Elettra
Sincrotrone
Trieste

CSR-Induced Emittance Growth and Related Design Strategis

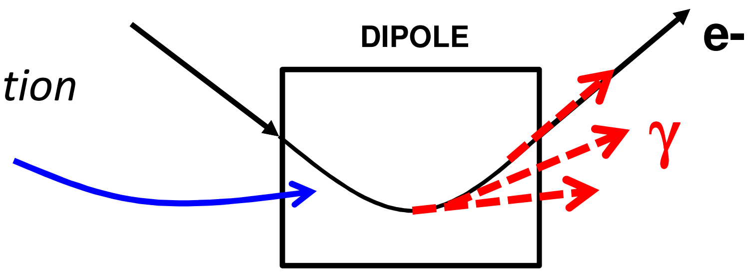
S. Di Mitri (60min.)

CSR Emission, 1-D Approximation

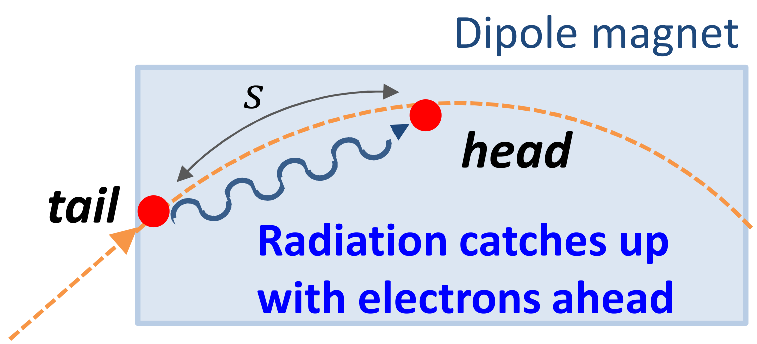
In the following we adopt a simplified picture for the CSR transverse effect. Experimental results suggest that it is accurate enough for describing *most* of the practical cases.

- *Photons* are *emitted* in the *beam direction of motion*, at any point along the curved trajectory in a dipole magnet ⇒ **CSR longitudinal effect**, $p_z(s) \rightarrow p_z(s) - \delta p_z(s)$. We thus neglect direct trasverse forces associated to the CSR field.

Remind: particle transv. motion is the linear superposition:
$x(s) = x_\beta(s) + x_\eta(s)$



- *As opposite to geometric wakefields in RF structures*, CSR shows up a tail-head effect, in which photons emitted by trailing electrons catch up with leading electrons.



CSR-Induced Energy Spread

- **Coherent emission** ($\lambda \geq \sigma_z$) dominates over incoherent by a **factor N_e**.
- Closed-form expression exists for the **electric field along direction of motion**:
  - two particles on same trajectory path,
  - uniform circular motion (steady-state),
  - use expressions for retarded-fields

ENERGY CHANGE ALONG BUNCH, per METER:

$$\Delta Ue(z) \cong -\frac{Ne^2}{4\pi\varepsilon_0}\frac{2}{3^{\frac{1}{3}}}\theta R^{\frac{1}{3}}\int_z^{\infty}\frac{dz'}{(z'-z)^{1/3}}\frac{d\lambda(z')}{dz'}$$

Current spikes or fast rises enhance the z-CSR field.

RELATIVE ENERGY SPREAD of GAUSSIAN bunch, per DIPOLE:

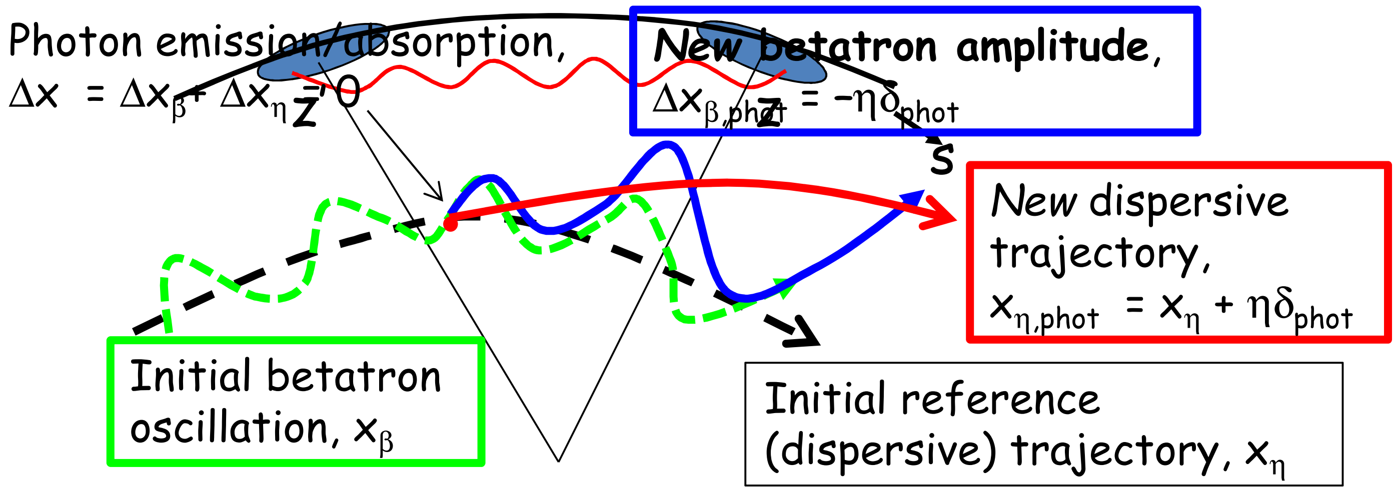
$$\sigma_{\delta,CSR} = 0.2459 \cdot r_e^2 \frac{N}{\gamma}\frac{\theta_B R^{1/3}}{\sigma_z^{4/3}}$$

- **high charge**
- **low beam energy**
- **short bunch length**
- **large bending angle**

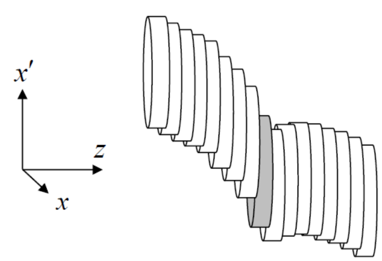
- 1D models accounting for transient effects are implemented in tracking codes.
- Codes with 2D CSR transverse forces exist; 3D effects are in progress.
- The most notable **macroscopic** effect of CSR is on the **transverse** dynamics.

CSR-Induced Emittance Growth: Naïve Picture

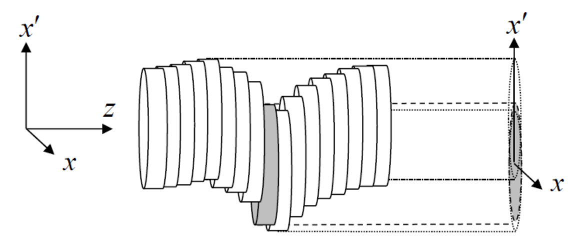
- At any **point of emission/absoprtion**, particle's ***transverse coordinates do not change***: $\Delta x=0$, $\Delta x'=0$. Since the emission happens in an energy-dispersive region, it implies $\Delta x_\beta=-\Delta x_\eta$. That is, the particle starts **β-oscillating** (Δx_β) around a ***new dispersive trajectory*** ($-\Delta x_\eta$).



- Once η_x is zeroed, e.g. At the exit of a symmetric chicane, the CSR-induced β-oscillation remains: the beam as «gained» a non-zero C-S amplitude which sums up to its initial emittance.



x' – x trace space of different slices (chirped)

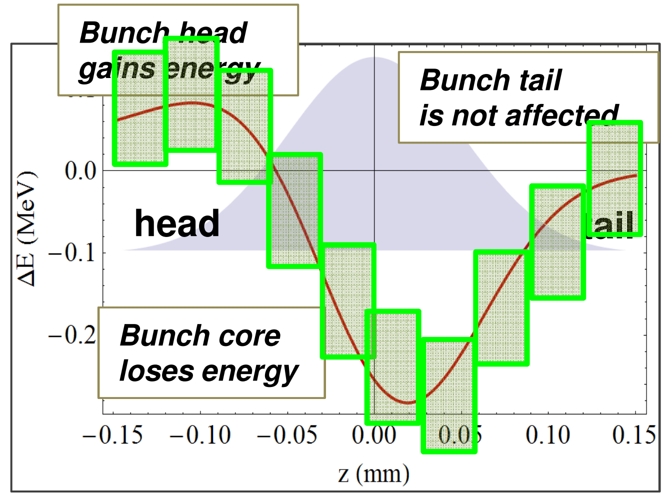


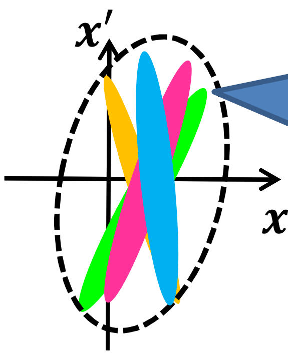
x' – x trace space of different slices (chirp removed)

CSR-Induced Emittance Growth: Estimate

$\delta p_{z,CSR}$ is correlated with z along the bunch (see previous expression for the energy change):

$\Rightarrow$ all particles at the same z-slice feel approximately the same CSR kick (we are assuming a slice much shorter than the bunch length, say 1/10 or even less).





*Different bunch slices feel different CSR kicks, thus move on different β-trajectories. If the slice ellipses are not concentric, the **projected** emittance is **larger**, although individual slices may have the **same slice emittance**.*

- Use the '***beam matrix***' to compute the CSR effect (single-kick approximation, average effect):

$$\langle x_\beta^2 \rangle \qquad \langle \Delta x_\beta^2 \rangle = \langle \Delta x_\eta^2 \rangle$$

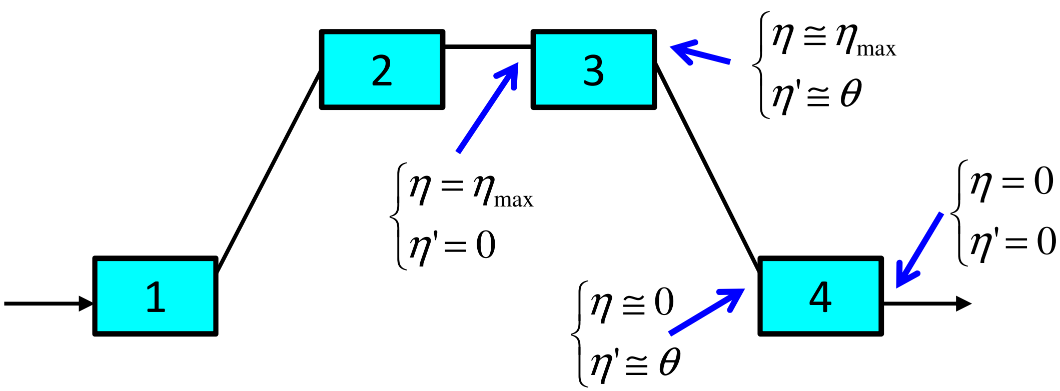
$$\varepsilon \cong \left[\det \begin{pmatrix} \varepsilon_0 \beta + \eta^2 \sigma_{\delta,CSR}^2 & -\varepsilon_0 \alpha + \eta\eta' \sigma_{\delta,CSR}^2 \\ -\varepsilon_0 \alpha + \eta\eta' \sigma_{\delta,CSR}^2 & \varepsilon_0 \dfrac{1+\alpha^2}{\beta} + \eta'^2 \sigma_{\delta,CSR}^2 \end{pmatrix} \right]^{1/2} = \varepsilon_0 \sqrt{1 + \frac{H}{\varepsilon_0} \sigma_{\delta,CSR}^2},$$

$$H = \left[\eta^2 + (\beta\eta' + \alpha\eta)^2\right] / \beta$$

takes care of the coupled betatron and dispersive motion.

CSR in a 4-Dipoles (Symmetric) Chicane

PRSTAB 16, 060703 (2013)
PR 539 (2014)

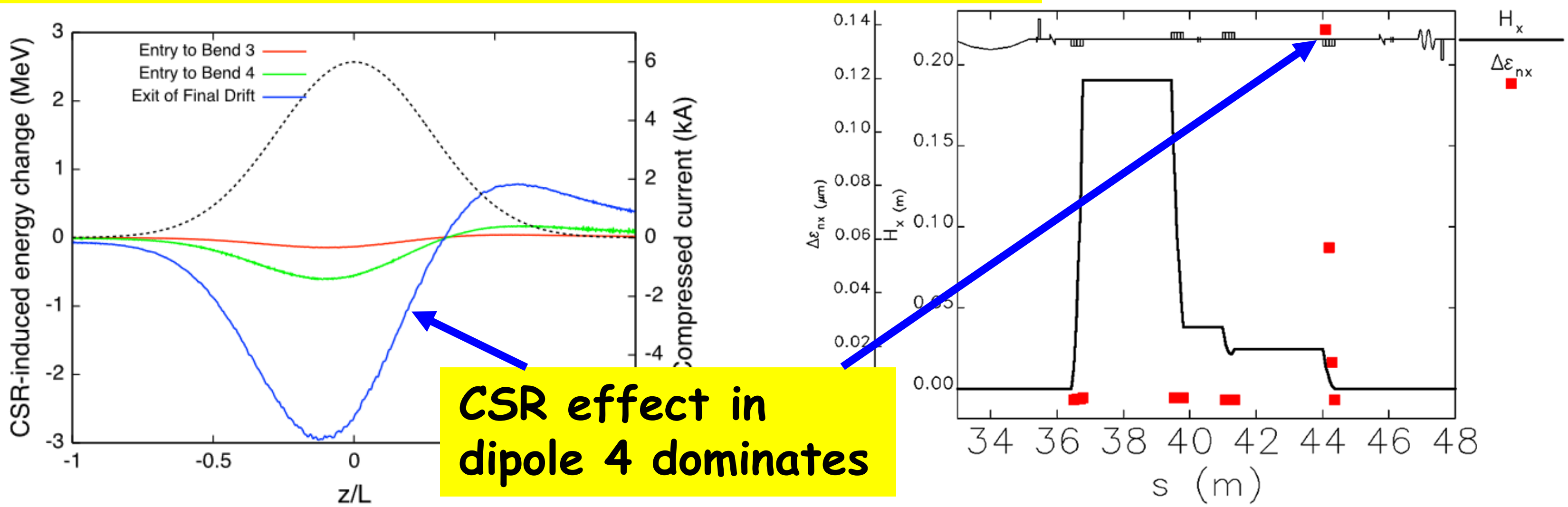


- Assume θ<<1.
- Assume $\alpha_x \approx 0$ between dipole 3 and 4.

- H-function is larger in proximity of dipole 4, and of the order of $\beta\theta^2$.
- Also, σ_z is shorter (CSR field is stronger) between dipole 3 and 4

$$\varepsilon = \varepsilon_0 \sqrt{1 + \frac{H}{\varepsilon_0}\sigma_{\delta,CSR}^2} \approx \varepsilon_0 \left(1 + \frac{\breve{\beta}\theta^2 \sigma_{\delta,CSR}^2}{2\varepsilon_0}\right)_{4th\ dipole}$$

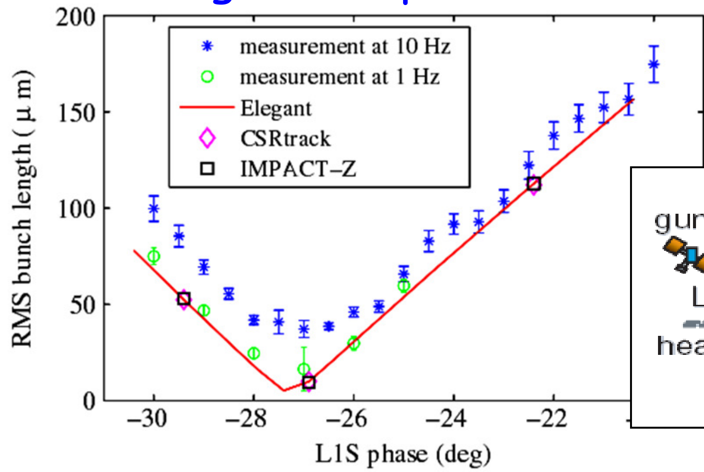
Warning! *CSR* propagation in *drifts* can be important, but it is *neglected here*!



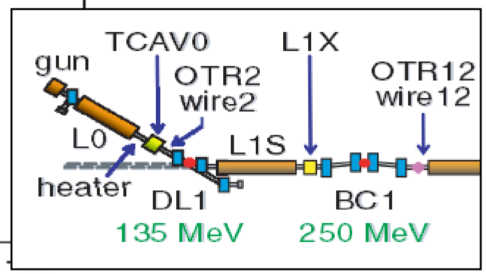
Projected Emittance and Bunch Length

PRSTAB 12, 030704 (2009)

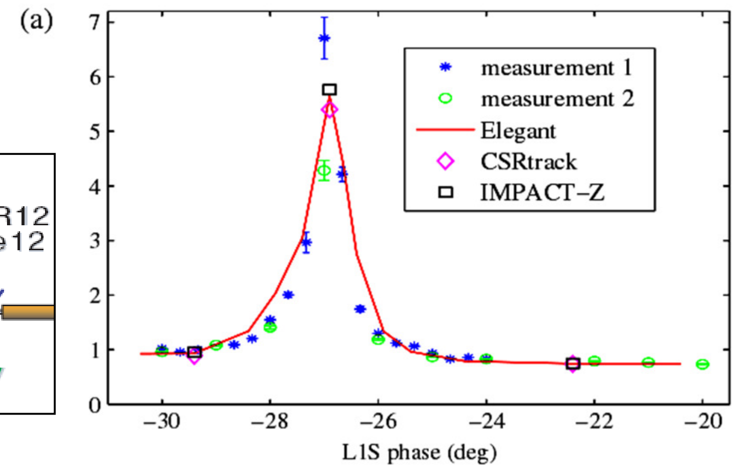
Bunch length vs. upstream linac phasing



LCLS BC1

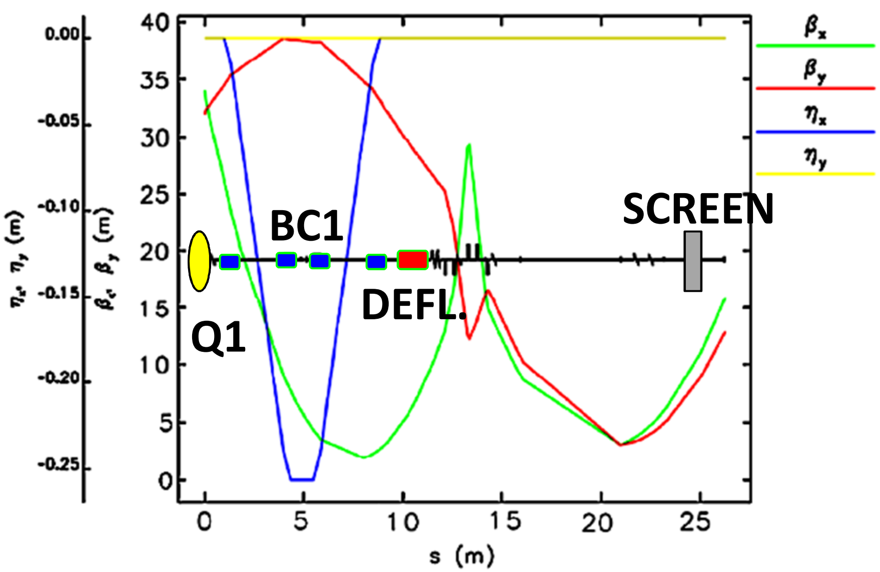


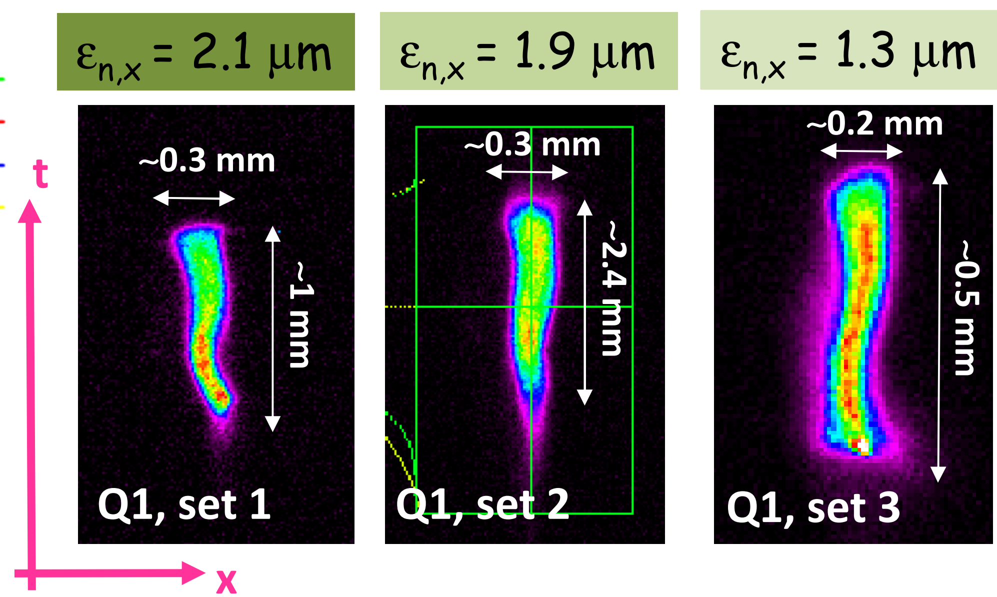
Horiz. proj. emittance vs. linac phase



Horiz. Proj. Emittance vs. upstream quad strength

FERMI BC1





Strategies for a 4-Dipoles Compressors Design

CSR-emittance can be **minimized in RMS sense** (along the bunch) with the following prescriptions (not exclusive), which apply to the **lattice design**:

$$R_{56} \cong -2\theta^2\left(L+\frac{2}{3}l_b\right)$$

- Design the chicane with the **lowest R_{56}** you may need *(this implies a larger energy spread for the same compression factor, thus high field quality to minimize chromatic aberrations).*

$$\sigma_{\delta,CSR} = 0.2459 \cdot r_e^2 \frac{N}{\gamma}\frac{\theta_B R^{1/3}}{\sigma_z^{4/3}}$$

- *For a given R_{56}, use* ***small bending angles*** *(in case, use longer dipoles and drifts).*

- Set the compressor **energy** as **high** as possible *(this requires more off-crest phasing for the same relative energy spread at the chicane).*

$$\varepsilon \cong \varepsilon_0\sqrt{1+\frac{H}{\varepsilon_0}\sigma_{\delta,CSR}^2}$$

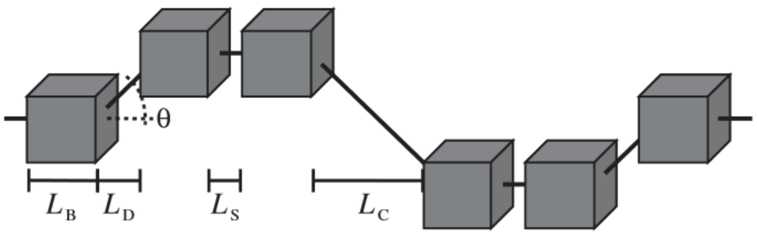
- *Minimize H-function in the second half of the chicane, e.g. squeeze β_x* ***to a minimum*** *in between dipole 3 and 4.*

Other Strategies

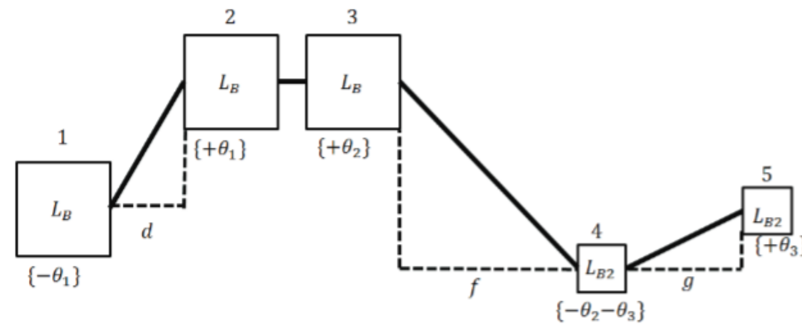
PRSTAB 10, 031001 (2007)
PRSTAB 16, 060703 (2013)
and courtesy D. Kahn

- We can even play with the chicane geometry, in order to minimize the cumulative effect of CSR kicks at the dipoles. This involves the **chicane geometry** and/or the **beam optics**:

4/6-bends symmetric chicane: the inner dipoles give twice the outer bending angle.

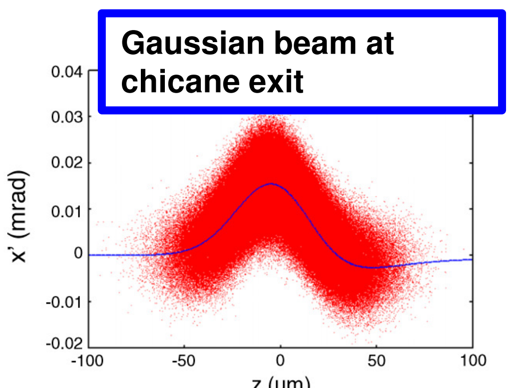


5-bends asymmetric chicane.

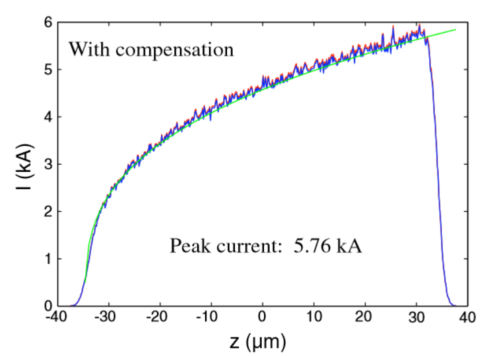


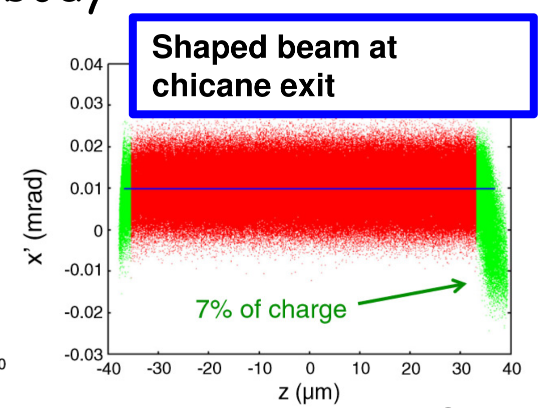
$$\Delta Ue(z) \propto \int_z^{\infty} \frac{dz'}{(z'-z)^{1/3}} \frac{d\lambda(z')}{dz'}$$

- Shape the **incoming bunch current profile** in order to induce a **uniform energy loss**, which would shift the bunch slices as a rigid body:



shaping





CSR-Emittance in a Transfer Line (σ_z=const.)

Problem.

When the **bunch length** is short and **constant** along a **multi-bend line** (e.g., high energy transfer line connecting linac to undulators), we cannot recognize any «dominant» point of CSR emission (e.g., dipole 4 in a chicane). Which design prescription, then?

Idea.

Adjust the optics along the line so that successive **CSR kicks cancel** each other (*~SBBU approach!*). For symmetric CSR-source points, *optics symmetry* and *π phase advance* between dipoles is a solution. More general optics schemes work as well if Twiss functions and phase advance are properly «balanced».

Warnings.

- This approach assumes identical CSR kicks *in module*, e.g. same bunch length emitting CSR in identical dipoles.
- The simplest analysis (see next slides) assume point-like optics functions in the dipoles (thin lens approximation). More accurate analysis implies dipoles' thick length.
- We neglect any transient CSR field at the dipoles' edges, and CSR in drift sections. These effects can be taken into account in tracking codes, e.g. ELEGANT.

Optics Balance & Courant-Snyder Invariant

A. Use the **Courant-Snyder formalism** for the particle coordinates. Initial invariant is zero.

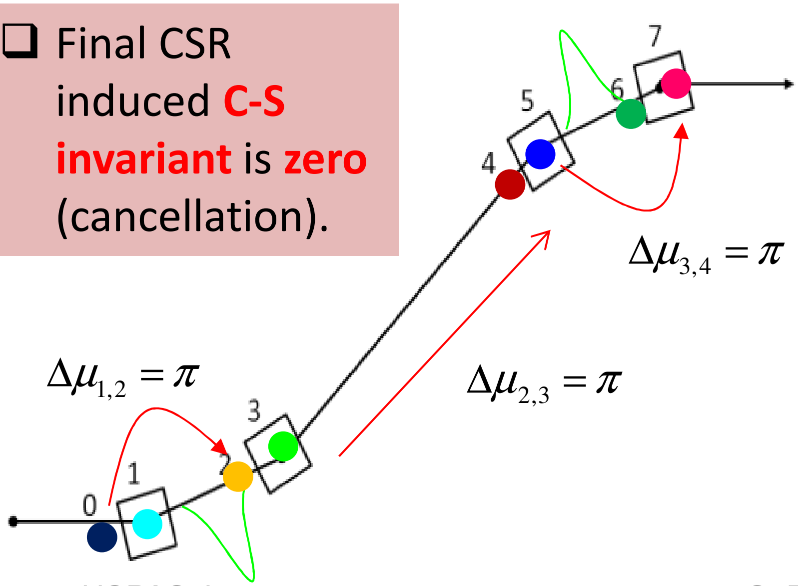
B. While traversing a dipole, add the CSR induced η-terms. This leads to an **increase** of the particle C-S **invariant**:

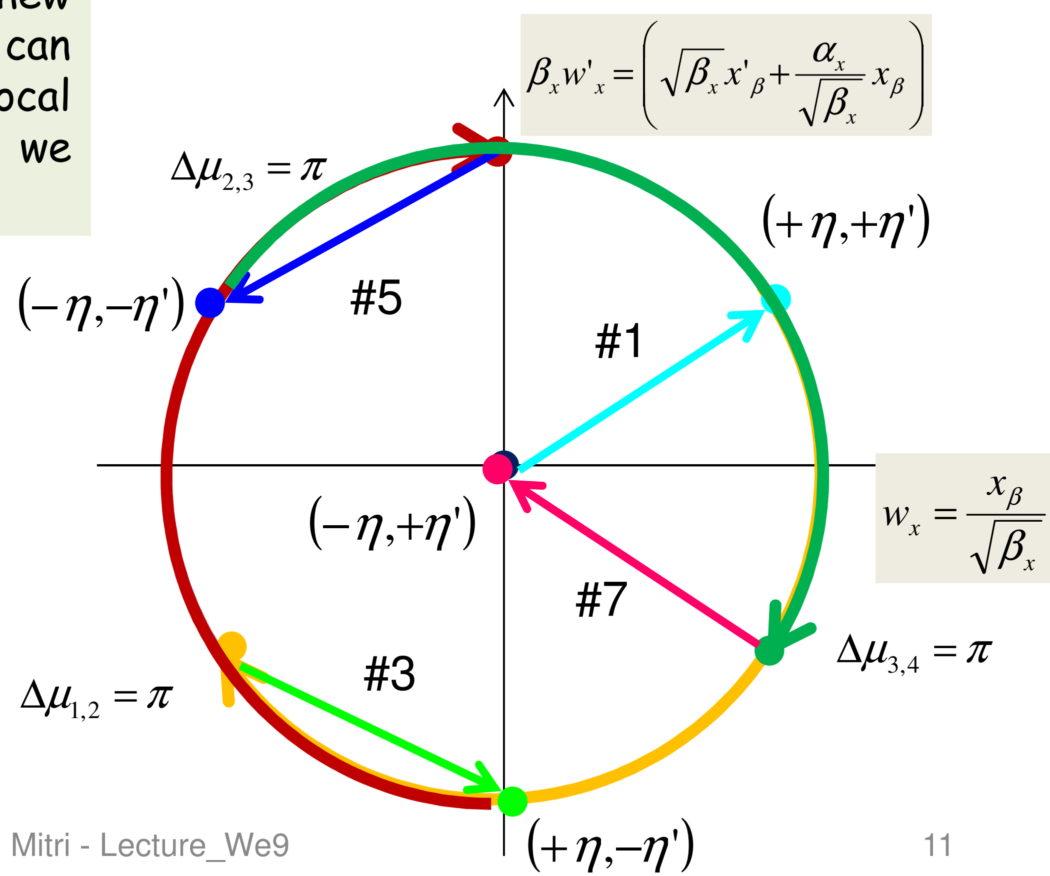
$$2J_1 = \gamma_1 x_1^2 + 2\alpha_1 x_1 x_1^{'} + \beta_1 x_1^{'2} = H_1 \delta_{CSR}^2$$

C. Repeat until the end of the line. Each new invariant after a CSR kick in a dipole, can be defined in terms of J_1 and of the local Twiss functions. After the **last dipole** we find:

$$\Delta\gamma\varepsilon = \gamma\varepsilon_0 \left[\sqrt{1 + \frac{H_1 \sigma_{\delta,CSR}^2}{\varepsilon_0} X_{17}} - 1 \right] < 0.1\mu m$$

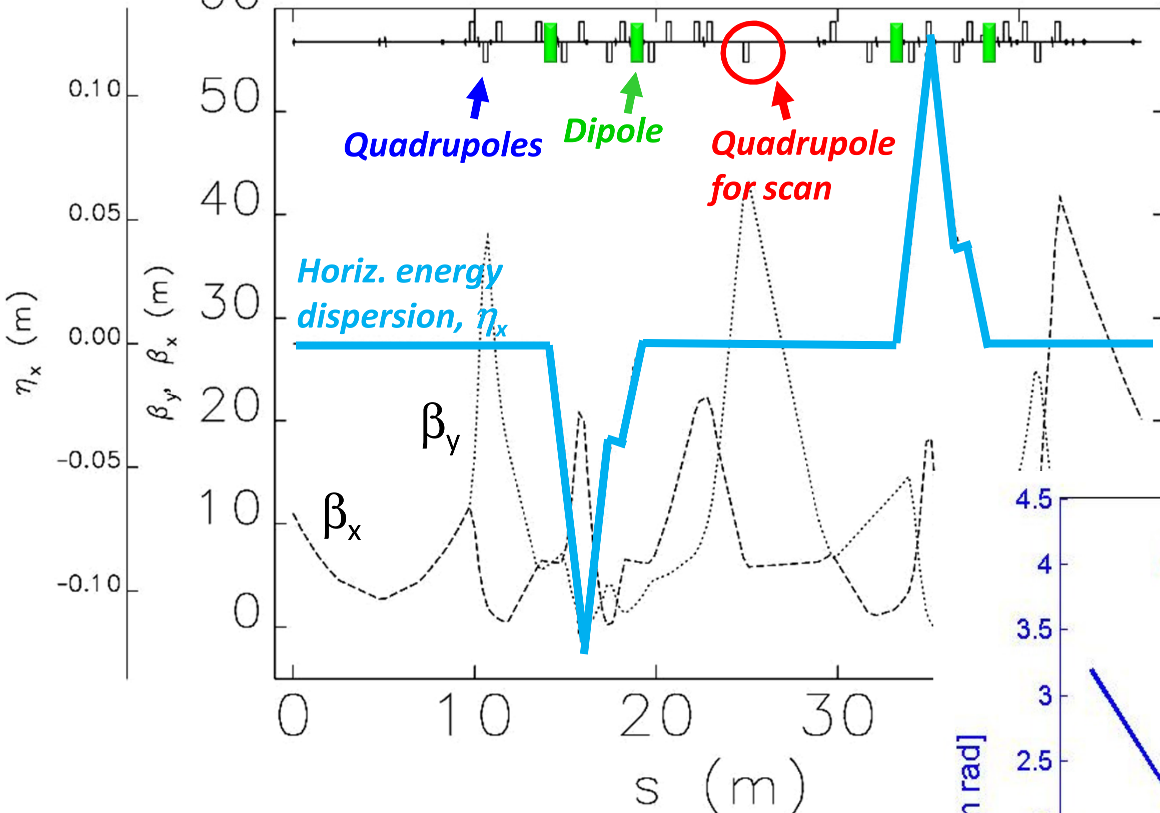
- Final CSR induced **C-S invariant** is **zero** (cancellation).





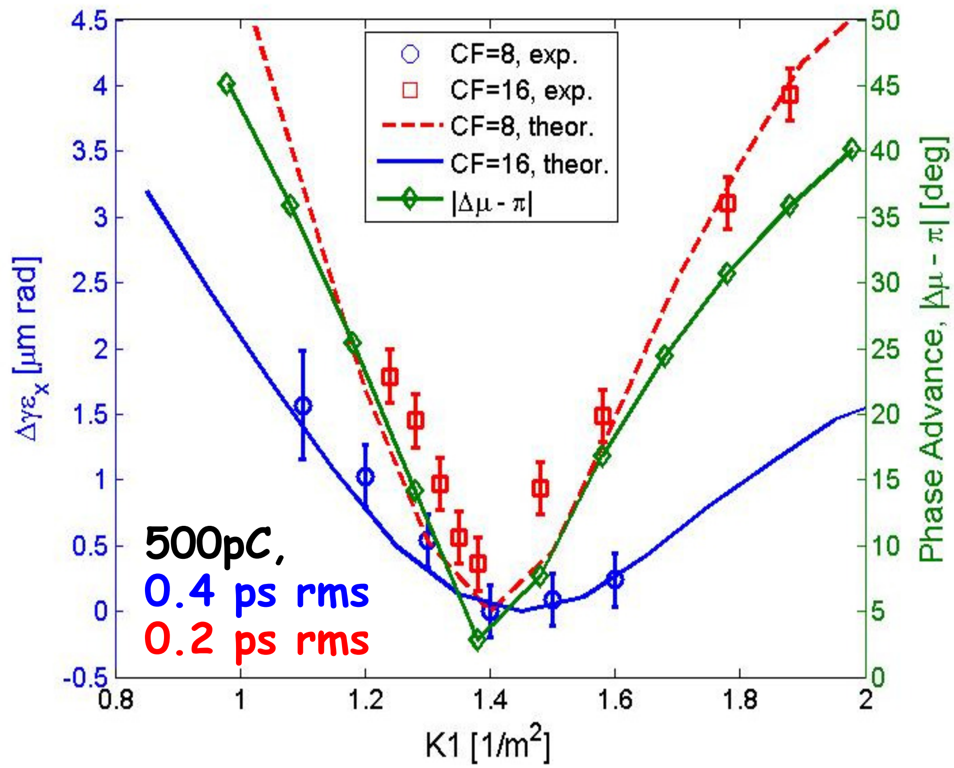
Experimental Results

PRL 109 (2013); PR 539, 1 (2014)



Many quadrupoles ensure π-phase advance between dipoles and proper values of β_x, α_x to cancel the final emittance growth.

One quadrupole's strength is varied in the experiment to scan the phase advance between the two achromats.



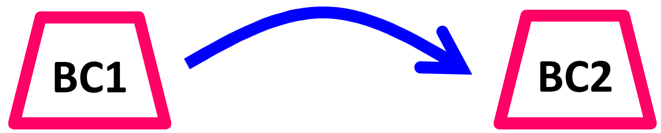
$\varepsilon_{n,x}$ is measured at the line end as function of the quadrupole strength:

- minimum $\varepsilon_{n,x}$ for nominal optics (π-phase advance)
- Larger $\varepsilon_{n,x}$-growth for shorter beam

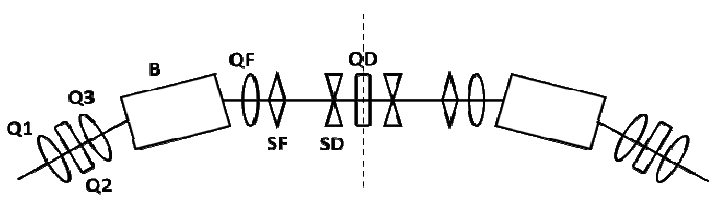
Potential Applications of Optics Balance

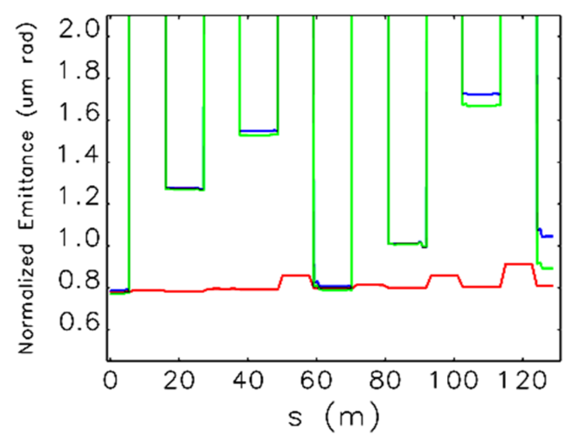
EPL 109 62002 (2015)

- ***Compensating CSR kicks produced in consecutive chicanes (BC1,BC2,…) ??***
  - Preliminarly investigated, with poor results. Difficulties arise from optics control in and between the chicanes. The scheme also limits the flexibility of beam transport between the chicanes.



- ***Compensating CSR kicks produced in long magnetic compressor, like a 180 deg arc ??***
  - Preliminarly investigated at GeV energies. Same principle than in a low-emittance storage ring lattice, where ε-control requires many, weak dipoles, thus a long line.





CSR-Induced Slice Emittance Growth

Courtesy M. Dohlus
NIM A 608 (2009)

Slice emittance is affected if the bunch becomes so short that particle cross over large portions of it, and, at the end of compression, lie in a slice different from the initial one ("phase mixing") ⇒ incoherent "sum" of C-S invariants.

This effect is more subtle than projected emittance growth and it is usually investigated with particle **tracking codes**.