Lecture 6

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Integral Solution to Poisson's Equation

Coil Design

System Design

Water Flow Calculations

Introduction

- This section develops the expressions for magnet excitation.
- The relationship between current density and magnet power is developed.
- An example of the optimization of a magnet system is presented in order to develop a logic for adopting *canonical* current density values.
- Engineering relationships for computing water flows for cooling magnet coils are developed.

Poisson’s Equation

- The *Poisson* equation is the nonhomogeneous version of the *LaPlace* equation and includes the term for the *current*.
- Both *Poisson’s* and *LaPlace’s* equations are two dimensional versions of *Maxwell’s* equations for magnetics.

- Application of Stoke’s Theorem results in the more familiar integral form of Poisson’s equation.
  - Stokes Theorem - The line integral of a potential function around a closed boundary is equal to the area integral of the source distribution within that closed boundary.

$$\oint \overline{H} \bullet \overline{dl} = NI$$

Dipole Excitation

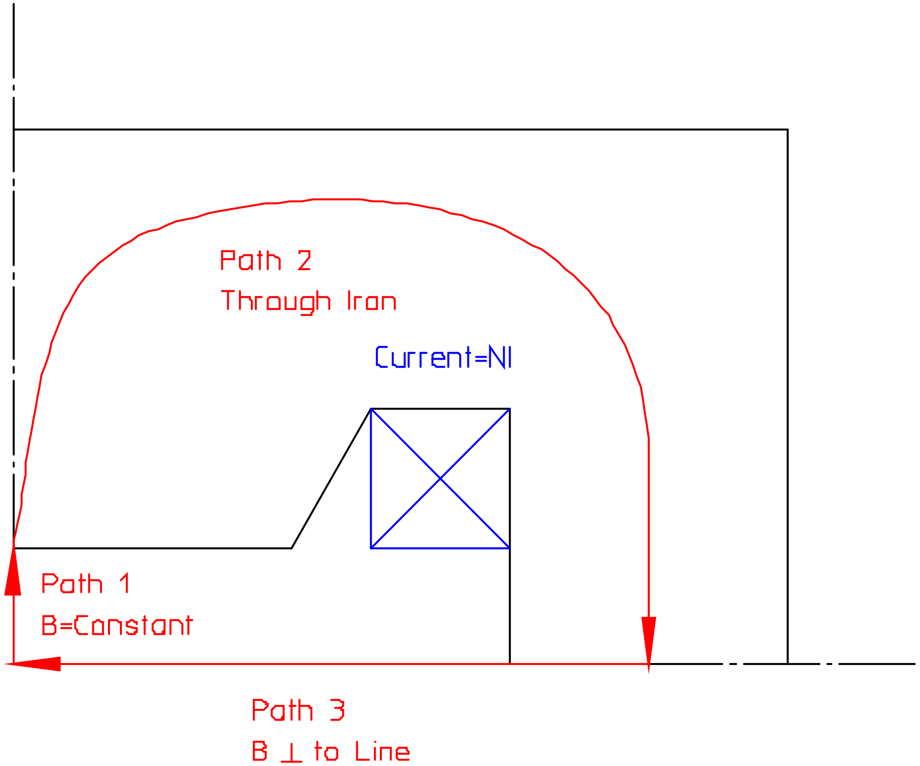
$$\oint \overline{H} \bullet \overline{dl} = \oint_{Path1} \overline{H} \bullet \overline{dl} + \oint_{Path2} \overline{H} \bullet \overline{dl} + \oint_{Path3} \overline{H} \bullet \overline{dl} = NI$$

Along Path 1

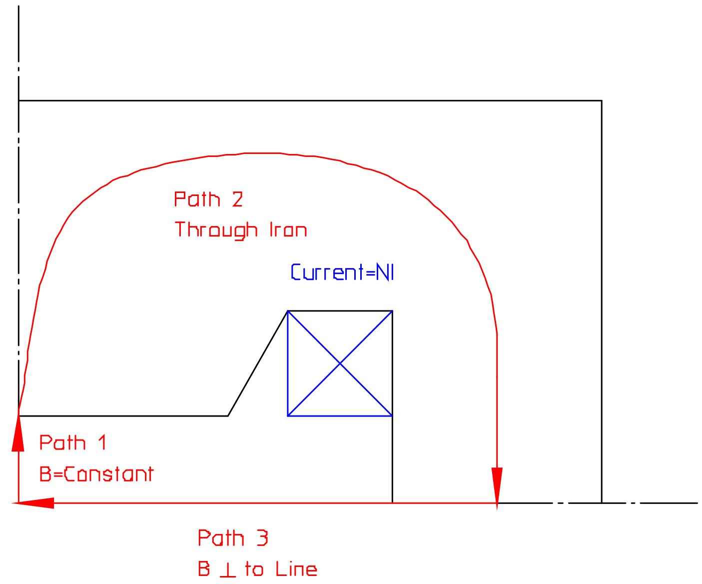
$$|H| = \frac{B}{\mu_0} \quad \text{and} \quad H \parallel l$$

Therefore;

$$\oint_{Path1} \overline{H} \bullet \overline{dl} = \frac{Bh}{\mu_0}$$



Along path 2, $|H| = \frac{B}{\mu \mu_0}$



For iron; $\mu \approx 1000$

Therefore; $\oint_{Path2} \overline{H} \bullet \overline{dl} = |H|_{iron} l_{iron} << \frac{Bh}{\mu_0} \approx 0$

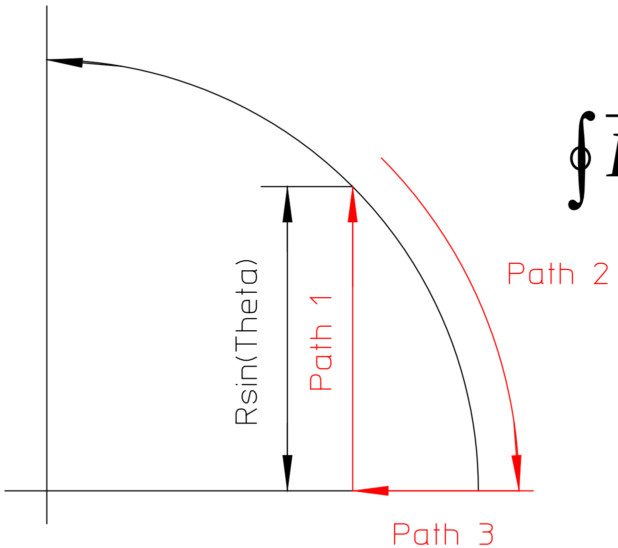
Along path 3, $\overline{H} \perp \overline{dl}$ and $\oint_{Path3} \overline{H} \bullet \overline{dl} = 0$

Therefore; $\overline{H} \bullet \overline{dl} = 0$

Finally; $\oint \overline{H} \bullet \overline{dl} = NI \approx \frac{Bh}{\mu_0}$

Current Dominated Magnets

- Occasionally, a need arises for a magnet whose field quality relies on the distribution of current. One example of this type of magnet is the superconducting magnet, whose field quality relies on the proper placement of current blocks.



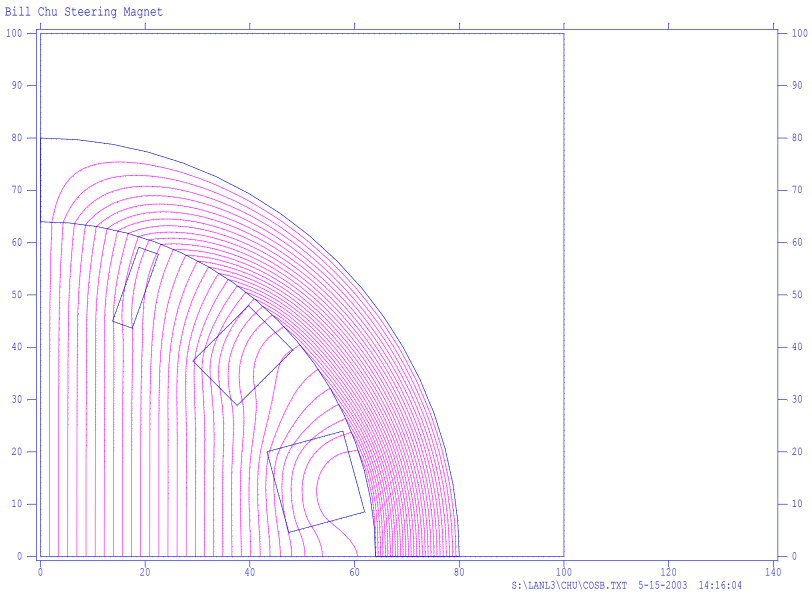
$$\oint \overline{H} \bullet \overline{dl} = I = \int j d\theta$$

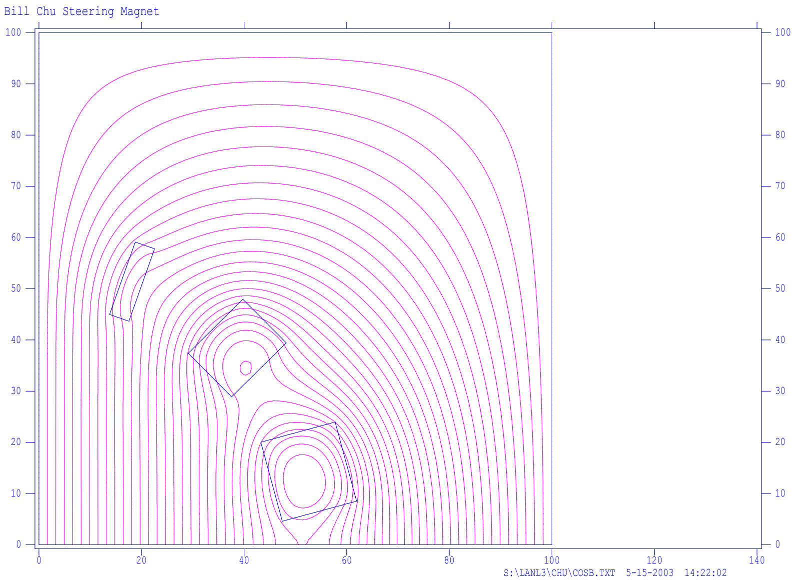
$$\oint \overline{H} \bullet \overline{dl} = \frac{BR \sin \theta}{\mu_0} + \int_{path2} \overline{H} \bullet \overline{dl} + \int_{path3} \overline{H} \bullet \overline{dl}$$

$$\int j d\theta \approx \frac{BR \sin \theta}{\mu_0} \quad \Rightarrow \quad j = \frac{BR \cos \theta}{\mu_0}$$

This is the cosine distribution of current.

- Two flux plots from a cosine block coil distribution are shown, one with a cylindrical iron yoke and the other without. The one without the shield appears to have poorer field uniformity. This is because the field along path2 is large. This is an artifact of the computation since this computed field would have been smaller had the boundaries been extended farther from the problem.





Cosine Block Distribution

- The distribution of the block areas approximate the cosine distribution of the current.
- The example shown illustrates a solution with three blocks.
- Three blocks provide three parameters so that the first three dipole multipole errors can be minimized.
- What are these multipole indices?

Quadrupole Excitation

Using arguments similar to those used for the dipole;

$$\oint_{Path2} \overline{H} \bullet \overline{dl} + \oint_{Path3} \overline{H} \bullet \overline{dl} \approx 0$$

Along Path 1, $B(r) = B'r$

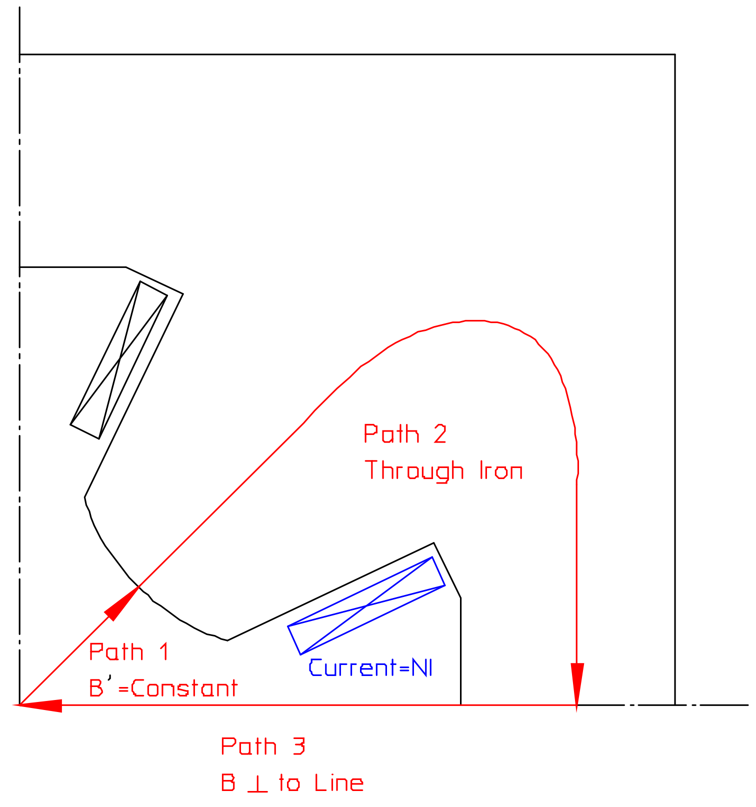
$H \| r$ and $|H(r)| = \dfrac{B'r}{\mu_0}$

Therefore;

$$\oint_{Path1} \overline{H} \bullet \overline{dl} = \int_0^h \frac{B'rdr}{\mu_0} = \frac{B'h^2}{2\mu_0}$$

Finally;

$$\oint \overline{H} \bullet \overline{dl} = NI \approx \frac{B'h^2}{2\mu_0}$$



Sextupole Excitation

Using arguments similar
to those used for the dipole;

$$\oint_{Path2} \overline{H} \bullet \overline{dl} + \oint_{Path3} \overline{H} \bullet \overline{dl} \approx 0$$

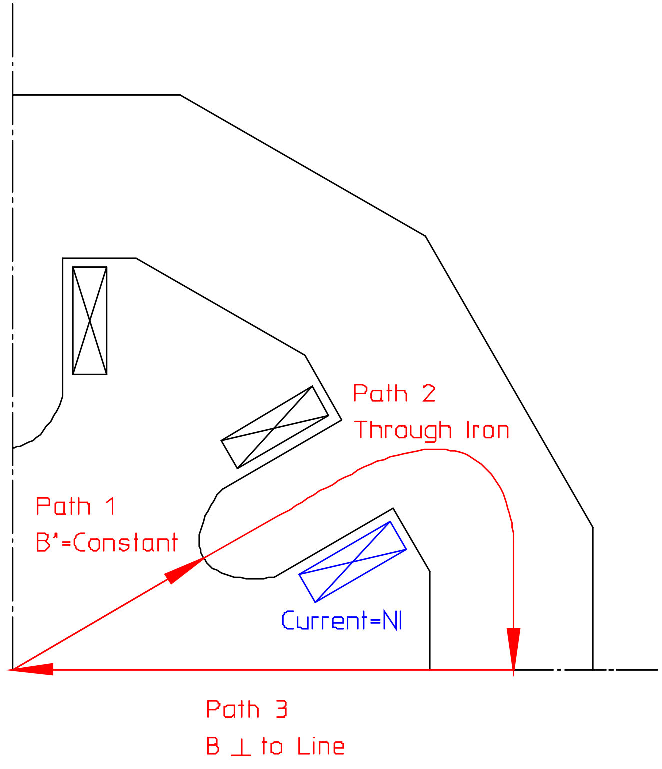
Along Path 1, $B'(r) = \int B'' dr = B'' r$

$$B(r) = \int B'' r dr = \frac{B'' r^2}{2}$$

$$H \| r \qquad \text{and} \qquad |H(r)| = \frac{B'' r^2}{2\mu_0}$$

$$\oint_{Path1} \overline{H} \bullet \overline{dl} = \int_0^h \frac{B'' r^2 dr}{2\mu_0} = \frac{B'' h^3}{6\mu_0}$$

Finally; $\oint \overline{H} \bullet \overline{dl} = NI \approx \frac{B'' h^3}{6\mu_0}$



Magnet Efficiency

- We introduce *efficiency* as a means of describing the losses in the iron. Use the expression for the dipole excitation as an example.

$$NI = \oint_{Path1} \overline{H} \bullet \overline{dl} + \oint_{Path2} \overline{H} \bullet \overline{dl} + \oint_{Path3} \overline{H} \bullet \overline{dl}$$

$$= \frac{Bh}{\mu_0} + small\ \ factor \times \frac{Bh}{\mu_0} + 0 \begin{pmatrix} \text{since} \\ B \perp l \end{pmatrix}$$

$$NI = (1 + small\ \ factor)\frac{Bh}{\mu_0} = \frac{Bh}{\eta\, \mu_0}$$

$\eta = efficiency \approx 0.98$ For magnets with well designed yokes.

Units

- For magnet excitation, we use the *MKS* system of units.

$$B = Tesla \qquad B' = \frac{Tesla}{meter} \qquad B'' = \frac{Tesla}{meter^2}$$

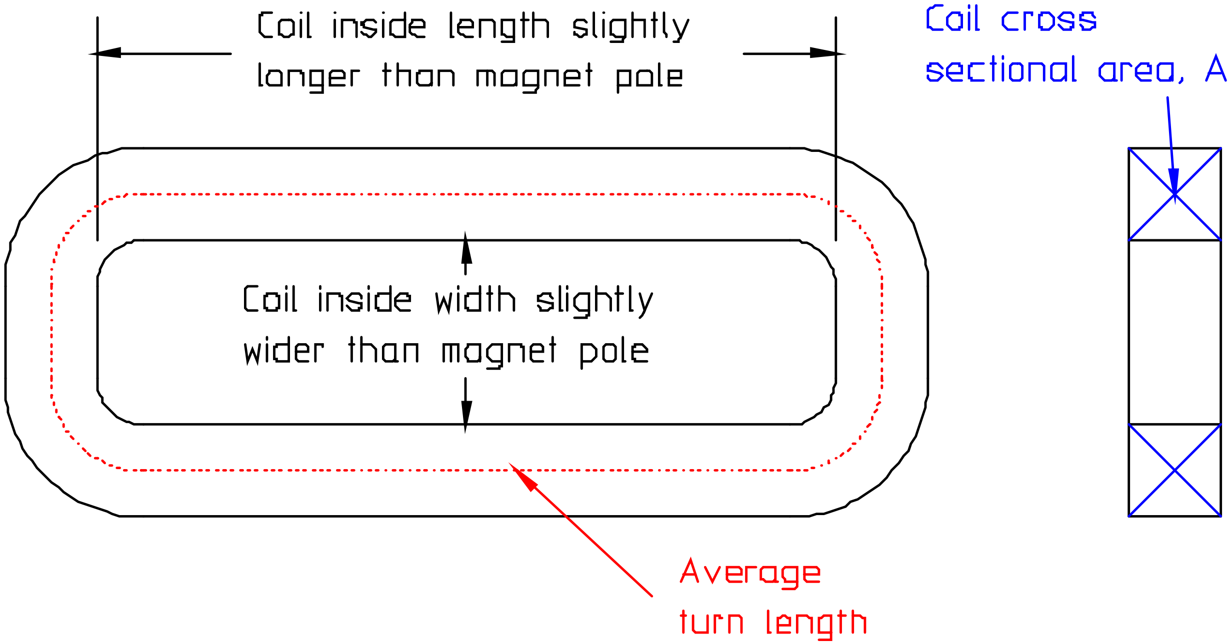
$$H = \frac{Amp}{meter}$$

$$h = meters$$

$$\mu_0 = 4\pi \times 10^{-7} \frac{Tesla - meter}{Amp}$$

Current Density

- One of the design choices made in the design of magnet coils is the choice of the coil cross section which determines the *current density*.
- Given the required *Physics parameters* of the magnet, the choice of the *current density* will determine the required magnet power.
  - Power is important because they affect both the cost of power supplies, power distribution (cables) and operating costs.
  - Power is also important because it affects the installation and operating costs of cooling systems.



$$Power = P = I^2 R$$

$$R = \frac{\rho L}{a} \quad \text{where} \quad \begin{aligned} \rho &= \text{resistivity } (\Omega\text{-m}) \\ L &= \text{conductor length (m)} \\ a &= \text{conductor net cross sectional area } (m^2) \end{aligned}$$

$L = N\ell_{ave}$ where N = number of turns in the coil.

$Na = fA$ where f = coil packing fraction.

Substituting; $R = \dfrac{\rho N \ell_{ave}}{\dfrac{fA}{N}} = \dfrac{\rho N^2 \ell_{ave}}{fA}$

Calculating the coil power; $P = I^2 R = \dfrac{\rho (NI)^2 \ell_{ave}}{fA} = \dfrac{\rho (NI) NI \ell_{ave}}{fA}$

Substituting, $Na = fA$ we get the expression for the power *per coil*,

$$P = \frac{\rho (NI) NI \ell_{ave}}{Na} = \frac{\rho (NI) I \ell_{ave}}{a} = \rho (NI) j \ell_{ave}$$

where, $j = \dfrac{I}{a}$ *the current density*.

But the required excitation for the three magnet types is,

$$(NI)_{dipole} = \frac{Bh}{\eta\,\mu_0}$$

$$(NI)_{quadrupole} = \frac{B'h^2}{2\eta\,\mu_0}$$

$$(NI)_{sextupole} = \frac{B''h^3}{6\eta\,\mu_0}$$

Substituting and multiplying the expression for the power per coil by 2 coils/magnet for the dipole, 4 coils/magnet for the quadrupole and 6 coils/magnet for the sextupole, the expressions for the power *per magnet* for each magnet type are,

$$P_{dipole} = \frac{2\rho\,Bhj\ell_{ave}}{\eta\,\mu_0}$$

$$P_{quadrupole} = \frac{2\rho\,B'h^2 j\ell_{ave}}{\eta\,\mu_0}$$

$$P_{sextupole} = \frac{\rho\,B''h^3 j\ell_{ave}}{\eta\,\mu_0}$$

Note that the expressions for the magnet power include only the resistivity ρ, gap h, the field values B, B', B'', current density j, the average turn length, the magnet efficiency and μ_0. Thus, the power can be computed for the magnet without choosing the number of turns or the conductor size. The power can be divided among the voltage and current thus leaving the choice of the final power supply design until later.

Reasonable magnet design can be obtained by using *canonical* values of some of the variables.

$$j = \frac{I}{a} \approx 10 \frac{Amps}{mm^2}$$

$$\eta = 0.98$$

$$Na = fA$$

$$f = \text{coil packing fraction} = 0.5$$

More Units

Using a consistent set of units, the power is expressed in Watts.

$$P_{dipole} = \frac{\rho\ Bhj\ell_{ave}}{\eta\ \mu_0}$$

$$= \frac{\rho(\Omega m)\ B(T)h(m)j\left(\frac{Amps}{m^2}\right)\ell_{ave}(m)}{\eta\ \mu_0\left(\frac{Tm}{Amp}\right)}$$

$$= Amps^2\Omega = Watts$$

Magnet *System* Design

- Magnets and their infrastructure represent a major cost of accelerator systems since they are so numerous.
- Magnet support infrastructure include;
  - Power Supplies
  - Power Distribution
  - Cooling Systems
  - Control Systems
  - Safety Systems

Power Supplies

- Generally, for the same power, a high current - low voltage *power supply* is more expensive than a low current - high voltage supply.
- *Power distribution* (cables) for high current magnets is more expensive. Power distribution cables are generally air-cooled and are generally limited to a current density of <1.5 to 2 Amps/mm^2. Air cooled cables generally are large cross section and costly.

- Dipole Power Supplies
  - In most accelerator lattices, the dipole magnets are generally at the same excitation and thus in series. Dipole coils are generally designed for high current, low voltage operation. The total voltage of a dipole string is the sum of the voltages for the magnet string.
    - If the power cable maximum voltage is > 600 Volts, a separate conduit is required for the power cables.
  - In general, the power supply and power distribution people will not object to a high current requirement for magnets in series since fewer supplies are required.

- Quadrupole Power Supplies
  - Quadrupole magnets are usually individually powered or connected in short series strings.
  - Since there are so many quadrupole circuits, quadrupole coils are generally designed to operate at low current and high voltage.

- Sextupole Power Supplies
  - Sextupole are generally operated in a limited number of series strings. Their effect is distributed around the lattice. In many lattices, there are a maximum of two series strings.
  - Since the excitation requirements for sextupole magnets is generally modest, sextupole coils can be designed to operate at either high or low currents.

Power Consumption

- The raw cost of power varies widely depending on location and constraints under which power is purchased.
  - In the Northwest US, power is cheap.
  - Power is often purchased at low prices by negotiating conditions where power can be interrupted.
  - The integrated cost of power requires consideration of the lifetime of the facility.
- The cost of cooling must also be factored into the cost of power.

Optimization

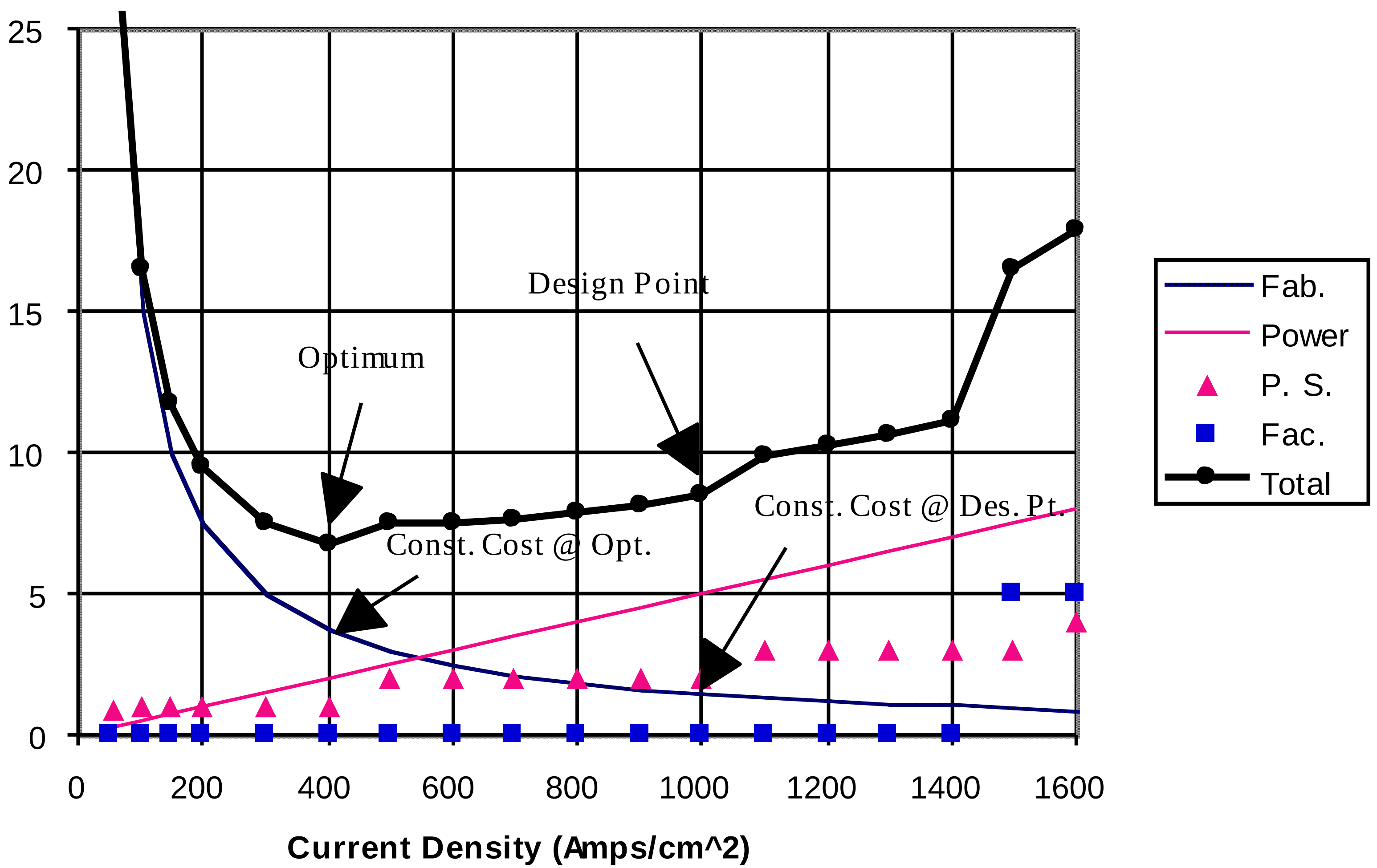
- As shown in a previous section, once the Physics requirements (field indices, *B, B', B'',* gaps and magnet lengths) have been determined, there is only one parameter which can be chosen in order to optimize the lifetime costs of the magnet system for a facility. That parameter is the current density, *j*. Therefore, based on the various cost parameters, a design current density can be selected for different magnet systems.

Optimization Example

- The following example is purely fictional and serves to illustrate the considerations which are included in the selection of a magnet system current density.
  - Magnet coil cost generally vary with the weight of the coil (its size) and thus varies inversely with the current density. For low current densities, the coil sizes and costs can increase exponentially.

- Power costs generally vary linearly with the power.
- Power supply costs vary at quantized levels and increase only when certain power thresh-holds are exceeded.
- Facility costs vary at quantized levels (substation costs) and take large increase increments at fairly high power levels.

Cost Optimization for XXX Magnet System



- For the illustrated example, the optimum is flat and appears to be *j=4 Amps/mm*^{2}. However, a higher design value (the *canonical j=10 Amps/mm*^{2} value) is generally chosen. This is because the integrated power cost is generally regarded as an operating expense.
- Construction estimates are normally kept as low as possible in order to secure funding.

Coil Cooling

- In this section, we shall temporarily abandon the MKS system of units and use the mixed engineering and English system of units.
- Assumptions
  - The water flow requirements are based on the heat capacity of the water and assumes no temperature difference between the bulk water and conductor cooling passage surface.
  - The temperature of the cooling passage and the bulk conductor temperature are the same. This is a good assumption since we usually specify good thermal conduction for the electrical conductor.

Pressure Drop

$$\Delta P = 0.433\, f \frac{L}{d} \frac{v^2}{2g}$$

where

$\Delta P = \text{pressure drop}\left(\text{psi}\right)$

$f = \text{friction factor (no units)}$

$L = \text{water circuit length (units same as d)}$

$d = \text{water circuit hold diameter (units same as L)}$

$v = \text{water velocity}\left(\frac{\text{ft}}{\text{sec}}\right)$

$g = \text{gravitational acceleration} = 32.2\left(\frac{\text{ft}}{\text{sec}^2}\right)$

Friction Factor, *f*

We are dealing with *smooth tubes*, where the *surface roughness* of the cooling channel is given by;

$$\varepsilon < 5 \times 10^{-6} \text{ ft}$$

Under this condition, the *friction factor* is a function of the dimensionless *Reynold's Number*.

$$\text{Re} = \frac{vd}{\nu} \quad \text{where} \quad \begin{aligned} &\text{Re} = \text{dimensionl ess number} \\ &v = \text{flow velocity} \left(\frac{ft}{\text{sec}} \right) \\ &d = \textit{hole} \text{ diameter } (\text{ft}) \\ &\nu = \textit{kinematic} \text{ viscosity} \end{aligned}$$

$$\nu = 1.216 \times 10^{-5} \left(\frac{\text{ft}^2}{\text{sec}} \right) \text{for water at } 20^\circ\text{C}$$

$$f = \frac{64}{\text{Re}} \text{ for laminar flow } (\text{Re} \le 2000)$$

For turbulent flow (Re>4000), the friction factor is gotten by solving a *transcendental* equation. Normally, this type of equation can be solved only by successive iterations. However some Algebra can be used to simplify the solution.

$$\frac{1}{\sqrt{f}} = -2\log_{10}\left(\frac{\varepsilon}{3.7d} + \frac{2.51}{\text{Re}\sqrt{f}}\right) \text{ for turbulent flow Re} > 4000.$$

Direct Solution of the Transcendental Equation

For *turbulent flow*, Re>4000;

$$\Delta P = 0.433\, f \frac{L}{d}\frac{v^2}{2g} \quad \Rightarrow \quad v = \sqrt{\frac{2g\Delta P}{0.433 f}\frac{d}{L}}$$

$$\text{Re} = \frac{vd}{\nu} = \frac{d}{\nu}\sqrt{\frac{2g\Delta P}{0.433 f}\frac{d}{L}}$$

Substituting into the expression for the the *friction factor*;

$$\frac{1}{\sqrt{f}} = -2\log_{10}\left(\frac{\varepsilon}{3.7d} + \frac{2.51}{\mathrm{Re}\sqrt{f}}\right)$$

$$= -2\log_{10}\left(\frac{\varepsilon}{3.7d} + \frac{2.51}{\frac{d}{\nu}\sqrt{\frac{2g\Delta P}{0.433f}\frac{d}{L}}\sqrt{f}}\right)$$

$$= -2\log_{10}\left(\frac{\varepsilon}{3.7d} + \frac{2.51}{\frac{d}{\nu}\sqrt{\frac{2g\Delta P}{0.433}\frac{d}{L}}}\right)$$

which is an equation that can be solved directly for *f*.

Water Flow

The equation for the pressure drop is,

$$\Delta P = 0.433\ f\ \frac{L}{d}\frac{v^2}{2g}$$

Solving for the water velocity,

$$v = \sqrt{\frac{2g\Delta P}{0.433\ f}\frac{d}{L}} = \frac{1}{\sqrt{f}}\sqrt{\frac{2g\Delta P}{0.433}\frac{d}{L}}$$

Substituting the expression derived for, $\frac{1}{\sqrt{f}}$

we get, finally, $v = -2\sqrt{\frac{2g\Delta P}{0.433}\frac{d}{L}}\log_{10}\left(\frac{\varepsilon}{3.7d} + \frac{2.51}{\frac{d}{\nu}\sqrt{\frac{2g\Delta P}{0.433}\frac{d}{L}}}\right)$

Water Flow - *Units*

The velocity is expressed in *ft/sec*. when *g=32.2ft/sec2* and ΔP in (*psi*).

$$v = -2\sqrt{\frac{2g\Delta P}{0.433}\frac{d}{L}}\log_{10}\left(\frac{\varepsilon}{3.7d} + \frac{2.51}{\frac{d}{\nu}\sqrt{\frac{2g\Delta P}{0.433}\frac{d}{L}}}\right)$$

In the expression, the units in the factors must be consistent.

ε, the surface roughness for a smooth tube, is $5x10^{-6}$ *ft*.

Therefore, in the term, $\frac{\varepsilon}{3.7d}$, *d* is expressed in *ft*.

More *Units*

Similarly, since the water kinematic viscosity,

$$\upsilon = 1.216 \times 10^{-5} \frac{ft^2}{\text{sec.}} @ 20^\circ C$$

for the term $\dfrac{d}{\upsilon}$, *d* is also expressed in *ft*.

Finally for the term $\dfrac{d}{L}$, *d* and *L* must be in the same units.

Coil Temperature Rise

Based on the heat capacity of water, the water temperature rise for a flow through a thermal load is given by,

$$\Delta T\left(^{\circ}C\right)=\frac{3.8P(kW)}{q(gpm)}$$

- Assuming good heat transfer between the water stream and the coil conductor, the maximum conductor temperature(at the water outlet end of the coil) is the same value.
- One more set of units has to be sorted out in order to compute the temperature rise.

$$q\left(\frac{gpm}{circuit}\right)=v\frac{\pi d^2}{4}=v\left(\frac{ft}{\sec}\right)\frac{\pi d^2}{4}\left(ft^2\right)\times\frac{gal}{0.1337\,ft^3}\times 60\frac{\sec}{\min}$$

Water Flow Spreadsheet

A CD is enclosed with the text. Among the files in the CD is a spreadsheet which can be used to reduce the drudgery involved in calculating the coil cooling. Two different spreadsheets are included in this file. One is written in metric units and the other is written in English units. Information in yellow are input data.

		Units		Units		Units
d			3.6	mm	0.011811	ft
L	40	m	40000	mm		
epsilon					0.000005	ft
nu	0.00001216	ft^2/sec				
Coil Power	0.62	kW				

		Units		Units		Units
d			3.6	mm	0.011811	ft
L	40	m	40000	mm		
epsilon					0.000005	ft
nu	0.00001216	ft^2/sec				
Coil Power	0.62	kW				

$$v = -2\sqrt{\frac{2g\Delta P}{0.433}\frac{d}{L}}\log_{10}\left(\frac{\varepsilon}{3.7d} + \frac{2.51}{\frac{d}{\nu}\sqrt{\frac{2g\Delta P}{0.433}\frac{d}{L}}}\right)$$

$$\frac{1}{\sqrt{f}} = -2\log_{10}\left(\frac{\varepsilon}{3.7d} + \frac{2.51}{\frac{d}{\nu}\sqrt{\frac{2g\Delta P}{0.433}\frac{d}{L}}}\right)$$

$$q\left(\frac{gpm}{circuit}\right) = v\frac{\pi d^2}{4} = v\left(\frac{ft}{sec}\right)\frac{\pi d^2}{4}\left(ft^2\right)\times\frac{gal}{0.1337\ ft^3}\times 60\frac{sec}{min}$$

$$\left(\frac{\varepsilon}{3.7d} + \frac{2.51}{\frac{d}{\nu}\sqrt{\frac{2g\Delta P}{0.433}\frac{d}{L}}}\right)$$

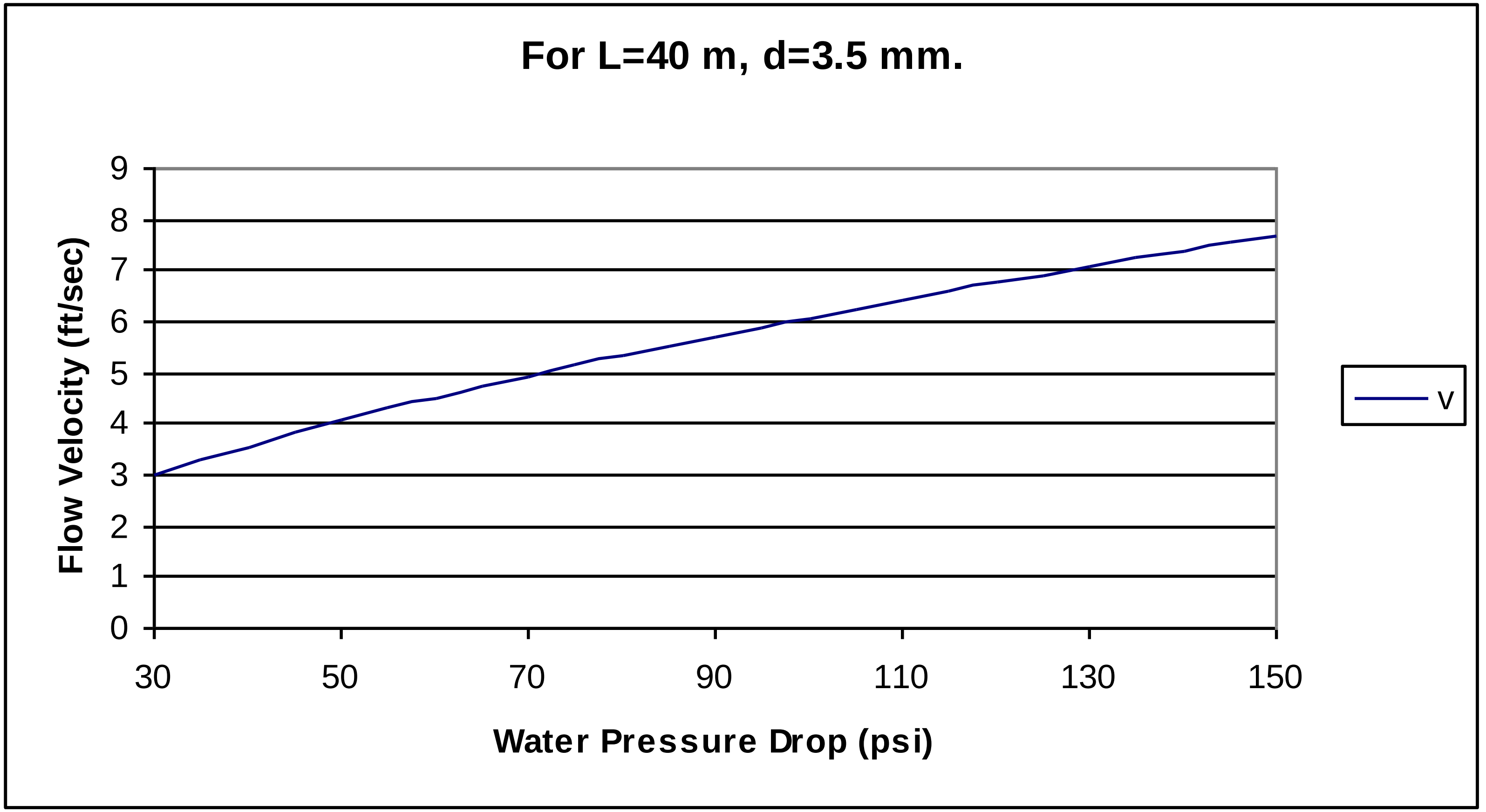
$$\mathrm{Re} = \frac{vd}{\upsilon}$$

$$\Delta T = \frac{3.8P}{q}$$

DeltaP	$\sqrt{\frac{2g\Delta P}{0.433}\frac{d}{L}}$			f	v	Re	q	DT
(psi)	(ft/sec)	(no units)	(no units)		(ft/sec)		(gpm)	(deg.C)
30	0.63369586	0.004192	4.755088	0.044227	3.013279	2926.802	0.148157693	15.90198
35	0.68446975	0.00389	4.820137	0.043041	3.299238	3204.554	0.162217788	14.52368
40	0.73172895	0.003646	4.876367	0.042054	3.568179	3465.777	0.175441135	13.42901
45	0.77611575	0.003444	4.925868	0.041213	3.823044	3713.327	0.187972383	12.53376
50	0.81809783	0.003273	4.970066	0.040483	4.066	3949.311	0.19991813	11.78482
55	0.85802825	0.003126	5.009978	0.039841	4.298702	4175.335	0.211359702	11.14687
60	0.89618127	0.002998	5.046354	0.039269	4.522448	4392.659	0.222360876	10.59539
65	0.93277504	0.002885	5.079764	0.038754	4.738277	4602.294	0.232972798	10.11277
70	0.96798641	0.002784	5.110649	0.038287	4.947039	4805.065	0.243237257	9.686016
75	1.00196113	0.002694	5.13936	0.03786	5.149439	5001.657	0.253188932	9.305304
80	1.034821	0.002612	5.16618	0.037468	5.346071	5192.646	0.262856992	8.963049
85	1.06666907	0.002537	5.191338	0.037106	5.53744	5378.522	0.27226626	8.653294
90	1.09759342	0.002469	5.215027	0.036769	5.723979	5559.708	0.281438073	8.371291
95	1.12767004	0.002406	5.237406	0.036456	5.906065	5736.569	0.290390935	8.113201
100	1.15696505	0.002348	5.25861	0.036162	6.084028	5909.424	0.299141025	7.875884
105	1.18553639	0.002294	5.278754	0.035887	6.258155	6078.554	0.307702585	7.656744
110	1.21343518	0.002244	5.297939	0.035628	6.428706	6244.21	0.316088237	7.453615
115	1.24070679	0.002197	5.316249	0.035383	6.595907	6406.613	0.324309228	7.264671
120	1.26739171	0.002153	5.333761	0.035151	6.759964	6565.962	0.332375634	7.088366
125	1.29352625	0.002112	5.350538	0.034931	6.921062	6722.436	0.340296525	6.923374
130	1.31914312	0.002073	5.36664	0.034721	7.079367	6876.198	0.348080103	6.768557
135	1.34427191	0.002037	5.382118	0.034522	7.23503	7027.394	0.355733812	6.62293
140	1.36893951	0.002002	5.397017	0.034331	7.38819	7176.159	0.363264435	6.485634
145	1.39317041	0.001969	5.411379	0.034149	7.538973	7322.615	0.370678175	6.355918
150	1.41698701	0.001938	5.42524	0.033975	7.687495	7466.874	0.377980718	6.233122

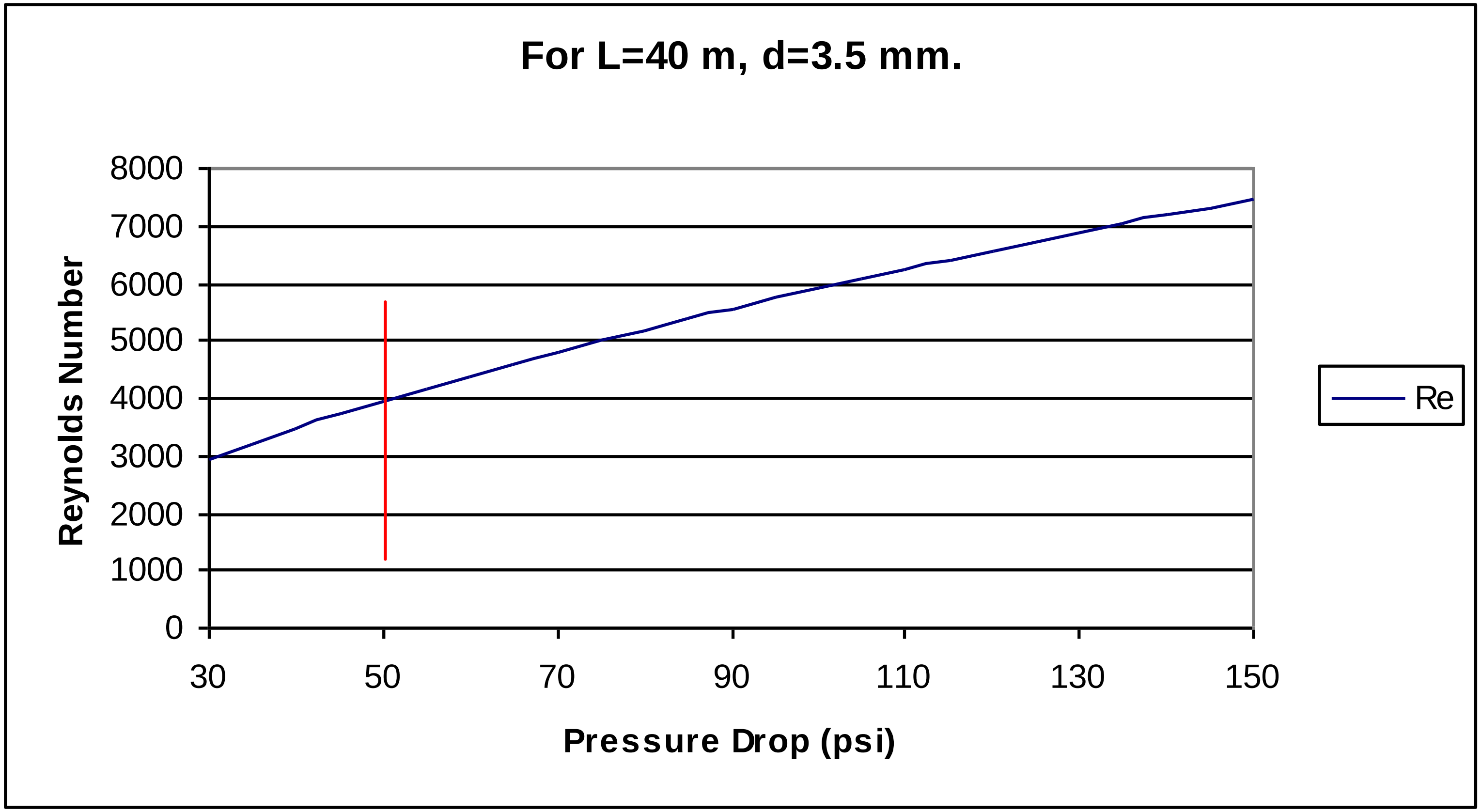
Results – *Water Velocity*

For water velocities > 15 fps, flow vibration will be present resulting in long term erosion of water cooling passage.



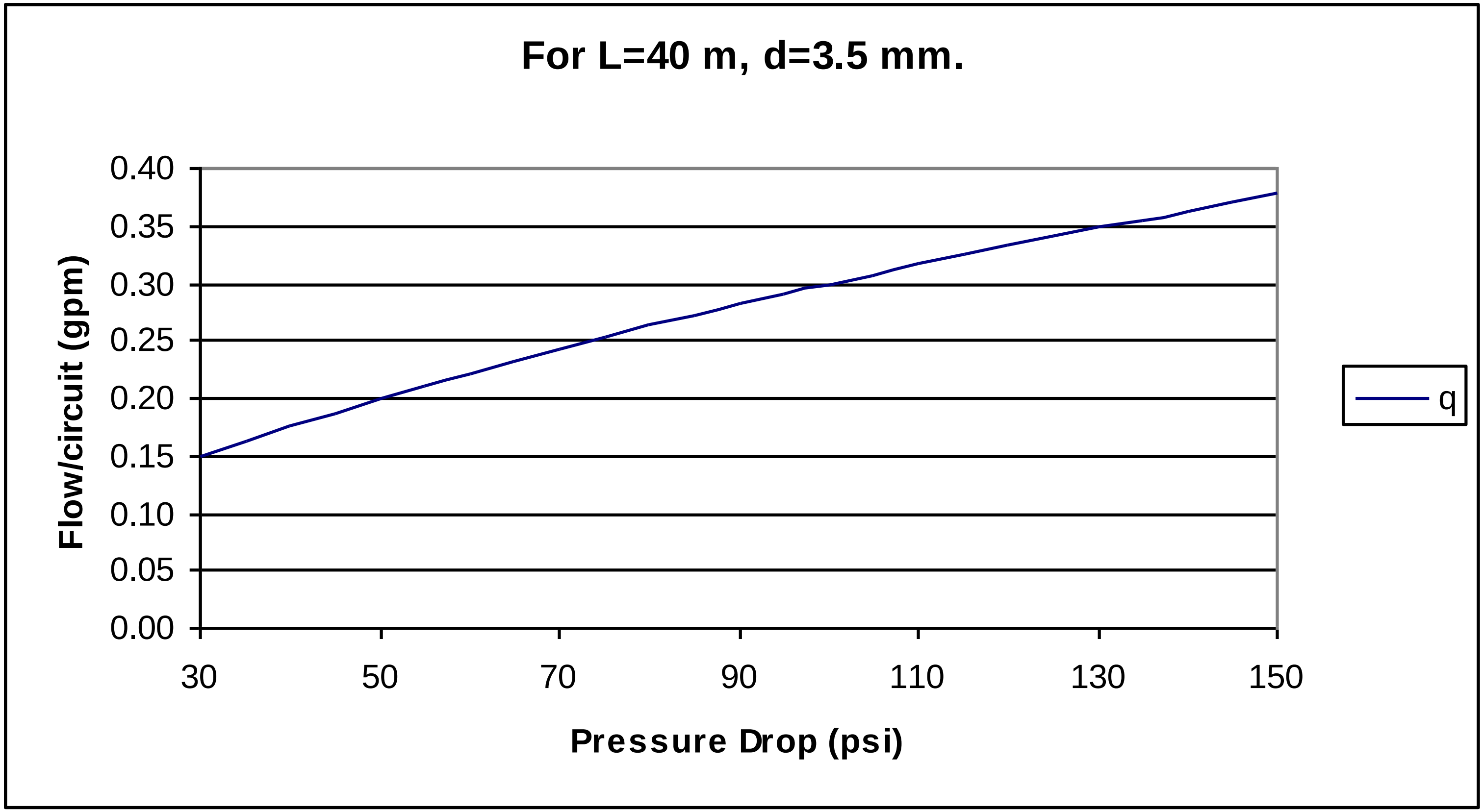
Results – *Reynolds Number*

Results valid only for Re > 4000 (turbulent flow).



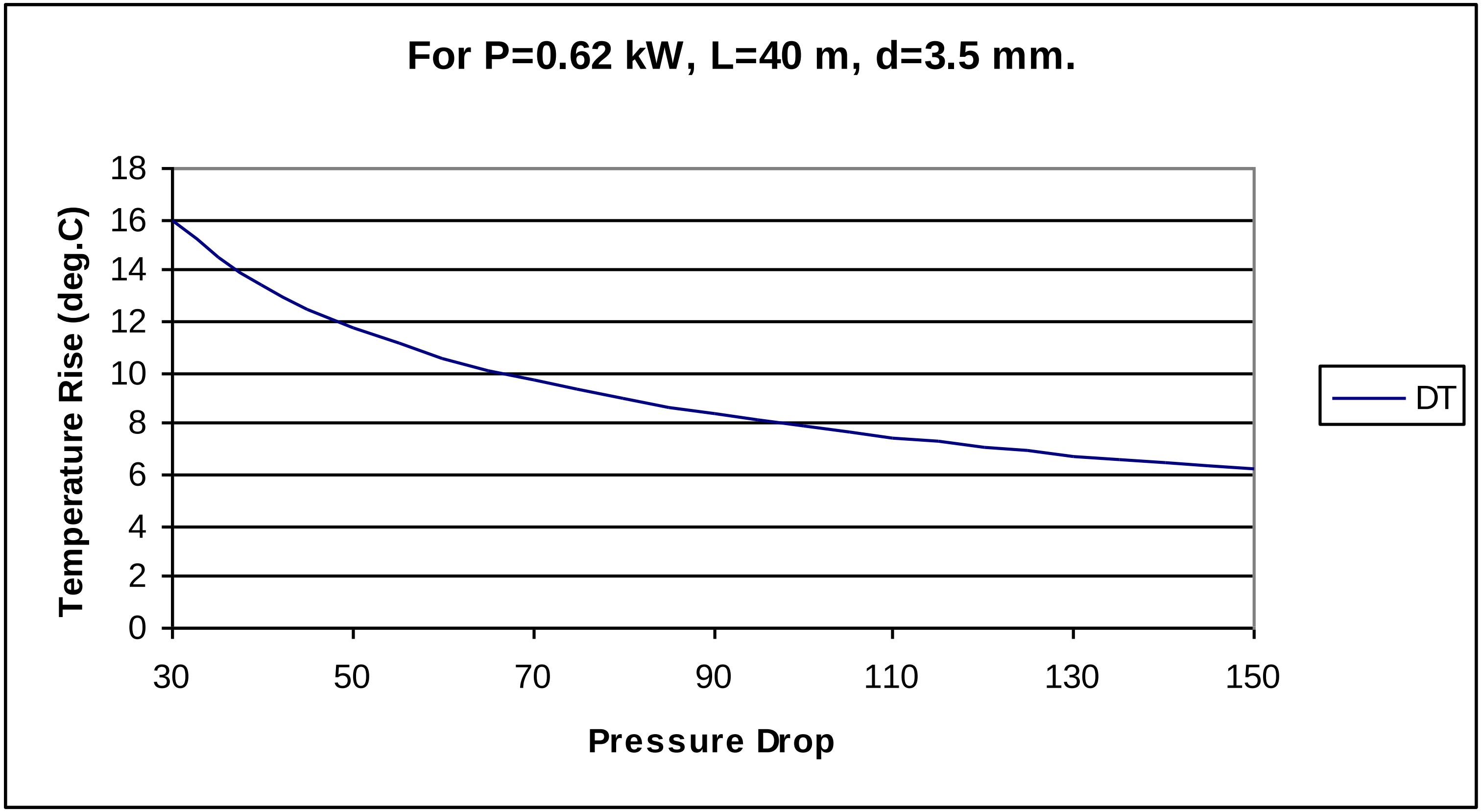
Results – *Water Flow*

Say, we designed quadrupole coils to operate at Δp=100 psi, four coils @ 0.30 gpm, total magnet water requirement = 1.2 gpm.



Results – *Water Temperature Rise*

Desirable temperature rise for Light Source Synchrotrons < 10 deg. C. Maximum allowable temperature rise (assuming 20 deg. C. input water) < 30 deg. C for long potted coil life.



Sensitivities

- Coil design is an iterative process.
- If you find that you selected coil geometries parameters which result in calculated values which exceed the design limits, then you have to start the design again.
  - Δp is too large for the maximum available pressure drop in the facility.
  - Temperature rise exceeds desirable value.
- The sensitivities to particular selection of parameters must be evaluated.

Sensitivities – *Number of Water Circuits*

The required pressure drop is given by,

$$\Delta P = 0.433 f \frac{L}{d}\frac{v^2}{2g} \propto Lv^2$$

where L is the water circuit length.

$$L = \frac{KN\ell_{ave}}{N_w}$$

K = 2, 4 or 6 for dipoles, quadrupoles or sextupoles, respectively. N = Number of turns per pole. N_W = Number of water circuits.

$$v \propto \frac{Q}{N_w}$$

Substituting into the pressure drop expression,

$$\Delta P \propto Lv^2 = \frac{KN\ell_{ave}}{N_w}\left(\frac{Q}{N_w}\right)^2$$

$$\Delta P \propto \frac{1}{N_w^3}$$

Pressure drop can be decreased by a factor of eight if the number of water circuits are doubled.

Sensitivities – *Water Channel Diameter*

The required pressure drop is given by,

$$\Delta P = 0.433 f \frac{L}{d}\frac{v^2}{2g} \propto \frac{v^2}{d}$$

where *d* is the water circuit diameter.

$$v = \frac{q}{hole\ Area} = \frac{q}{\pi \frac{d^2}{4}} \propto \frac{1}{d^2}$$

where *q* is the volume flow per circuit.

Substituting,

$$\Delta P \propto \frac{v^2}{d} \propto \frac{1}{d}\left(\frac{1}{d^2}\right)^2 = \frac{1}{d^5}$$

If the design hole diameter is increased, the required pressure drop is decreased dramatically.

If the fabricated hole diameter is too small (too generous tolerances) then the required pressure drop can increase substantially.

Homework

- Do problems 5.1 and 5.2 on page 128 of the text.
- Study problem 5.4. The answer is given at the end of the text. Practice inputting the parameters in the problem and observe the results.
- Change some of the parameters and observe the changes in the results.

Lecture 7

- Magnetic measurements is a specialized area.
  - Few (if any) of you will be involved intimately in this area and will need to understand the concepts in detail.
- However, the field is important since the quality of the magnets manufactured using the design principles covered in this course cannot be evaluated without a good magnetic measurements infrastructure.
  - Few institutions maintains this infrastructure and often has to resurrect this capability whenever the needs arise.
- Some time will be invested in covering the material so that the student can gain some appreciation of the electronics which must be gathered and connected to take measurements and the mathematical rigor which underpins this field. The electronics required in the area of a small area of magnetic measurements, rotating coil measurements, and the mathematics used for the data reduction is covered in chapter 8.