the machine and the beam transported downstream.

Transverse Phase Space ~ Multiple Screens

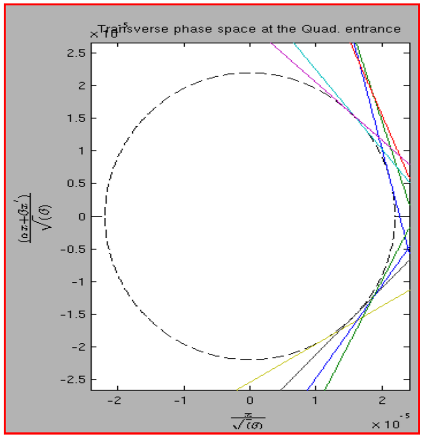
- Force the beam **phase space rotating** along the diagnostic line. Look at **different screens** (fixed optics) or vary a quadrupole's strength.
- Collect all **phase space projections** onto the spatial coordinate (projected beam size). Then apply the ***MENT algorithm*** to reconstruct, for the given transport matrix, the phase space at the source point.
- In general, ***tomographic reconstruction*** is as accurate as many experimental points are used. But, redundancy is avoided as the phase space is sampled at different rotation angles. **Minimum reconstruction error is for 45° phase advance** between samples.

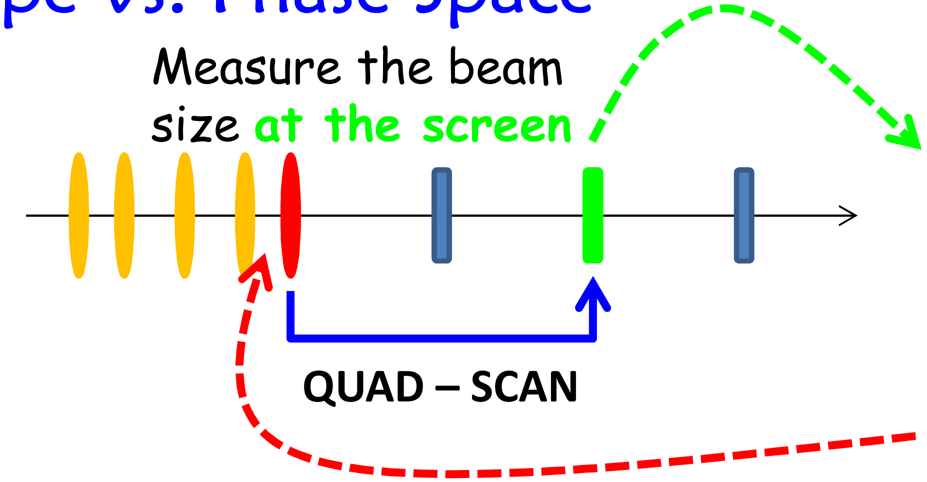


~ phase space at the detector(s)

~ projections

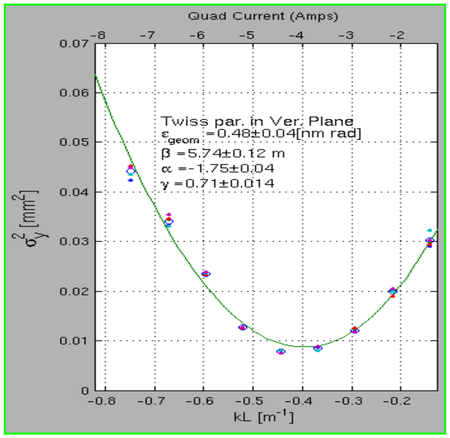
Beam Enevelope vs. Phase Space



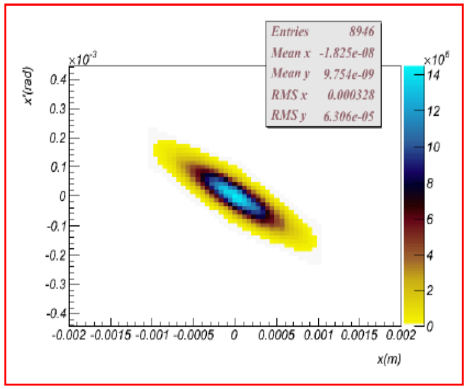


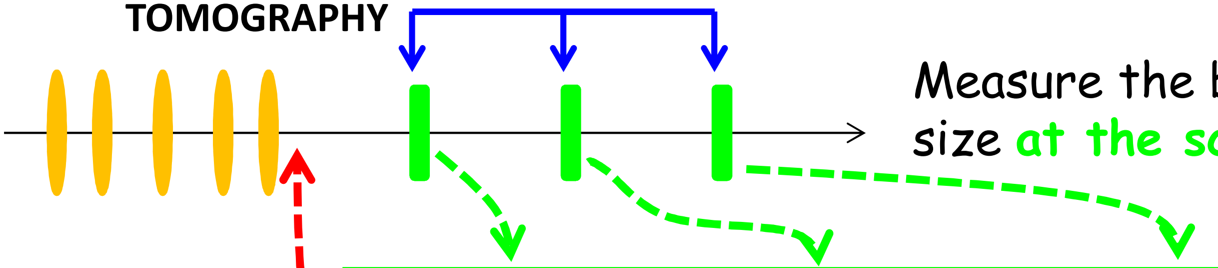
Measure the beam size **at the screen**

QUAD – SCAN



Trace the phase *space ellipse* back **at the quadrupole entrance**. This *approximates* the ***beam envelope***.

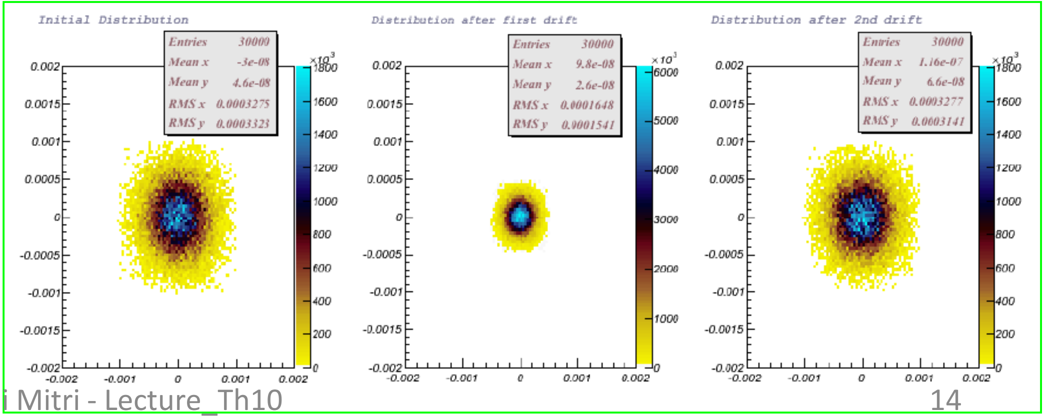




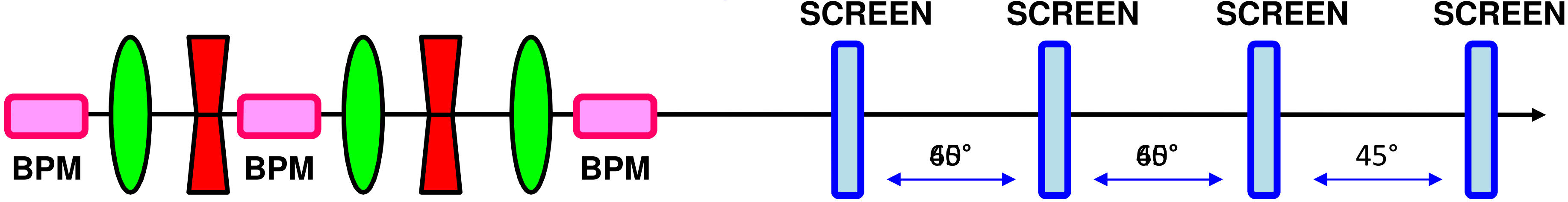
TOMOGRAPHY

Measure the beam size **at the screens**

Trace back the ***phase space density***.



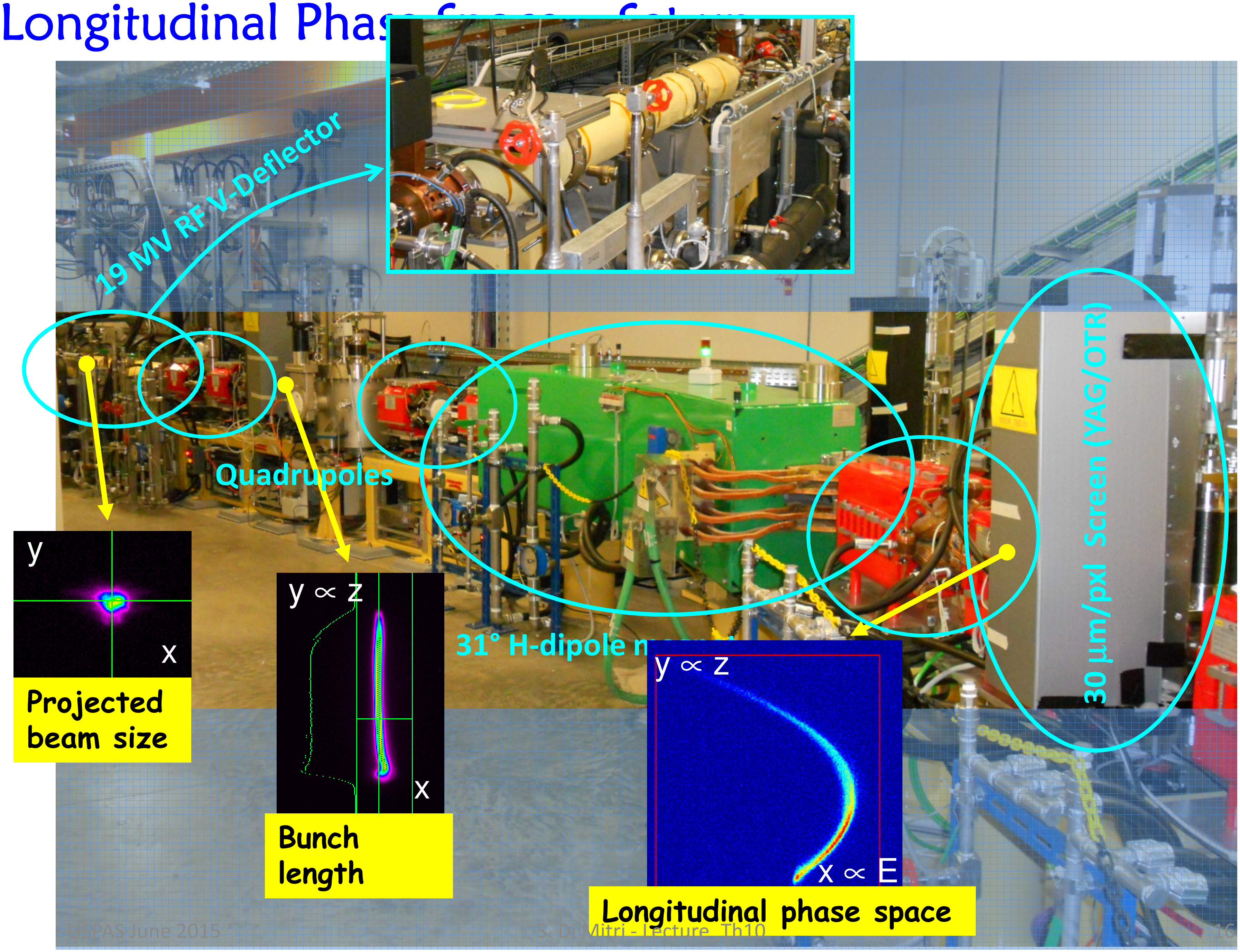
Impact on the Layout Design



- ➢ At leats one quad + one screen, for the "**quad-scan**".
- ➢ **More quads** can be tuned *simultaneously* to make the scan in one plane while the **beam size** is mantained almost constantin the other (multi-quad scan).
- ➢ By adding at least **two more screens**, *e.g.* at 60° phase advance ⇒ "multiple screens – fixed optics", **without touching** the optics for **beam production**.
- ➢ With "**mutiple screens**", the emittance measurement is less sensitive to erros with (at least) 4 screens at 45° phase advance. This would also permit "phase space **tomography**" with four points, at fixed optics.

- ❑ **BPMs near quadrupoles** allow the beam to be centered into the magnets, so avoiding trajectory steering when the strengths are varied.
- ❑ **Beam size detectors** can be either *metallic targets* (*e.g.*, ~100μm thick Yttrium Aluminum Garnet target producing visible fluorescence, or ~10μm thick Aluminum foil producing Optical transition Radiation). Alternatively, *wire scanners* ed to intercept the beam.

Longitudinal Phas



19 MV RF V-Deflector

Quadrupoles

31° H-dipole magnet

30 μm/pxl Screen (YAG/OTR)

Projected beam size

Bunch length

Longitudinal phase space

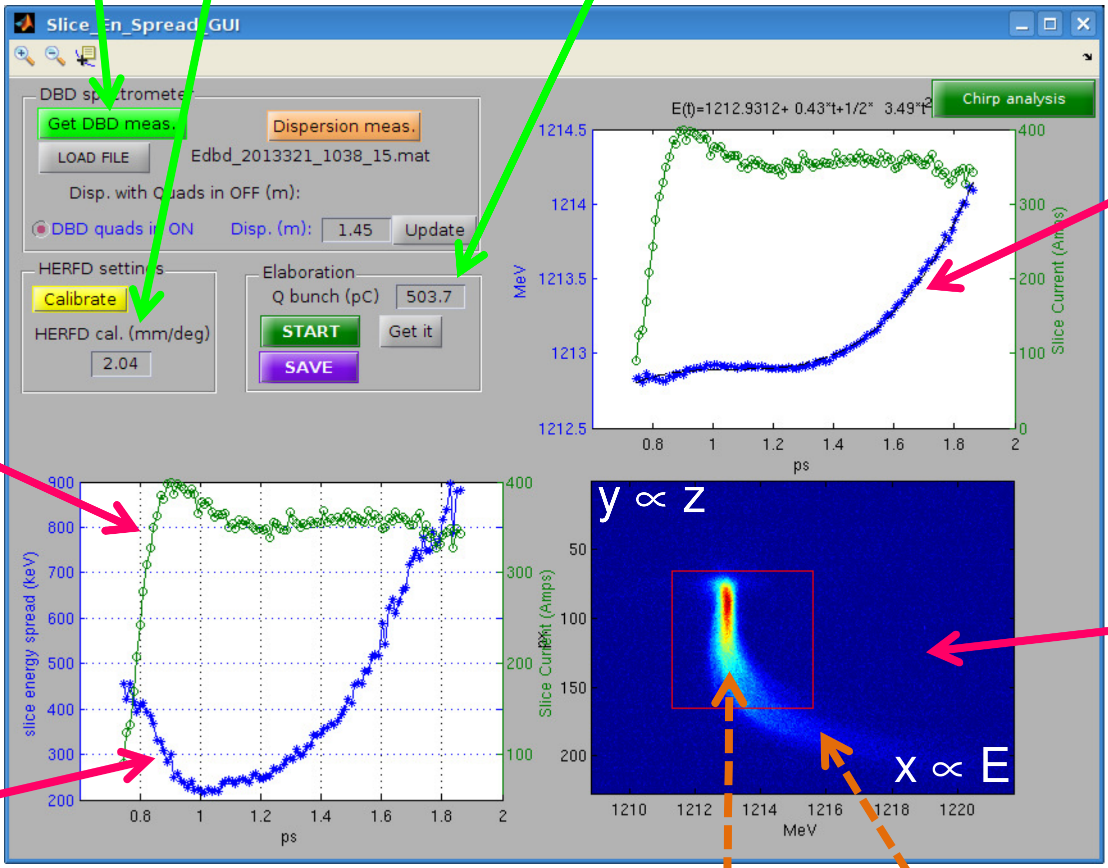
Longitudinal Phase Space – Post-processing

Picture courtesy
of G. Penco

This panel inputs the dispersion for energy spread measurement

This panel inputs the RF deflector calibration for bunch length measurement.

This panel inputs the beam charge for peak current measurement.



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Disp. with Quads in OFF (m):

● DBD quads in ON Disp. (m): 1.45

HERFD cal. (mm/deg) 2.04

Q bunch (pC) 503.7

E(t)=1212.9312+ 0.43*t+1/2* 3.49*t^2

X-position of each slice centroid provides the **slice mean energy**.

Charge density of each slice provides the **slice peak current**.

X-width of each slice provides the **slice energy spread**.

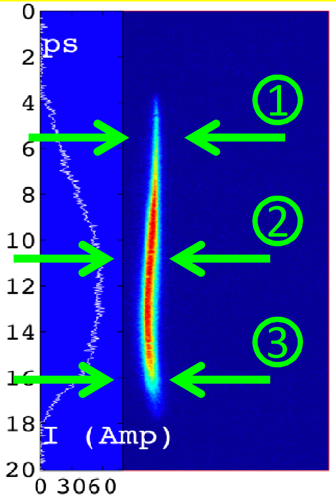
The **screen image** is post-processed for slice analysis. Optics was tuned for energy and bunch length max. resolution.

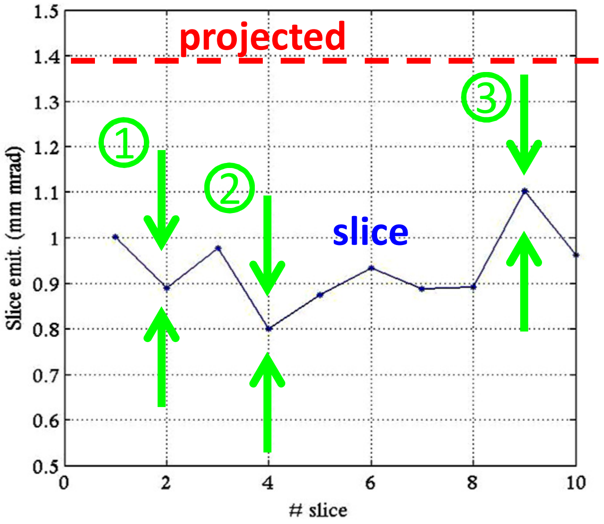
Note: *slice energy spread* is the sum of *uncorrelated* and *correlated one*

Pictures courtesy
S. Spampinati et al.

Slice Emittance, Slice Energy Spread

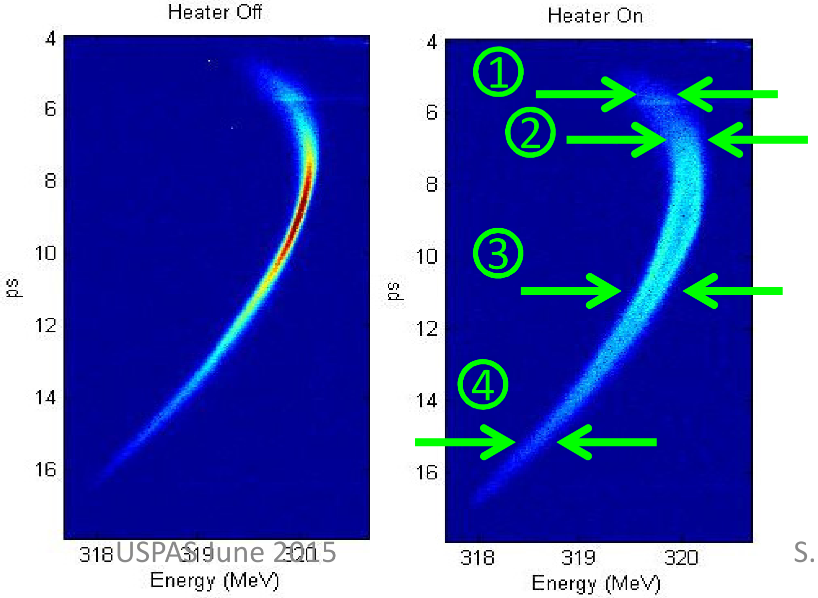
❖ RF **V-deflector** + beam goes **straight**:



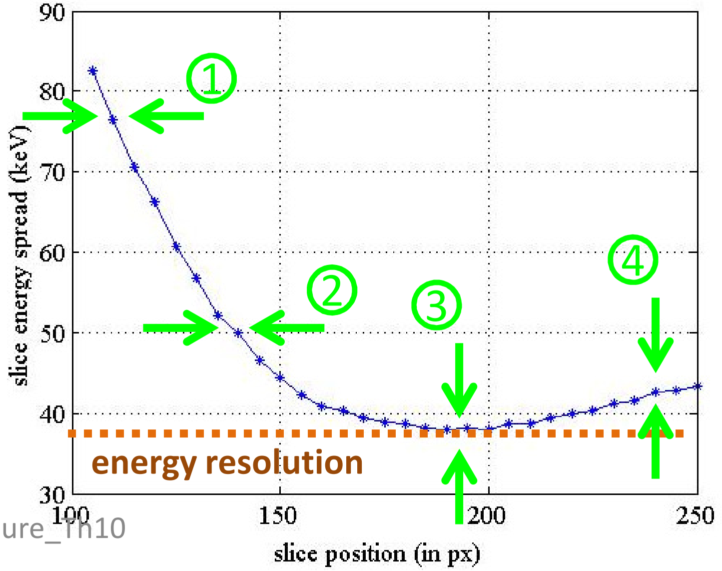


- **Quad-scan** is applied to the stretched beam. The image is then sampled and the emittance is computed for each slice.

❖ RF **V-deflector** + **H-bent** beam:



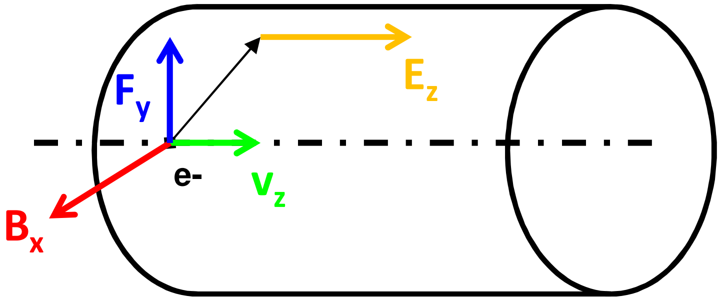
- The **slice energy spread** is dominated by the RF curvature (x-E correlation) at the bunch edges, and by the optics resolution in the core.



RF Deflector-Induced Energy Spread

- ❑ A **time-varying transverse magnetic field B_x** for deflection implies an ***off-axis longitudinal electric field $E_z \propto r$*** (Panofsky-Wenzel theorem).

RF DEFLECTOR (MAGNETIC)



- ❑ Owing to the particles longitudinal motion, **E_z(r)** changes the longitudinal momentum → **RF deflector induced energy spread, δ.**

$$\delta \cong \frac{1}{L}\int_0^L ds\frac{\Delta p_z(s)}{p_z} = \frac{1}{L}\int_0^L ds\int_0^{s'}\left(-\frac{e}{p_z c}\right)E_z(y,s')ds' = -\frac{e}{p_z c}\frac{1}{L}\int_0^L ds\int_0^{s'} E_{z,0}k_{RF}\,y(s)\cos\varphi_{RF}ds' = \ldots$$

- ❑ Why do particles sample E_z(y) *off-axis* ?
  - They travel off-axis due to non-zero **beam size** and **divergence**.
  - They are (vertically) displaced by ***deflection*** inside the cavity.

Estimate of Beam Heating

❑ For an <u>RMS beam size</u> $\sigma_y = \sqrt{\beta_{y,D}\varepsilon_y}$, the previous equation gives:

$$\sigma_{\delta,\beta} \cong \ldots = -\frac{eE_{z,0}k_{RF}\cos\varphi_{RF}}{p_z c}\frac{1}{L}\int_0^L ds\int_0^{s'} ds'\sqrt{\beta_{y,D}\varepsilon_y} \cong -\frac{eE_{z,0}k_{RF}\cos\varphi_{RF}}{p_z c}\frac{1}{L}\cdot\frac{L^2}{2} \cong \left(\frac{eV_{RF}k_{RF}\cos\varphi_{RF}}{p_z c}\right)\frac{\sqrt{\beta_{y,D}\varepsilon_y}}{2}$$

❑ For a <u>cavity length</u> L, we obtain:

$$\sigma_{\delta,L} \cong \ldots = -\frac{eE_{z,0}k_{RF}\cos\varphi_{RF}}{p_z c}\frac{1}{L}\int_0^L ds\int_0^s ds'\int_0^{s'} y'(s)ds'' = -\frac{eE_{z,0}k_{RF}\cos\varphi_{RF}}{p_z c}\frac{1}{L}\cdot\frac{eV_{RF}k_{RF}\cos\varphi_{RF}}{p_z c}\frac{L^3}{6}z \cong$$

$$\cong \left(\frac{eV_{RF}k_{RF}\cos\varphi_{RF}}{p_z c}\right)^2\frac{L}{6}z$$

At the zero-crossing RF phase:

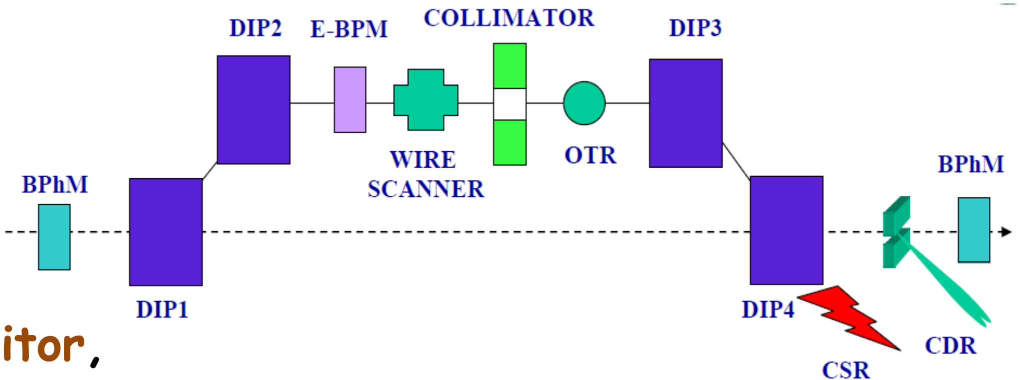
$$\boxed{\sigma_\delta \approx \frac{eV_{RF}k_{RF}}{2p_z c}\sqrt{\beta_{y,D}\varepsilon_y + \sigma_z^2\left(\frac{eV_{RF}k_{RF}}{p_z c}\right)^2\frac{L^2}{9}}}$$

This is *uncorrelated* with z, but correlated with y.

This is *correlated* with, and averaged over *z*.

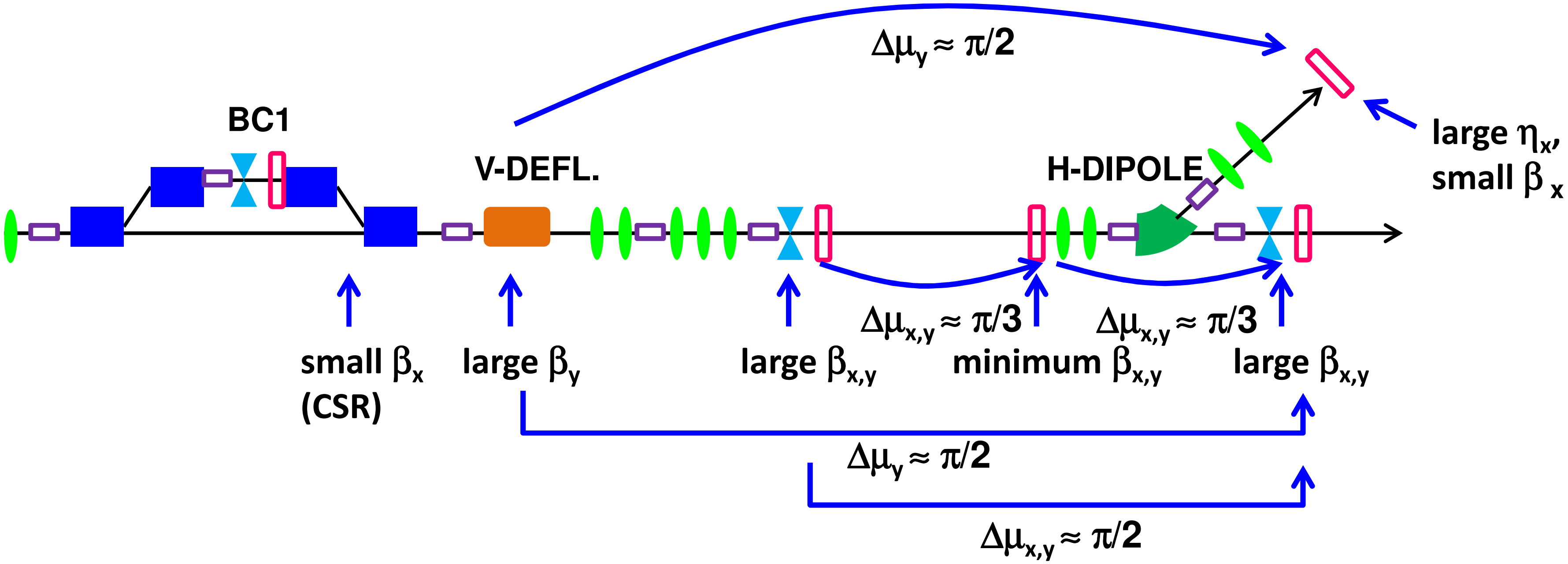
Other Diagnostic Elements (BC Region)

A dispersive region (BC) allows to profit of diagnostics for time, energy and transverse beam characterization, both on-line and invasive.



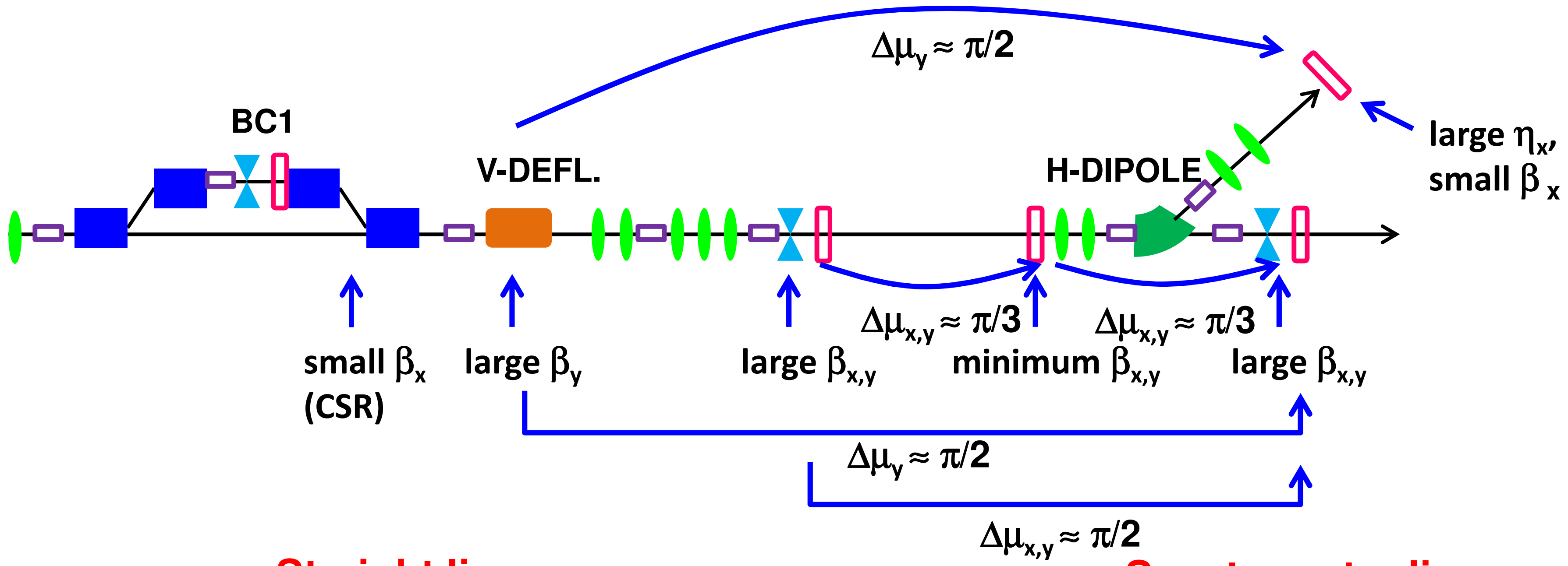
- ✓ **BPhM = Bunch PHase (Arrival) Monitor**,
  → e-beam arrival time respect to the machine clock. On-line → *arrival time jitter feedback*.
  → When used before and after a chicane, it gives the beam time-delay across it.
- ✓ **E-BPM = Energy–Beam Position Monitor**,
  → e-beam mean energy. On-line → *energy-feedback*.
- ✓ **Wire Scanner** (alternatively, YAG/OTR screen),
  → beam energy spread. Invasive.
- ✓ **Collimator** (or Scraper),
  → in the presence of a linear t-E correlation (energy chirp), it selects *longitudinal bunch slices*, to be characterized downstream with *no need of a deflector*.
- ✓ **CRM = Coherent Radiation Monitor**,
  → bunch length variation. On-line → *bunch length (peak current) feedback*.

Summing Up



1. Mean energy
2. Energy spread
3. Quadrupole scan
4. Transverse tomography
5. Bunch length
6. Slice transverse emittance
7. Slice energy spread
8. Geometric collimation
9. Slicing

Summing Up



BC1

V-DEFL.

H-DIPOLE

$\Delta\mu_y \approx \pi/2$

large η_x, small β_x

small β_x (CSR)

large β_y

large $\beta_{x,y}$

$\Delta\mu_{x,y} \approx \pi/3$

minimum $\beta_{x,y}$

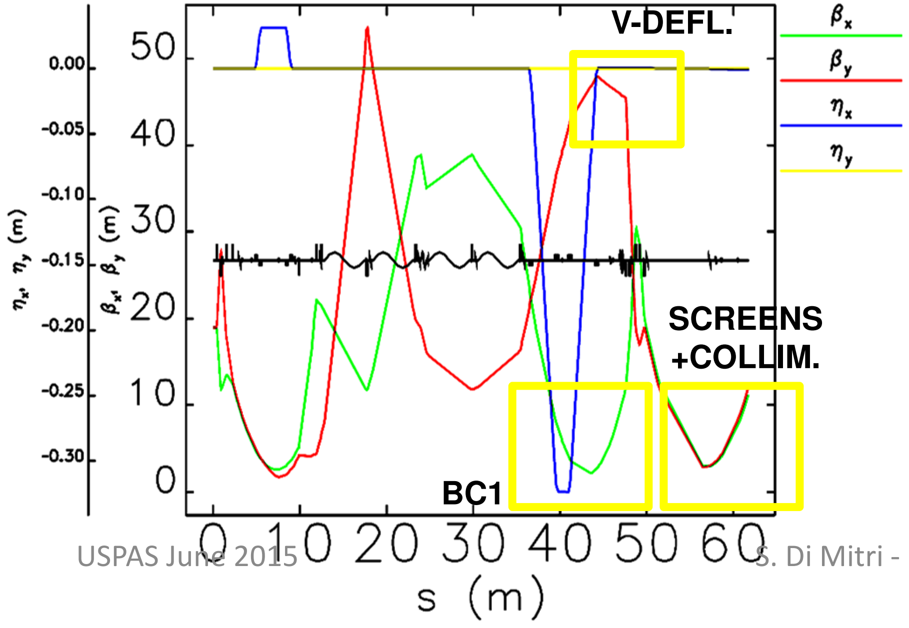
$\Delta\mu_{x,y} \approx \pi/3$

large $\beta_{x,y}$

$\Delta\mu_y \approx \pi/2$

$\Delta\mu_{x,y} \approx \pi/2$

Straight line

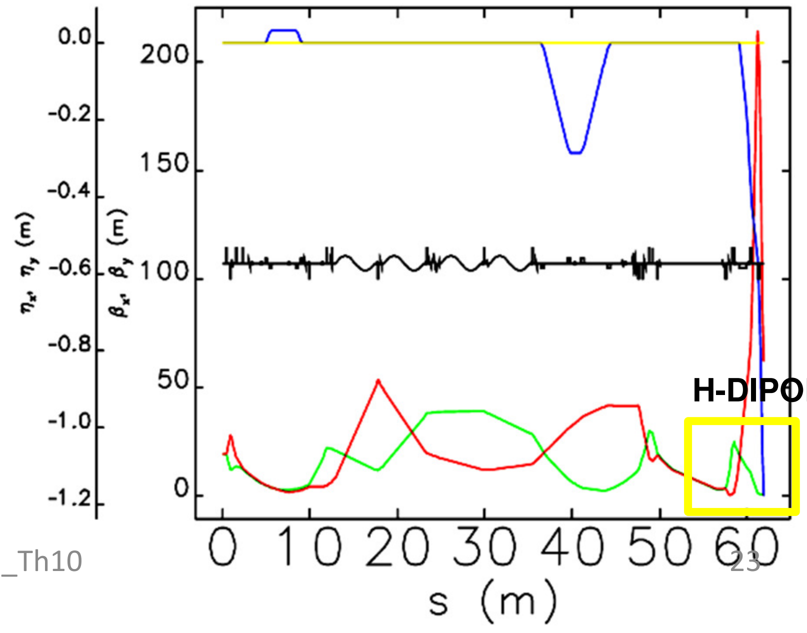


V-DEFL.

SCREENS +COLLIM.

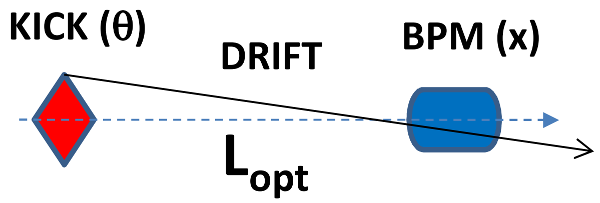
BC1

Spectrometer line



H-DIPOLE

Trajectory Steering (1-to-1)

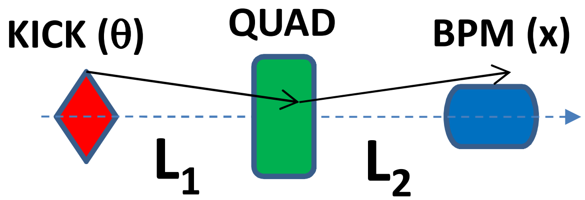


$$x \sim \vartheta\sqrt{\beta_k\beta_b}\,\sin(\Delta\mu_{kb})$$

☐ Look for an optimum distance L_{opt} ($\Delta\mu_{k\beta}=\pi/2$) that *minimizes the steerer's kick* θ, for any given displacement x at the end:

$$\Delta\mu_{kb} \approx \frac{s_k - s_b}{\bar{\beta}} \Rightarrow L_{opt} = \frac{\pi}{2}\bar{\beta}$$

☐ Now consider a quadrupole magnet in between:



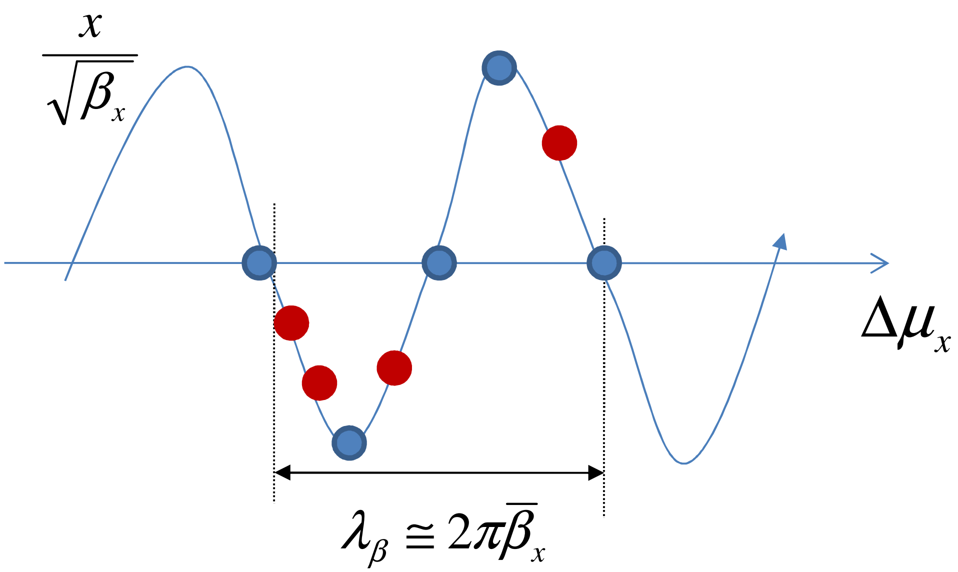
$$M_{kb} = M_{BPM}M_D M_Q M_D M_{KICK} = \begin{pmatrix} 1-\frac{L_2}{f_q} & L_1+L_2-\frac{L_1L_2}{f_q} \\ -\frac{1}{f_q} & 1-\frac{L_1}{f_q} \end{pmatrix}, \qquad \frac{1}{f_q} = kl_q$$

$$\frac{1}{2}Tr(M_{kb}) = 1 - \frac{L_1+L_2}{2f_q} \approx \cos\Delta\mu_x \equiv 0 \Rightarrow L_{opt} = L_1 + L_2 = 2f_q$$

☐ In reality, spatial constraints force to compromises:

- A BPM ***too close*** to steering magnets makes the correction largely inefficient – poor sensitivity to trajectory control.
- A BPM ***too far*** from steering magnets leads to poor reconstruction of the beam trajectory – poor correction.

Trajectory Sampling



4 BPMs per β-period can sample and reconstruct the trajectory.

Over-sampling may lead to:

- improvement of the correction sensitivity,
  → weaker corrector strengths
- redundancy (degeneracy),
  → stronger corrector strengths

Optimize the correctors and BPMs layout with *codes*.