3
Perturbations

Mauricio Lopes – FNAL

Introduction

Random multipole errors are introduced if the poles are improperly excited or assembly errors which displace poles are introduced. If one can identify these errors, one can predict the multipole content of the magnet. The means for calculating these errors are summarized in two papers published by Klaus Halbach. The first paper describes the derivation of the relationships, the second computes and tabulates the coefficients used to calculate the multipole errors from the perturbations derived in the first paper.

[1] Halbach, K., FIRST ORDER PERTURBATION EFFECTS IN IRON-DOMINATED TWO-DIMENSIONAL SYMMETRICAL MULTIPOLES, “Nuclear Instruments and Methods”, Volume 74 (1969) No. 1, pp. 147-164.

[2] Halbach, K., and R. Yourd, TABLES AND GRAPHS OF FIRST ORDER PERTURBATION EFFECTS IN IRON-DOMINATED TWO-DIMENSIONAL SYMMETRICAL MULTIPOLES, LBNL Internal Report, UCRL-18916, UC-34 Physics, TID 4500 (54thEd.), May 1969.

Effect of Mechanical Fabrication Errors on Error Multipole Content

In the previous lecture, we showed that the field distribution in a magnet can be characterized by a function of the complex variable, *z*. In particular;

$$F = C_N z^N + \sum_{n \neq N} C_n z^n$$

$C_N z^N$ Fundamental field component

$\sum_{n \neq N} C_n z^n$ Error fields

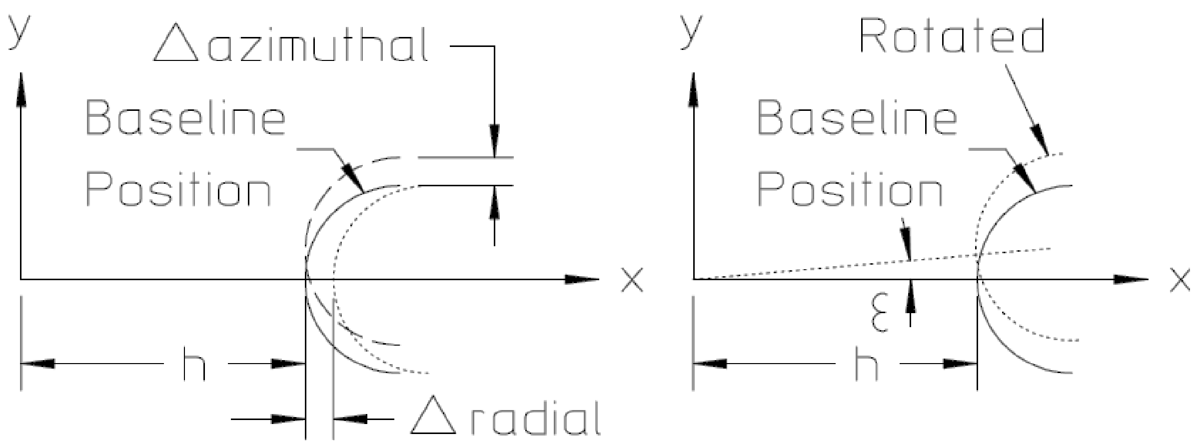
$n = (2m+1)$ for $m = 1, 2, 3, 4 \ldots$ Allowed harmonics $\longrightarrow$ Symmetric magnet

Non-Symmetric magnet $\longrightarrow$ $n = 1, 2, 3, 4 \ldots$

Random Multipole Errors Due to Pole Excitation and Pole Placement Errors

$$\varepsilon_{curr} = \frac{\Delta\,\text{excitation}}{\text{excitation}} \qquad \varepsilon_{ad} = \frac{\Delta\,\text{azimuthal position}}{\text{h}}$$

$$\varepsilon_{rd} = \frac{\Delta\,\text{radial position}}{\text{h}} \qquad \varepsilon_{\rho} = \Delta pole\,\text{rotation}\,(\text{radians})$$



Random Multipole Errors Due to Pole Excitation and Pole Placement Errors

$$\left(\frac{B_n^*}{B_N^*}\right)_{@r=h} = \frac{n}{N}\Delta C_n e^{-in\beta}$$

where β is the perturbed pole

$$\left(\frac{B_{nx} - iB_{ny}}{-iB_{Ny}}\right)_{@r=h} = \frac{n}{N}\Delta C_n e^{-in\beta}$$

$$\left(\frac{B_{ny} + iB_{nx}}{B_{Ny}}\right)_{@r=h} = \frac{n}{N}\Delta C_n e^{-in\beta}$$

$$\left(\frac{B_{ny} + iB_{nx}}{B_{Ny}}\right)_{@r=h} = \left(i\varepsilon_{curr}E_{curr_n} + i\varepsilon_{Rd}E_{Rd_n} + \varepsilon_{Az}E_{Az_n} + \varepsilon_{Rot}E_{Rot_n}\right)e^{-in\beta}$$

$$\boxed{\left(\frac{B_{ny} + iB_{nx}}{B_{Ny}}\right)_{@r=h} = \left(i\varepsilon_{curr}E_{curr_n} + i\varepsilon_{Rd}E_{Rd_n} + \varepsilon_{Az}E_{Az_n} + \varepsilon_{Rot}E_{Rot_n}\right)(\cos n\beta - i\sin n\beta)}$$

Table with errors

Quadrupole

n	Excitation (i)	Radial (i)	Azimuthal	Rotational
1	1.99E-01	-4.25E-01	7.46E-02	1.76E-01
2	2.50E-01	-5.16E-01	2.14E-01	5.00E-01
3	1.57E-01	-2.88E-01	2.88E-01	6.60E-01
4	0	6.76E-02	2.31E-01	5.00E-01
5	-2.05E-02	1.08E-01	1.08E-01	1.91E-01
6	0	4.45E-02	2.87E-02	0
7	-1.61E-02	-1.04E-02	1.04E-02	-3.06E-02
8	0	1.28E-02	1.56E-02	0
9	-1.90E-03	1.25E-02	1.25E-02	7.53E-03
10	0	6.37E-03	5.81E-03	0
11	3.15E-03	-2.44E-03	2.44E-03	-3.62E-03
12	0	2.66E-03	2.79E-03	0
13	-2.45E-04	2.27E-03	2.27E-03	9.28E-04
14	0	1.26E-03	1.23E-03	0
15	6.69E-04	-5.55E-04	5.55E-04	-6.66E-04

Sextupole

n	Excitation (i)	Radial (i)	Azimuthal	Rotational
1	9.79E-02	-3.14E-01	5.09E-02	8.47E-02
2	1.56E-01	-4.95E-01	1.71E-01	2.84E-01
3	1.67E-01	-5.15E-01	3.03E-01	5.00E-01
4	1.33E-01	-3.90E-01	3.90E-01	6.39E-01
5	7.09E-02	-1.73E-01	3.97E-01	6.43E-01
6	0	6.55E-02	3.18E-01	5.00E-01
7	-1.34E-02	1.08E-01	1.95E-01	2.88E-01
8	-1.07E-02	9.03E-02	9.03E-02	1.08E-01
9	0	4.16E-02	2.51E-02	0
10	9.13E-03	-1.93E-03	1.90E-03	-3.38E-02
11	9.72E-03	-1.45E-02	5.49E-03	-2.05E-02
12	0	1.05E-02	1.31E-02	0
13	-1.01E-03	1.07E-02	1.36E-02	7.34E-03
14	-1.18E-03	9.85E-03	9.85E-03	5.82E-03
15	0	5.06E-03	4.56E-03	0

$$\frac{n}{N}\frac{\Delta C_n(curr)}{i\varepsilon} = iE_{curr_n} \qquad \frac{n}{N}\frac{\Delta C_n(rd)}{i\varepsilon} = iE_{Rd_n} \qquad \frac{n}{N}\frac{\Delta C_n(ad)}{\varepsilon} = E_{Az_n} \qquad \frac{n}{N}\frac{\Delta C_n(r)}{\varepsilon} = E_{Rot_n}$$

Example

Suppose we construct a 35 mm radius quadrupole (N=2) whose first pole ($\beta=\pi/4$) is radially offset by 1 mm. What is the effect on the n=3 multipole error?

$$\left(\frac{B_{ny}+iB_{nx}}{B_{Ny}}\right)_{@r=h}=\left(i\varepsilon_{curr}E_{curr_n}+i\varepsilon_{Rd}E_{Rd_n}+\varepsilon_{Az}E_{Az_n}+\varepsilon_{Rot}E_{Rot_n}\right)(\cos n\beta-i\sin n\beta)$$

$$\left(\frac{B_{3y}+iB_{3x}}{B_{2y}}\right)_{@r=h}=\left(i\varepsilon_{Rd}E_{Rd_n}\right)(\cos n\beta-i\sin n\beta) \qquad \varepsilon_{rd}=\frac{\Delta\,\text{azimuthal position}}{\text{h}}=\frac{1}{35}$$

$$\left(\frac{B_{3y}+iB_{3x}}{B_{2y}}\right)_{@r=35}=\left(i\frac{1}{35}(-0.288)\right)\left(\cos 3\frac{\pi}{4}-i\sin 3\frac{\pi}{4}\right)\begin{cases}\left(\frac{B_{3y}}{B_{2y}}\right)_{@r=35}=\left(\frac{1}{35}(-0.288)\right)\sin 3\frac{\pi}{4}=-5.82\cdot 10^{-3} & \text{(Normal term)}\\[2ex] \left(\frac{B_{3x}}{B_{2y}}\right)_{@r=35}=\left(\frac{1}{35}(-0.288)\right)\cos 3\frac{\pi}{4}=+5.82\cdot 10^{-3} & \text{(Skew term)}\end{cases}$$

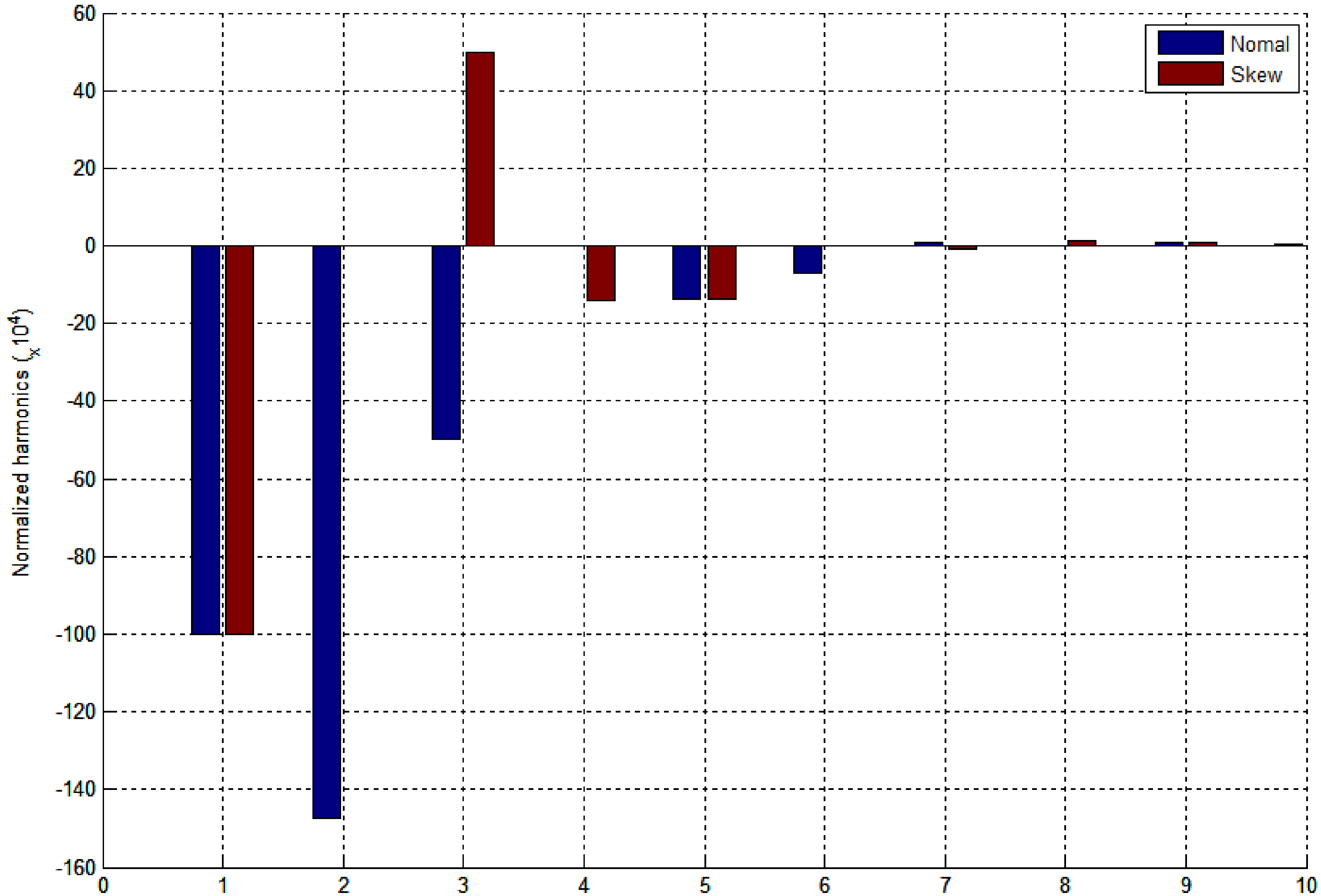
Evaluation at the Required Good Field Radius

$$\boxed{\left(\frac{B_n}{B_N}\right)_{@r=r_o} = \left(\frac{B_n}{B_N}\right)_{@r=h} \left(\frac{r_o}{h}\right)^{n-N}}$$

Assume that the good field radius is r_o=30 mm

$$\left(\frac{B_{3y}}{B_{2y}}\right)_{@r=30} = \left(\frac{B_n}{B_N}\right)_{@r=35} \left(\frac{30}{35}\right)^{3-2} = -4.99 \cdot 10^{-3}$$

Full Spectrum



Lesson to be learned…

The lesson from this sample calculation is *not* the detailed calculation of the multipole error, but the estimate of the mechanical assembly tolerances which must be met in order to achieve a required field quality.

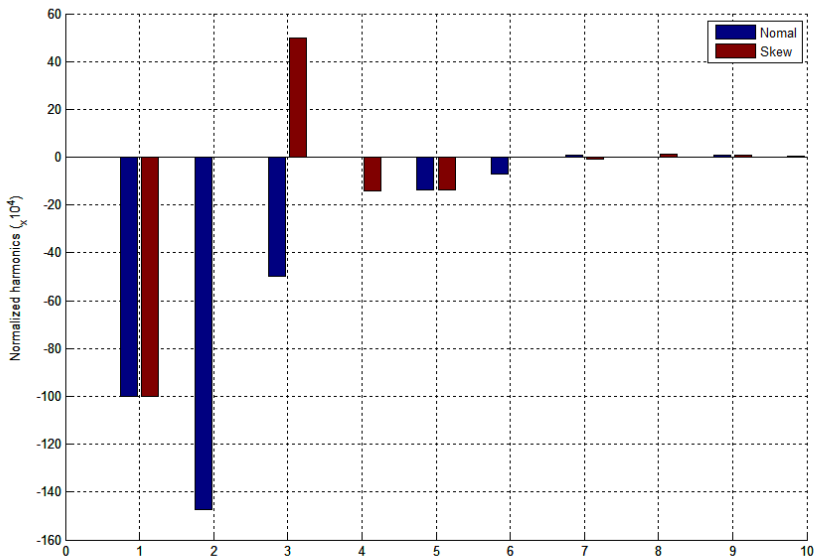
In general, the coefficient is <0.5. Therefore in order to achieve a field error at the pole radius of 5 parts in 10000 (a typical multipole error tolerance), the following tolerance illustrated in the calculation must be maintained.

$$\left|\frac{B_n^*}{B_N^*}\right| = E \times \frac{\Delta}{h} \qquad \Delta \le \frac{h}{E} \cdot \left|\frac{B_n^*}{B_N^*}\right|_{required}$$

$$\Delta \le \frac{35}{0.5} 5 \cdot 10^{-4} = 0.035mm$$

Very small error!

The Magnet Center



We note that *all* the multipole errors are introduced by mechanical assembly errors. In particular, we look in detail at the *dipole* error term introduced by assembly errors.

For the *pure* quadrupole field, the expression for the complex function is; $F(z) = C_2 z^2$

If the magnet center is shifted by an amount Δz, the expression becomes; $F(z) = C_2 (z + \Delta z)^2$

$$B^* = iF'(z)$$

$$B^* = iF'(z) = i2C_2(z + \Delta z)$$

$$B^* = i2C_2 z + i2C_2 \Delta z$$

Quadrupole term   Dipole term

Rewriting the expression as the sum of two fields;

$$B^* = B_2^* + B_1^* = i2C_2 z + i2C_2(\Delta x + i\Delta y)$$

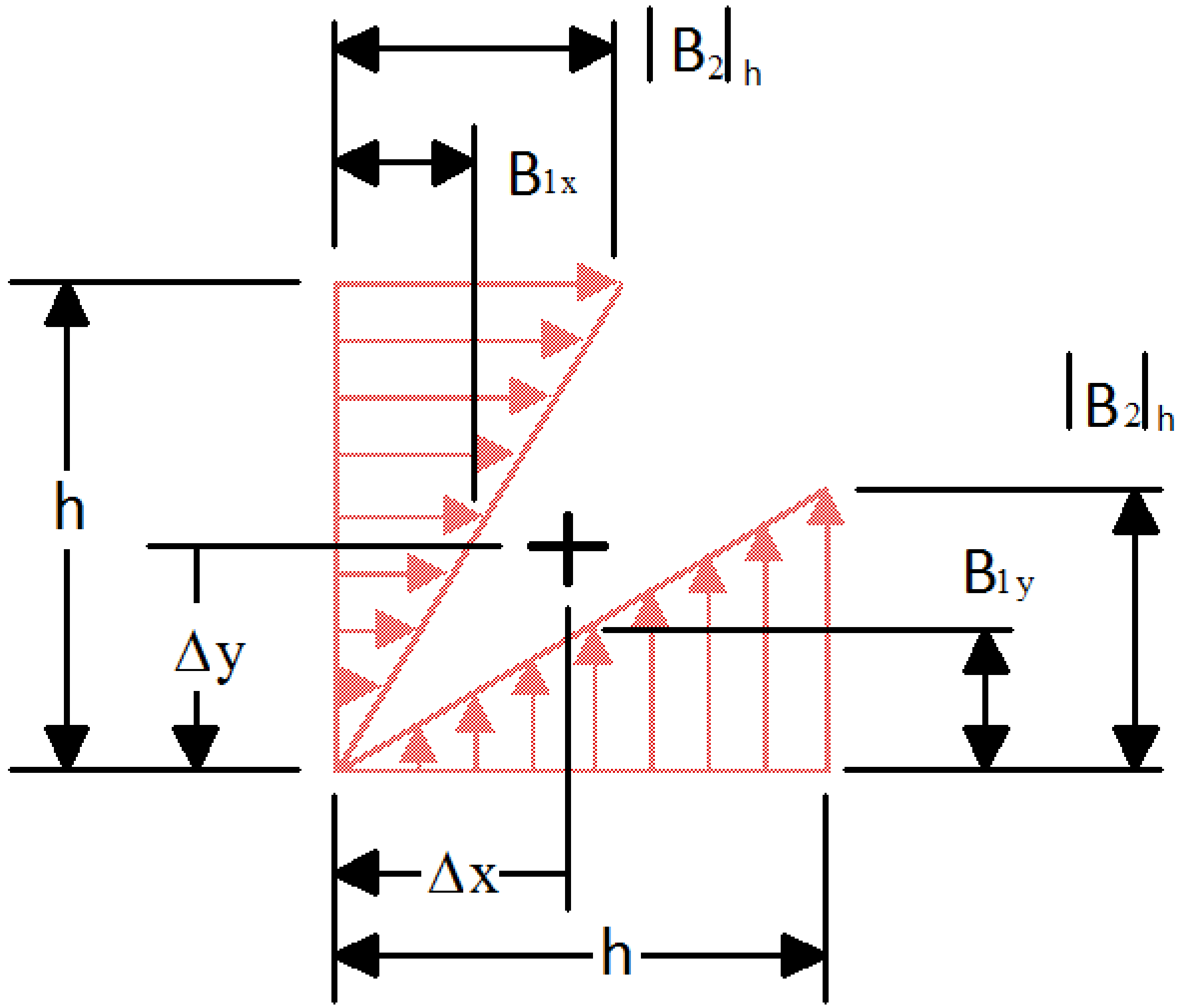
$$B_1^* = B_{1x} - iB_{1y} = i2C_2(\Delta x + i\Delta y)$$

$$\frac{B_{1x} - iB_{1y}}{\left|B_2^*\right|} = \frac{i2C_2(\Delta x + i\Delta y)}{2C_2|z|}$$

Evaluating the quadrupole field at the pole radius, *h*;

$$\frac{B_{1x} - iB_{1y}}{\left|B_2^*\right|_{@h}} = \frac{B_{1x}}{\left|B_2^*\right|_{@h}} - i\frac{B_{1y}}{\left|B_2^*\right|_{@h}} = \frac{i(\Delta x + i\Delta y)}{h} = i\frac{\Delta x}{h} - \frac{\Delta y}{h}$$

$$\boxed{\Delta x = -h\frac{B_{1y}}{\left|B_2\right|_h}} \qquad \boxed{\Delta y = -h\frac{B_{1x}}{\left|B_2\right|_h}}$$



|B2|h
B1x
h
Δy
|B2|h
B1y
Δx
h

Effect of a Pole Excitation Error on the Magnetic Center

The magnetic center is an important parameter for linear colliders. The requirement for the magnetic center for the ILC linac quadrupoles is 100μm.

A sample calculation is made to compute the required pole excitation precision.

$$\left(\frac{B_n^*}{B_N^*}\right)_{@r=h} = \left(i\varepsilon_{curr} E_{curr_n}\right) e^{-in\beta} \qquad E_{curr_n} = 0.199 \quad \text{for n=1}$$

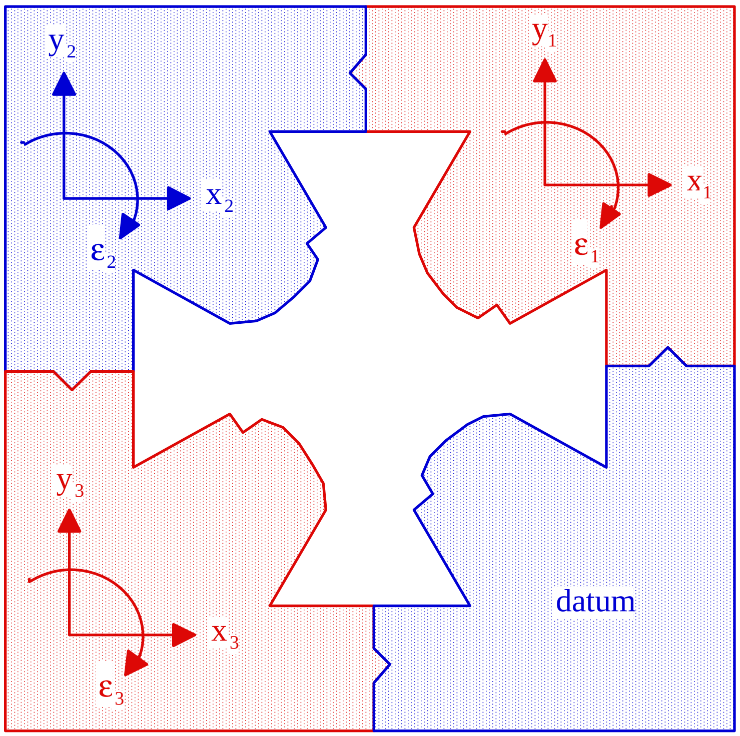
$$\varepsilon_{curr} = \frac{\Delta\,\text{excitation}}{\text{excitation}}$$ suppose 1% error on the excitation of one pole $\varepsilon_{curr} = 0.01$

$$\left(\frac{B_n^*}{B_N^*}\right)_{@r=h} = \left(i\varepsilon_{curr} E_{curr_n}\right) e^{-in\beta}$$

$$\left(\frac{B_{1y} + iB_{1x}}{B_{2y}}\right)_{@r=h} = i0.01 \cdot 0.199 \left(\cos\frac{\pi}{4} - i\sin\frac{\pi}{4}\right) \left\{ \begin{array}{l} \dfrac{H_{1y}}{H_{2y}} = 0.0014 \\[2ex] \dfrac{H_{1x}}{H_{2y}} = 0.0014 \end{array} \right. \Rightarrow \begin{array}{l} \Delta x = -h\dfrac{H_{1y}}{\left|H_2\right|_h} = \text{-}\,0.0014h \\[2ex] \Delta y = -h\dfrac{H_{1x}}{\left|H_2\right|_h} = -0.0014h \end{array}$$

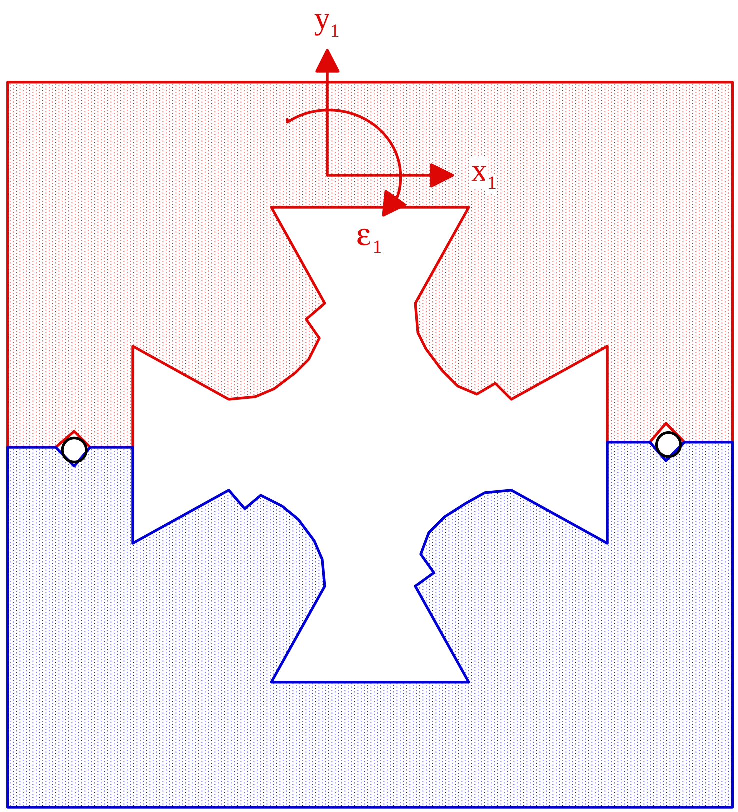
The Four Piece Magnet Yoke

- The ideal assembly satisfies the rotational symmetry requirements so that the only error multipoles are allowed multipoles, n=6, 10, 14...
- However, *each* segment can be assembled with errors with three kinematic motions, x, y and ε (rotation).
- Thus, combining the possible errors of the three segments with respect to the datum segment, the core assembly can be assembled with errors with 3x3x3=27 degrees of freedom.

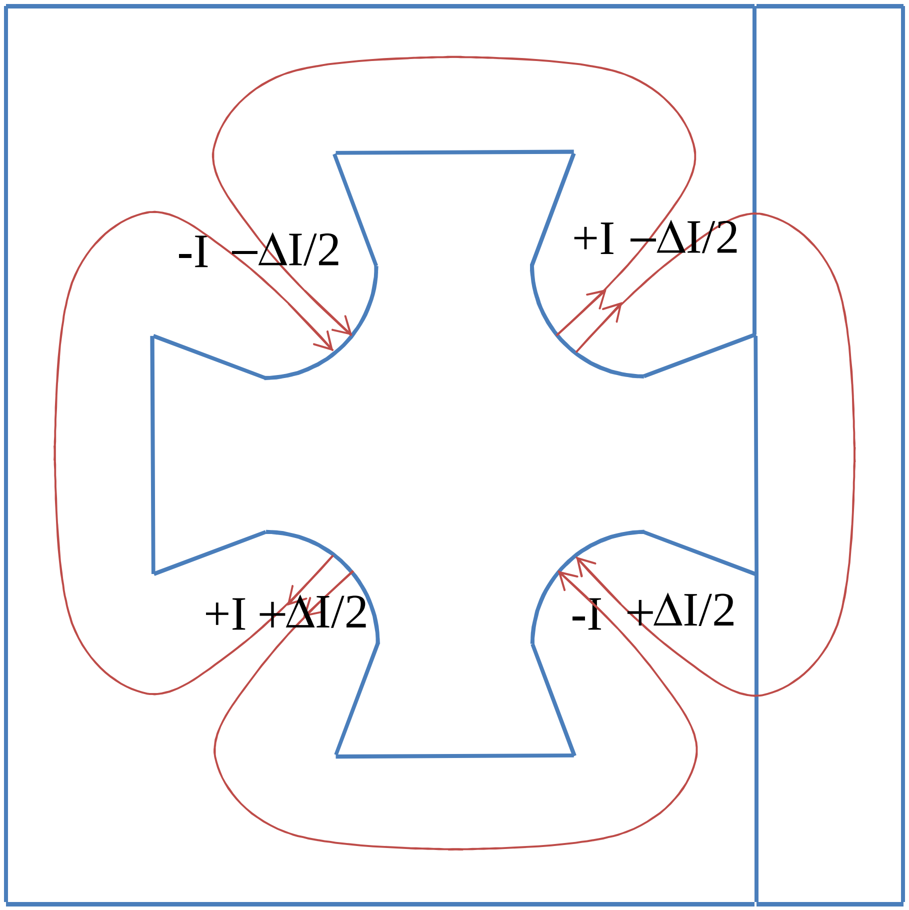


The Two Piece Magnet Yoke

- This assembly has the advantage that the two core halves can be assembled with only <u>three</u> degrees of freedom for assembly errors. Thus, assembly errors are more easily measured and controlled



Two Piece Asymmetric Quadrupole



$$\Delta I = \int_{path} H.dl$$

$$\left|\varepsilon_a\right| = \frac{\int_{path} H.dl}{I}$$

Two Piece Asymmetric Quadrupole

$$\left(\frac{B_{ny}+iB_{nx}}{B_{Ny}}\right)_{@r=h} = \left(i\varepsilon_{curr}E_{curr_n}\right)e^{-in\beta}$$

$$\left(\frac{B_{ny}+iB_{nx}}{B_{Ny}}\right)_{@r=h} = \left(-i\frac{\varepsilon_a}{2}e^{-in\frac{\pi}{4}} - i\frac{\varepsilon_a}{2}e^{-in\frac{3\pi}{4}} + i\frac{\varepsilon_a}{2}e^{-in\frac{5\pi}{4}} + i\frac{\varepsilon_a}{2}e^{-in\frac{7\pi}{4}}\right)E_{curr_n}$$

$$\left(\frac{B_{ny}+iB_{nx}}{B_{Ny}}\right)_{@r=h} = \left(-e^{-in\frac{\pi}{4}} - e^{-in\frac{3\pi}{4}} + e^{-in\frac{5\pi}{4}} + e^{-in\frac{7\pi}{4}}\right)i\frac{\varepsilon_a}{2}E_{curr_n}$$

$$\left(\frac{B_{ny}+iB_{nx}}{B_{Ny}}\right)_{@r=h} = \left(\begin{array}{l} -\cos n\frac{\pi}{4} + i\sin n\frac{\pi}{4} - \cos n\frac{3\pi}{4} + i\sin n\frac{3\pi}{4} \\ +\cos n\frac{5\pi}{4} - i\sin n\frac{5\pi}{4} + \cos n\frac{7\pi}{4} - i\sin n\frac{7\pi}{4} \end{array}\right)i\frac{\varepsilon_a}{2}E_{curr_n}$$

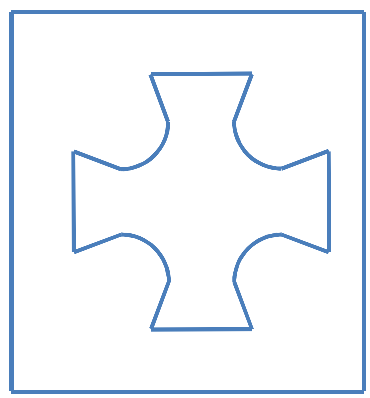
$$\left(\frac{B_{ny}+iB_{nx}}{B_{Ny}}\right)_{@r=h} = \left(\begin{array}{l} -i\cos n\frac{\pi}{4} - \sin n\frac{\pi}{4} - i\cos n\frac{3\pi}{4} - \sin n\frac{3\pi}{4} \\ +i\cos n\frac{5\pi}{4} + \sin n\frac{5\pi}{4} + i\cos n\frac{7\pi}{4} + \sin n\frac{7\pi}{4} \end{array}\right)\frac{\varepsilon_a}{2}E_{curr_n}$$

Two Piece Asymmetric Quadrupole

$$\left(\frac{B_{ny}}{B_{Ny}}\right)_{@r=h} = \left(-\sin n\frac{\pi}{4} - \sin n\frac{3\pi}{4} + \sin n\frac{5\pi}{4} + \sin n\frac{7\pi}{4}\right)\frac{\varepsilon_a}{2} E_{curr_n}$$

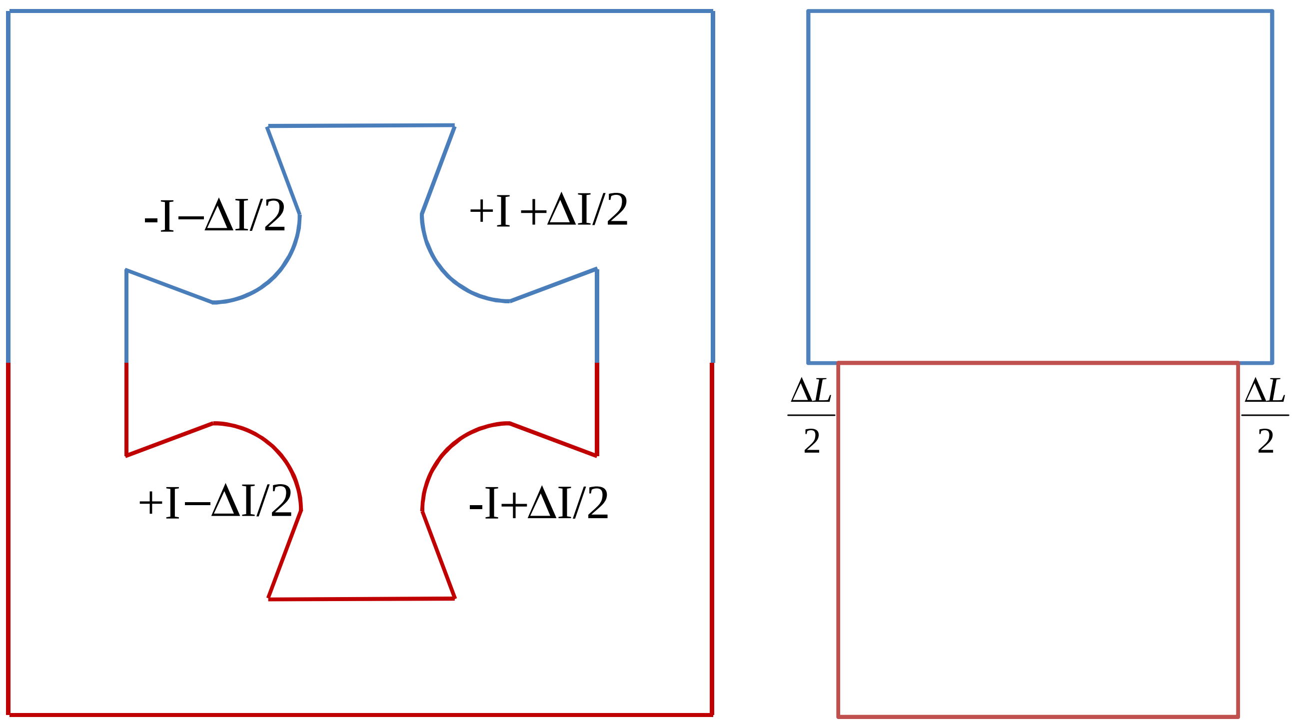
$$\left(\frac{B_{nx}}{B_{Ny}}\right)_{@r=h} = \left(-\cos n\frac{\pi}{4} - \cos n\frac{3\pi}{4} + \cos n\frac{5\pi}{4} + \cos n\frac{7\pi}{4}\right)\frac{\varepsilon_a}{2} E_{curr_n}$$

0



n	Asymmetry
1	-2.81E-01
2	0
3	-2.22E-01
4	0
5	-2.90E-02
6	0
7	-2.28E-02
8	0
9	2.69E-03
10	0
11	-4.45E-03
12	0
13	-3.46E-04
14	0
15	9.46E-04
16	0

Differences in Lengths of the Upper and Lower Halves



$$\left|\varepsilon_l\right| = \frac{\Delta L}{L}$$

Differences in Lengths of the Upper and Lower Halves

$$\left(\frac{B_{ny}+iB_{nx}}{B_{Ny}}\right)_{@r=h} = \left(i\varepsilon_{curr}E_{curr_n}\right)e^{-in\beta}$$

$$\left(\frac{B_{ny}+iB_{nx}}{B_{Ny}}\right)_{@r=h} = \left(+i\frac{\varepsilon_l}{2}e^{-in\frac{\pi}{4}} - i\frac{\varepsilon_l}{2}e^{-in\frac{3\pi}{4}} - i\frac{\varepsilon_l}{2}e^{-in\frac{5\pi}{4}} + i\frac{\varepsilon_l}{2}e^{-in\frac{7\pi}{4}}\right)E_{curr_n}$$

$$\left(\frac{B_{ny}+iB_{nx}}{B_{Ny}}\right)_{@r=h} = \left(+e^{-in\frac{\pi}{4}} - e^{-in\frac{3\pi}{4}} - e^{-in\frac{5\pi}{4}} + e^{-in\frac{7\pi}{4}}\right)i\frac{\varepsilon_l}{2}E_{curr_n}$$

$$\left(\frac{B_{ny}+iB_{nx}}{B_{Ny}}\right)_{@r=h} = \left(\begin{array}{l} +\cos n\frac{\pi}{4} - i\sin n\frac{\pi}{4} - \cos n\frac{3\pi}{4} + i\sin n\frac{3\pi}{4} \\ -\cos n\frac{5\pi}{4} + i\sin n\frac{5\pi}{4} + \cos n\frac{7\pi}{4} - i\sin n\frac{7\pi}{4} \end{array}\right)i\frac{\varepsilon_l}{2}E_{curr_n}$$

$$\left(\frac{B_{ny}+iB_{nx}}{B_{Ny}}\right)_{@r=h} = \left(\begin{array}{l} +i\cos n\frac{\pi}{4} + \sin n\frac{\pi}{4} - i\cos n\frac{3\pi}{4} - \sin n\frac{3\pi}{4} \\ -i\cos n\frac{5\pi}{4} - \sin n\frac{5\pi}{4} + i\cos n\frac{7\pi}{4} + \sin n\frac{7\pi}{4} \end{array}\right)\frac{\varepsilon_l}{2}E_{curr_n}$$

Differences in Lengths of the Upper and Lower Halves

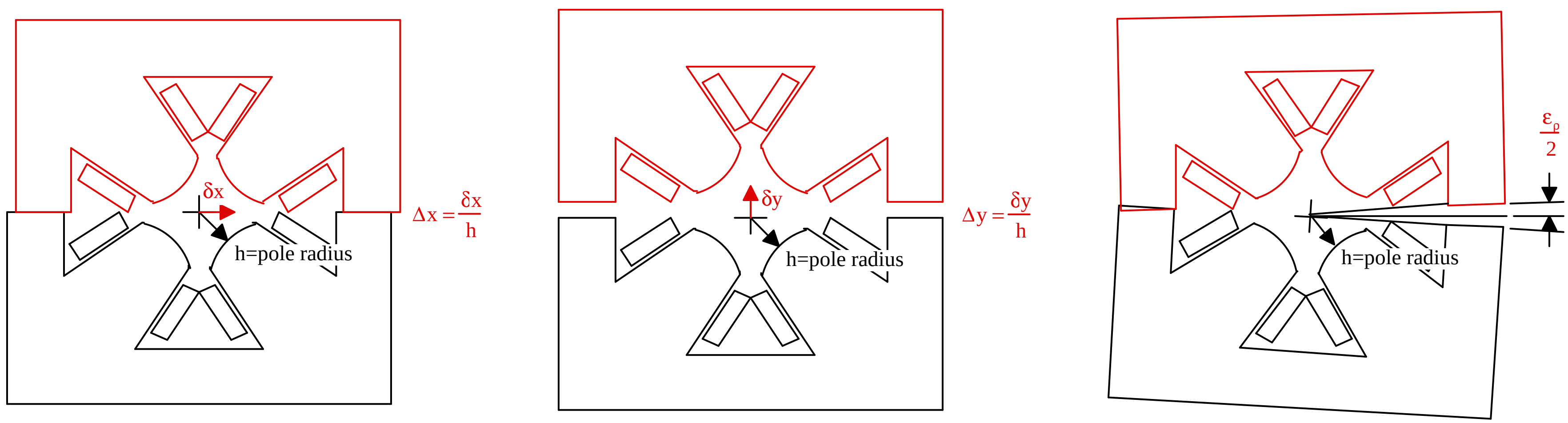
$$\left(\frac{B_{ny}}{B_{Ny}}\right)_{@r=h} = \left(+\sin n\frac{\pi}{4} - \sin n\frac{3\pi}{4} - \sin n\frac{5\pi}{4} + \sin n\frac{7\pi}{4}\right)\frac{\varepsilon_l}{2}E_{curr_n}$$

0

$$\left(\frac{B_{nx}}{B_{Ny}}\right)_{@r=h} = \left(+\cos n\frac{\pi}{4} - \cos n\frac{3\pi}{4} - \cos n\frac{5\pi}{4} + \cos n\frac{7\pi}{4}\right)\frac{\varepsilon_l}{2}E_{curr_n}$$

n	Different length (i)
1	2.81E-01
2	0
3	-2.22E-01
4	0
5	2.90E-02
6	0
7	-2.28E-02
8	0
9	-2.69E-03
10	0
11	-4.45E-03
12	0
13	3.46E-04
14	0
15	9.46E-04
16	0

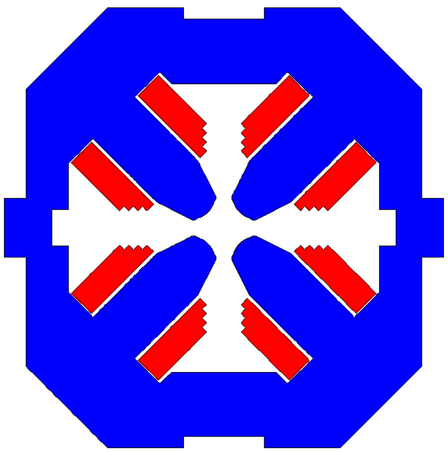
Two Piece Quadrupole Errors



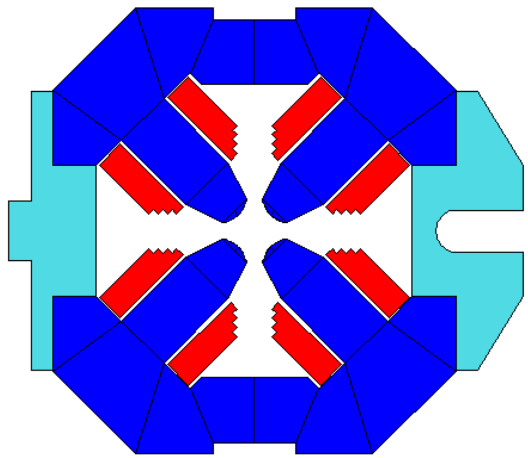
Two Piece Quadrupole Errors

n	Asymmetry	ΔL (i)	ΔX (i)	ΔY	Rot
1	-2.81E-01	2.81E-01	0	0	2.49E-01
2	0	0	0.302	-7.30E-01	0
3	-2.22E-01	-2.22E-01	0	0	-9.33E-01
4	0	0	-9.56E-02	-3.27E-01	0
5	-2.90E-02	2.90E-02	0	0	-2.70E-01
6	0	0	-4.06E-02	-6.29E-02	0
7	-2.28E-02	-2.28E-02	0	0	-4.33E-02
8	0	0	1.81E-02	2.20E-02	0
9	2.69E-03	-2.69E-03	0	0	1.06E-02
10	0	0	8.22E-03	9.01E-03	0
11	-4.45E-03	-4.45E-03	0	0	5.12E-03
12	0	0	-3.77E-03	3.94E-03	0
13	-3.46E-04	3.46E-04	0	0	-1.31E-03
14	0	0	-1.74E-03	-1.78E-03	0
15	9.46E-04	9.46E-04	0	0	-9.41E-04
16	0	0	8.14E-04	8.23E-04	0

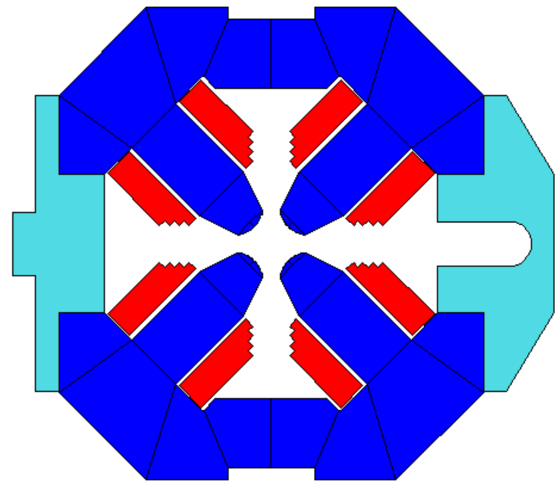
Examples: Alba SR-Quadrupoles



CX

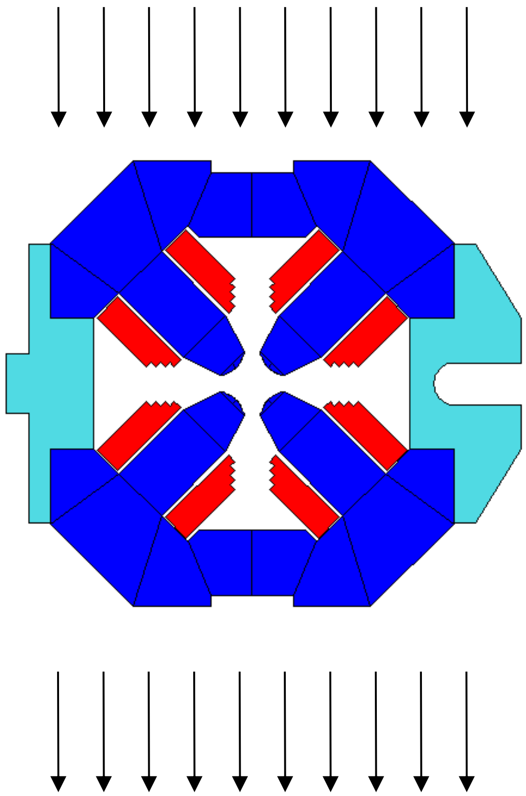


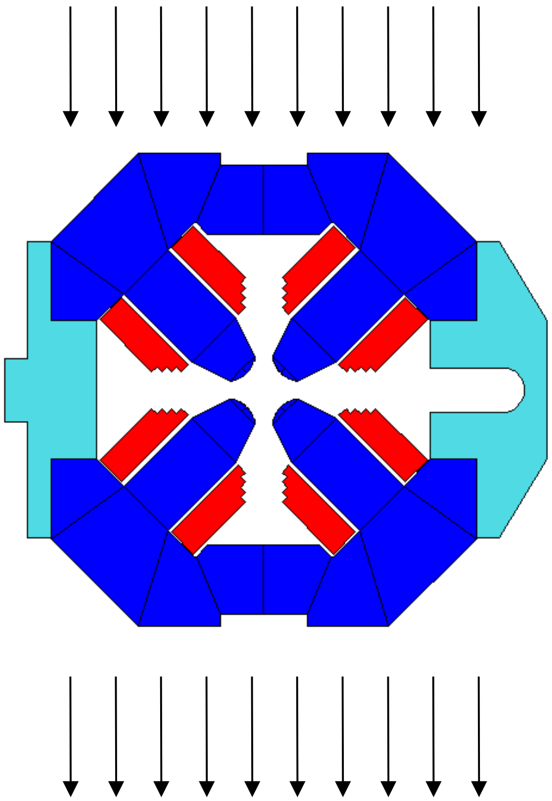
OC



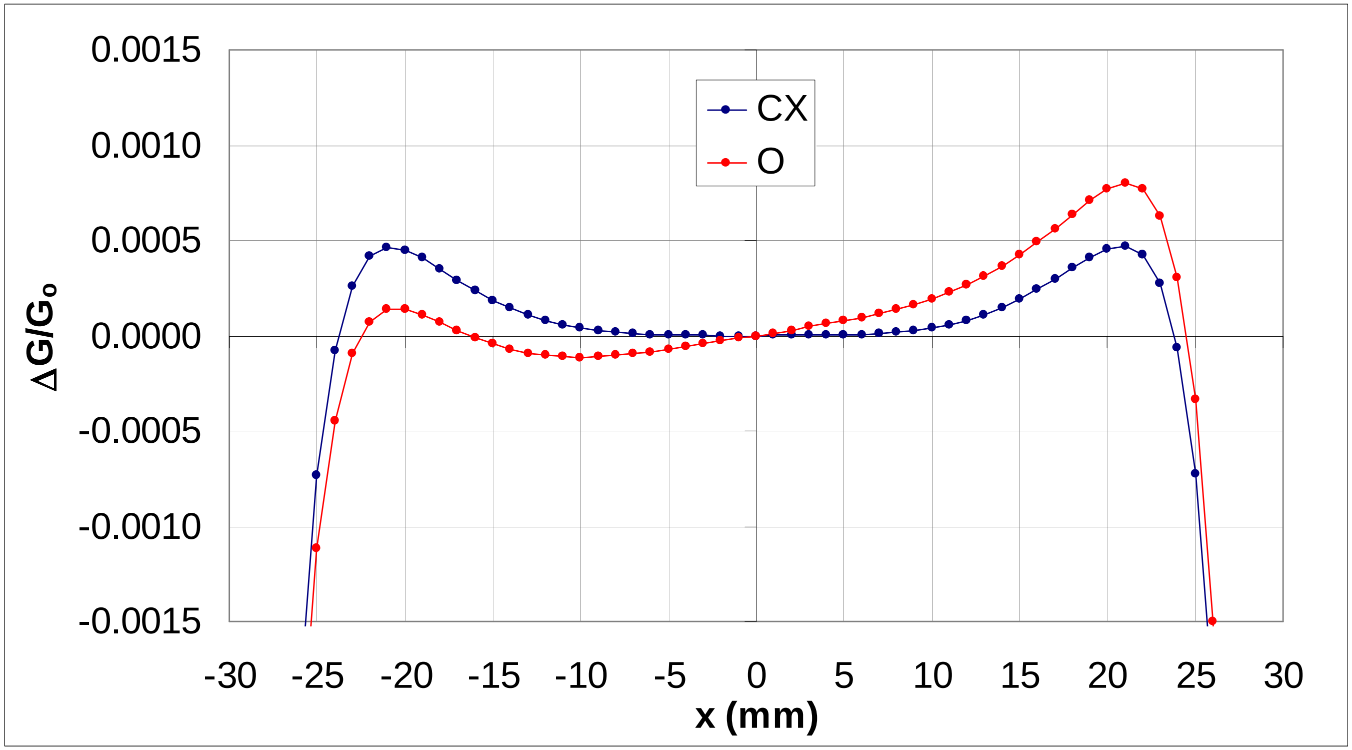
OI

External field

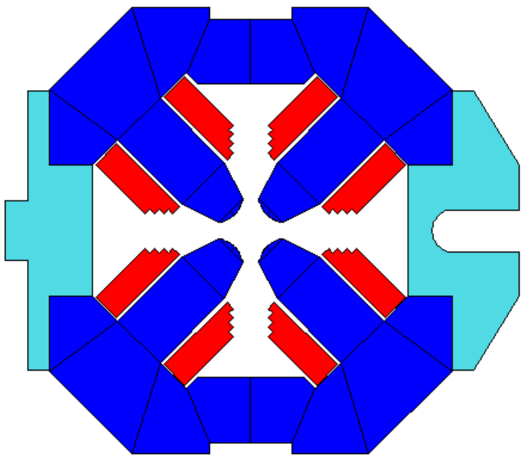




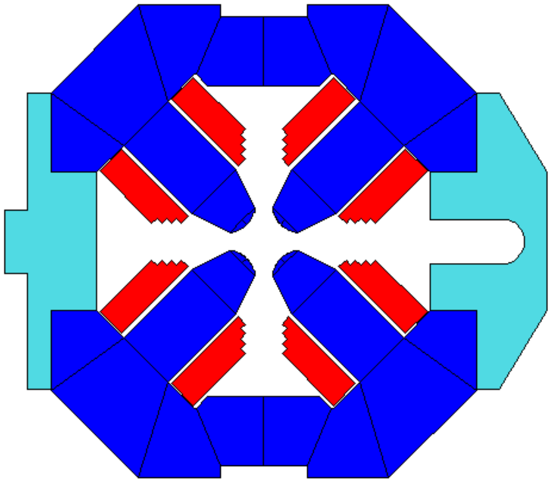
Gradient Uniformity



Magnetic Material Spacers

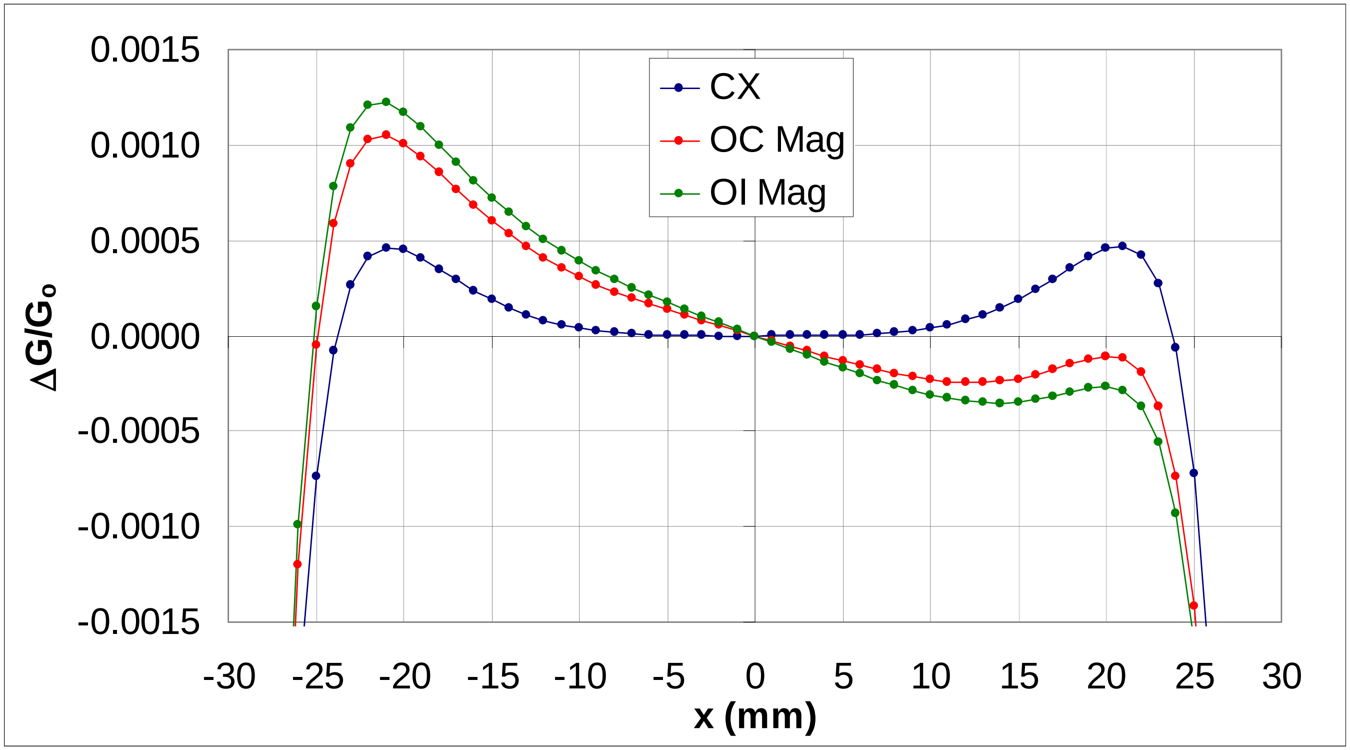


OC

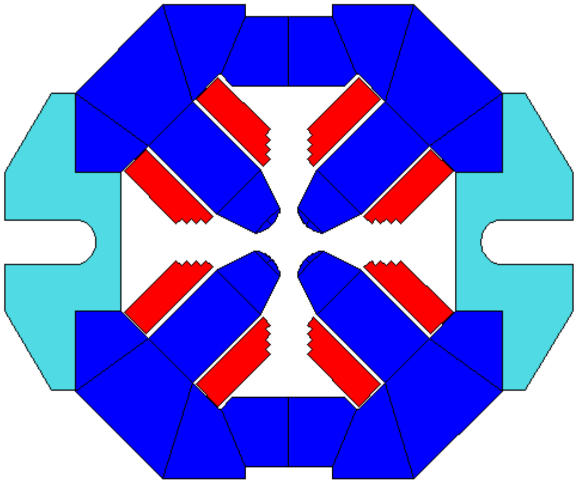


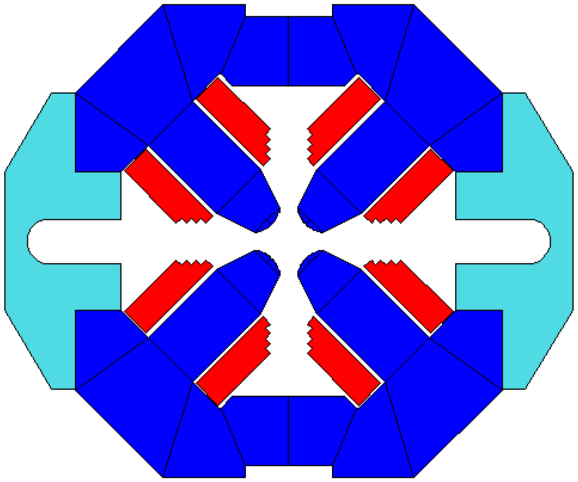
OI

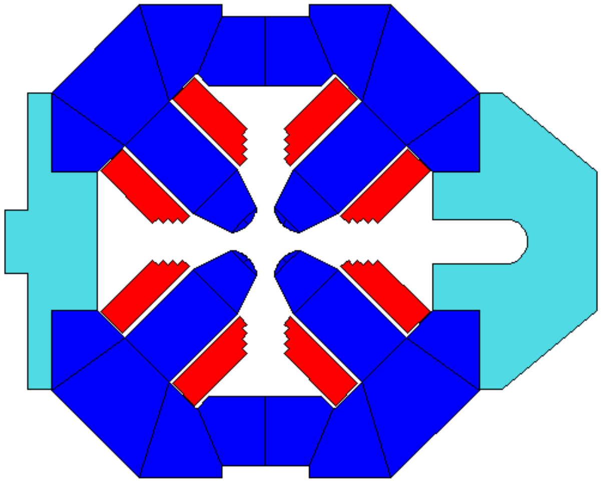
Gradient Uniformity



Solutions

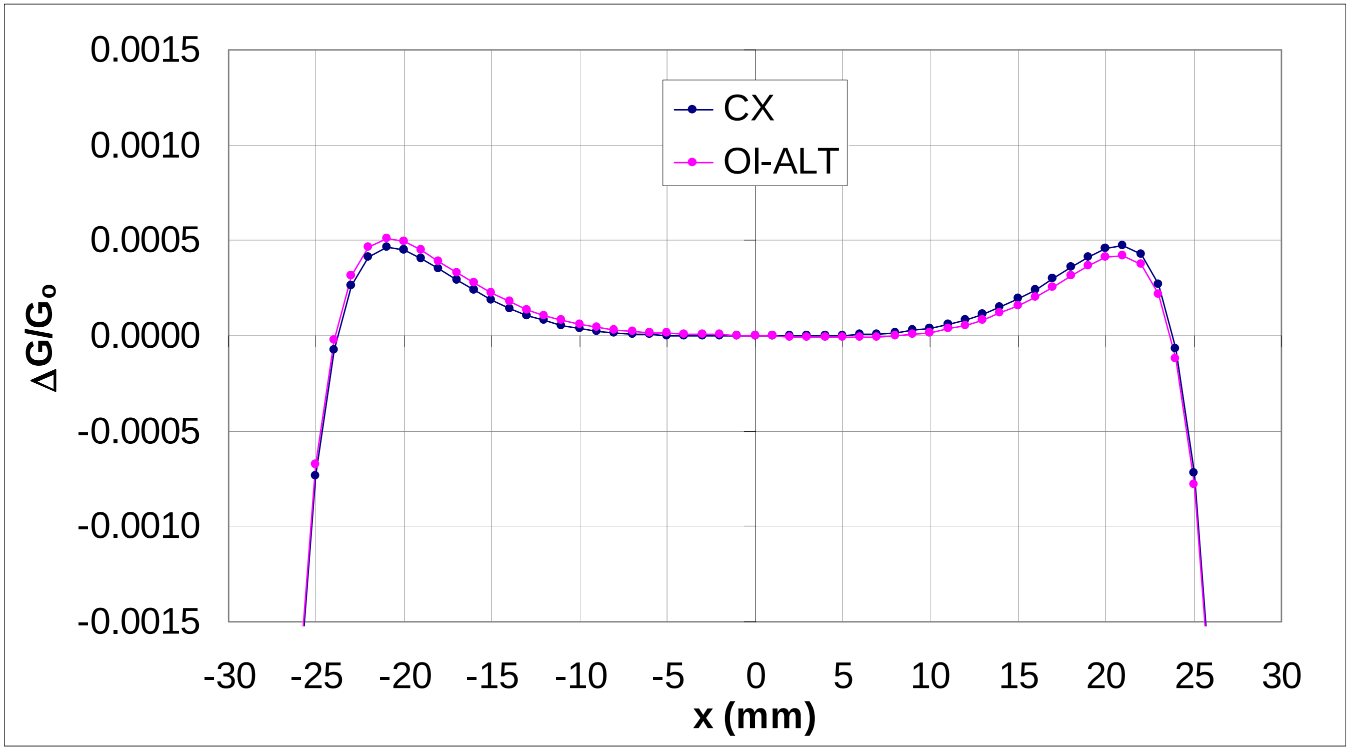




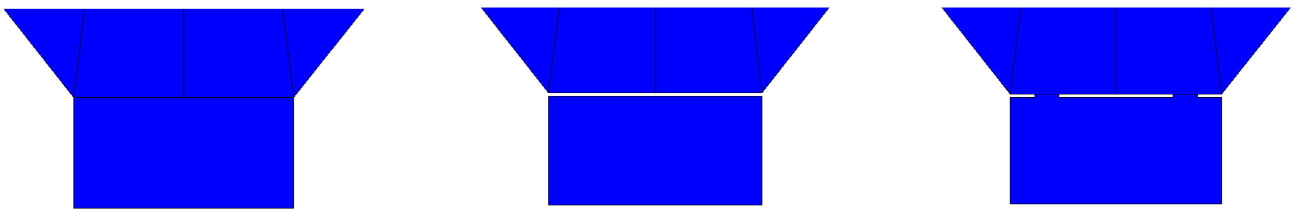


Magnetic Material Spacers
with the same thickness

Gradient Uniformity



Magnetic Base



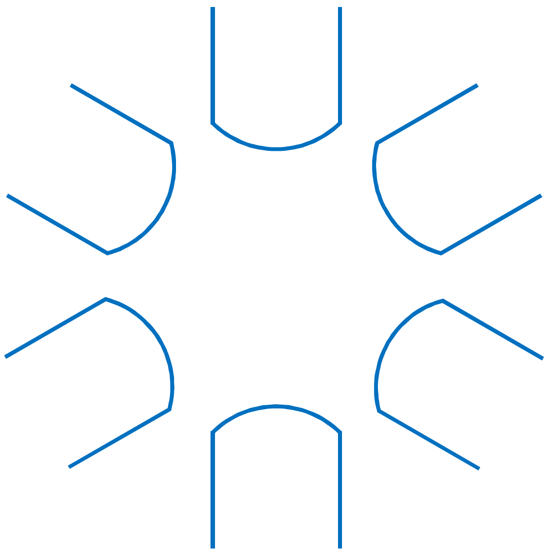
Quadrupole

n	reference	iron feet	iron feet + air space	iron feet + air space + contact points
1	0	0	0	0
2	10000	10000	10000	10000
3	0	0	0	0
4	0.0	**0.8**	0.0	**0.4**
5	0	0	0	0
6	3	3	3	3
7	0	0	0	0
8	0	0	0	0
9	0	0	0	0
10	-1	-1	-1	-1

Sextupole

n	reference	iron feet	iron feet + air space	iron feet + air space + contact points
1	0	**-1**	0	0
2	0	0	0	0
3	10000	10000	10000	10000
4	0	0	0	0
5	0	0	0	0
6	0	0	0	0
7	0	0	0	0
8	0	0	0	0
9	-2	-2	-2	2
10	0	0	0	0

Sextupole Trim Coils



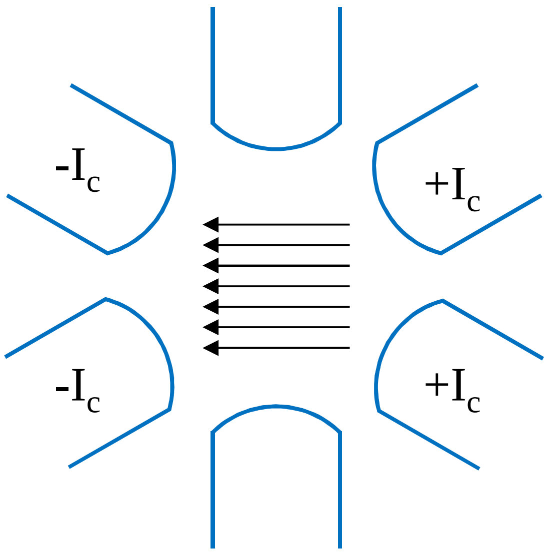
n	Excitation (i)
1	9.79E-02
2	1.56E-01
3	1.67E-01
4	1.33E-01
5	7.09E-02
6	0.00E+00
7	-1.34E-02
8	-1.07E-02
9	0.00E+00
10	9.13E-03
11	9.72E-03
12	0.00E+00
13	-1.01E-03
14	-1.18E-03
15	0.00E+00

$$\left(\frac{B_{ny}+iB_{nx}}{B_{Ny}}\right)_{@r=h} = \left(i\varepsilon_{curr}E_{curr_n}\right)(\cos n\beta - i\sin n\beta)$$

Vertical Corrector

$$\left(\frac{B_{ny}+iB_{nx}}{B_{Ny}}\right)_{@r=h} = \left(i\varepsilon_{curr}E_{curr_n}\right)\left(\cos n\beta - i\sin n\beta\right)$$

$$\left(\frac{B_{ny}+iB_{nx}}{B_{Ny}}\right)_{@r=h} = \left(iE_{curr_n}\right)\left(\begin{array}{l}\cos n\frac{\pi}{6} - i\sin n\frac{\pi}{6} - \cos n\frac{5\pi}{6} + i\sin n\frac{5\pi}{6} \\ -\cos n\frac{7\pi}{6} - i\sin n\frac{7\pi}{6} + \cos n\frac{11\pi}{6} - i\sin n\frac{11\pi}{6}\end{array}\right)$$



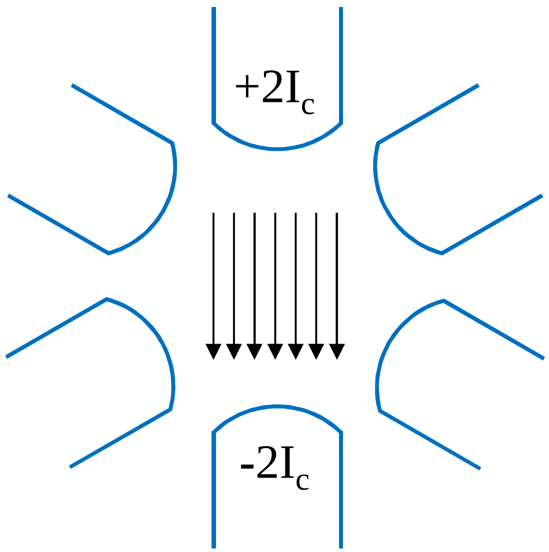
$$\left(\frac{B_{ny}}{B_{3y}}\right)_{@r=h} = \left(E_{curr_n}\right)\left(\cos n\frac{\pi}{6} - \cancelto{0}{\cos n\frac{5\pi}{6}} - \cos n\frac{7\pi}{6} + \cos n\frac{11\pi}{6}\right)$$

$$\left(\frac{B_{nx}}{B_{3y}}\right)_{@r=h} = \left(E_{curr_n}\right)\left(-\sin n\frac{\pi}{6} + \sin n\frac{5\pi}{6} + \sin n\frac{7\pi}{6} - \sin n\frac{11\pi}{6}\right)$$

n	Bnx/B3y
1	0.3391
2	0
3	0
4	0
5	-0.2456
6	0
7	0.0464
8	0
9	0
10	0
11	0.0337
12	0
13	-0.0035
14	0
15	0

Horizontal Corrector

$$\left(\frac{B_{ny}+iB_{nx}}{B_{Ny}}\right)_{@r=h} = \left(i\varepsilon_{curr}E_{curr_n}\right)\left(\cos n\beta - i\sin n\beta\right)$$



$$\left(\frac{B_{ny}+iB_{nx}}{B_{Ny}}\right)_{@r=h} = \left(i2E_{curr_n}\right)\left(\cos n\frac{\pi}{2} - i\sin n\frac{\pi}{2} - \cos n\frac{3\pi}{2} + i\sin n\frac{3\pi}{2}\right)$$

$$\left(\frac{B_{ny}+iB_{nx}}{B_{Ny}}\right)_{@r=h} = \left(2E_{curr_n}\right)\left(i\cos n\frac{\pi}{2} + \sin n\frac{\pi}{2} - i\cos n\frac{3\pi}{2} - \sin n\frac{3\pi}{2}\right)$$

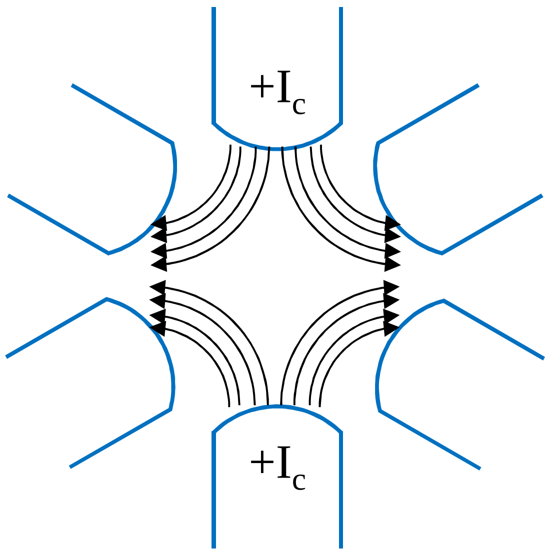
$$\left(\frac{B_{nx}}{B_{3y}}\right)_{@r=h} = \left(2E_{curr_n}\right)\left(\cos n\frac{\pi}{2} - \cos n\frac{3\pi}{2}\right)$$

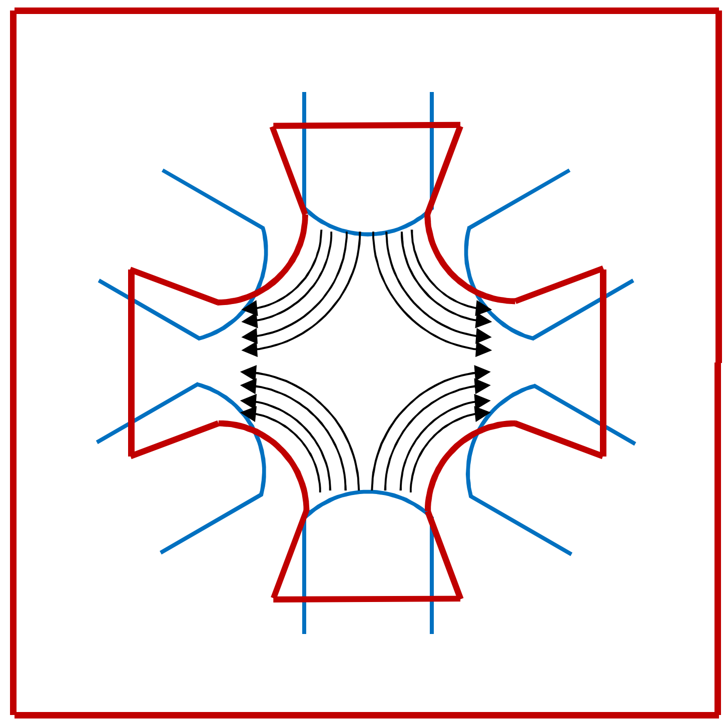
0

$$\left(\frac{B_{ny}}{B_{3y}}\right)_{@r=h} = \left(2E_{curr_n}\right)\left(+\sin n\frac{\pi}{2} - \sin n\frac{3\pi}{2}\right)$$

n	Bny/B3y
1	0.3916
2	0
3	-0.6680
4	0
5	0.2836
6	0
7	0.0536
8	0
9	0
10	0
11	-0.0389
12	0
13	-0.0040
14	0
15	0

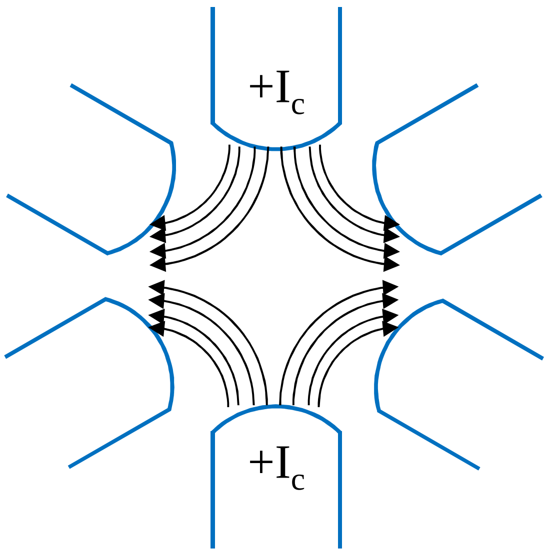
Skew Quadrupole Corrector





Skew Quadrupole Corrector

$$\left(\frac{B_{ny}+iB_{nx}}{B_{Ny}}\right)_{@r=h} = \left(i\varepsilon_{curr}E_{curr_n}\right)\left(\cos n\beta - i\sin n\beta\right)$$



$$\left(\frac{B_{ny}+iB_{nx}}{B_{Ny}}\right)_{@r=h} = \left(iE_{curr_n}\right)\left(\cos n\frac{\pi}{2} - i\sin n\frac{\pi}{2} + \cos n\frac{3\pi}{2} - i\sin n\frac{3\pi}{2}\right)$$

$$\left(\frac{B_{ny}+iB_{nx}}{B_{Ny}}\right)_{@r=h} = \left(E_{curr_n}\right)\left(i\cos n\frac{\pi}{2} + \sin n\frac{\pi}{2} + i\cos n\frac{3\pi}{2} + \sin n\frac{3\pi}{2}\right)$$

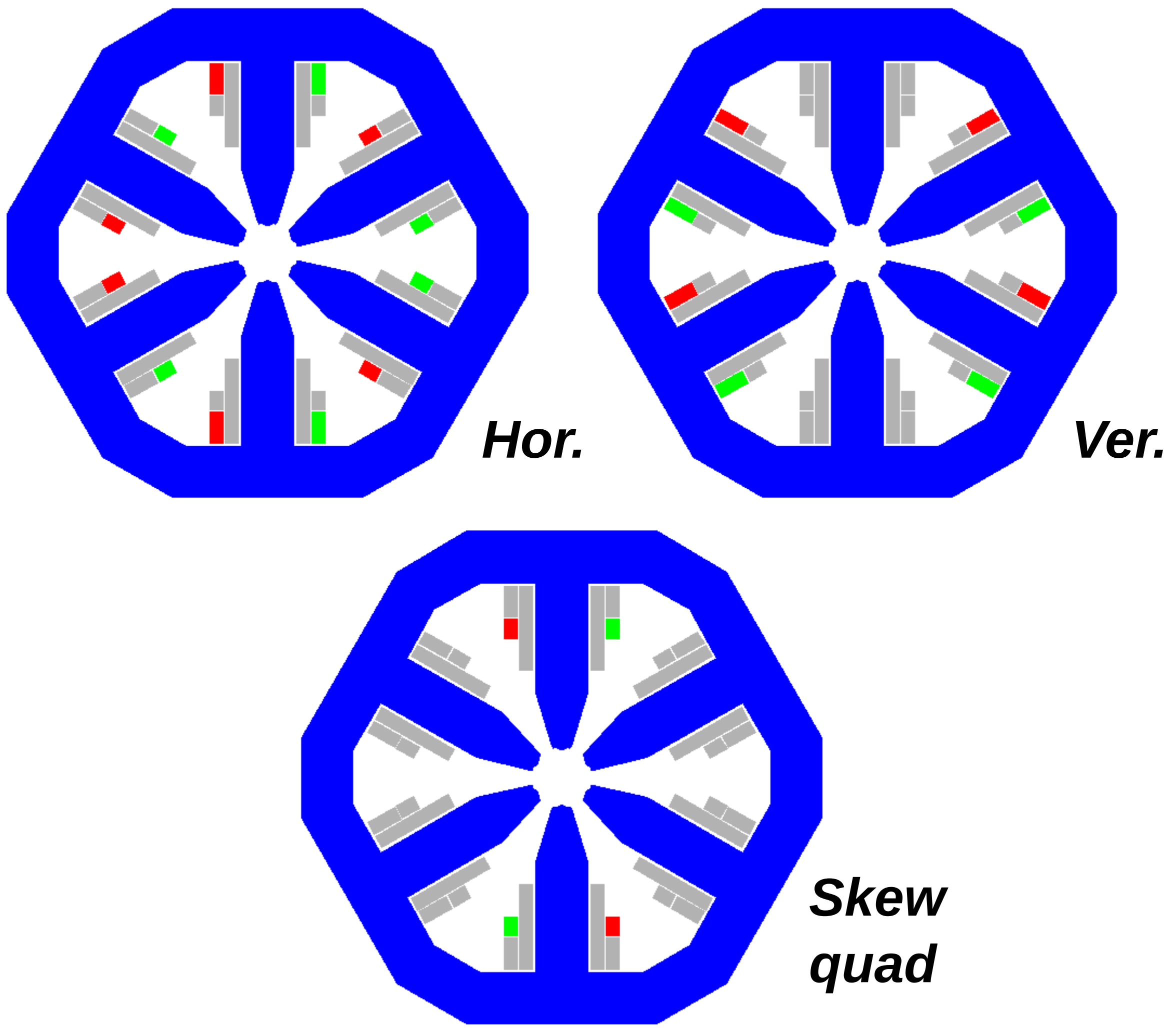
$$\left(\frac{B_{ny}}{B_{3y}}\right)_{@r=h} = \left(E_{curr_n}\right)\left(\sin n\frac{\pi}{2} + \cos n\frac{3\pi}{2}\right)$$

0

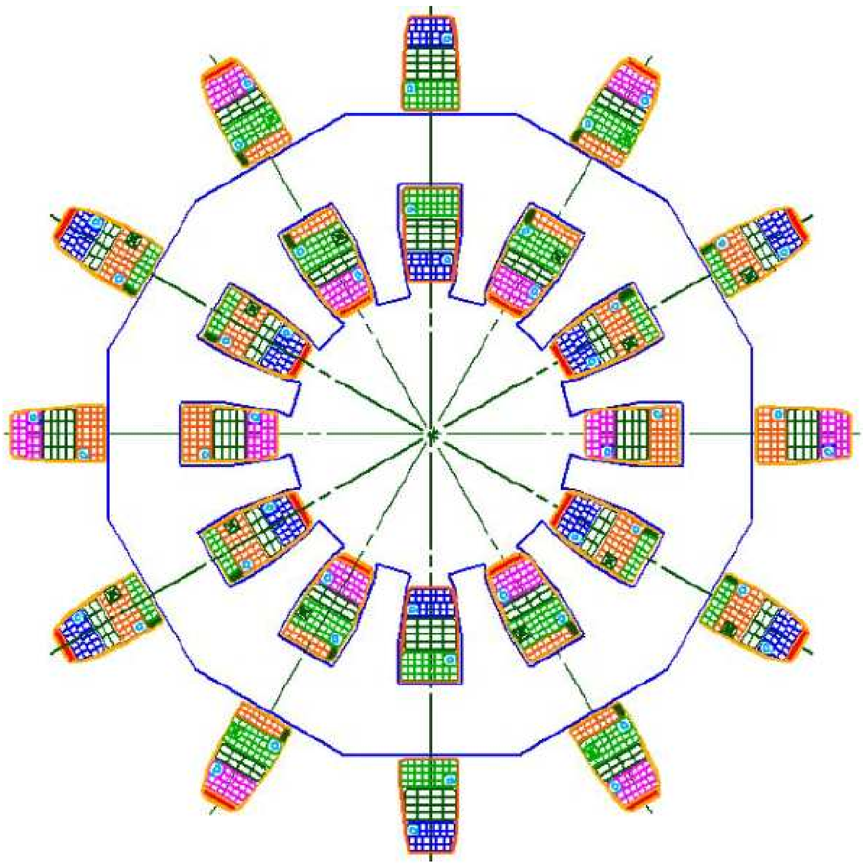
n	Bnx/B3y
1	0
2	-0.3120
3	0
4	0.2660
5	0
6	0
7	0
8	-0.0214
9	0
10	-0.0183
11	0
12	0
13	0
14	0.0024
15	0

$$\left(\frac{B_{nx}}{B_{3y}}\right)_{@r=h} = \left(E_{curr_n}\right)\left(i\cos n\frac{\pi}{2} + i\cos n\frac{3\pi}{2}\right)$$

Sextupole Trim Coils

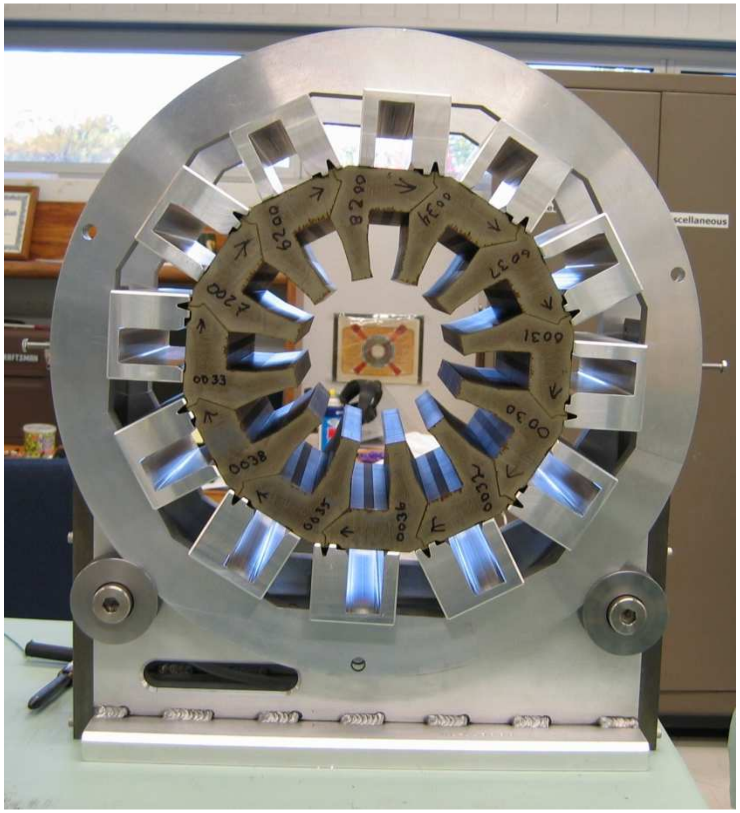


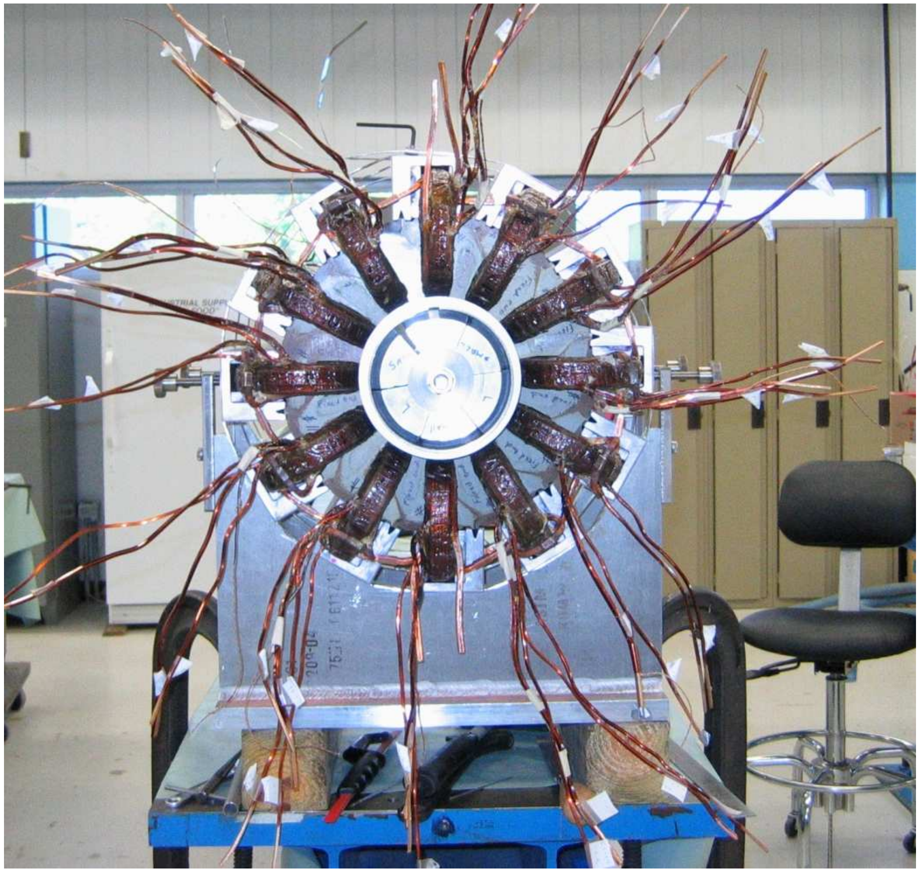
Fermilab Booster Corrector



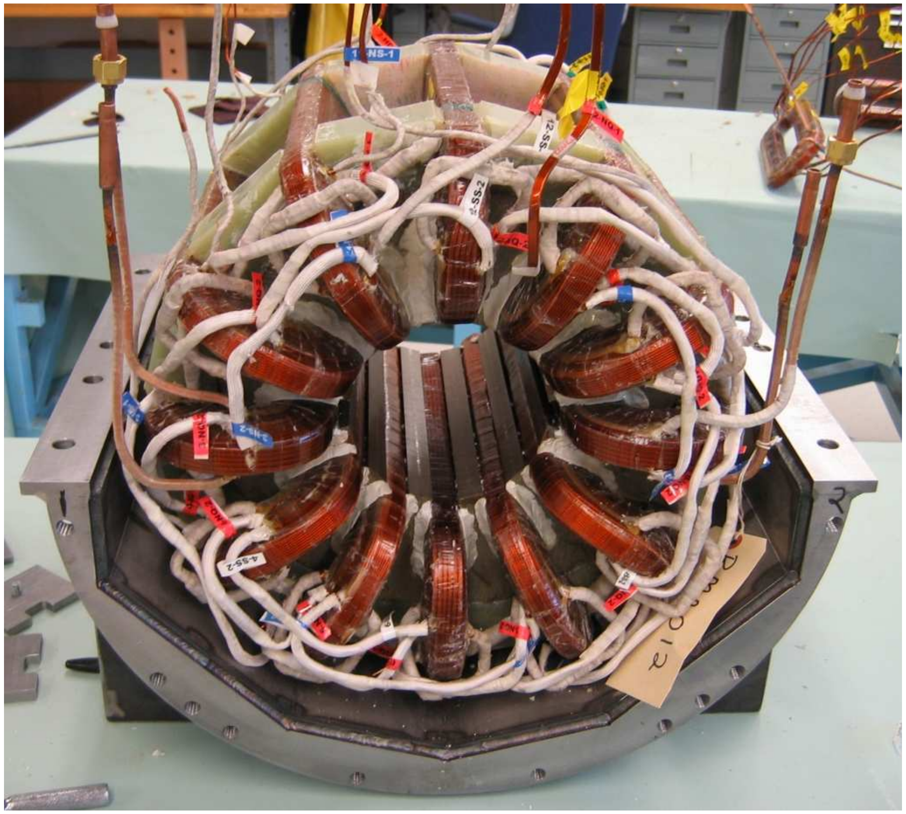
- Vertical dipole
- Horizontal dipole
- Normal Quadrupole
- Skew Quadrupole
- Normal Sextupole
- Skew Sextupole

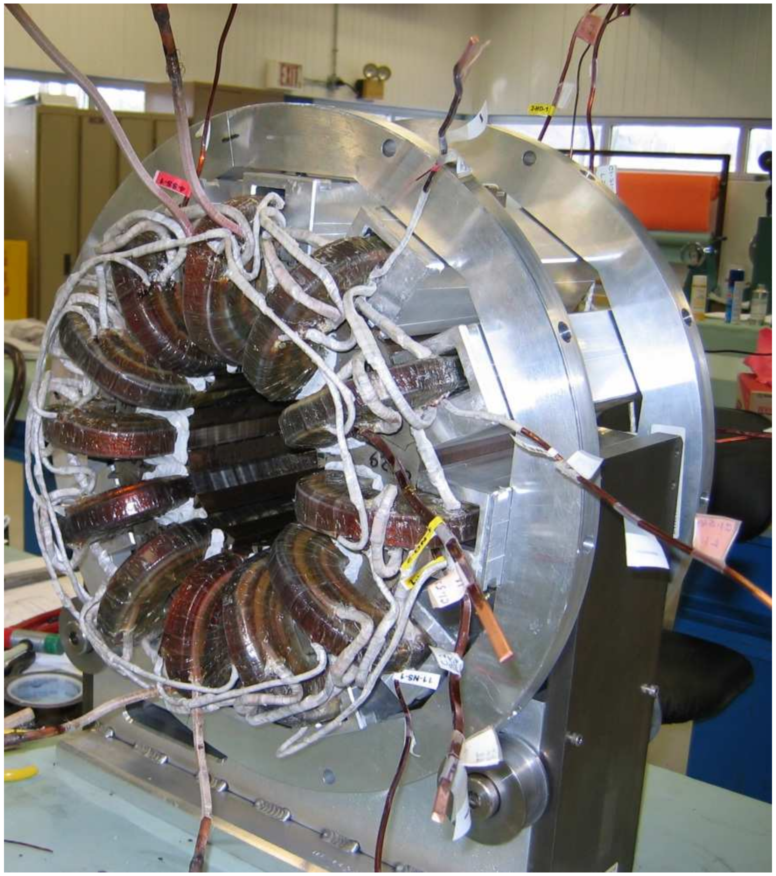
Fermilab Booster Corrector





Fermilab Booster Corrector





Fermilab Booster Corrector

Summary

- In many ways, this is one of the most important lectures. It is important to understand the chapter on perturbations since successfully translating the performance of the mathematical design to the magnets manufactured and installed in a synchrotron requires that mechanical manufacturing and assembly errors translates into field errors which can threaten the performance of the synchrotron.

- Understanding the impact of mechanical fabrication and assembly errors on the magnet performance and thus, the physics impacts of these errors, can provide the understanding so that mechanical tolerances can be properly assigned.

Next…

- Magnet Excitation
- Iron saturation effects
- Coil design