4
Magnet Excitation and Coil Design

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Introduction

- This section develops the expressions for magnet excitation.
- The relationship between current density and magnet power is developed.
- Iron saturation is discussed.
- An example of the optimization of a magnet system is presented in order to develop a logic for adopting canonical current density values.
- Engineering relationships for computing water flows for cooling magnet coils are developed.

Maxwell's Equations

(in media)

Gauss's law

$$\nabla . \boldsymbol{D} = \rho_f \qquad \oiint \boldsymbol{D} . d\boldsymbol{A} = Q_f$$

$$\nabla . \boldsymbol{B} = 0 \qquad \oiint \boldsymbol{B} . d\boldsymbol{A} = 0$$

Faraday's law

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t} \qquad \oint \boldsymbol{E} . d\boldsymbol{l} = -\iint \frac{\partial \boldsymbol{B}}{\partial t} . d\boldsymbol{A}$$

Ampere's law

$$\nabla \times \boldsymbol{H} = \boldsymbol{J_f} + \frac{\partial \boldsymbol{D}}{\partial t} \qquad \oint \boldsymbol{H} . d\boldsymbol{l} = \boldsymbol{I_f} + \iint \frac{\partial \boldsymbol{D}}{\partial t} . d\boldsymbol{A}$$

Ampere's Law - Integral Form

$$\oint \boldsymbol{H}.d\boldsymbol{l} = IT$$

$$\boldsymbol{H} = \frac{\boldsymbol{B}}{\mu_o.\mu_r}$$

$$\mu_o = 4\pi.10^{-7}$$

$$\mu_{r\_air} = 1$$

$$\mu_{r\_Iron} \approx 1000^{\,*}$$

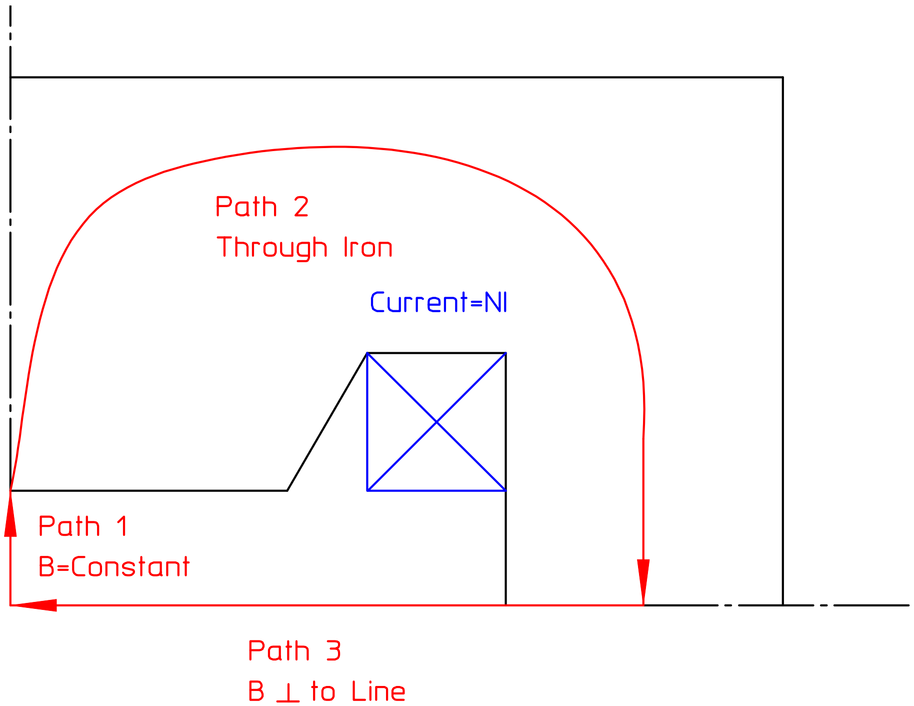
$$I_T = NI$$

I_T = Total current

N = Number of turns

I = Current

Dipole Excitation



$$\oint H \cdot dl = \oint_{Path1} H \cdot dl + \oint_{Path2} H \cdot dl + \oint_{Path3} H \cdot dl = NI$$

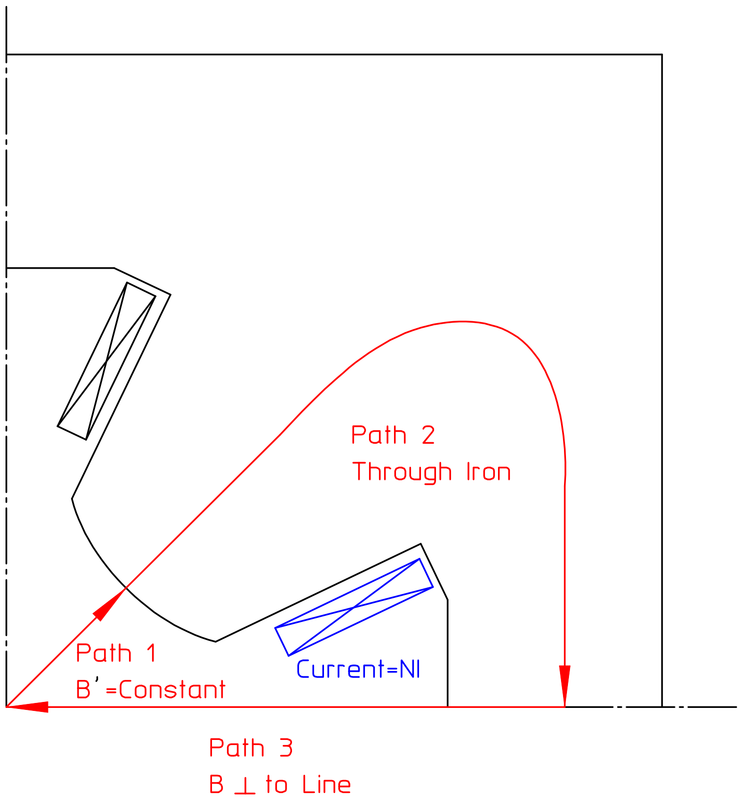
$$\oint_{Path1} H \cdot dl = \frac{B.h}{\mu_o \mu_{r\_air}} = \frac{B.h}{\mu_o}$$

$$\oint_{Path2} \frac{B}{\mu_o \mu_{r\_Iron}} \cdot dl \approx 0$$

$$\oint_{Path3} H \cdot dl = 0$$

$$NI = \frac{B.h}{\mu_o}$$

Quadrupole Excitation



$$\oint H \cdot dl = \oint_{Path1} H \cdot dl + \oint_{Path2} H \cdot dl + \oint_{Path3} H \cdot dl = NI$$

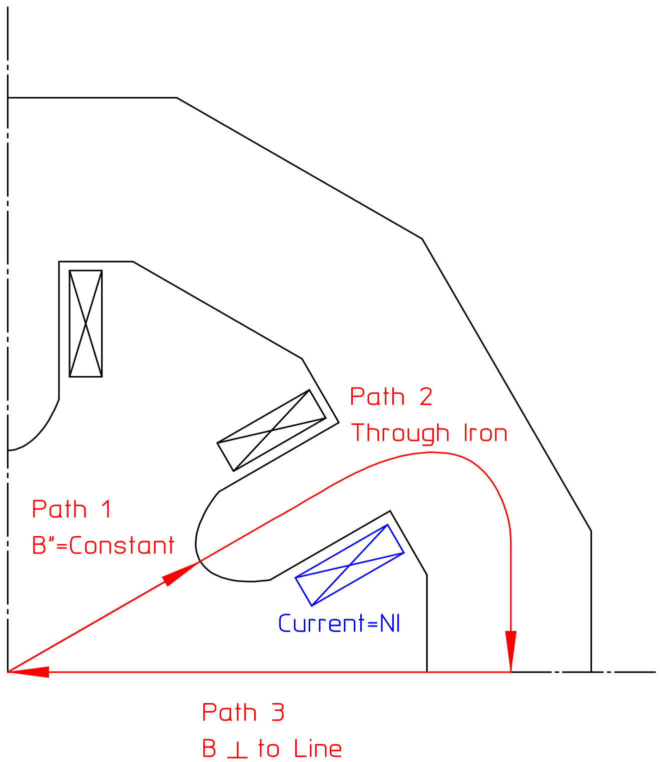
$$\oint_{Path2} \frac{B}{\mu_o \mu_{r\_Iron}} \cdot dl \approx 0$$

$$\oint_{Path3} H \cdot dl = 0$$

$$\oint_{Path1} H \cdot dl = \oint_{Path1} \frac{(B'.r)}{\mu_o \mu_{r\_air}} \cdot dr = \frac{B' h^2}{2\mu_o}$$

$$NI = \frac{B' h^2}{2\mu_o}$$

Sextupole Excitation



$$\oint H \cdot dl = \oint_{Path1} H \cdot dl + \oint_{Path2} H \cdot dl + \oint_{Path3} H \cdot dl = NI$$

$$\oint_{Path2} \frac{B}{\mu_o \mu_{r\_Iron}} \cdot dl \approx 0 \qquad \oint_{Path3} H \cdot dl = 0$$

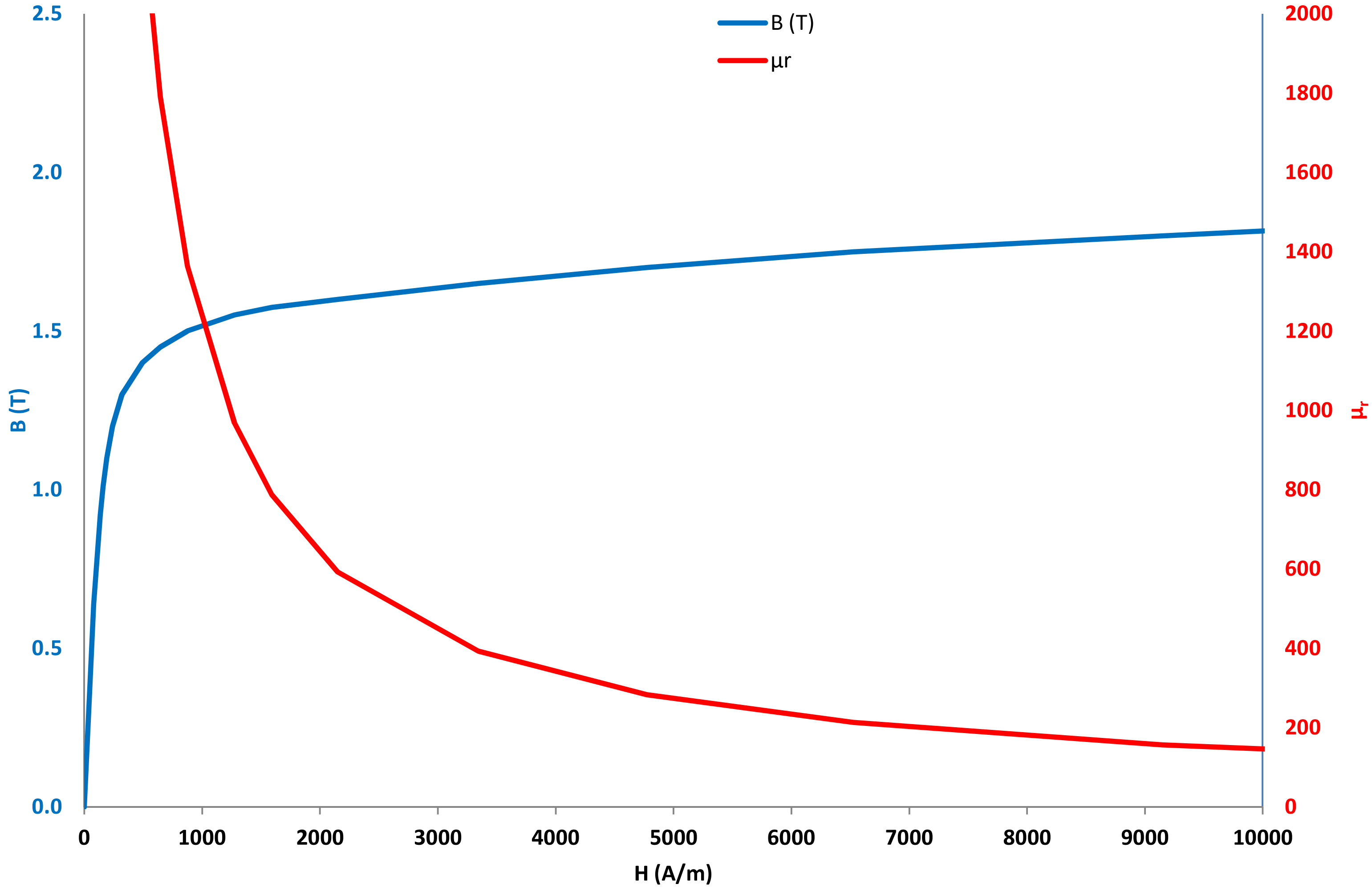
$$B'(r) = \int B'' dr = B'' r$$

$$B(r) = \int B' dr = \int B'' r dr = \frac{B'' r^2}{2}$$

$$\oint_{Path1} H \cdot dl = \oint_{Path1} \frac{(B''.r^2)}{2\mu_o \mu_{r\_air}} \cdot dr = \frac{B'' h^3}{6\mu_o}$$

$$NI = \frac{B'' h^3}{6\mu_o}$$

B-H Curve



B (T)
µr
2.5
2.0
1.5
1.0
0.5
0.0
B (T)
2000
1800
1600
1400
1200
1000
800
600
400
200
0
μ_r
0
1000
2000
3000
4000
5000
6000
7000
8000
9000
10000
H (A/m)

Magnet Efficiency

We introduce *efficiency* as a means of describing the losses in the iron. USe the expression for the dipole excitation as an example.

$$NI = \oint_{Path1} H \cdot dl + \oint_{Path2} H \cdot dl + \oint_{Path3} H \cdot dl$$

$$= \left(\frac{Bh}{\mu_0}\right)_{Path1} + \left(small\ \ factor \times \frac{Bh}{\mu_0}\right)_{Path2} + \left(0\begin{pmatrix}\text{since}\\ B \perp l\end{pmatrix}\right)_{Path3}$$

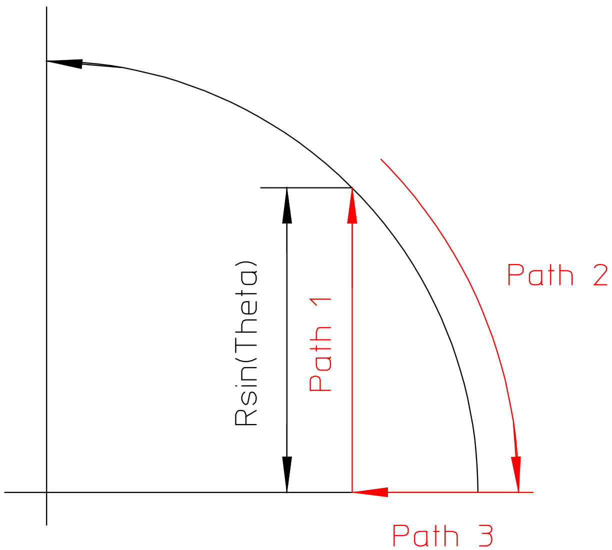
$$NI = (1 + small\ \ factor)\frac{Bh}{\mu_0} = \frac{Bh}{\eta \mu_0}$$

$\eta = efficiency \approx 0.98$ For magnets with well designed yokes.

Current Dominated Magnets

Occasionally, a need arises for a magnet whose field quality relies on the distribution of current. One example of this type of magnet is the superconducting magnet, whose field quality relies on the proper placement of current blocks.

$$\oint H \cdot dl = I = \int J d\theta$$



$$\oint H \cdot dl = \oint_{Path1} H \cdot dl + \oint_{Path2} H \cdot dl + \oint_{Path3} H \cdot dl = NI$$

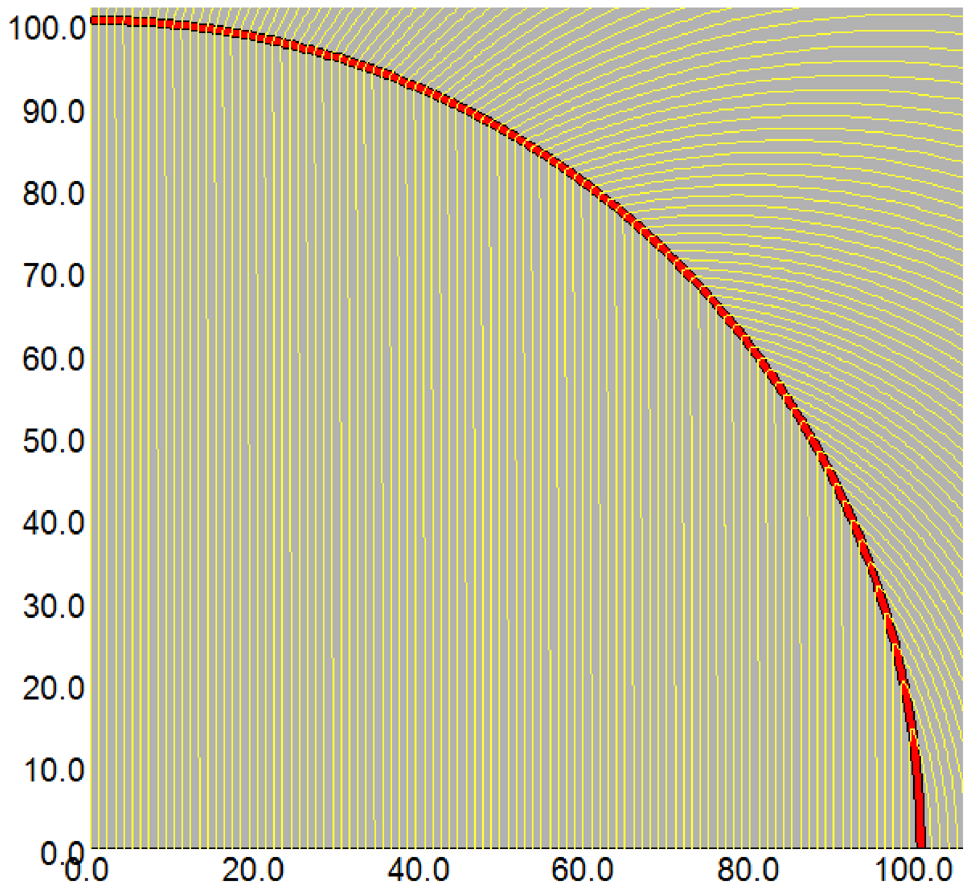
$$\oint_{Path3} H \cdot dl = 0 \qquad \oint_{Path2} H \cdot dl \approx 0$$

$$\oint_{Path1} H \cdot dl = \frac{B.R \sin\theta}{\mu_o}$$

$$\frac{B.R \sin\theta}{\mu_o} = \int J d\theta \Rightarrow J = \frac{B.R \cos\theta}{\mu_o}$$

Cosine theta distribution

Cosine Theta Current Distribution

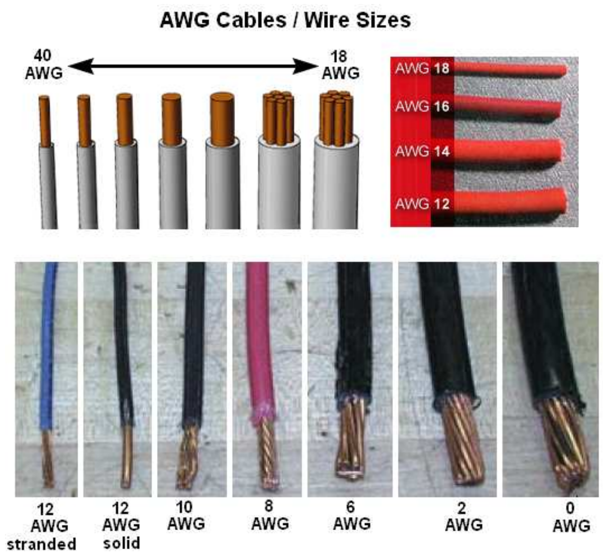


Current Density

- One of the design choices made in the design of magnet coils is the choice of the coil cross section which determines the current density.
- Given the required Physics parameters of the magnet, the choice of the current density will determine the required magnet power.
  - Power is important because they affect both the cost of power supplies, power distribution (cables) and operating costs.
  - Power is also important because it affects the installation and operating costs of cooling systems.

Canonical Current Densities

Solid conductor



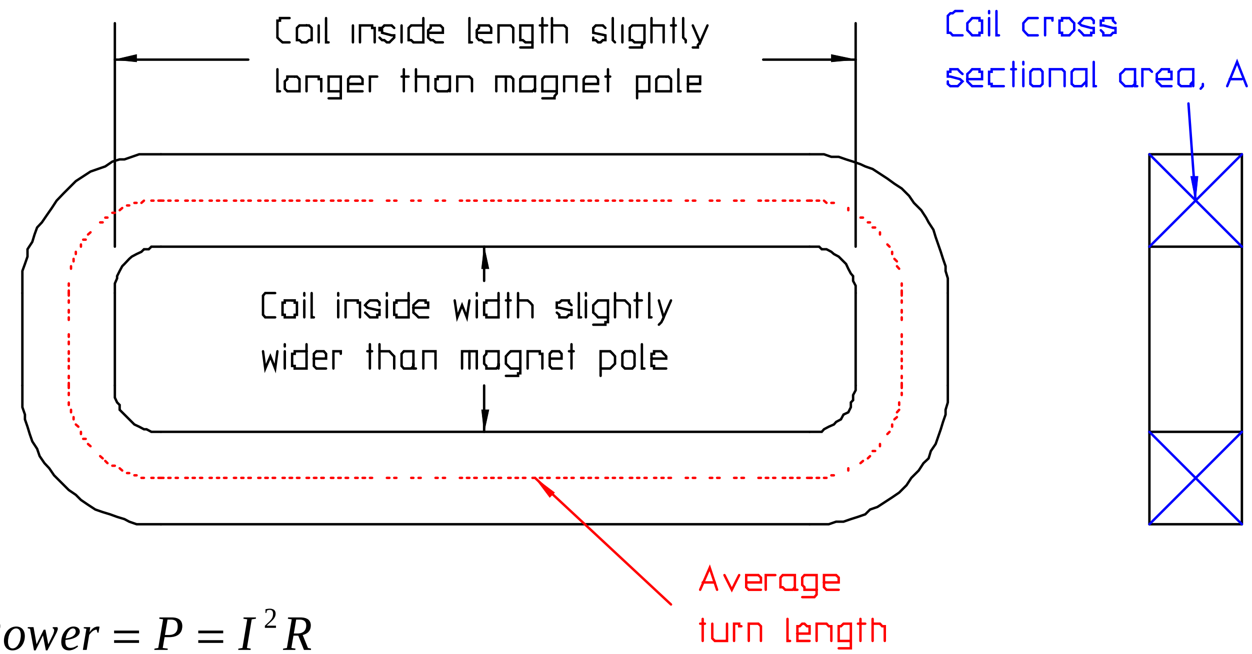
AWG	Diameter (mm)	Area (mm^2)	ρ (Ω/km)	Max Current (A)	Max Current Density (A/mm^2)
1	7.348	42.4	0.406392	119	2.81
2	6.543	33.6	0.512664	94	2.80
3	5.827	26.7	0.64616	75	2.81
⋮	⋮	⋮	⋮	⋮	⋮
38	0.102	0.00797	2163	0.0228	2.86
39	0.089	0.00632	2728	0.0175	2.77
40	0.079	0.00501	3440	0.0137	2.73

Hollow conductor

→ ~ 10 A/mm^2

(with proper cooling)

Coil main parameters



$$Power = P = I^2 R$$

$$R = \frac{\rho L}{a} \quad \text{where} \quad \begin{aligned} \rho &= \text{resistivity } (\Omega\text{-m}) \\ \text{L} &= \text{conductor length (m)} \\ \text{a} &= \text{conductor net cross sectional area } (\text{m}^2) \end{aligned}$$

$$L = N\ell_{ave} \quad \text{where } N = \text{number of turns in the coil.}$$

$$Na = fA \quad \text{where } f = \text{coil packing fraction.}$$

Substituting;

$$R = \frac{\rho N\ell_{ave}}{\frac{fA}{N}} = \frac{\rho N^2\ell_{ave}}{fA}$$

Calculating the coil power;

$$P = I^2 R = \frac{\rho (NI)^2 \ell_{ave}}{fA} = \frac{\rho (NI) NI\ell_{ave}}{fA}$$

Substituting, $Na = fA$ we get the expression for the power *per coil*,

$$P = \frac{\rho (NI) NI\ell_{ave}}{Na} = \frac{\rho (NI) I\ell_{ave}}{a} = \rho (NI) j\ell_{ave}$$

where, $j = \frac{I}{a}$ *the current density.*

Magnet power

$$(NI)_{dipole} = \frac{Bh}{\eta \mu_0} \quad (NI)_{quadrupole} = \frac{B'h^2}{2\eta \mu_0} \quad (NI)_{sextupole} = \frac{B''h^3}{6\eta \mu_0}$$

Substituting and multiplying the expression for the power per coil by 2 coils/magnet for the dipole, 4 coils/magnet for the quadrupole and 6 coils/magnet for the sextupole, the expressions for the power *per magnet* for each magnet type are,

$$P_{dipole} = \frac{2\rho Bhj\ell_{ave}}{\eta \mu_0} \quad P_{quadrupole} = \frac{2\rho B'h^2 j\ell_{ave}}{\eta \mu_0} \quad P_{sextupole} = \frac{\rho B''h^3 j\ell_{ave}}{\eta \mu_0}$$

General guideline

Note that the expressions for the magnet power include only the resistivity *ρ*, gap *h*, the field values *B*, *B'*, *B''*, current density *j*, the average turn length, the magnet efficiency and μ_0. Thus, the power can be computed for the magnet without choosing the number of turns or the conductor size. The power can be divided among the voltage and current thus leaving the choice of the final power supply design until later.

Canonical values

$$j = \frac{I}{a} \approx 10 \frac{Amps}{mm^2}$$

$$Na = fA$$

$$f = \text{coil packing fraction} = 0.5$$

$$\eta = 0.98$$

Magnet *System* Design

- Magnets and their infrastructure represent a major cost of accelerator systems since they are so numerous.
- Magnet support infrastructure include:
  - Power Supplies
  - Power Distribution
  - Cooling Systems
  - Control Systems
  - Safety Systems

Power Supplies

- Generally, for the same power, a high current - low voltage power supply is more expensive than a low current - high voltage supply.

- Power distribution (cables) for high current magnets is more expensive. Power distribution cables are generally air-cooled and are generally limited to a current density of < 1.5 to 2 A/mm^2. Air cooled cables generally are large cross section and costly.

Dipole Power Supplies

- In most accelerator lattices, the dipole magnets are generally at the same excitation and thus in series. Dipole coils are generally designed for high current, low voltage operation. The total voltage of a dipole string is the sum of the voltages for the magnet string.

- If the power cable maximum voltage is > 600 Volts, a separate conduit is required for the power cables.

- In general, the power supply and power distribution people will not object to a high current requirement for magnets in series since fewer supplies are required.

Quadrupole Power Supplies

- Quadrupole magnets are usually individually powered or connected in short series strings (families).

- Since there are so many quadrupole circuits, quadrupole coils are generally designed to operate at lower current and higher voltage.

Sextupole Power Supplies

- Sextupole are generally operated in a limited number of series strings (families). Their effect is distributed around the lattice. In many lattices, there are a maximum of two series strings.

- Since the excitation requirements for sextupole magnets is generally modest, sextupole coils can be designed to operate at either high or low currents.

Power Consumption

- The raw cost of power varies widely depending on location and constraints under which power is purchased.
    - In the Northwest US, power is cheap.
    - Power is often purchased at low prices by negotiating conditions where power can be interrupted.
    - The integrated cost of power requires consideration of the lifetime of the facility.
- The cost of cooling must also be factored into the cost of power.

Coil Cooling

- In this section, we shall temporarily abandon the MKS system of units and use the mixed engineering and English system of units.
- Assumptions
  - The water flow requirements are based on the heat capacity of the water and assumes no temperature difference between the bulk water and conductor cooling passage surface.
  - The temperature of the cooling passage and the bulk conductor temperature are the same. This is a good assumption since we usually specify good thermal conduction for the electrical conductor.

Pressure Drop

$$\Delta P = 0.433\, f \frac{L}{d} \frac{v^2}{2g}$$

where

ΔP = pressure drop (psi)

f = friction factor (no units)

L = water circuit length (units same as d)

d = water circuit hole diameter (units same as L)

v = water velocity $\left(\frac{\text{ft}}{\text{sec}}\right)$ 1 ft/s = 0.3048 m/s

g = gravitational acceleration = $32.2\left(\frac{\text{ft}}{\text{sec}^2}\right)$ 9.8 m/s^2

Friction Factor, *f*

We are dealing with *smooth tubes*, where the *surface roughness* of the cooling channel is given by;

$$\varepsilon < 5\times 10^{-6}\ \text{ft} \quad < 1.524\text{x}10^{-3}\ \text{mm}$$

Under this condition, the *friction factor* is a function of the dimensionless R*eynold's Number*.

$$\text{Re} = \frac{vd}{\nu_k} \quad \text{where} \quad \begin{aligned} &\text{Re} = \text{dimensionless number} \\ &v = \text{flow velocity}\left(\frac{ft}{\text{sec}}\right) \quad 1\ \text{ft/s} = 0.3048\ \text{m/s} \\ &d = \textit{hole}\ \text{diameter}\ (\text{ft}) \quad 1\ \text{ft} = 304.8\ \text{mm} \\ &\nu = \textit{kinematic}\ \text{viscosity} \end{aligned}$$

$$\nu_k = 1.216\times 10^{-5}\left(\frac{\text{ft}^2}{\text{sec}}\right) \text{for water at } 20°\text{C} \qquad 1.1297\times 10^{-6}\ \text{m}^2/\text{s}$$

Laminar vs. Turbulent Flow

for laminar flow Re ≤ 2000

$$f = \frac{64}{\mathrm{Re}}$$

for turbulent flow Re > 4000

$$\frac{1}{\sqrt{f}} = -2\log_{10}\left(\frac{\varepsilon}{3.7d} + \frac{2.51}{\mathrm{Re}\sqrt{f}}\right) \qquad \frac{1}{\sqrt{f}} = -2\log_{10}\left(\frac{\varepsilon}{3.7d} + \frac{2.51}{\frac{d}{\nu_k}\sqrt{\frac{2g\Delta P}{0.433}\frac{d}{L}}}\right)$$

Water Flow

The equation for the pressure drop is,

$$\Delta P = 0.433\, f\, \frac{L}{d}\frac{v^2}{2g}$$

Solving for the water velocity,

$$v = \sqrt{\frac{2g\Delta P}{0.433\, f}\frac{d}{L}} = \frac{1}{\sqrt{f}}\sqrt{\frac{2g\Delta P}{0.433}\frac{d}{L}}$$

Substituting the expression derived for, $\frac{1}{\sqrt{f}}$

we get, finally,

$$v = -2\sqrt{\frac{2g\Delta P}{0.433}\frac{d}{L}}\log_{10}\left(\frac{\varepsilon}{3.7d} + \frac{2.51}{\frac{d}{\nu_k}\sqrt{\frac{2g\Delta P}{0.433}\frac{d}{L}}}\right)$$

Coil Temperature Rise

Based on the heat capacity of water, the water temperature rise for a flow through a thermal load is given by,

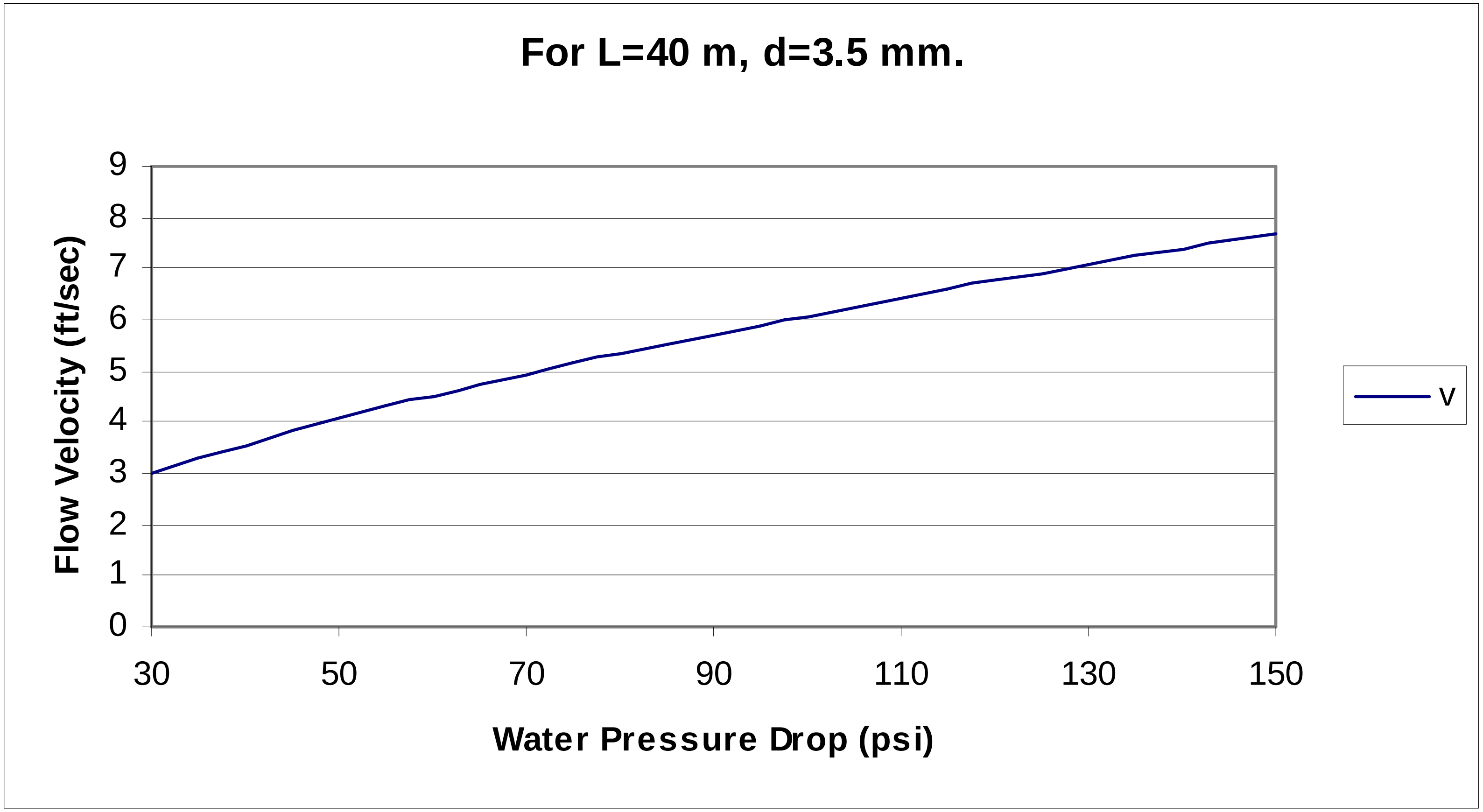
$$\Delta T\left(^{\circ}C\right)=\frac{3.8P\left(kW\right)}{q\left(gpm\right)}$$

1 gpm = 0.0630901 liter/s
1 gpm = 3.78541 liter/s

Assuming good heat transfer between the water stream and the coil conductor, the maximum conductor temperature (at the water outlet end of the coil) is the same value.

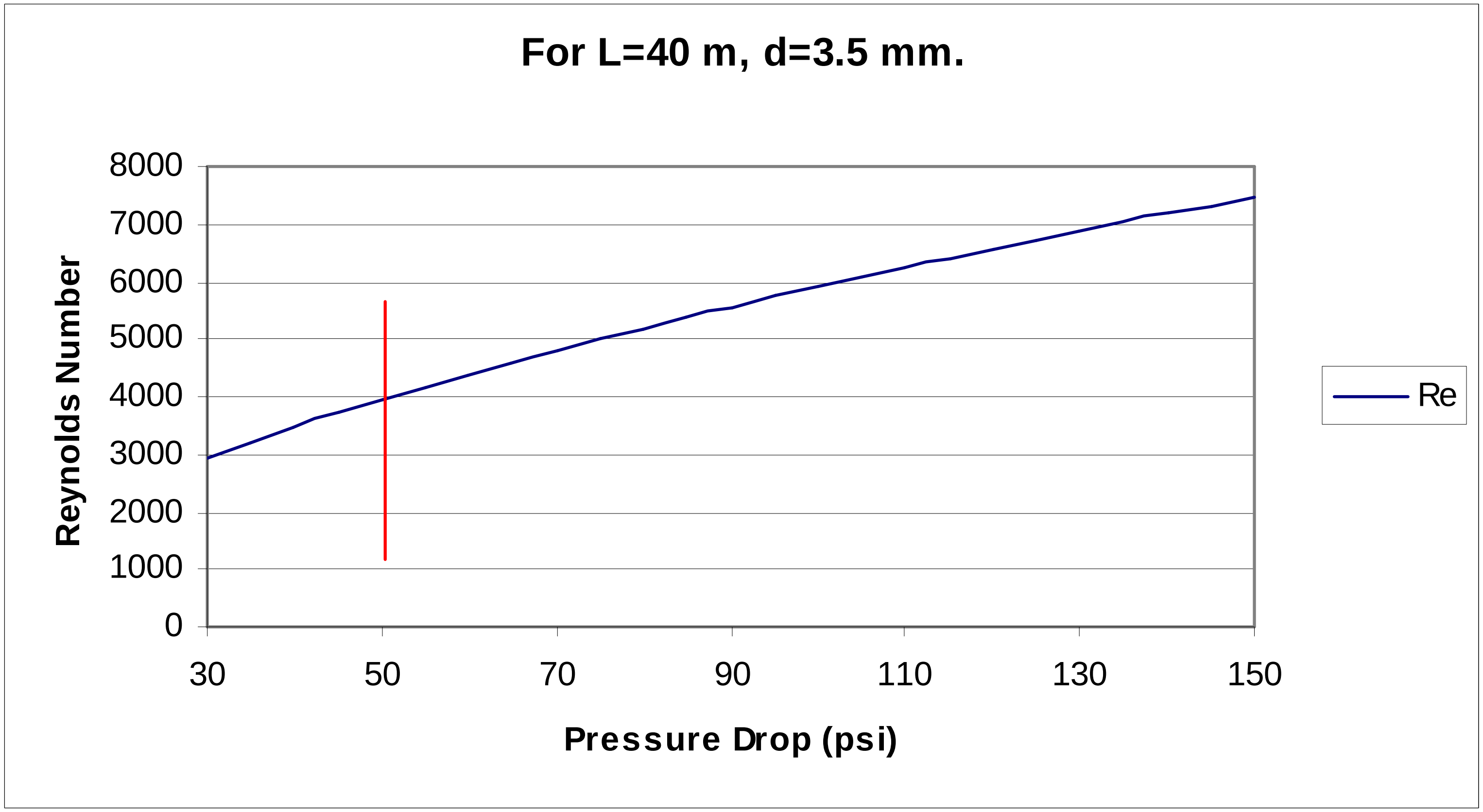
Results – *Water Velocity*

For water velocities > 15 fps, flow vibration will be present resulting in long term erosion of water cooling passage.



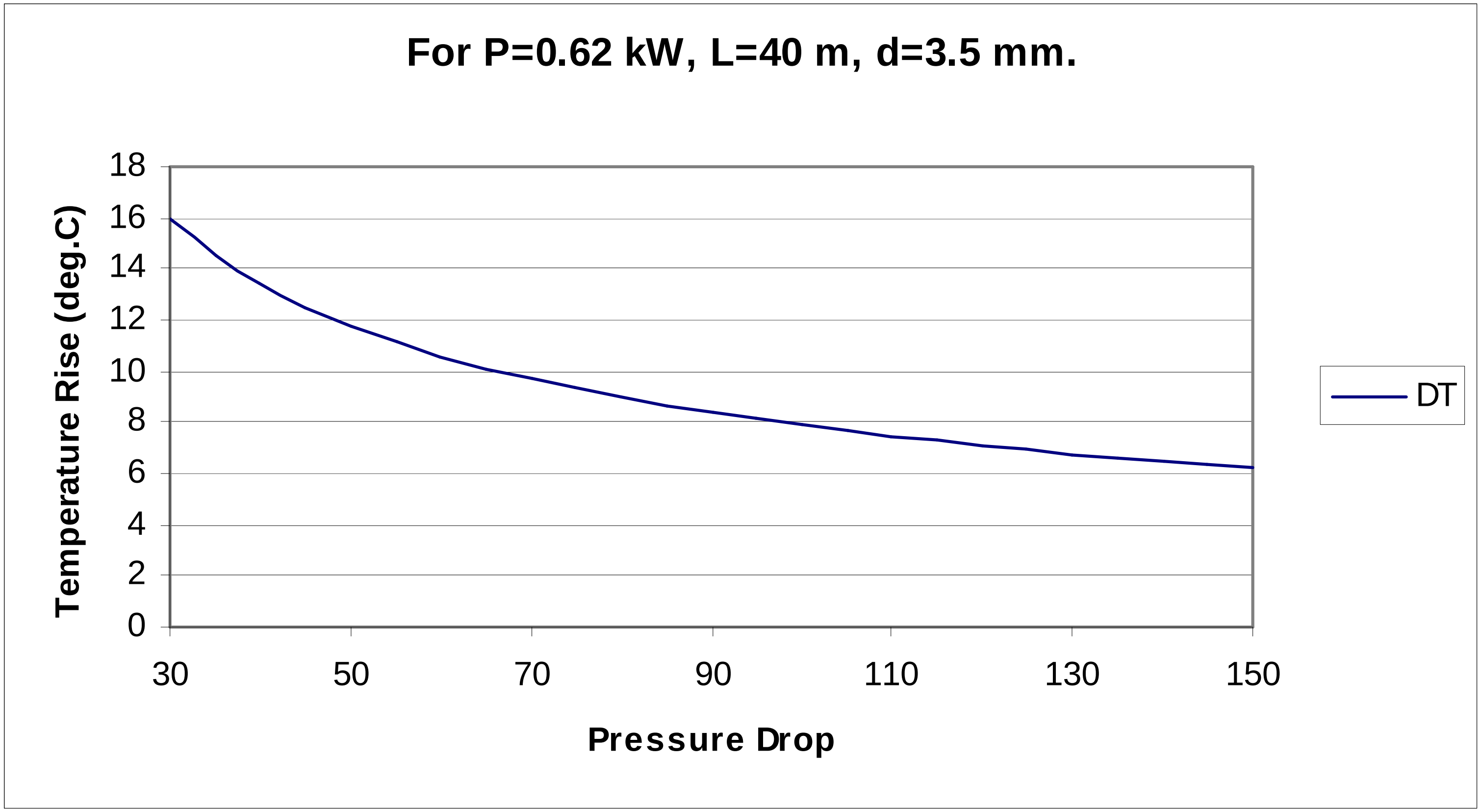
Results – *Reynolds Number*

Results valid only for Re > 4000 (turbulent flow).



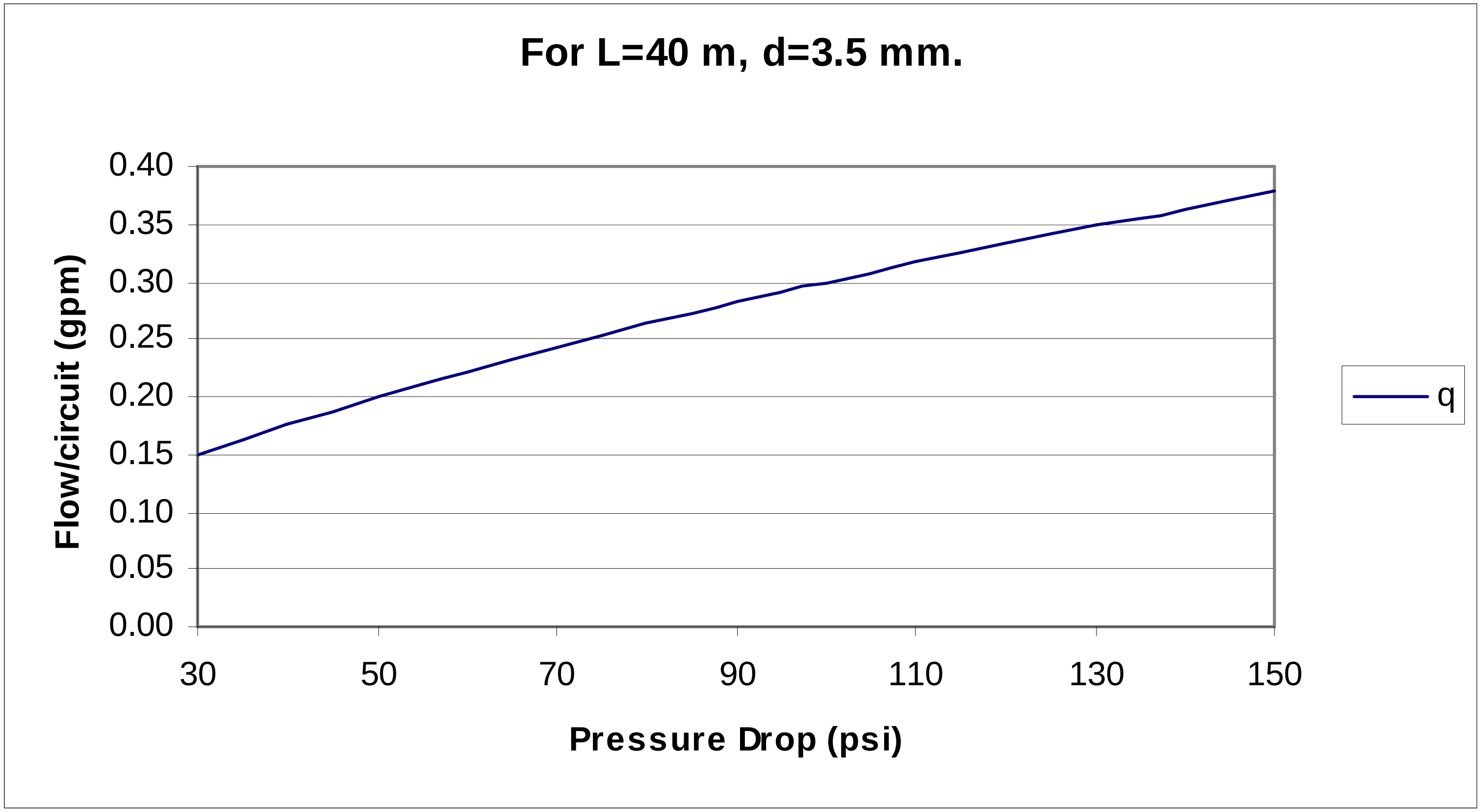
Results – *Water Temperature Rise*

Desirable temperature rise for Light Source Synchrotrons < 10°C. Maximum allowable temperature rise (assuming 20°C. input water) < 30°C for long potted coil life.



Results – *Water Flow*

Say, we designed quadrupole coils to operate at Δp=100 psi, four coils @ 0.30 gpm, total magnet water requirement = 1.2 gpm.



Sensitivities

- Coil design is an iterative process.
- If you find that you selected coil geometries parameters which result in calculated values which exceed the design limits, then you have to start the design again.
  - ΔP is too large for the maximum available pressure drop in the facility.
  - Temperature rise exceeds desirable value.
- The sensitivities to particular selection of parameters must be evaluated.

Sensitivities – *Number of Water Circuits*

The required pressure drop is given by,

$$\Delta P = 0.433 f \frac{L}{d}\frac{v^2}{2g} \propto Lv^2$$

where L is the water circuit length.

$$L = \frac{KN\ell_{ave}}{N_w}$$

K = 2, 4 or 6 for dipoles, quadrupoles or sextupoles, respectively. N = Number of turns per pole. N_W = Number of water circuits.

$$v \propto \frac{Q}{N_w}$$

Substituting into the pressure drop expression,

$$\Delta P \propto Lv^2 = \frac{KN\ell_{ave}}{N_w}\left(\frac{Q}{N_w}\right)^2$$

$$\Delta P \propto \frac{1}{N_w^3}$$

Pressure drop can be decreased by a factor of eight if the number of water circuits are doubled.

Sensitivities – *Water Channel Diameter*

The required pressure drop is given by,

$$\Delta P = 0.433\, f\,\frac{L}{d}\frac{v^2}{2g} \propto \frac{v^2}{d}$$

where *d* is the water circuit diameter.

$$v = \frac{q}{hole\ Area} = \frac{q}{\pi\frac{d^2}{4}} \propto \frac{1}{d^2}$$

where *q* is the volume flow per circuit.

Substituting,

$$\Delta P \propto \frac{v^2}{d} \propto \frac{1}{d}\left(\frac{1}{d^2}\right)^2 = \frac{1}{d^5}$$

If the design hole diameter is increased, the required pressure drop is decreased dramatically.

If the fabricated hole diameter is too small (too generous tolerances) then the required pressure drop can increase substantially.

Summary

- Excitation current for several kinds of magnets were derived.
- Saturation must be avoided ($\eta \geq 0.98$).
- Current densities canonical numbers where presented.
- Magnet power and its implications with the facility was discussed.
- Coil cooling parameters was shown.
- Coil design is an iterative process.

Next…

- Stored Energy
- Magnetic Forces
- Dynamic effects (eddy currents)