Orbit Correction Basics

Orbit correction is one of the most fundamental processes used for beam control in accelerators. Whether steering beams into collision for high-energy physics, steering photon beams in a synchrotron light source or locating beam on target in a medical application, it is essential to control the beam trajectory. This section investigates closed orbit control for storage ring applications. Extension to steering in transport lines requires only minor modification.

CORRECTOR-TO-BPM RESPONSE MATRIX

The corrector-to-BPM response matrix is an vital piece of information for both orbit control and optics analysis. The response matrix can either be calculated (betafunction theory or numerical 'tracking' solutions) or measured directly on the accelerator. The basic format of the response matrix equation is

$$x = R\theta$$

where column vector *x* contains the orbit *shift* produced by *incremental changes* in the corrector magnets, θ. The response of BPMS to the corrector magnets is contained in R.

An accelerator with m-BPMS and n-correctors produces an *m x* n dimensional response matrix. It is worth noting that the response matrix is really a sampled version of a continuous function that describes the effect of dipole perturbations at all points in the storage ring on all other points in the storage ring. The linear response of a single dipole perturbation is the well-known closed orbit formula:

$$x_o = \theta\sqrt{\beta_o\beta_k}\cos(\pi\upsilon - |\phi_o - \phi_k|) + \frac{\eta_o\eta_k}{\alpha L}$$

In some sense, the closed orbit formula can be thought of as the Greens function or impulse response to a δ-function corrector impulse. The impulse is in position, not time.

We often work with differential orbit and corrector changes rather than the absolute orbit and corrector values. The process of orbit control involves defining a differential orbit vector x and solving for the set of differential correctors, θ.

LINEAR TRANSPORT THEORY (R_{12} PARAMETERS)

From linear transport theory for a <u>transmission line</u>, particle position and angle evolve as

$$\begin{bmatrix} x \\ x' \end{bmatrix}_{final} = \left[R_{transport}\right]\begin{bmatrix} x \\ x' \end{bmatrix}_{initial} = \begin{bmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{bmatrix}\begin{bmatrix} x \\ x' \end{bmatrix}_{initial}$$

where $R_{transport}$ is a *2 x 2 transport matrix.* The full *6 x 6* transport matrix includes motion in both transverse planes and in the longitudinal plane. The R_{12} elements describe beam motion at the ith BPM (x) in response to a corrector kick at the jth corrector position (x'). Each element of our 'response matrix' is also an R_{12} element of a transport matrix connecting corrector kicks to BPM readings but in this case for the closed orbit, not an open transmission line. The reason for using the R_{12} elements is that we can physically 'kick' with corrector magnets (x') and 'observe' position at BPM sites (x). Throughout this course, we will use 'θ' for the angular deflection x' imparted by correctors.

UNITS

In MKS each element of the response matrix has physics units of m/radian, equivalently mm/mrad. It is not uncommon to make mistakes mixing mm/radian or m/mrad when trying to keep the units for the orbit and corrector vectors consistent. For online applications, the response matrix can have hardware units such as mm/amp, mm/bit, volt/bit, etc. The trick is to keep units consistent. In the orbit control section of this course we use mm/mrad exclusively.

ORBIT CORRECTION ALGORITHMS

Throughout the years, many orbit correction schemes have been devised. This course is concerned with the SVD approach presently in use at many accelerator laboratories worldwide.

In principle, orbit correction seeks to invert the response matrix equations:

$$x = R\theta$$
$$\theta = R^{-1}x$$

When a BPM is malfunctioning, not used in the problem, or requires weighting, we strike out or weight the corresponding rows of x and R. A similar process is used for corrector magnets and the columns of R and the rows of θ.

1. *Harmonic correction* – the orbit is decomposed onto a sinusoidal set of basis functions and each component is corrected independently. This method has a solid physical basis because the orbit typically contains strong sinusoidal harmonic components centered near the betatron tunes.

2. *Most effective corrector (MICADO)* – the Householder transformation is used to find the most effective corrector. After each correction cycle, the next most effective corrector is found. Since corrector magnets produce strong harmonics centered near the betatron tunes, MICADO has proven and effective means to correct global orbit distortions.

3. *Eigenvector correction* – the orbit is decomposed onto a set of eigenvectors for the response matrix. Since the response matrix contains significant structure near the betatron tunes, the dominant eigenvectors have sinusoidal structure near the betatron tunes. Similar to harmonic correction, the operator can select specific components for

correction. The drawback of the eigenvector correction technique is that it requires a square response matrix.

4.*Least-squares* – when the number of BPM readings exceeds the number of corrector magnets the response matrix becomes 'tall' (more rows than columns). In this case least squares or weighted least-squares can be used to solve for the corrector pattern. Weighting of individual rows of R corresponds to weighting individual BPMS.

5. *Singular value decomposition* – singular value decomposition performs the same functions as eigenvector or weighted least square corrections but is much more general, mathematically manageable, and numerically robust. In particular, where least-squares breaks down (non-invertible matrix R, singular value decomposition and the associated psuedoinverse produce the full set of eigenvector/eigenvalue pairs found in the general theory of linear algebra.

6. *constrained problems* – Lagrange multipliers and linear programming techniques have been used to constrain errors in the solution vector and corrector magnet strengths. This is a proven, yet still active field of research.

GLOBAL AND LOCAL ORBIT CORRECTION

Global orbit correction refers to calculations that bring the global closed orbit to a desired position. A global orbit correction correction usually involves many BPMS and corrector magnets distributed throughout the storage ring. The desired orbit may be the 'golden' orbit used for operations or a variant for experimental purposes.

Local orbit corrections or 'bumps' attempt to move the beam orbit in a restricted region. The smallest 'bump' is a 2-corrector bump when the magnets separated by 180 degrees in betatron phase. Otherwise 3 correctors are required to 'close the bump'. In terms of degrees of freedom, in a 3-magnet bump the first magnet deflects the beam and the next two are used to correct position and angle. Three magnet bumps are popular because the analytical solution can be written in terms of betafunction parameters. Pure position and angle bumps require four magnets (two to establish position/angle, two to restore position/angle).

TYPICAL ORBIT CORRECTION ALGORITHM

1. establish reference orbit (beam based alignment)
2. measure response matrix
3. select BPMs, correctors and weights and number of singular values
4. measure actual orbit - check for bad readings
5. compute difference orbit
6. compute corrector strength from $\Delta\theta = R^{-1}\Delta x$
7. check for corrector currents in range
8. apply corrector currents