Microwave Physics and Techniques

IU/UPSAS P671D – Homework 1
Due Tuesday 6-17-03

Problem1. Consider the set of equations $x(t) = 2b\cos t$
$y(t) = b\sin t$

(b>0 is a parameter) describes a family of ellipses with the same ratio of semi-major to semi-minor axis. (i) Give an implicit equation $F(x,y) = C = const$ and an exact differential equation for this family of curves. (ii) Show that the differential equation

$$2x\sin y\dot{x} + \left(x^2\cos y + \sin y\right)\dot{y} = 0$$

is exact. Express the general solution in implicit form $F(x,y) = C = const$.

Problem2. Use the method of separation of variables to solve $u_t = u_{xx}$ *in* $-1 < x < 1$ with boundary conditions

$$u(-1,t) = u(1,t)$$
$$u_x(-1,t) = u_x(1,t)$$

and initial condition $u(x,0) = 1 + 4\sin 2\pi x + \frac{1}{2}\cos 5\pi x$.

Problem3. Find the Fourier series representation for $f(x) = x^2$ in $-\pi < x < \pi$ and plot the sum obtained for $-2\pi < x < 2\pi$.

Problem4. Find the function given by $H(z) = \int_0^z f(x)dx, z \geq 0$ where

$$f(x) = \begin{cases} x \ \ if \ x < 1 \\ 1 \ if \ x \geq 1 \end{cases}$$

Problem5.Find c, such that the function $f(x) = ce^{-x^2}, x \geq 0$ becomes a density function.

Problem6.The magnetic field of a TV broadcast signal propagating in air is given by

$$\overline{\mathrm{H}}(x,t) = \hat{x}0.1\sin(\omega t - 9.3z)mA - m^{-1}$$

(a) Find the wave frequency. (b) Find the corresponding $\overline{\mathrm{E}}(z,t)$.

Problem7. The electric field component of an electromagnetic wave in free space is given by

$$\overline{\mathrm{E}}(y,z,t) = \hat{x}E_0\cos(ay)\cos(\omega t - bz)$$

(a)Find the corresponding magnetic field. (b)Find the relationship between the constants a,b,c, and ω such that all the Maxwell's equations are satisfied. (c) Assuming that this wave may be regarded as a sum of two uniform plane waves, determine the direction of propagation of the two component waves.

Problem8. The electric field component of a communication satellite signal traveling in free space is given by $E(z) = \left[\hat{x} - \hat{y}(1-j)\right]12e^{j50\pi}V/m$. (a) Find the corresponding H(z). (b) Find the total time-average power carried by this wave. (c) Determine the polarization (both type and sense) of the wave.